STUDENT CENTERED MATHEMATICS INSTRUCTION: DEVELOPING A TEACHER AND STUDENT SURVEY AND UNDERSTANDING STUDENTS' EXPERIENCES

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Over the past thirty years, policies have been enacted at the local and state levels to reform mathematics instruction to be student-centered (see Cohen & Ball, 1990; Resnick, Stein, & Coon, 2008). Despite the wave of instructional reforms, several critical gaps remain in our understanding of student-centered mathematics instruction. First, the field lacks a conceptual framework that relates the underlying theories with student-centered instructional practice in mathematics. Second, there has been very little systematic and large-scale research on the implementation and effects of student-centered mathematics instruction. Third, we know very little about how students experience and respond to the implementation of student-centered instructional reform.

The dissertation studies aimed to bridge these gaps with two mixed methods studies. Study 1 proposes a conceptual framework of student-centered mathematics instruction and uses a combination of literature review, feedback from experts, and data from a large sample of urban and suburban youth (n= 2,536 students) and their mathematics teachers (n = 34) to validate a student and teacher survey of student-centered mathematics instruction.

Study 2 investigates seventh grade adolescents' experiences of their mathematics teachers implementing a reform to student-centered instruction. Specifically, the study examines the emotional experience of getting stuck in their algebra coursework and how the frequency and nature of these emotions vary by student characteristics and how students' emotional experiences influence their sense of competence in math.

The studies have important implications for our understanding of student-centered mathematics instruction. In particular, the studies suggest that students' perspective and experiences could be important to both the implementation and effects of student-centered mathematics instruction. The studies also suggest mechanisms for differential experiences and effects of student-centered instruction for minority, low-income, and female students. Study findings are discussed in terms of implications for education research, practice, and theory.

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PREFACE

ACKNOWLEDGEMENTS

"We all carry the seeds of greatness within us, but we need an image as a point of focus in order that they may sprout." ~ Epictetus

"If I have seen further it is by standing on the shoulders of giants." ~ Isaac Newton

For college, I attended a small state University in central Wisconsin. In the beginning of my second year, a professor asked me, "What are your plans after college? Do you want to go to graduate school?" I had no idea what she was talking about. I asked her, "What's graduate school?" We spoke for a long while. She explained what graduate school was and why she thought I might want to go. In many ways, that conversation changed my life – not least of all by initiating the journey to pursue a doctoral degree and an academic career. I did not know it at the time, but she was my first mentor.

Since that time and throughout my doctoral studies, I have been blessed with several mentors who have supported, advised, encouraged, and inspired me. I would like to take the time to express my gratitude to some of those folks here.

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1. INTRODUCTION

1.1. BACKGROUND CONTEXT

1.1.1. Mathematics achievement

Supporting adolescents' engagement and achievement in mathematics coursework is a high priority for education researchers, policy makers, and educators in the United States. Far from when mathematics was considered an intellectual luxury and of relatively little value to the average person in the early 20th century (see Klein, 2007; Osborne & Crosswhite, 1970), success in mathematics coursework has become essential for adolescents' success in life. The mathematical knowledge and skills attained in secondary mathematics coursework are important for participating productively in school and beyond (Evan, Gray, & Olchefske, 2006; U.S. Department of Labor, 2007). Not only do students need to pass required mathematics coursework in order to graduate from high school, their success in that coursework is also a strong predictor of their ability to enroll in and complete a postsecondary degree (Evan et al., 2006), which has become increasingly important for attaining well-paying jobs (Achieve, 2006; Carnevale & Desroches, 2003). Youth who complete an associate's or bachelor's degree earn 20 to 40 percent more than those who earn a high school diploma and are eligible for a wider range of stable professional careers throughout adulthood (Grubb, 1996; U.S. Department of

Education, 2015). In order to be prepared to complete a postsecondary degree, adolescents need to successfully acquire the knowledge and skills taught in four years of challenging mathematics coursework throughout secondary school (Achieve, 2006).

Success in mathematics coursework is also critical for supporting youth in pursuing highly skilled and well-paying careers in the fields of science, technology, engineering, and mathematics (STEM). Students with postsecondary degrees in STEM fields have significantly higher incomes than peers with degrees in social sciences, education, or humanities (Grubb, 1996). Furthermore, there are growing opportunities in STEM fields. Between 1998 and 2008, jobs that require training in STEM increased by 51%, which is four times the overall job growth rate (Department of Labor, 2007; Evan et al., 2006). As of 2015 there were more openings in STEM careers than there were qualified applicants to fill them (National Science Foundation, 2015). Adolescents' engagement and achievement in their mathematics coursework is crucial for developing the skills and motivation needed to be qualified and motivated to take advantage of these opportunities (Evan et al., 2006; Maltese & Tai, 2010; Singh, Granville, & Dika, 2002; Taningco, 2008).

Unfortunately, national assessments reveal that many American adolescents' achievement in mathematics declines after the transition to middle school (Gonzales, Williams, Jocelyn, Roey, Kastberg, & Brenwald, 2008). In the National Report Card for 2007, the U.S. Department of Education reported that only 39% of American eighth graders and only 23% of twelfth graders scored at or above "proficient" in math on the National Assessment of Educational Progress (NAEP) assessments. By 2015, the number of eighth graders scoring proficient on the NAEP assessment in mathematics dropped to 33% (U.S. Department of Education, 2015).

Underachievement in mathematics means that students have to catch up after high school, creating a burden for students and educators down the road. On average, one-fifth of college freshman are required to take remedial mathematics classes in order to be eligible to take college-level mathematics courses (Evan et al., 2006). Remedial mathematics courses add to the financial cost and time cost of post-secondary education, creating additional challenges to making good progress and being academically successful in college (Achieve, 2006). Indeed, two-thirds of students who need to take post-secondary remedial mathematics coursework do not complete their college education (Evans et al., 2006).

Data from international assessments reveal another challenge for American youth who are interested in pursuing STEM degrees and careers. Not only do American adolescents rank close to the bottom in mathematics literacy compared to their peers in other countries (Baldi et al., 2007; Evan et al., 2006), they are also disproportionately less likely to rank among the top performers in mathematics on international assessments (OECD, 2012). Students scoring at the advanced level (Level 6) on assessments like PISA have the skills of students who have achieved ambitious learning goals: they can develop and work with models for complex situations and work strategically using broad, well-developed thinking and reasoning skills (OECD, 2012). Unfortunately, American youth in Advanced Placement Calculus – generally considered to be the type of mathematics coursework taken by our best and brightest students – finish below the international average in mathematics (Evan et al., 2006). Given the increasingly complex and competitive global marketplace, this results in fewer American youth being eligible for the highest skilled and best paying careers in STEM fields.

1.1.2. Student engagement in mathematics

Adolescents' achievement in mathematics coursework and aspirations to pursue mathematics-related college majors and careers is related to their engagement in mathematics coursework. Engagement refers to the *quality* of adolescents' involvement in their academic coursework (Wang & Eccles, 2012; Skinner, Kindermann, Connell & Wellborn, 2009), such as the energy, purpose, and durability that they expend towards their academic work and participation in mathematics classes (Skinner & Pitzer, 2012). Specifically, engagement is considered to be multi-dimensional and to consist of cognitive, behavioral, and emotional components (Fredricks, Blumenfeld, & Paris, 2004; Skinner & Pitzer, 2012). Behavioral engagement refers to adolescents' overt participation. Cognitive engagement refers to the level of mental investment and effort that adolescents expend in class and on academic work. Emotional engagement consists of adolescents' interest in, value of, and positive and negative affective responses in mathematics class.

Recently, some researchers have conceptualized social engagement as another dimension that is related to adolescent outcomes in mathematics (e.g., Wang, Fredricks, Hofkens, Schall, & Parr, 2016; Rimm-Kaufman et al., 2014). Social engagement refers to the quality of social interactions and interactive participation in mathematics coursework (Wang et al., 2016; Rimm-Kaufman et al., 2014). High quality social interactions can support high levels of engagement in mathematics coursework (Vygotsky, 1978; Michaels, O'Connor, & Resnick, 2008; Resnick, & Nelson-Le Gall, 1997), and social engagement has been shown to predict unique variance in math achievement and STEM-related aspirations (Wang et al., 2016).

Adolescents' behavioral, cognitive, emotional, and social engagement shapes multiple education outcomes for youth. Adolescents' engagement is linked to their depth of understanding (Nystrand & Gamoran, 1991), academic achievement (Wang & Holcombe, 2010; Sinclair, Christenson, Lehr, & Anderson, 2003; Klem & Connell, 2004; Connell, Spencer, & Aber, 1994; Marks, 2000; National Research Council, 2004), achievement trajectory over time (Alexander, Entwisle, & Dauber, 1993; Alexander, Entwisle, & Horsey, 1997), and educational aspirations (Wang & Eccles, 2012). Engagement is also considered by many to be content specific (see Wang et al., 2016), and engagement in mathematics is associated with mathematics achievement and aspirations to pursue a college major in a STEM field (Wang et al., 2016).

Unfortunately, adolescents' engagement in mathematics coursework declines throughout secondary school (Martin, Way, Bobis, & Anderson, 2014; Wigfield, Byrnes, & Eccles, 2006). Starting at the transition from elementary to middle school, adolescents become increasingly less engaged in their academic coursework (Eccles & Roeser, 2011). Since algebra coursework begins in earnest in middle school, this means that many adolescents are becoming less engaged in mathematics at the same time that they are embarking on content that is a gateway for multiple aspects of their academic and professional trajectories.

1.1.3. Achievement emotions

Adolescents' engagement and achievement in mathematics could be related to the achievement emotions that they experience in mathematics coursework. Students experience achievement emotions in response to how much control they feel they have over their success and how much they value doing well in mathematics coursework (see Pekrun, Goetz, & Perry, 2002). In turn, these emotions shape engagement and achievement by affecting the cognitive resources that are available for students to devote to learning tasks, students' motivation to learn, the learning strategies that students use, and their level of self-regulation while learning. For example, students who experience positive achievement emotions, such as enjoyment, hope, and pride, experience less task-irrelevant thinking (Pekrun et al., 2004), increased motivation (Pekrun et al., 2004), and application of effective learning strategies like elaboration and organization of material (Pekrun, Frenzel, Goetz, & Perry, 2007). Students who experience negative achievement emotions, on the other hand, such as anxiety, shame, and hopelessness, experience more task-irrelevant thinking (Pekrun et al., 2004), and less motivation (Pekrun et al, 2004). Research on math anxiety suggests that achievement emotions may be content specific (Pekrun, 2002) and that achievement emotions may play a particularly important role in how students engage and achieve in mathematics throughout secondary school.

1.1.4. Student characteristics

There is evidence that motivational and academic trajectories in mathematics are related to student characteristics. Specifically, students' race, socioeconomic status, and gender have been linked to disproportionate declines in engagement, achievement, and mathematics-related aspirations in secondary school. Education policies targeting increased excellence and equity in mathematics outcomes need to consider the extent to which student-centered instruction interacts to shape mathematics outcomes for students based on these demographic characteristics. In addition, students' overall level of academic achievement could influence how students engage and learn in student-centered mathematics classrooms.

1.1.4.1. Race

Since the United States started tracking student achievement data in mathematics, data has consistently revealed that minority youth experience disproportionately negative outcomes in mathematics (U.S. Department of Education, 2015). Specifically, minority youth disengage from and underperform in mathematics throughout secondary school (Martin et al., 2015; U.S. Department of Education, 2015). At the start of secondary school, minority youth are less engaged and lower achieving than their peers, and these gaps widen over the course of secondary school (Bacharach, Baumeister, & Furr, 2003; Riegle-Crumb & Grodsky, 2010). They represent a major obstacle for minority students developing the skills they need to succeed in school and to participate in the workforce (Baldi et al., 2007; Evan et al., 2006). In particular, achievement gaps in mathematics contribute to minority students becoming unqualified and unmotivated to pursue STEM careers, which in turn contributes to the lack of diversity in those professions (National Science Foundation, 2015; U.S. Department of Labor, 2007).

1.1.4.2. Socioeconomic status

Similar to minority students, low-income students disproportionately disengage from and underachieve in their algebra coursework (U.S. Department of Education, 2015). In 2015, students who were eligible for free or reduced price lunch scored 28 points lower on state standardized mathematics tests than their higher-income peers (U.S. Department of Education, 2015). Like minority students, gaps for low-income students widen as they progress through secondary school (Bacharach et al., 2003; Riegle-Crumb & Grodsky, 2010) and represent obstacles for them in secondary school, postsecondary school, and in the workplace (Achieve, 2006; Evan et al., 2006).

1.1.4.3. Gender

Female students also experience a disproportionate decline in mathematics engagement throughout secondary school (Wigfield et al., 2006), and they are under-represented in mathematics-intensive STEM college majors and careers (National Science Foundation, 2015). Female students report lower perceptions of their competence in mathematics (Andre, Wingham, Hendrickson & Chambers, 1999; Marsh, Trautwein, Lüdtke, Köller, & Baumert, 2005; Watt et al., 2012), experience more mathematics-related anxiety (e.g., Lau & Roeser, 2002), and are less interested in mathematics than their male peers (Frenzel, Goetz, Pekrun, & Watt, 2010; Hyde, Fennema, Ryan, Frost & Hopp, 1990). Thus, it is not surprising that female students who are as academically successful in their mathematics coursework as their male peers are less likely to choose a mathematics-intensive STEM college major or career (Clewell & Campbell, 2002; Wang, Eccles, & Kenny, 2013).

1.1.5. Summary of the background context

There is a pressing need to support adolescents' engagement and achievement in mathematics coursework. Students who are disengaged are more likely to underachieve, and students who underachieve in algebra are at an increased risk of not graduating high school and not completing their postsecondary education, which can have a significant impact on their professional opportunities and financial stability throughout adulthood. Students also need to be supported in attaining high levels of achievement in mathematics coursework so that they can be competitive for the wide range of opportunities in STEM fields that require mathematical fluency and academic success. Minority, low-income, and female students in particular need targeted support, as they are more likely to experience low levels of engagement and/or achievement in mathematics coursework. Research on achievement emotions suggests that students' affective experiences in mathematics coursework could play an important role in engagement and achievement trajectories.

1.2. STATEMENT OF THE PROBLEM

To support adolescents' engagement and achievement in mathematics coursework, education research and policy have focused on how mathematics is taught. Instruction is the primary policy lever used by states and school districts to shape education outcomes for youth (see Cohen & Ball, 1990). Instruction structures the nature of academic work and the factors that support adolescents' motivation to engage in it (see Doyle, 1988; Eccles & Roeser, 2009; OECD, 2012). In the United States, mathematics instruction tends to focus on students acquiring a specific set of discrete mathematics knowledge and skills. Increasingly, mathematics educators argue that students need to go beyond the memorization of facts and procedures in order to develop a deeper understanding of mathematics (National Council of Teachers of Mathematics, 2014). Furthermore, as education and workplace environments become more complex and technical (Fulton, 2012), educators argue that it is important to teach students how to teach themselves by coming up with their own strategies for understanding and solving complex problems (Clinton & Rieber, 2010; Land, Hannafin, & Oliver, 2012; Gijselaers, 2000). By having students think critically and reason about mathematical problems, students can improve their achievement in mathematics while also learning how to be effective learners and thinkers in school and beyond (Anderson, Greeno, Reder, & Simon, 2000; Kuhn, 2007).

To support students in becoming more engaged in mathematics coursework and in becoming more effective thinkers and learners, education policy has pushed to reform mathematics instruction to be more student-centered. Teacher-centered instruction and student-centered instruction are two pedagogical approaches to teaching mathematics that have fundamentally different views about what students need to learn and how mathematics should be taught. In addition to featuring more cognitively challenging and open-ended tasks that emphasize metacognitive skills, student-centered instruction also differs in terms of the role of the teacher and student. Education policy has echoed research and theory from the fields of the learning sciences and motivational psychology that argue that the nature of academic work and the transformation of the teacher and student roles inherent in student-centered instruction supports adolescents' engagement and achievement in mathematics (described in Chapter 3). As a result, there has been a wave of instructional reforms at local and state levels that call on mathematics teachers to implement student-centered instructional practices (e.g., Common Core State Standards, 2015; National Research Council, 2012; Resnick et al., 2008).

As these reforms progress, there is an urgent need for more research on exactly how student-centered instruction shapes outcomes for youth. While working under pressure to improve mathematics education outcomes for American youth, instructional policies have surged ahead, but relatively little research has been done to systematically study the effects of student-centered mathematics instruction on adolescents' engagement and achievement in mathematics. Given the amount of time and resources devoted to implementing instructional reform and the pressing need to improve mathematics outcomes for youth, it is imperative to study and understand the effects of student-centered mathematics instruction. Specifically, we need to know for whom student-centered mathematics instruction is effective, in what ways, and how it works. Is student-centered mathematics instruction associated with adolescents' engagement and achievement in mathematics? Are there differential associations for minority, low-income, or female students? What mechanisms or processes explain the relationship – or differential association – of student-centered instruction with mathematics outcomes for youth?

In order to assess its effects, we first need to operationalize student-centered mathematics instruction. In the academic literature and in education policy, student-centered mathematics instruction refers broadly to instructional practices that shift the locus of classroom activity, responsibility for learning, and cognitive effort from the teacher to the student (e.g., see Lee & Hannafin, 2016; Slavich & Zimbardo, 2012). While there is relatively broad consensus on the overall conceptualization, more work is needed to identify, articulate, and examine potential components or dimensions of student-centered mathematics instruction. Research and policy tend to either describe student-centered instruction as an overall instructional strategy (i.e., one dimensional) or equate the implementation of a specific component of student-centered instruction (e.g., students' responsibility for learning) with teachers' implementation of student-centered instruction overall (described in Chapter 3). Different components of student-centered mathematics instruction could have different effects on engagement and achievement and could even contribute to differential effects by student characteristics. For example, students taking on more cognitively challenging work could support their cognitive engagement in mathematics coursework. However, the increase in cognitive demand could have differential effects on students' emotional engagement, depending on students' perceptions of their level of competence, which can differ by race, socioeconomic status, and gender (described in more detail in the section "Student Characteristics" in Chapter 2).

Operationalizing and examining the dimensions of student-centered mathematics instruction can inform the development of a survey that can be validated for use in large-scale studies of student-centered instruction. Currently, there are few well-validated measures of student-centered instruction (extant measures described in Chapter 3). Survey measures can enable systematic research into student-centered instructional practice at a scale that is needed to understand student-centered instruction and to inform instructional policy (instructional policy around student-centered mathematics instruction is described in more detail in Chapter 2).

Finally, we would benefit from an on-the-ground understanding of adolescents' experience of student-centered mathematics instruction. Efforts to study and understand the effects of student-centered instruction with large-scale survey studies could be informed by grounded insight on what student-centered mathematics instruction is like for adolescents. In particular, it could be valuable to understand how adolescents experience student-centered instruction in the context of an instructional reform. Even though the aim of instructional reforms is to transform student learning by fundamentally changing how mathematics is taught, we have not studied how these changes are experienced and taken in by students. Most adolescents in the United States have been socialized into the norms and expectations of teacher-centered mathematics classrooms. The transition from teacher- to student-centered instruction could profoundly impact their experiences of mathematics learning and their views of themselves as learners.

In particular, the transition from teacher- to student-centered mathematics instruction could influence the types of achievement emotions that adolescents experience in mathematics class. According to Control Value Theory of emotions (CVT) instructional practice can influence academic outcomes by shaping the ways in which the academic environment and academic work trigger students' cost-value appraisals (see Pekrun et al., 2007; described in Chapter 4). Student-centered instructional reforms call on teachers to fundamentally change the nature of academic work and the role of the student in ways that could, at least initially, clash with the sense of control and value of academic work that students developed in their experiences in teacher-centered classrooms, which could significantly impact students' emotional experiences, attentional resources, motivation, and learning behaviors in their mathematics coursework (Pekrun et al., 2007).

1.3. DISSERTATION OVERVIEW

This dissertation aims to address these limitations with two mixed methods studies: the first study aims to develop a measure of student-centered mathematics instruction; and the second examines 7th grade students' emotional experiences of getting stuck in the context of a school-wide instructional reform from teacher- to student-centered instruction.

To lay the groundwork for the dissertation, Chapter 2 reviews the underlying theories of teacher- and student-centered instruction, differences in teacher- and student-centered instructional practice, and the policy context for the rise of student-centered instruction.

Chapter 2 also reviews what we know about the effects of student-centered mathematics instruction and the gaps in theory and research to which the dissertation studies aim to contribute.

Chapter 3 describes Study 1, a mixed method study to develop a measure of student-centered instruction. The chapter reviews current conceptualizations of student-centered instruction and corresponding measures and research on its effects. After reviewing the limitations of the extant conceptualizations, measures, and research, the chapter proposes a multi-dimensional conceptualization that synthesizes research on a variety of student-centered instructional approaches and integrates the theoretical foundations of student-centered mathematics instruction reviewed in Chapter 2. Study 1 includes six specific research questions that describe the analytic steps for developing the measure and examining validity by measurement invariance by student race, socioeconomic status, and gender. The study also tests the association of the dimensions of the student-centered mathematics with mathematics engagement and achievement and differential associations based on student characteristics. After identifying the research questions, Chapter 3 includes a detailed description of the study sample and analytic strategy for addressing each research question. Finally, the results are described, followed by a brief summary of the study.

Chapter 4 describes Study 2, a mixed method investigation of 7th grade students' emotional experiences of getting stuck while working on mathematics problems in the context of a school-wide reform to student-centered mathematics instruction. The chapter begins with a definition of what it means to be stuck while working on mathematics problems. Then I use the Control-Value Theory of Achievement Emotions to explain the ways in which student-centered mathematics instruction can affect engagement and achievement in mathematics coursework by influencing the emotions that arise when students get stuck while working on challenging problems. The chapter identifies qualitative research questions that aim to describe adolescents' emotional experiences when they are stuck and quantitative research questions that intend to: 1) describe the prevalence of getting stuck and various types of emotional responses; 2) examine the relation of getting stuck and emotional responses with student characteristics; and 3) examine the extent to which emotional experiences predict changes in perceived competence in mathematics over the course of the academic year. Similar to Chapter 3, after identifying the research questions, Chapter 4 includes a detailed description of the study sample and analytic strategy for addressing each research question, which is followed by the results and a brief chapter summary. Chapter 5 provides an overview of the conceptual landscape of the significance, contributions, and implications of the two studies for theory, research, policy and practice.

2. LITERATURE REVIEW

Teacher-centered instruction and student-centered instruction are two philosophical approaches to teaching mathematics. Each instructional approach is based on different theories of learning and correspondingly different views on: 1) the nature of mathematical tasks, 2) the role of the teacher, and 3) the role of the student. In addition, student-centered instruction has been informed by developmental theories of adolescent motivation and engagement in school. The following two sections describe the two main theoretical frameworks that are cited in literature describing teacher- and student-centered mathematics instruction (2.1) and how these theoretical foundations contribute to differences in the nature of academic work and the roles of the teacher and student (2.2).

2.1. THEORETICAL BACKGROUND OF STUDENT-CENTERED INSTRUCTION

2.1.1. Theories of learning

2.1.1.1. Behavioral theories of learning

Traditionally, mathematics instruction has been based on a behavioral or acquisition theory of learning (see Jonassen, 1991; Lave, 1997). The behavioral perspective argues that mathematics expertise can be broken down into specific knowledge and procedural skills that students learn through repeated practice (Bonk & Cunningham, 1998; Kember & Gow, 1994). In this view, mathematical knowledge is known and teachers are the experts who possess the knowledge and skills that students need to acquire (Nowell, 1992; Porter, 1989; Stoldosky, 1988). "Mastery" is attained when students can apply mathematical knowledge and skills with accuracy and efficiency (Ackerman, 2003).

2.1.1.2. Constructivist theories of learning

Unlike the acquisition perspective that emphasizes memorization and procedural fluency, constructivist views of learning emphasize the importance of the learning process (Ziegler & Yan, 2001). Specifically, constructivism argues that adolescents attain the deepest level of understanding when they develop their own knowledge from personal experience and through social interactions with others.[1] The constructivist perspective is based on a number of fundamental perspectives about how children learn, including:[2]

1) Learning is a developmental process of actively constructing logical structures and systems of meaning that are established when children integrate new information with previous knowledge and experience (Piaget, 1960, 1972; Vygotsky, 1978);
2) Active construction of meaning is supported by children's first hand experiences and reflecting on those experiences (Piaget, 1972; Dewey, 1925);
3) Language plays a central role in the development of logical structures and understanding (Vygotsky, 1978; Bruner, 1990; Resnick et al., 2008); and
4) (Extending from 2 and 3) Learning and development cannot be separated from the real world and social contexts (Dewey, 1933, 1998; Vygotsky, 1978).

In summary, the constructivist view defines learning as cognitive development that is highly individualized and that occurs through social interaction. Figure 1 illustrates how learning occurs in what Vygotsky described as the "Zone of Proximal Development" (ZPD; Vygotsky, 1978), which is defined as "the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as

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[1] Some literature distinguishes between cognitive constructivism and social constructivism, wherein the actual construction of understanding from experience is cognitive construction (i.e. the Piagetian perspective) and embedding experience in language through social interaction is social construction (i.e., Bruner's work on the centrality of language). In the dissertation, "constructivism" refers to both of these two components (i.e., Vygotsky's, 1978).

[2] The elements of constructivism outlined here are described in a number of original and contemporary sources. For the purpose of describing the fundamental principles of constructivism, I am focusing on the foundational texts and scholars.

determined through problem-solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 85). In other words, it describes knowledge and skills that are attainable but that currently fall outside of the learner's reach. Teachers "scaffold" learning by facilitating a student's inquiry in ways that ensure that the academic tasks stay within his or her zone of proximal development (ZPD), which continually evolves throughout learning and can even shift within a given lesson (Vygotsky, 1978).

While conceptualized as a general theory of learning, constructivism has been applied specifically to mathematics to articulate how students learn mathematics. Mathematics educators and education researchers argue that children learn mathematics best when they play an active role in constructing their knowledge (De Kock, Sleegers, & Voeten, 2004) while working on relevant and real world mathematical tasks (e.g., Elen, Clarebout, Leonard, & Lowyck 2007) that are designed and implemented to be at the appropriate level of challenge for students (Smith & Stein, 2011) in the context of interacting with teachers and peers (Bruner & Haste, 2010; Resnick & Nelson-Le Gall, 1997). In this view, mathematical expertise is re-conceptualized (from a behavioral view of learning) as consisting of a range of complex cognitive and metacognitive skills that students use to understand, apply, and make connections between mathematical concepts and procedures and that can be generalized (or "transfer") to learning in other subjects (e.g., Michaels et al., 2008; Resnick, & Nelson-Le Gall, 1997). "Mastery" is attained when students have developed a deep understanding of mathematical concepts and procedures that they can translate into solving challenging and open-ended problems.

2.1.2. Self-determination theory of motivation

Student-centered instruction is also informed by self-determination theory of motivation (SDT). In order for students to be successful in their academic work, they need to maintain a high level of quality engagement (Lee & Hannafin, 2016; Fredricks et al., 2004; Ryan & Deci, 2000). SDT argues that the type of intrinsic motivation that sustains student engagement is fostered when instruction supports adolescents' psychological needs for competence, autonomy, and relatedness (Deci & Ryan, 1987, 2000). Competence refers to students' sense of self-efficacy; autonomy refers to students perceiving that they have psychological freedom and can make meaningful and personally relevant choices in their learning; and relatedness refers to students experiencing a sense of belonging and feeling respected by others (Eccles & Midgley, 1989; Niemic & Ryan, 2009; Sierens, Vansteenkiste, Goossens, Soenens, & Dochy, 2009).

The psychological needs are distinct but dynamically related in their effect on student motivation; self-determined motivation is sustained in contexts in which all three psychological needs are met (Deci & Ryan, 2000). Adolescents whose psychological needs are met are intrinsically motivated to learn, are more persistent while working on challenging tasks, and are resilient in the face of setbacks or failure (Deci & Ryan, 2000; Niemic & Ryan, 2009). Figure 2 illustrates that intrinsic motivation results when students attain an integrated sense of competence, relatedness, and autonomy (Ryan & Deci, 2000). Figure 3a illustrates how the attainment of those psychological needs is hypothesized to mediate the association between instruction and engagement (Skinner & Chi, 2012).

While psychological needs are sustained throughout the lifespan, the contextual and relational factors that contribute to their attainment change throughout development. Specifically, psychological needs are met in environments that offer activities and supports that are aligned with children's developmental stage. Stage environment fit theory – based on SDT – explains that adolescents' psychological needs are inextricably linked with their developmental abilities and tasks (Eccles & Roeser, 2009). Throughout secondary school, adolescents develop increasingly complex cognitive skills, including an increased capacity for considering others' perspectives (Blakemore & Choudhury, 2006) and an intrinsic desire to explore meaningful connections with the real world (Assor, Kaplan, & Roth, 2002). Adolescents also have a significant need for successful relationships in school (Ryan & Patrick, 2001), and there is evidence that the quality of social interactions significantly impacts their achievement (Kiefer & Ryan, 2011; Shin & Ryan, 2012; Wentzel & Battle, 1991; Wigfield et al., 2006). Thus, instruction that features tasks that require a wide range of cognitive skills (e.g., memorizing, comprehending) and metacognitive skills (e.g., considering multiple representations, thinking about and explaining your thinking) and that supports adolescents' developmental needs to experience productive social interactions can fuel the type of intrinsic motivation that drives deep cognitive, emotional, behavioral, and social engagement in academic work (Ryan & Deci, 2000; Fredricks et al., 2004; Wang et al., 2016).[3]

2.2. TEACHER- AND STUDENT-CENTERED MATHEMATICS INSTRUCTION

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3 Often, literature uses *either* a constructivist *or* a motivational lens when describing student-centered mathematics instruction. Because I am studying mathematics engagement and learning, I am including both of these frameworks. In Chapter 3, I review literature that has integrated the two and propose another way of thinking about how the two can be integrated into a single conceptual framework describing how student-centered mathematics instruction shapes engagement and learning.

2.2.1. Differences based on theories of learning

2.2.1.1. Teacher-centered mathematics instruction

Also referred to as direct instruction (e.g., Stein & Smith, 2011), explicit instruction (e.g., Kroesbergen, Van Luit, & Maas; 2004), lecture-based instruction (e.g., Thompson, 2009), or school mathematics (Richards, 1991), teacher-centered instruction is a complex pedagogical strategy that positions teachers as at the "center" of knowledge and classroom activity. As a discipline, mathematics has a particularly strong tradition of teacher-centered instruction in large part because it supports a behavioral or acquisition theory of learning that is a hallmark of mathematics disciplinary traditions (see Lave, 1997). According to this view, the goal of academic work or tasks is to teach essential information (Nowell, 1992), and the focus of mathematics instruction is to support memorization and the development of procedural fluency. Teachers break mathematical concepts and procedures into manageable pieces that students can acquire through lecture and repeated practice (Porter, 1989; Stoldosky, 1988). Lectures are used to elicit relevant previous knowledge, state the learning objectives for the lesson, and describe how and why a skill or procedure works by modeling the procedures needed to solve mathematical problems (see Lampert, 1990). The tasks are relatively routine and removed from their real world application, focusing on developing fundamental mathematical skills that can be applied across contexts. There is a focus on efficiency and only using the instructional time necessary to successfully transfer essential information (Ackerman, 2003).

The role of the teacher in teacher-centered instruction is to transfer his or her knowledge to students. As the mathematical expert, this means that teachers maintain control over information and the activities in the classroom as they ensure students learn the basic computational facts and skills of mathematics (Capraro, 2001). Teachers supervise students' application of the mathematical procedures described in the lecture, answer students' questions, and assess the accuracy of students' statements, strategies, and solutions to mathematical problems (Banilower, Smith, Weiss, & Pasley, 2006; Cogan, Schmidt, & Wiley, 2001; Stigler & Hiebert, 2004; Thompson, 1992, in Fulmer & Turner, 2014).

For their part, the role of students in teacher-centered classrooms is to acquire the knowledge that their mathematics teachers impart. They are expected to be active listeners during lecture and to participate fully in repeated practice of mathematical procedures. There is a strong emphasis on following instructions and accuracy (Daniels, Kalkman, & McCombs, 2001). Students are expected to use the procedures described and modeled by the teacher to solve problems, and they are encouraged to check the accuracy of their work by consulting with the teacher or mathematical texts. They understand that there is one correct answer, and it is the one described by their teacher or text (Schoenfeld, 1992).

2.2.1.2. Student-centered mathematics instruction

A wide range of terminology is used to describe instructional practice that is considered to be student-centered, including inquiry instruction (Richards, 1991), teaching for understanding (Hiebert et al., 1997), student-activated instruction (Struyven, Dochy, Janssens, & Gielen, 2006), learner-centered instruction (e.g., Meece, 2003; American Psychological Association, 1995), standards-based instruction (Tarr et al., 2008), constructivist teaching (Jonassen 1991; Struyven et al., 2006), student-directed learning (Oser, & Baeriswyl, 2001; Zimmerman, 2002), self-regulated instruction (Paris & Paris, 2001), and instruction that supports student-centered learning environments (Cannon & Newble, 2000; Savery & Duffy, 1995), to name a few. Each of these terms refers to an instructional philosophy that aims to position students at the "center" of inquiry and problem solving (see Slavich & Zimbardo, 2012; Schuh, 2004). Teachers accomplish this by changing the nature of academic work and the role of the teacher and the student in ways that: a) require students to construct a deep and personalized understanding of mathematics; and b) support their engagement by meeting adolescents' psychological needs and developmental skills.

In student-centered instruction, the purpose of academic work or tasks is to go beyond memorizing mathematical concepts and procedures. Adolescents in student-centered classrooms are supported in constructing their own understanding by engaging in mathematical thinking through problem solving (Schoenfeld, 1992). Cognitively challenging and open-ended tasks for which there are more than one solution provide students the opportunity to come up with their own strategies based on previous experience and the application of critical thinking skills (Stein, Grover, & Henningsen, 1996). Teachers provide tasks that are at the appropriate level of challenge for the class and use scaffolding to ensure that tasks are at the appropriate level of challenge for each student (i.e., in their zone of proximal development; Vygotsky, 1978). A significant challenge for teachers is to provide support without providing the procedures for solving the problem, which devolves the level of cognitive challenge and inquiry of the task. Teachers also capitalize on the constructivist principle that learning is embedded in personal experience and prior knowledge by using tasks that are meaningful and relevant to adolescents and that involve explaining and justifying their thinking (Elen et al., 2007; Resnick et al., 2008; Stein et al., 1996). Finally, teachers situate learning in the classroom social context primarily through discourse and collaboration. Specifically, teachers use a range of specific interactive strategies to structure adolescents' opportunities to learn through social interaction (discussed in "student roles" below National Council of Teachers of Mathematics, 1991; Stein et al., 1996).

In order to support students constructing their own understanding of mathematics, the role of the teacher is transformed from being the source of classroom activity and knowledge to facilitating students' intellectual authority and responsibility for their learning (Fuller & Johnson, 2001; Smith & Stein, 2011). Teachers work as coaches by monitoring the level of challenge and engagement of each learner, by facilitating productive interactions (e.g., with questioning, re-phrasing, summarizing, and questioning techniques to support dialogue and considering multiple perspectives), and by ensuring that the social interactions are focused on the mathematics as intended (e.g., by addressing misconceptions and by keeping the discussion focused on the goals of the lesson) (Michaels et al., 2008; Resnick, & Nelson-Le Gall, 1997; Stein & Smith, 2011; Smith & Stein, 2011). The role of the teacher is also to establish and maintain classroom norms and routines that support a productive and safe environment for students to share their ideas and take intellectual risks, especially when there is a focus on students collaborating to solve tasks.

For their part, the role of the student in student-centered instruction is to assume a position of intellectual authority and take responsibility for their learning by actively constructing their mathematical understanding (Claxton 1996; Cobb, 1994; Zimmerman 1990). This means that students use critical thinking and reasoning to solve open-ended and cognitively challenging problems (Brooks & Brooks, 1999). They are in charge of identifying a strategy or multiple strategies and knowing when and how to get more information to move their work forward (Lesh, Doerr, Carmona, & Hjalmarson, 2003). Students are also charged with co-constructing knowledge with their instructor and/or with their peers, which requires listening to others' ideas and explaining their ideas to others in order to construct a shared understanding. In some classes, teachers emphasize shared responsibility among students, meaning students have intellectual authority and share responsibility for learning among their peers (e.g., Michaels et al., 2008). Students in student-centered classrooms are expected to explain and justify their thinking, listen to and make sense of others' explanations, and share their level of agreement or disagreement with the teacher and peers (Wilson, Abbott, Joireman, & Stroh, 2002; Yackel & Cobb, 1994).

2.2.2. Motivational differences of student-centered mathematics instruction

2.2.2.1. Hypothesized indirect effects on student engagement

In addition to supporting learning from a constructivist perspective, the changes to the nature of academic tasks and to the role of the teacher and student can indirectly foster adolescents' engagement in mathematics coursework by meeting their psychological needs and developmental skills. Adolescents who have their psychological needs met experience the type of intrinsic motivation that fuels persistent engagement in academic work (see Deci & Ryan, 2000; Figures 2 and 3a). Teacher-centered instruction tends to limit opportunities for student autonomy and decision-making and can undermine students' sense of competence and relatedness by emphasizing performance and social comparison (Eccles et al., 1993). In contrast, student-centered instruction meets these developmental needs by providing developmentally aligned and motivating academic tasks and by supporting adolescents' need for self-determination, while also being supported in their learning (Smit, de Brabander, &

Martens, 2014). For example, competence is supported by teacher scaffolding and teacher support (Urdan & Turner, 2005) and by tasks that require adolescents to apply their increasingly complex cognitive skills (Eccles & Roeser, 2009). Autonomy is supported by providing relevant tasks (Assor & Roth, 2002), by giving students meaningful choices in their academic work and working styles (Savery & Duffy, 2001), and by fostering and respecting adolescents' ideas (Eccles & Roeser, 2009). Relatedness is supported by providing individualized support, by teachers facilitating supportive and cooperative interactions with students, and by establishing a climate within which students can initiate and participate in interactions that support their learning (Savery & Duffy, 2001). Thus, theoretically, student-centered instruction has the potential to support adolescents' engagement in mathematics coursework by meeting adolescents' developmental needs (Deci & Ryan, 2000). Figure 3b (Figure 3a modified based on the self-determination model of motivational development that appears in Skinner & Chi, 2012) illustrates the way in which student-centered mathematics instruction could indirectly influence overall engagement by meeting adolescents' psychological needs.

2.2.2.2. Hypothesized direct effects on student engagement

In addition to indirectly targeting adolescents' engagement in mathematics, there are also specific ways in which student-centered instruction could have a direct effect on adolescent engagement by structuring the ways in which adolescents engage in mathematics coursework. For example, the open-ended and cognitively challenging nature of academic tasks, the need for students to come up with their own solution strategies to these problems, explaining their thinking, and presenting and considering multiple solution strategies could require students to sustain a significant level of cognitive engagement. Tasks in teacher-centered classrooms tend to be focused on memorizing a specific set of procedures and discrete facts, which requires a relatively low level of mental investment or effort that is indicative of students' cognitive engagement (i.e., memorization does not require students to apply complex cognitive strategies or skills).

Similarly, student-centered instruction shapes adolescents' behavioral engagement by structuring the way in which they participate in class. Due to the focus on lectures and repeated practice, behavioral engagement in teacher-centered classrooms largely consists of active listening, answering questions, and completing independent work in class. In contrast, adolescents in student-centered mathematics classrooms are required to participate in a wide range of academic behaviors, including asking questions, participating in discussions, and presenting their work to the class. Of note, a greater proportion of these behaviors are student-initiated as adolescents take responsibility for their learning as they work to solve problems and understand complex mathematical ideas.

Student-centered instruction can also directly influence adolescents' emotional engagement by shaping their interest in, value of, and positive and negative responses in mathematics class. Constructivist and SDT theories both argue that offering tasks that are at the appropriate level of challenge can help to stave off the boredom and frustration associated with tasks that are too easy or too difficult (Smith & Stein, 2011; Eccles & Roeser, 2009). Adolescents are also more likely to value and be interested in tasks that they perceive as being relevant to their lives (Assor, Kaplan, & Roth, 2002).

Of the four proposed dimensions of engagement, the largest difference on hypothesized direct effects could be on students' social engagement in their mathematics coursework. Due to its focus on lecture and independent practice, social interaction is relatively limited in teacher-centered instruction. In student-centered instruction, learning is embedded in the classroom social context by positioning adolescents as responsible for their learning and by teachers facilitating adolescents' thinking through social interaction (e.g. questioning, discussion, students explaining their thinking, considering others' perspectives, working with peers, etc.).

Taken together, the hypothesized indirect and direct effects of student-centered mathematics instruction on students' engagement distinguish it from teacher-centered instruction by creating multiple pathways for shaping student outcomes in mathematics. Figure 4 illustrates the direct and indirect pathways between student-centered mathematics instruction, engagement, and outcomes are discussed in Chapter 3.

2.3. INSTRUCTIONAL REFORM

Mathematics instruction in the United States has a strong tradition of being teacher-centered. Beginning in the 1970's and 1980's, there was a growing sense among many in education policy and research that American society, education, and the workplace were becoming more complex, requiring more advanced mathematical skills (Gijselaers, 2000). At the same time, national achievement tests revealed vast mathematical underachievement among American youth (Klein, 2003). As a result, many believed that our education system needed to change how much mathematics students learn and how they learn it.

In 1983, the National Commission on Educational Excellence released the report "Nation at Risk: The Imperative for Educational Reform," which called for more work to be done to understand teaching and learning. In particular, the report asked for mathematics guidelines that would inform what students need to learn in order to be successful after high school. Historically, education policy did not mandate instruction per se; how subjects are taught was largely left up to districts, schools, or teachers (see Resnick & Resnick, 1992). However, the policy response that followed the Nation at Risk report addressed what students needed to know and the types of instructional practices or classroom activities that supported students in meeting the stated learning goals. For example, in 1989 the National Council of Teachers of Mathematics (NCTM) responded by articulating ambitious learning goals for adolescents. Falling under the name "standards," the guidelines included instructions on how to attain ambitious learning goals by adapting pedagogical practices that are largely student-centered. For example, the guidelines include:

- Students are, expected to be more active in their learning, should be asked to pursue open-ended problems and extended problem-solving projects where they investigate and formulate questions from problem situations;
- Students should discuss, write, read, and listen to mathematical ideas, rather than provide short answers, or purely numerical solutions;
- More attention should be spent on reasoning, placing the student in the role of mathematical authority rather than the teacher; and
- Students should engage in these topics in order to model, describe, analyze, evaluate, and make decisions about problem situations (National Council of Teachers of Mathematics, 2014)

By calling on teachers to position students as having intellectual authority as they actively engage in solving complex mathematical problems, the NCTM guidelines paved the road for instructional reform.

The movement to reform mathematics instruction gained traction in earnest when the majority of states adopted the Common Core State Standards (2015). Initiated by a bi-partisan organization that aimed to raise learning standards across content areas (Achieve, 1996), the CCSS articulated specific learning targets for students at each grade level, as well as eight Mathematical Practices that outline a student-centered approach to mathematics instruction. These included several key elements of a transition of intellectual authority and responsibility for learning from the teacher to the student, such as having students come up with their own way to solve a problem and having students explain and justify their thinking and strategies (see CCSS, 2011).

The development and adoption of the CCSS also saw several other policies emerge at local and state levels that further supported the implementation of student-centered instruction. Many school districts – some coordinated by efforts at the State level – invested in reform-based mathematics curriculum, hired content coaches, and implemented professional development to support teachers' transition from teacher- to student-centered approaches (see Cohen & Ball, 1990). In addition, several states developed and implemented state-mandated teacher evaluation policies that focused on teachers' use of student-centered practices (Danielson, 2012; Cohen & Ball, 1990; Resnick & Resnick, 1992). In the state of Pennsylvania, for example (the context for the dissertation studies), Act 83 required that teachers were evaluated in part by principals or central office administrators observing the extent to which their instructional practice supported a student-centered approach (Danielson, 2012). Taken together, the swell in policies and resources for training, supporting, and evaluating student-centered mathematics instruction contributed to a shift in the way that the education community at large thought about what we understand to be "high quality" or effective mathematics instruction to a view of mathematics instruction that is largely student-centered.

2.4. STUDENT CHARACTERISTICS

In addition to aiming to improve overall mathematics achievement, student-centered instructional policy aims to improve equity in mathematics outcomes. While education policies have focused on improving outcomes among minority and low-income youth (National Center for Education Statistics, 2015; U.S. Department of Education, 2015), there is also evidence that students' gender and overall level of achievement could influence how they engage in mathematics coursework and their subsequent level of mathematics achievement.

2.4.1. Race

Improving the underachievement of minority youth has been an impetus for numerous reforms at local and state levels (see Cohen & Ball, 1990; Common Core, 2015; National Council of Teachers of Mathematics, 2014). In their executive summary in 2014, NCTM argued that in order for mathematics instruction to support students' learning, teachers need to change their traditional practice to one that "engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically" (pg. 24). Algebra coursework has been identified as being particularly critical for creating equitable opportunities for success in secondary school and the workplace for low-income and minority youth (Silva, Moses, Rivers, & Johnson, 1990).

Student-centered mathematics instruction could influence engagement and achievement among minority youth in complex ways. On the one hand, African-American youth could be concerned that their high level of active and social participation in mathematics classes could activate others' stereotypes about African-Americans being bad at mathematics (Steele, 1997). African-American adolescents could fail to fully engage in student-centered instruction out of fear that they could activate those beliefs in others, or they could fully engage and experience complex achievement emotions out of concern of experiencing the negative stereotype about their mathematical ability. Indeed, research has found that teachers are more likely to perceive minorities as having less mathematical ability in schools with a high concentration of minorities (e.g. Flores, 2007). Minority youth also decrease their engagement to avoid stereotype threat in mathematics (Aronson, Fried, & Good, 2002), and experiencing stereotype threat can trigger emotions that are detrimental to learning (Mangels, Good, Whiteman, Maniscalco, & Dweck, 2012).

On the other hand, being positioned as an intellectual authority could empower minority youth to engage in their mathematics class. The perception or effect of a stereotype threat is reduced when students have a way of experiencing a positive academic self-image (Croizet, Désert, Dutrévis, & Leyens, 2000; Van Loo & Rydell, 2013) and believe that all students can improve their ability through effort (Boaler, 2013). In this way, student-centered instruction could further support minority students' engagement in their mathematics coursework by offering tasks that are personally meaningful and culturally relevant.

2.4.2. Socioeconomic status

Low-income students have also been the focus of instructional reform due to a disproportionate underperformance in mathematics coursework (National Center for Education Statistics, 2015; U.S. Department of Education, 2015). Student-centered instruction could either enhance or hinder mathematics learning for low-income youth. Students from low-income families may not be as socialized as their peers to participate in academic discourse, especially with their teachers (e.g., Hart & Risley 1995). Teachers supporting low-income students in academic discourse could provide rich learning opportunities for these youth, or they could put them at a disadvantage to participate productively. Furthermore, the inability or unwillingness to participate in academic discourse could disproportionately disadvantage low-income youth from eliciting high leverage teaching practice from their teachers (see Nurmi & Kiuru, 2015).

2.4.3. Gender

Student-centered mathematics instruction is also considered a tool for improving the relatively low mathematics-related motivational beliefs and aspirations of female students (e.g., Frenzel et al., 2010; Watt et al., 2012; Wigfield et al., 2006) and the under-representation of women in STEM fields (National Science Foundation, 2015), especially STEM fields that are math-intensive (Wang et al., 2013). There is some evidence that student-centered mathematics instruction is beneficial for both boys and girls (Brotman & Moore, 2008). However, some research suggests that particular components of student-centered mathematics instruction may have differential effects. For example, some research has found that girls may benefit more from instruction that includes relevant and meaningful tasks (Baker & Leary, 1995; Geist & King, 2008; Burkam, Lee, & Smerdon, 1997) and social interaction (Gilligan, 1982; Zohar,

2006) and that boys benefit more from autonomy-supportive instructional practices (Lietaert, Roorda, Laevers, Verschueren, & DeFraine, 2015).

2.4.4. Achievement level

The level of students' academic achievement in mathematics could influence their experience of and engagement in student-centered mathematics instruction in complex and interesting ways. Research shows that teachers adapt their instructional strategies and the amount of individualized support they offer in response to students' academic skills (Kiuru et al., 2015) and classroom behavior (Nurmi & Kiuru, 2015). Mathematics teachers, in particular, are more likely to use teacher-centered practices when they perceive that students are not working hard, are off task (Kiuru et al., 2015; Nurmi & Kiuru, 2015), or resist working on challenging tasks (Fulmer & Turner, 2014). Therefore, lower achieving students may elicit less frequent or less effective high leverage student-centered instructional practices, while higher achieving students may elicit more of them. If true, this could result in high achieving students being afforded additional advantages with student-centered mathematics instruction.

The fact that mathematics teachers' perceptions of student ability and effort shape their use of student-centered instruction could disproportionately affect how they teach to classes with predominately minority or low-income students, potentially compounding the relationship between race, socioeconomic states (SES), and achievement. Teachers are more likely to perceive minorities as having less mathematics ability in schools with a high concentration of minorities (e.g., Flores, 2007), and minority students who are motivated to do well in mathematics may, in turn, opt out of fully participating in student-centered classrooms in order to avoid invoking a negative performance stereotype (Sackett, Hardison, & Cullen, 2004; Steele

& Aronson, 1998). These complex interpretations of student effort, achievement level, and student characteristics eliciting specific types of instructional practice could contribute to perpetuating the disproportionate outcomes for minority or low-income youth that instructional reforms are intended to remedy.

2.5. GAPS IN THE RESEARCH

Despite the large-scale implementation of student-centered instructional practices, there is a need to systematically study the effects on mathematics engagement and achievement in secondary schools and how students experience it. In particular, it is important to investigate the effectiveness of student-centered mathematics instruction and potential differential effects for students with different demographic characteristics. Despite decades of instructional reform, evidence indicates that declines in engagement and achievement persist (e.g., Gonzales et al., 2008). When a Nation at Risk was released in 1983, the report cited the fact that only one-third of high school graduates completed intermediate algebra coursework, resulting in an undue burden to post-secondary education institutions and businesses to provide remedial education and training (Evans et al., 2006). While achievement has risen nationally overall, the disproportionate underachievement of minorities and low-income students in the United States (U.S. Department of Education, 2015) and the relatively low performance of the United States on the international stage persist (OECD, 2012). Thus, it appears that we are in critical need of better understanding how to support adolescents' mathematics engagement and achievement and the role of student-centered mathematics instruction.

Studying student-centered mathematics instruction is also necessary to address and understand critiques of the instructional approach. While student-centered instruction has gained momentum in education policy, there are educators and researchers who question when and/or whether student-centered practices effectively support learning. Some teachers are concerned that implementing student-centered instructional practice will undermine their ability to maintain classroom order. Teachers have reported that young students respond to student-centered instruction with high energy that is difficult to manage (Polly, Margerison, & Piel, 2014). Teachers report that students can resist engaging in challenging and open-ended tasks indicative of student-centered mathematics instruction with negative emotions and behaviors (Felder & Brent, 1996; Garrett, 2008; Lasry, Charles, Whittaker, 2014; Pedersen & Liu, 2003), and teachers worry that student-centered approaches do not provide the structure needed to enforce discipline (Polly et al., 2013). Furthermore, some teachers argue that students lack the fundamental skills to reason independently (Felder & Brent, 1996; Pedersen & Liu, 2003) and believe that in order for students to learn something, teachers should convey it to them explicitly and directly (Kirschner, Sweller, & Clark, 2006). Without guidance, opponents question how students ever come to construct the concepts that are necessary for success in higher-level mathematics, which can contribute to feelings of incompetence and underachievement (Assor, Kaplan, & Roth, 2002). Some argue that the level of cognitive demand in student-centered instruction could tax students to the point of becoming demotivated (Jitendra, 2013; Kirschner et al., 2006; Klahr & Nigam, 2004). Indeed, some students experiencing new forms of learning report difficulties in finding effective strategies to tackle authentic tasks because they are used to tasks that are conceptually and procedurally simplistic (Mayer 2004).

Education research and policy would benefit from studying student-centered mathematics instruction in secondary schools, since the context of instructional reform is adolescents' declining engagement in mathematics throughout secondary school and the fact that disproportionate achievement outcomes for low-income and minority youth begin to widen during this time (see "Background Context" in Chapter 1). The importance of academic success in mathematics coursework also changes during this time. Algebra coursework is considered a gateway for success in secondary school mathematics (Evans et al., 2006). For most adolescents, two years of required algebra coursework begins in earnest in seventh grade, which can also mark the transition from elementary to middle school.

2.5.1. Survey measure of student-centered mathematics instruction

To better understand how and when student-centered instruction influences adolescents' engagement and learning in mathematics, we first need to develop a well-validated measure of student-centered mathematics instruction. In particular, we need a well-validated measure that: a) uses the literature and feedback from teachers and experts to develop consensus around what student-centered mathematics instruction is; and b) uses information from a large and diverse sample of adolescents to establish predictive validity with student engagement and achievement in mathematics. SDT posits that student-centered instruction could enhance adolescents' engagement and achievement in their mathematics coursework by meeting their psychological needs and by being well aligned to adolescents' developmental skills. Constructivist theories argue that student-centered mathematics instruction supports adolescents' learning and achievement by structuring opportunities for adolescents to engage deeply in mathematics coursework. A well-validated measure of student-centered mathematics instruction should relate to and can be used to help us better understand adolescents' cognitive, behavioral, emotional, and social engagement in mathematics coursework.

In particular, a measure of student-centered mathematics instruction could help us examine social engagement and its role in mathematics learning. Social engagement is featured in student-centered mathematics instruction and is a relatively new construct in engagement research. Social engagement has been validated as a central component of adolescents' engagement in mathematics coursework (Wang et al., 2016) and constructivist learning theory, and SDT argues that the quality of social interactions in mathematics can contribute to adolescents' achievement and aspirations to continue to pursue mathematics in secondary school, higher education, and in the workplace. However, the relationship between student-centered instruction and social engagement has not been studied. Examining the extent to which student-centered instruction influences adolescents' social engagement in mathematics coursework could help us understand how instruction influences the quality of social interactions.

A well-validated measure would also enable us to collect more evidence about the effects of student-centered instruction on adolescents' achievement in mathematics and potential disproportionate effects based on student characteristics. Policies calling for teachers to implement student-centered instruction have two aims: 1) improve overall mathematics achievement among American youth; and 2) address disproportionate underachievement, particularly among low-income and minority youth, by providing high quality opportunities to learn to all students. Chapter 3 describes the research on student-centered mathematics instruction and the need to understand its effects at scale and among different student populations. The increasing number of policies calling for widespread implementation of student-centered mathematics instruction represents a substantial investment in mathematics education – an investment that often requires school leaders and teachers to transform how mathematics is taught to students. In many cases, mathematics teachers are asked to abandon teacher-centered instructional practice altogether and to develop a student-centered instructional practice. Given the amount of time, energy, and resources required to support teachers in transforming their instructional practice, it befits education researchers and policy makers to understand the effects of student-centered instruction on math outcomes.

A measure of student-centered mathematics instruction would also enable us to study the complex ways in which student-centered instruction relates to a mathematics teacher's overall instructional practice. Generally, education research, policy, and practice describe teacher-centered to student-centered instruction as a paradigm shift that calls on teachers to transfer intellectual authority, responsibility for learning, and the cognitive effort to solve problems to students (e.g., Vermunt & Verloop, 1999). It is possible, however, that effective mathematics teaching involves a more complex relationship between the two pedagogical approaches. For example, teacher- and student-centered instruction could be related through transactional processes in the classroom, according to which there is a continuous renegotiation of classroom activities and student and teacher roles (Cooper & McIntyre, 1993). In practice, teachers using student-centered instruction incorporate teacher-centered instruction into their teaching practice, sometimes with positive results for students' achievement (e.g., Tarr et al., 2008). For example, mathematics teachers may include repeated practice and memorization of procedures fundamental to students' success on a specific open-ended task.

For their part, students report that teacher-centered instruction is a key component of effective mathematics teaching. In a study of teacher- and student-centered instruction, Elen et al. (2007) found that factor analysis of secondary students' report of what makes for a "good" mathematics teacher revealed that students feel they learn best when the teacher monitors the learning process, the capabilities of students, and the willingness of students to regulate their own learning and uses this information to continuously assess and reorient the responsibilities and tasks between the teacher and student. In this view, students are supported in taking on more responsibilities (Vermunt & Verloop, 1999), but the gradual assumption of intellectual authority and responsibility for learning is coached and monitored by the teacher. In other words, from students' perspective, teacher- and student-centered practices are not necessarily mutually exclusive, but instead can be mutually reinforcing features of high quality mathematics instruction (Elen, Clarebout, Leonard, Loweyck, 2007).

However, not all combinations of teacher- and student-centered instruction result in effective instruction. Some ways in which teachers integrate teacher- and student-centered instruction can undermine student learning (e.g., Cohen, 1990). Furthermore, teachers who implement cognitively challenging and relevant tasks often reduce the cognitive demand of the task throughout the lesson, reverting to explaining the mathematical concepts and describing how to solve the problem (Stein, Grover, & Henningsen, 1996). Finally, many teachers implementing student-centered instruction revert to teacher-centered instruction specifically when students are struggling, which can undermine the quality of the learning task and students' opportunities to learn (Reinhart 2000; Stein, Engle, Smith, & Hughes, 2008). We need to be able to measure and understand the difference between effective integration of instructional strategies and failure to implement student-centered mathematics instruction.

Thus, in order to lay the foundations for the needed research on student-centered mathematics instruction, I proposed developing a well-validated measure in Study 1. After describing extant measures, Study 1 describes a mixed method investigation that uses a review of the literature and current measures, expert validation, and psychometric analyses to propose a multi-dimensional conceptualization of student-centered instruction. The measure is tested and validated with data from a large-scale study of adolescents' engagement in mathematics and science coursework. Specifically, the analyses explore: factor structure, measurement invariance by student race, gender, socioeconomic status, and achievement level; predictive validity of students' cognitive, behavioral, emotional and social engagement and mathematics course grades; and differential predictive validity by student characteristics.

2.5.2 Student emotional experience of student-centered instructional reform

In addition to developing a well-validated measure, the field would also benefit from studying adolescents' experiences of student-centered instruction. We do not yet understand if adolescents require or would benefit from specific supports to do well with student-centered practices, especially in the transition from teacher- to student-centered instruction. The transition from teacher- to student-centered instruction – or the relative increase in student-centered practices – is a significant source of concern and challenge for teachers during district or school-wide instructional reform (Fulmer & Turner, 2014; Felder & Brent, 1996; Garrett, 2008; Lasry et al., 2014; Pedersen & Liu, 2003). The challenges for students adapting to student-centered mathematics instruction has not been systematically studied. Student-centered instruction involves a transformation of student and teacher roles and the nature of academic work that can be motivating and that can support deep learning, but that is also dissonant with the norms and expectations to which students have been socialized in their mathematics classes. Policy and practice maintain student-centered instructional practices as high leverage teaching practices in their own right; i.e., if you implement student-centered instruction, then student engagement and achievement will improve. However, we know very little about the challenges that students experience as they adapt to student-centered instruction. An on-the-ground study of how students experience the transition from teacher- to student-centered mathematics instruction could help us better understand how to support their productive and successful engagement in instructional reform.

In particular, students may grapple with the experience of working on challenging and open-ended tasks. In teacher-centered instruction, academic work flows in cycles of repeated practice of procedures that are well described and supported by the teacher. In student-centered instruction, students are encouraged to productively struggle to solve mathematical tasks (Smith & Stein, 2011), which could include periods of being stuck and not knowing how to move forward. Research on achievement emotions suggests that getting stuck could be an emotionally salient experience that shapes adolescents engagement and achievement by triggering students' cost-value appraisals (see Pekrun et al., 2007; described in Chapter 4).

In order to gain a better on-the-ground perspective of what student-centered mathematics instruction is like for students, I proposed studying middle school students' emotional experience of getting stuck in the context of transitioning from teacher-centered instruction. Specifically, Study 2 describes an in-depth mixed method study of students' emotional experiences of getting stuck in reform mathematics classes. Using the control-value theory of emotions, the study examines the prevalence of getting stuck, emotional experiences of getting stuck, differences based on student characteristics, and effects of getting stuck on perceived competence.

2.5.3 Bringing it all together: developing a measure and studying student experiences

Taken together, Study 1 and Study 2 lay the groundwork for studying and supporting mathematics engagement and achievement from the perspective of adolescent development in schools. Student-centered mathematics instruction describes features of the classroom environment that have the potential to shape adolescents' motivation and engagement in learning mathematics. In this way, conceptualizing, operationalizing, and measuring student-centered instruction can help us study the effects of the classroom context on education outcomes for youth. However, in order to fully understand *how* student-centered instruction shapes outcomes for youth, we need to identify *specific* ways in which student-centered instruction interacts with adolescent development. Study 2 studies a specific mechanism that is both prevalent in student-centered instruction and salient to adolescents' developmental needs, which can help us better understand how to study and support student engagement and achievement in classrooms implementing student-centered instruction.

2.6. STUDY CONTEXT

Both studies use data recently collected from teachers and students in the state of Pennsylvania. The achievement gap based on race and socioeconomic status in Pennsylvania exceeds the national average and widened between 2013 and 2015 (U.S. Department of Education), the period during which the data for this study was collected. Student-centered instructional practices are seen as being a critical part of reversing this trend. In this context, over the past five years Pennsylvania has enacted multiple education policies that support the implementation of student-centered instruction. For instance, the Pennsylvania Core Standards for Mathematics has a mathematics practice standard that states, "[Students] construct viable arguments and critique the reasoning of others" (Pennsylvania State Academic Standards, 2017). The standard aligns with reform, or constructivist, approaches to mathematics instruction. To meet this standard, students must be presented with open-ended problems to which they construct responses and consider other students' ideas and solutions. Similarly, the mandatory teacher evaluation system (Act 82 Educator Effectiveness) describes that students should be positioned as sources of knowledge for one another instead of exclusively for the teacher (Danielson, 2012). Finally, statewide tests (the Pennsylvania System of School Assessments and the Keystone Exams) promote student-centered instruction by assessing student skills that align with the standards. For example, the Algebra Keystone exam contains constructed response questions for which students have to explain their mathematical reasoning. To prepare for constructed response items on the exam, students need to explain their reasoning in math class, a practice that supports students' intellectual authority. Thus, students and teachers in the state of Pennsylvania are likely to be familiar with student-centered instructional practices, and student-centered instruction is likely to be used in some degree in mathematics classrooms, making the phenomena under study observable (Eisenhardt, 1989).

3. STUDY 1: DEVELOPING A MEASURE OF STUDENT-CENTERED MATHEMATICS INSTRUCTION

3.1. INTRODUCTION

Developing a reliable and valid measure of student-centered mathematics instruction is urgently needed in order to inform mathematics education research, policy, and practice. More and more, local and state polices are calling on teachers to implement student-centered mathematics instruction in their classrooms. The call is reflected in standards, teacher evaluations, and curricular reforms, and is becoming a central part of teacher-education programs. Despite the increasing prevalence of student-centered mathematics, we still know very little about how it is implemented or how it affects students' experiences of and outcomes in mathematics. The majority of research on student-centered instruction (as an overall pedagogical philosophy) focuses on higher education contexts (e.g., Severiens, Meeuwisse, & Born, 2015). Extant research in secondary schools focuses on cases studies (e.g., Knight, Parker, Zimmerman, & Ikhlief, 2014) or relatively small-scale studies of the implementation of student-centered instruction (e.g., Saragih & Napitupulu, 2015), each of which tend to characterize student-centered mathematics instruction through qualitative assessment or assignments (i.e., through the implementation of a specific curriculum or reform, or through assigning teachers or schools to receive training on or implementation of student-centered instruction in a randomized control trial).

Having a well-validnewated measure of student-centered mathematics instruction would enable us to study the extent to which student-centered instruction is implemented in secondary mathematics classes, challenges to implementation, and between- and within-teacher variability in implementation. In addition, we could systematically study the short- and long-term effects of student-centered instruction on student motivation, engagement, and achievement. Operationalizing the components of student-centered instruction would also enable us to study what components of student-centered instruction work for whom and under what circumstances. Finally, an appropriate measure would enable us to study the complexity of instructional practice by examining how student-centered mathematics instruction compares to and interacts with other aspects of instructional practice. Taken together, this knowledge could inform ongoing instructional reforms and their implementation in policy and practice and could inform the development of targeted interventions to support mathematics education for youth.

This paper addresses the need for a well-validated measure with a mixed method approach that begins with a qualitative assessment of the components and indicators of student-centered instruction, and that is followed up with a quantitative study of the psychometric properties of the newly developed measure. The conceptualization and operationalization of student-centered mathematics instruction is informed by: literature on student-centered instruction and corresponding literature in constructivist views of learning and self-determination theory of motivation (SDT); systematic review, selection, and revision of items from existing scales; and a review of items by experts in mathematics instruction and the learning sciences. The quantitative study is informed by two waves of survey data that are used to provide evidence that support the reliability and validity of the measure.

3.2. LITERATURE REVIEW

3.2.1. Defining student-centered mathematics instruction

Student-centered instruction has been described in several ways. The *Encyclopedia of Mathematics Education* explains, "Student-centered teaching… has grown so prominent in both research and teaching venues over the decades that many differences have emerged, rendering one, unified approach difficult to describe" (Lerman, 2014, p. 339). In a similar spirit, Newman (2016) expressed, "Instead of being a simple concept, 'student-centered learning' is actually a complicated and messy idea that has encompassed a wide range of sometimes fundamentally different meanings, each holding important implications for education" (pg. 161). One of the challenges of defining student-centered mathematics instruction is that a variety of teaching methods have been developed to bring a student-centered approach into classrooms. For example, dialogic instruction is a specific variant of student-centered instruction that focuses on the role of academic discourse for situating learning in social interaction and for supporting students as intellectual authorities responsible for their learning (see Resnick et al., 2008). Collaborative instruction is another method that emphasizes collaborative work with peers as a mechanism for situating learning in social interaction and establishing shared intellectual authority and responsibility for learning (Michaels et al., 2008; Resnick & Nelson-Le Gall, 1997). Problem- or project-based learning foregrounds the role of real world and open-ended tasks and student responsibility for coming up with novel and multiple strategies for solving problems (e.g., Barrows & Tamblyn, 1980; Dole, Bloom, & Kowalske, 2016). In some cases, students in problem- or project-based classrooms select authentic problems or challenges to address, work with peers to solve them, and present their solutions to real audiences (Barron & Darling-Hammond, 2008). Inquiry-based pedagogy focuses on the learning process by having students identify questions that need to be answered, analyzing and interpreting data, and considering others' solutions (e.g., Wilhem & Wilhelm, 2010). These methods exemplify strategies teachers use to implement a student-centered instructional philosophy, each with features that are indicative of student-centered instructional practice which rely on changes to the nature of academic tasks and the role of the teacher and student in student-centered classrooms. However, it is not clear that this conceptualization of student-centered instruction is an effective framework for developing a measure of student-centered instruction. Furthermore, it is not clear what type of measure would contribute to the evidence base and deepen our understanding of student-centered instruction. To inform the development of a measure of student-centered mathematic instruction, I first review the effects of student-centered instruction from literature that focuses on student-centered instruction as a general instructional philosophy. Then, I describe the measures used in that research and review the ways in which these measures contribute to a gap in the research on student-centered instruction. Finally, I describe the limitations of the general conceptualization of student-centered mathematics instruction and propose a conceptualization that synthesize research on the nature and effects of key mechanisms of student-centered instruction that serve as the foundation for the development of a measure.

3.2.2. Effects of student-centered mathematics instruction

To develop an understanding of the value and purpose of a measure to assess student-centered mathematics instruction, I began a review of the literature that explains the effects of student-centered instruction, in general, as an overall instructional philosophy. For this search, I used the database Scopus to search literature that describes research or meta-analyses on student-centered instruction or learning, active instruction or learning, reform-based instruction, and ambitious math instruction. In order to inform the understanding of mathematics outcomes for adolescents, I focused the search by content and educational context. Specifically, I searched studies examining engagement, learning, or achievement in mathematics coursework among students in secondary school.

The various types of instructional methods that are used to bring certain aspects of student-centered instruction into the classroom were not included here for two reasons. First, the specific methods, such as inquiry instruction, dialogic instruction, and collaborative learning, do not, on their own, represent "student-centered instruction" as an instructional philosophy. Instead, they tend to foreground specific components of student-centered instruction, which could change the mechanism that is shaping engagement and achievement. Student-centered instruction as an instructional philosophy broadly describes characteristics of academic work and student and teacher roles. For the purpose of understanding the effectiveness of this instructional philosophy, I begin with literature that examines this broad view. The specific methods of student-centered mathematics instruction and literature on their known effects are reviewed later in the chapter when I propose a conceptual framework in which specific methods are integrated.

Research on the effects of student-centered mathematics instruction on academic outcomes for youth in secondary schools is relatively limited. While many instructional reforms target secondary schools, the majority of the research of student-centered mathematics instruction is from its implementation in early elementary schools or in post-secondary settings, leaving the effects of student-centered mathematics instruction on adolescents relatively unexplored. The evidence that is available suggests that student-centered mathematics instruction may support adolescents' motivation, engagement, and achievement in their mathematics coursework. Students report higher levels of intrinsic motivation in academic work when their teachers are using student-centered instructional practices (Baeten, Dochy, & Struyven, 2013; Hänze & Berger, 2007; Meece, 2003; Smit et al., 2014; Turner, Thorpe, & Meyer, 1998). Middle and high school students whose mathematics teachers used student-centered instruction reported higher perceived competence, autonomy, and relatedness (Smit et al., 2014), and a greater enjoyment of mathematics (Noyes, 2012). In contrast, middle school students report the highest rates of boredom when doing passive academic work, such as listening to lectures, that is common in teacher-centered instruction (Larson, 2000). Student-centered instructional practices have also been shown to increase adolescents' understanding of mathematical concepts and practices (Saraghi & Napitulu, 2015), achievement in mathematics coursework (Wilson et al., 2002; Ziegler and Yan 2001), and better performance on standardized tests (Cornelius-White, 2007; Friedlaender, Burns, Lewish-Charp, Cook-Harvey, & Darling-Hammond, 2014; Lasry, Charles, & Whittaker, 2014; Polly, McGee, Wang, Lambert, Pugalee, & Johnson, 2013; Tarr et al., 2008).

3.2.3. Current measurement of student-centered mathematics instruction

In these studies, student-centered instruction is measured mostly through observational tools, qualitative assessment of classroom artifacts or teacher interviews, by assignment in a randomized control trial or in a school implementing student-centered mathematics instruction, or by a combination of these strategies. The least common measurement tools are self-reports or survey instruments. All measurement strategies are reviewed briefly below, followed by a more detailed review of extant survey measures.

3.2.3.1. Observational tools

Several observational assessments measure the presence of specific characteristics of student-centered instruction. Polly et al. (2014) used a Mathematics Teaching Scale (MTS) in combination with teacher interviews to determine if teachers espoused teacher- or student-centered instruction. The Mathematics Teaching Scale consists of 5 sets of items that assess the teachers' emphasis on problem-solving, small group instruction with differentiation, knowledge focus, and overall teacher- and student-centered instruction. Videos of teachers' lessons were coded with the MTS. Teachers were categorized as either teacher- or student-centered.

Other observational assessments have been developed for teacher professional development and teacher evaluation that measure student-centered instructional practice as part of or reform-oriented or ambitious instruction. For example, the Instructional Quality Assessment (IQA) (Matsumura, Slater, Junker, & Peterson, 2006) measures a range of high-leverage teaching practices, including the use of instructional dialogue (Resnick et al., 2008) and the extent to which the level of cognitive demand is initiated and maintained in classwork (Smith & Stein, 2011; Stein & Smith, 2011). The Danielson Framework for Teaching (Danielson, 2012) is an observational tool that is based on constructivist views of teaching and learning and is aligned to content standards. The IQA is most commonly used in research and the Danielson in teacher professional development and evaluation. The IQA and other observational assessments have also been combined with other qualitative data, like teacher interviews and classroom artifacts. Smit et al. (2014), for example, determined if teachers in a school were using student- or teacher-centered instruction by examining their curricula, interviewing team managers, observing lessons, and consulting school policy documents.

3.2.3.2. Assignment

Student-centered mathematics instruction is also measured through assignment, either by being assigned to receive training as part of a research study or through the implementation of student-centered reform. In randomized control trials, teachers or schools can be assigned to receive training on student-centered mathematics instruction, which is then measured and compared to a control group. Researchers also study the implementation of school-wide or district-wide reforms, in which case teachers may receive professional development and/or may use a new curriculum that helps teachers use student-centered instructional tasks and practices. Students, teachers, schools, or districts participating in reforms can then be studied to examine experiences, implementation, and effects. In this case, the "measure" of student-centered instruction is the assignment to receive training and/or a mandate to implement student-centered practices.

3.2.3.3. Self-report surveys

There are very few student- or teacher-report surveys that measure student-centered instruction specifically. In the literature I found evidence of one survey measure, called the Scale on Student-Centered Learning Environments (QSCLE; Acat, 2006, cited in Cubukcu, 2012). The measure includes 50 items that assess 4 dimensions: the psychosocial environment, infrastructure-hardware, place, and time. It was developed for a foreign sample and is not available in English. There are, however, several measures that are conceptually consistent with and/or include components of student-centered mathematics instruction. In particular, extant student and teacher reports of classroom climate, related instructional practices, and constructs of constructivist learning (e.g. inquiry or investigations) and SDT theories of motivation (e.g. autonomy support) include several relevant indicators of student-centered instruction that could be used to develop an effective survey measure. The following sections describe the limitations of extant measures of student-centered mathematics instruction, identify the potential benefits of developing a student and teacher survey, and explain how articulating a clear conceptualization of student-centered mathematics instruction can inform the development of a survey measure.

3.2.4. Limitations of extant research and measures

While promising, evidence for the effects of student-centered mathematics instruction is relatively limited. Despite the large-scale implementation of student-centered instructional practices, there have been few large-scale studies of its effects. The majority of research available does not directly study student-centered mathematics instruction, but instead focuses on the implementation of instructional reform, which includes student-centered instruction as a key feature, but also includes the adoption of specific content standards and standardized tests. The few studies that focus directly on student-centered instruction focus on relatively small-scale implementation. As a result, student-centered instruction remains largely informed and fueled by a combination of: 1) large-scale student achievement data from national and international assessments over the past 40-50 years; and 2) theories about how students learn mathematics and what motivates them to stay engaged in learning tasks. More research is needed to help us understand when and how student-centered mathematics instruction influences adolescents' engagement and achievement in mathematics class.

The limitations in the research are related to limitations of the tools and strategies available to measure student-centered instruction. Observational tools, for example, are generally used to generate an overall assessment of a teacher's practice. The primary benefit is that they generate a third-person perspective of instruction and so they are less susceptible to reporter bias. There are multiple significant limitations, however. In particular, observations are time-consuming and tend to assess instruction by averaging observations of instructional practice. For example, research and practice often focus on assessing teachers' overall practice by observing a single class period. This could work well if teachers' use of various instructional practices was relatively consistent within classes, across classes, and over time. Research shows, however, that teachers vary their instructional practices (Raudenbush, Rowan, & Cheong, 1993), suggesting that "overall" assessments of instruction may obscure important variation in students' instructional experiences. Second, teachers may vary their instructional practice in important ways within classrooms, especially in regards to student characteristics (see 2.5 Gaps in the research). By averaging instructional practice within a class period, observations miss potentially important variation in an individual student's experiences. While research based on observation can provide a third person perspective on instruction and insight into student-centered instruction as it unfolds in classrooms, its generalizability is limited by the scope of classrooms that can be included.

Other types of qualitative assessment of instructional practice have similar limitations. Using multiple sources of data can provide rich information about classroom processes while triangulating assessment with multiple types of data. The limitation is that it is burdensome, and so tends to focus on one class or generalizing across a teacher's practice by assessing one class period. Another significant limitation of this strategy is that despite the availability of rich data, the purpose of the analysis can be to "bin" teachers into "teacher-centered" or "student-centered" instruction. Without necessarily an assessment of the extent to which a teacher used teacher- or student-centered practices, this approach to studying student-centered mathematics instruction can oversimplify and misrepresent instructional practice.

Studying student-centered instruction as it unfolds in classrooms selected to participate in instructional reform (i.e., through assignments) has the benefit of providing data about how student-centered instruction is implemented and its effects when student-centered instruction is what is intended. This work can be limited, however, by the tendency to rely on the teaching practice that was intended and not measure teachers' practice as enacted (for example, in state-wide comparisons of the effects of the implementation of the CCSS; U.S. Department of Education, 2015). Another significant limitation is that randomized control trials can be based on extreme versions of student-centered instructional practice. For example, in a study comparing the effects of direct instruction with discovery learning in science, Klahr and Nigam (2004) created an extreme version of the student-centered method and its teacher-centered counterpart. In the discovery learning arm of the study, students had no instructions and little or no direction or help from teachers. In the direct instruction arm of the study, teachers controlled all aspects of academic activity and classroom interactions. The result is that the two caricatures of instruction do not reflect good instructional practice of either kind (Kuhn, 2007). Student-centered teachers provide feedback and assistance, and teacher-centered instructors usually do not demand control of all aspects of classroom work.

Survey measures might be a particularly good fit for the pressing need to build the evidence for the effects of student-centered mathematics instruction. Survey measures of instructional practice are not without their limitations. They are susceptible to social desirability bias and can be limited in their ability to accurately and adequately capture the scope of instructional practice in mathematics. Instructional practice involves the integration and coordination of a number of behaviors that may not be represented well in survey items about instruction. However, a well-validated survey measure would enable the study of student-centered mathematics instruction on a large scale relatively more efficiently and cost-effectively than with observations or other types of qualitative assessment. It could be beneficial to add a well-validated survey to the tools available to study and understand the implementation and effects of the types of instructional practices lauded by learning and motivational theory and put forth in instructional reforms.

3.2.5. Student and teacher self-report surveys

In particular, the field could benefit from developing a teacher and student report. Existing measures favor observer report, which can provide a third-person (i.e., relatively objective) overview of a teacher's practice. Similarly, teachers can report on their average use of student-centered instruction in a given class or overall in the classes that they teach. On the one hand, teachers have unique insight into their instructional practice across classes and their pedagogical knowledge can provide an informed perspective. On the other hand, teacher reports of their instructional practice can be inaccurate (Rowan, Correnti, & Miller, 2002) and can obscure potentially important variation in students' experiences of instruction across classrooms (Meyer & Eley, 1999).

For their part, students may be uniquely situated to report on student-centered pedagogy (Ferguson, 2012). Unlike teacher-centered instruction that focuses on teacher behaviors, student-centered instruction emphasizes teacher behaviors that support specific types of *student* thoughts and behaviors. In other words, student-centered instruction is about students. Students might be in an appropriate position to report on the extent to which tasks were relevant and challenging, their intellectual authority, and the extent to which their teachers provide flexible and responsive support. In addition, students' perspectives may be particularly important to ascertain in mathematics wherein their engagement and achievement tends to decline throughout secondary school (McPhan, Morony, Pegp, Cooksey, & Lynch, 2008). In their study to develop a measure of student and teacher report of student engagement in mathematics and science, Wang et al. (2016) found that student and teacher reports were related to the dimensions of engagement and student outcomes in different ways. Specifically, while teacher and student reports were highly correlated on behavioral and cognitive engagement in mathematics (*rs* = .57-.45), teachers' reports were less correlated with students' reports of their emotional and social engagement (math: *rs* = .34-.21). This indicates that students and teachers may have unique perspectives on instruction and that their perspectives may predict different types of important mathematics outcomes.

These findings support the possibility that students' perspectives of student-centered instruction may provide unique insight into their experiences or perceptions of their mathematics teachers' instructional practice and/or the relationship between student-centered mathematics instruction and education outcomes. Student reports may also help us better understand differential experiences of students with different demographic characteristics. Given the aim to improve underachievement in mathematics among minority and low-income students and low aspirations to pursue math-intensive STEM college majors and careers among female students, it would be valuable to see if these groups of students experience a differential amount or quality of student-centered instructional practice and if there are differential effects for these groups. Student reports may reveal variation in teaching across and within classrooms that teacher reports may not reveal.

3.2.6. Conceptualizing the components of student-centered instruction

Developing a validated student and teacher report of student-centered mathematics begins by articulating a clear view of what student-centered mathematics instruction is. Given the volume of perspectives and the tendency to group these perspectives under a broad view of what student-centered instruction *is*, I begin by defining the conceptual landscape. In Chapter 2 student-centered instruction was described in terms of the theory of learning upon which it is based, the nature of academic tasks, and the role of the teacher and students. This conceptualization is useful for understanding the differences between teacher- and student-centered instruction. However, in order to operationalize student-centered instruction for the purpose of measuring its implementation and effects, I need to identify, describe, and categorize the types of student-centered instructional practices that teachers use to establish a student-centered learning environment. What does a mathematics teacher do when he or she implements student-centered instructional strategies, and what instructional practices do teachers use to create and maintain a student-centered learning environment? Do student-centered instructional practices share common features or purposes that enable them to be categorized into components or dimensions of student-centered teaching? Finally, what are the indicators of each of these types of practices?

This section lays the groundwork for an a priori framework that is used to develop a survey by: 1) identifying the types of practices that are described in literature on student-centered instruction; 2) exploring the ways in which these practices are supported in literature on adolescent development and motivation in school; and 3) integrating research on methods of student-centered instruction that align with the components of student-centered mathematics instruction. A priori constructs can serve as conceptual guides for survey development that can be examined, confirmed, and/or revised with additional evidence (Eisenhardt, 1989; Gehlbach & Brinkworth, 2011). In this study, I used an a priori framework of student-centered instruction to: a) identify indicators of student-centered mathematics instruction; and b) inform a review of extant measures for items that measure these indicators.

3.2.6.1. Current conceptualizations

While several descriptions and conceptual frameworks have been presented for instruction in postsecondary settings, there are very few well-articulated conceptual frameworks of student-centered instruction in secondary schools.[4] Research on student-centered instruction in

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[4] Given the focus on instructional reforms in secondary education and given important differences between teachers, learners, and teaching and learning in the secondary compared to postsecondary education, I focus on conceptualizations and research in K-12 education.

secondary schools generally describes the theoretical foundations (usually constructivism and/or SDT) and the structural components (nature of academic work and the role of the teacher and student, reviewed in Chapter 2). The few conceptualizations that have been articulated focus on student-centered learning. However, none are specific to mathematics teaching or learning. In education research and practice, pedagogical practices are considered to be at least partly discipline-specific. For example, mathematics teachers' pedagogical content knowledge (Shulman, 1987) has been associated with multiple measures of mathematics learning and achievement (see Staub & Stern, 2002). There is also evidence that student engagement in mathematics coursework could be distinct from their engagement in other content areas (Wang et al., 2016). Thus, it could be beneficial to consider how conceptualizations of student-centered learning, in general, could inform the development of a survey for student-centered mathematics instruction.

One such model is Lee and Hannafin's (2014) conceptualization of student-centered learning in K-12 education. Lee and Hannafin (2014) define student-centered learning as "a learning approach during which students generate learning opportunities and reconstruct knowledge dynamically in an open-ended learning environment" (p. 707). In this way, they describe student-centered learning similar to how it is described in Chapter 2, as based in constructivist theories of learning and SDT theories of motivation and as differing from direct instruction in terms of the theoretical perspective of learning, the nature of academic work, and the role of the teacher and student. They also describe differences in locus of control. The focus of the conceptual paper, however, is on articulating design principles for supporting student-centered learning. They use the phrase "own it, learn it, share it" to frame the types of teacher practices that support student-centered learning. For example, to support students in "owning" their learning, teachers can provide choices that matter and opportunities to set personal goals. To support students "learning it," teachers support students' varying needs and support students as they monitor progress. To support student interaction that supports learning ("sharing it"), teachers can promote dialogue and provide opportunities to review student work. This framework was used along with structural components described in Chapter 2 to inform the coding of literature that examined or described student-centered mathematics instruction.

3.2.6.2. Proposed conceptualization

To examine the components of student-centered instruction from the point of view of literature and theory, I completed two literature searches. The first literature search examined literature in mathematics education, the learning sciences, educational psychology, and developmental psychology that described student-centered instruction.[5] Unlike the previous literature search, which focused on conceptual frameworks of student-centered instruction, this literature search examined the implicit perspectives of student-centered instruction described in literature reviews examining student-centered practices. Specifically, articles about student-centered mathematics instruction from these fields were reviewed and coded for features of student-centered instruction.[6] Themes from this coding were reduced, and an

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5 Search terms included the terminology for student-centered instruction described on page 18.

6 In order to focus the literature search, I included only literature about student-centered instruction in general or in mathematics. Literature that was specific to student-centered instruction in other content areas was omitted. Similarly, the secondary literature search that

additional search was done to identify literature that supported the themes as components of instruction that are salient to adolescents' engagement and achievement in mathematics. Specifically, the second literature search examined whether and how the features of student-centered instruction were independently supported as mechanisms that support adolescents' engagement and achievement in math in ways that align with the constructivist perspectives of learning and self-determination theory perspectives of motivation upon which student-centered instruction is based or supported.

The two rounds of searching and reviewing the literature revealed four primary components of student-centered instructional practice: 1) relevant and cognitively challenging tasks; 2) supporting adolescents' intellectual authority; 3) flexible and responsive support of student understanding; and 4) situating learning in social interaction. [7]

3.2.6.2.1. Providing meaningful and cognitively challenging tasks

Literature about student-centered instruction states that student engagement is higher in classes where tasks are relevant and meaningful to adolescents (Bransford, Brown, &

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followed up on themes revealed in the primary literature search focused on salience to engagement and achievement in academics, in general, or in mathematics in particular.

[7] Some literature described two additional components of student-centered mathematics instruction: 1) the use of engaging learning formats, like technology, manipulatives, and/or other interactive materials; and 2) the use of formative assessment. They were omitted because they did not appear in the literature as core tenants of student-centered mathematics instruction. Rather, they are components of instruction that are used in both teacher- and student-centered instruction to support student learning. It is not whether they are used but how they are used that indicates the extent to which the instruction is student-centered.

Cocking 1999; Cubukcu, 2012; Elen et al., 2007; Slavich & Zimbardo, 2012; Smit et al., 2014). Students construct their own knowledge best when classroom tasks facilitate the connection of their learning experiences to other experiences in their lives (Piaget, 1960; Vygotsky, 1978) and the historical and social context of the world around them (Brunner & Haste, 2010). In this way, attending to students' interests and incorporating meaningful tasks is a key component of student-centered tasks (Cubucku, 2012; Dunlap & Grabinger, 1996; Lunenberg & Korthagen, 2007). Research in adolescent development and motivation confirms that feelings of autonomy are supported by tasks and choices that are perceived by adolescents as being related to their interests and goals (Assor et al., 2002). The relevance of choices given to students in schools predicts their positive affective responses to and engagement in academic work (Assor, Kaplan, & Roth, 2002). There is also evidence that task relevance can predict achievement. In one study, students in student-centered classrooms that featured real world problems and focused on developing higher order thinking skills outperformed their peers on NAEP assessments (1999)[8].

In order to attain a deep understanding of mathematics, however, tasks also need to be cognitively challenging. Specifically, students must engage in activities or exercises that require them to think critically about mathematics and to examine and explain their thinking and the thinking of others (Jensen & Lawson, 2011; McCombs & Whisler, 1997; Slavich & Zimbardo, 2012; Stein, Grover, Henningsen 1996). Solving open-ended problems is particularly valuable because it supports students' use of higher order cognitive strategies and skills (Smith, Hughes, Engle, & Stein, 2009). Studies have found that solving problems that

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8 Note that these studies varied in the extent to which they made a direct comparison between teacher- and student-centered mathematics instruction. See Chapter 3 for a more detailed discussion of the nature and limitations of the research.

require higher order thinking without strong direction by the teacher is related to higher achievement in mathematics (Cornelius-White 2007). Tasks that encourage students to think deeply about their work and to reflect on and to explain their thinking to others can also motivate adolescents to stay engaged in math coursework by meeting their needs for competence and autonomy and by being a good fit to their increasingly complex developmental skills (Eccles & Roeser, 2009). This could be why working on challenging, open-ended problems has been shown to build students' perseverance and openness to problem-solving in general and improves learning and achievement in math, in particular (OECD, 2012).

3.2.6.2.2. Supporting student intellectual authority and responsibility for learning

A central component of student-centered instruction is transforming students' dispositions towards learning (Slavich & Zimbardo, 2012). Instead of thinking of learning as a passive process that focuses on following the teacher's instructions (i.e., the teacher as the "sage on the stage," Weimer 2002), student-centered instruction aims to shift adolescents' perspective to focus on developing their own ideas about mathematics and finding their own solutions to problems (Çubukçu, 2012). Referred to as "intellectual authority" in the student-centered instruction literature (e.g., Brown 2015; Zimmerman, 1990), students who are in charge of constructing their own knowledge become actively engaged and self-regulated in their learning (Smit et al., 2014), which contributes to their persistence and success at solving open-ended problems and performing challenging tasks and contributes to the development of cognitive and metacognitive skills that make students effective lifelong learners (Engle, 2011; Engle & Conant, 2002; Boaler & Greeno, 2000; Hiemstra, 2004; Lave & Wenger, 1991).

Intellectual authority is strongly related to students' autonomy in class. Autonomy refers to students' ability to make decisions for themselves (Skinner & Belmont, 1993) and to have control over their behavior (Reeve, Bolt, & Cai, 1999). According to stage-environment fit theory, adolescents have a developmental need for feeling autonomous and the extent to which school environments support autonomy translates into adolescents' motivation to engage in school. Students' perceptions of autonomy have been linked to a number of positive academic outcomes for youth, including perceptions of competence (e.g., Black & Deci, 2000; Deci, Vallerand, Pelletier, & Ryan, 1991; Williams, Wiener, Markakis, Reeve, & Deci, 1994), learning (e.g., McGraw & McCullers, 1979), and academic performance (e.g., deCharms 1976). In student-centered mathematics instruction, teachers can support autonomy by using a range of instructional practices that can be described as student-centered, including considering students' perspectives, welcoming students' positive and negative thoughts and feelings, and supporting students' capacity for self-regulation (Reeve, 2002).

In addition, intellectual or cognitive autonomy develops from providing adolescents opportunities to execute and evaluate work from a self-referent standard (Kamii, 1982; Stefanou, Perencevich, DiCintio, Turner, 2004). Cognitive autonomy is supported by providing opportunities for adolescents to discover and discuss multiple approaches and strategies, explain and justify their thinking, solve problems independently, and debate ideas freely. Unlike procedural and organizational autonomy, cognitive autonomy supports adolescents' deep investment in learning because the "ownership and justification of ideas, the construction of meaning, and the intentional self-reliance used in critical thinking are at the heart of learning and motivation in the classroom" (Stefanou et al., 2004, p. 109). Thus, while student-centered instruction includes support of adolescents' autonomy in general, instructional practices aim to support cognitive autonomy in order to deepen levels of engagement and achievement.

3.2.6.2.3. Flexible and responsive support of student understanding

In addition to making tasks and instruction relevant and appropriately challenging, student-centered instruction calls on teachers to offer flexible and responsive support to students as they work to construct their own understanding of mathematical concepts and tasks (see Slavich & Zimbardo, 2012). Flexible and responsive support of student understanding refers to teachers' persistent monitoring and scaffolding of student comprehension and progress in ways that maintain students' intellectual authority and an appropriate level of cognitive challenge of the task. Adolescents vary in regards to their developmental skills related to learning, background knowledge and skills, learning preferences, and cultural, social, and historical context that shape adolescents' experiences in mathematics (see American Psychological Association, 1995). In order to ensure that the challenging tasks fall within each student's ZPD (Zone of Proximal Development; Vygotsky, 1978), teachers take students' individual skills and needs into account when planning and facilitating tasks (Cubukcu, 2012). They also support adolescents' effective engagement in academic work by monitoring and responding to the level of difficulty that is appropriate for each student (Cubukcu, 2012; Slavich & Zimbardo, 2012; Smith & Stein, 2008). Specifically, teachers using student-centered mathematics instruction offer students the guidance that they each need in order to attain and maintain a high level of self-direction (Vermunt & Verloop, 1999; Lunenberg & Korthagen, 2005). A meta-analysis studying the features of student-centered instruction, Cornelius-White (2007) found that teachers' adapting their instruction to individual differences was moderately associated with positive education outcomes.

The relationship of teachers' flexible and responsive support of student understanding with student engagement and achievement has been developed in multiple measures and types of high quality, reform-oriented instruction. For example, the Danielson Framework describes how effective instruction includes teachers' ability to adapt lessons when needed, monitor and respond to student difficulties, and offering the needed assistance until the student is able to make progress (Danielson, 2012). Similarly, the Responsive Classroom approach is an instructional strategy that aims to "improve classroom social environments and facilitate more positive and instructionally productive interactions among teachers and peers" (Griggs, Rimm-Kaufman, Merritt, & Patton, 2013, p. 4). Responsive Classroom embraces the student-centered principles that learning occurs through social interaction, the learning process is as important as its products, and students learn best when their needs for social and emotional support are met. It also includes practices at the middle school level that support student intellectual authority and responsibility for learning, such as having students contribute to establishing classroom rules, relevant and interesting tasks, and having students take the lead (with teachers working as a coach) to practice class work. Teacher practices -- such as morning meetings, interactive modeling, guided discovery, academic choice, rule creation, and logical consequences, and academic competencies – support students' effective engagement in relevant and challenging tasks and taking on intellectual authority and responsibility for learning (see Table 1 in Griggs et al., 2013). The implementation of Responsive Classroom has been linked to higher academic achievement and higher quality instruction (Rimm-Kaufman et al., 2014).

Unfortunately, flexible and responsive support of student understanding, through the implementation of instructional strategies like Responsive Classroom or just in general, decline after the transition from elementary to middle school. Indeed, the decrease in teacher responsiveness or support is related to adolescents' decline in engagement after the transition from elementary to middle school (see Eccles & Roeser, 2009; Wigfield et al., 2006). Declines in teacher support can thwart adolescents' ability to meet their psychological needs for autonomy, competence, and relatedness, which can negatively impact their engagement and achievement in mathematics (Niemiec & Ryan, 2009; Zimmer-Gembeck, Chipuer, Hanisch, Creed, & McGregor, 2006). To support adolescents' engagement and achievement in mathematics coursework, student-centered mathematics instruction targets continued individualized support for adolescents in their mathematics coursework throughout secondary school.

3.2.6.2.4. Situating learning in social interaction

A cornerstone of student-centered instruction is the perspective that students attain the deepest levels of engagement and learning when academic activity and inquiry are situated in social interaction (e.g., Vygotsky, 1978; Lave & Wenger, 2001; Resnick et al., 2008). A number of strategies are used to embed academic work into social interactions, including whole group discussion and collaborative peer work. However, the use of these "activity types" varies across the methods used to bring student-centered instruction into the classroom.[9] The most common

[9] Collaborative work with peers is also a component that varies significantly between and within teachers. Teachers who are concerned about losing control over class activity or students not focusing on their academic work can vary in their support of peer interaction and

practices are interactive strategies that teachers use throughout the various phases and activity structures that occur with each method of student-centered instruction. For example, strategic questioning, asking students to explain their thinking, having students present their work, and considering multiple solution strategies used by peers are implemented ubiquitously across the methods of student-centered mathematics instruction. These strategies occur throughout in the design and presentation of relevant and challenging tasks, supporting student intellectual authority and responsibility for their learning, and responding to students' individual needs. For example, relevant and cognitively challenging tasks can include asking students to consider their classmates' points-of-view. Supporting students' assumptions of intellectual authority and responsibility for learning can include asking students to share their ideas with the class, asking students to explain their answers/thinking, and doing group work for portions of the class. Flexible and responsive support of student understanding includes teachers' active listening to student questions and problems, explaining work in a new way, and understanding how students feel about the work they are doing in class. Given the fundamental nature of social interactions to student-centered mathematics instruction and the distribution of social interaction across the other components, I conceptualize social interaction as a sub-component of all of the other three main components of student-centered mathematics instruction.

3.2.7. Summary

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collaborative work. In the pilot study for the longitudinal study on adolescents' engagement in mathematics, student and teacher reports of the extent to which teachers supported student interaction had low reliability and were removed. This could be related to the variation in the interpretation and use of "encouraging students to interact with one another."

In the context of a wave of instructional reform in mathematics, there is a pressing need to study the implementation and effects of student-centered mathematics instruction. While improving achievement among low-income and minority youth and the mathematics-related aspirations of female students are the focuses of many of these reforms, there have been few systematic studies on how mathematics teachers implement student-centered mathematics instruction, how this variation relates to the demographic characteristics of students, and the effects of the implementation of student-centered mathematics instruction on student achievement. A significant barrier to this research is the lack of well-validated measures to support the wide-scale study of student-centered mathematics instruction. In particular, developing student- and teacher-reports would enable the examination of the variation in implementation and effects at various levels (student, classroom, and teacher) and in relation to relevant student characteristics (race, socioeconomic status, gender, and achievement level). Thus, I proposed developing and testing student- and teacher-reports of student-centered mathematics instruction.

3.2.8. Theoretical integration

To inform the development of the measure and the tests for predictive validity, I present a few figures that illustrate a proposed integration of constructivist and SDT perspectives in an integrated conceptual framework for student-centered mathematics instruction. Over the course of its rise in education policy (described in more detail in section 2.3 that describes the history of instructional reform in Chapter 2), constructivism and SDT have been used to inform student-centered mathematics instruction. Generally, the two theoretical approaches are conceptualized as making independent conceptual contributions to student-centered instruction. For example, Figure 5 illustrates the ways in which each of these components is related to constructivist theories of learning and SDT. While helpful in summarizing the (one-directional) association of each component with elements of each theoretical perspective, a summary of this sort does not integrate the two theoretical perspectives. In this section, I use the proposed components of student-centered instruction to develop a framework that integrates the two perspectives, with two aims and advantages: 1) to provide a unified conceptualization; and 2) to situate the current survey validation study in the conceptual framework.

A significant challenge in integrating the two theoretical approaches is that SDT focuses on psychological process of individuals, and constructivism situates engagement and learning in a social context, which can conceptualize them as being distributed among the members of a group. Figure 4 illustrates the ways in which integrating constructivist perspectives on learning and SDT shapes learning by creating direct and indirect pathways to student engagement in mathematics coursework (the individual lines from each component of student-centered mathematics instruction to psychological needs and engagement are replaced with a single line to simplify the illustration; refer to Figure 5 for specific hypothesized relations). Figure 6 illustrates how these direct and indirect pathways could unfold in a classroom (shown here as consisting of students *x* and y). In addition to representing the psychological needs and engagement of more than one student, this model proposes four additional theoretical components. First, social engagement is situated conceptually at the overlap or intersection of an individual student's engagement. Second, learning is represented as construction of knowledge, which is presented as distinct from and antecedent to achievement. Third, the co-construction of knowledge is represented as the overlap between individual students' construction of knowledge. Fourth, there is a set of independent and direct pathways from social engagement to co-construction of knowledge to achievement. To illustrate the way in which this model integrates constructivist perspectives and SDT, the arrows connecting the various components of student-centered mathematics instruction are coded based on whether they are part of constructivism, SDT, or are pathways that resulted from integrating the two.

3.3. STUDY 1 AIM AND RESEARCH QUESTIONS

The aims of the study are to operationalize student-centered mathematics instruction and to validate student- and teacher-centered instructional scales. The study includes qualitative and quantitative components that are used to explore the following research questions:

3.3.1. Qualitative Study

1. Based on the literature and extant measures of student-centered instruction and related constructs, what are the potential indicators and items of student-centered mathematics instruction?

3.3.2. Quantitative Study

2. How do the psychometric properties of the student-centered instruction scale assessed with a large sample of secondary school students compare to/inform the conceptualization of SCI?

a. What is the factor structure of the student and teacher reports, as revealed by confirmatory factor analytic procedures?

b. What is the reliability of student and teacher reports?

c. Is there measurement invariance of student reports by gender, race, or SES? Is there measurement invariance of teacher reports by teacher [x]?

3. Do the student- or teacher-reported dimensions of SCI predict student engagement or achievement in mathematics?

4. How correlated are student and teacher reports of each of the dimensions of student-centered mathematics instruction?

Figure 7 illustrates how the research questions exploring predictive validity are positioned in the proposed conceptual framework. The blue variables and arrows illustrate the analyses examining predictive associations between student-centered mathematics instruction and student engagement. The orange boxes illustrate the analyses examining potential indirect pathways between student-centered mathematics instruction and achievement.

3.4. METHODS

3.4.1. Mixed methods

The survey study employed a mixed methods sequential exploratory design (Creswell, Plano Clark, Gutmann, & Hanson, 2003) that used qualitative methods to develop the survey informed by literature and extant measures and quantitative methods to examine the psychometric properties of the survey. The qualitative component followed guidelines suggested by Gehlbach & Brinkworth (2011), which began by using the literature to develop a preliminary conceptualization of student-centered instruction (described in Chapter 3). The literature-based conceptualization was then used to identify items from available measures to measure student-centered instructional practice. Specifically, measures of student-centered instruction and related constructs (reform-based instruction, classroom climate, autonomy support – described in detail below) were reviewed for items that could measure indicators of student-centered mathematics instruction based on the review of the literature. Then, the list of potential items was reviewed by experts in the learning sciences and applied developmental psychology. Experts provided feedback on the conceptualization of student-centered instruction, the choice of indicators, and the wording of items.

The quantitative study began by using factor analytic procedures to explore the dimensions of student-centered instruction and confirm dimensions identified through a review of the literature and extant measures. Then the quantitative study examined scale reliability and measurement invariance by student race, socioeconomic status, gender, and level of achievement, and predictive validity of student cognitive, behavioral, emotional, and social engagement in mathematics and achievement in mathematics coursework (mathematics course grades).

Mixed methods are effective for developing robust instruments (Collins, Onwuegbuzie, & Jiao, 2006). Using a sequential exploratory approach enabled me to develop a psychometrically sound measure that is grounded in student-centered mathematics instruction, as it is understood and described in the literature (Creswell & Plano Clark, 2011), that is triangulated with experts in the field (Greene, Caracelli & Graham, 1989) and validated with a large and diverse urban sample of adolescents.

3.4.2. Qualitative study

The qualitative study includes two components. First, I reviewed items from relevant survey measures that are included in the parent survey study "Assessing Student Engagement in Math and Science in Middle School: Classroom, Family, and Peer Effects on Engagement" (NSF Grant Number 1315943) in order to identify items that measure indicators of the dimensions of student-centered instruction. Second, experts in the learning sciences and applied developmental psychology reviewed the items for construct validity and appropriate use of language or wording.

3.4.2.1. Review of extant scales from the literature

In order to build a framework to operationalize student-centered mathematics instruction for the purposes of developing a survey, I first reviewed extant measures of student-centered mathematics instruction as well as well-validated scales that measure constructs related to student-centered mathematics instruction or the components of student-centered instruction. In order for the search to be comprehensive, I completed four rounds of literature search with EBSCO Host and PyschInfo databases. First, in developing the literature review for the dissertation, I searched for scales of student-centered mathematics instruction or student-centered instruction in general using terminology that the literature identified as describing the overall approach or instructional philosophy described by student-centered instruction. Search terms included: student-centered/centred instruction, teaching for understanding, student-activated instruction, learner-centered instruction, standards-based instruction, student-directed learning or instruction, and self-regulated learning or instruction. Second, I used terminology that identifies student-centered instructional *methods*, including: dialogic instruction; collaborative instruction; problem- and project-based learning; competency based learning; and differentiated or personalized instruction; and adaptive instruction or learning. Third, I searched for instructional practices related to the underlying framework, including: autonomy support, constructivist teaching and learning, metacognitive tasks, relevant tasks, and quality of teacher feedback or support. Finally, since student-centered instruction is relatively amorphous in the extant literature, in order to be comprehensive, I also searched for the effects of instruction or teacher practice on secondary school students' engagement and achievement in mathematics.

Given the focus of the dissertation on instructional reforms over the past 30 years, the search focused on literature and measures from 1990 to the present. Searches were also focused on research and measures for secondary school students. In each round, I first specified "mathematics" and then searched for measures or literature that related to academic content, in general. Thus, measures specific to other content areas (e.g. English Language Arts and science) were excluded.

Surveys were reviewed and coded for items that featured indicators of each of the proposed dimensions of student-centered mathematics instruction. The coded surveys were organized into tables in order to: 1) assess how the conceptualization of student-centered instruction compares to instructional constructs in existing surveys; 2) organize the types of items and indicators that comprise each dimension in order to inform 2a) how each of the dimensions of student-centered mathematics instruction are operationalized and 2b) the selection of items from the parent study for the quantitative study. The items review is summarized in the results of Study 1 (Chapter 7).

3.4.2.2. Review of items available in the parent study

I used the results of the item review from the literature to identify constructs and items from the parent study that could be pooled to measure the dimensions of student-centered mathematics instruction. The parent study included some measures that were in the item review from the literature, some of which were adapted to study student engagement in mathematics, including autonomy support, dialogic instruction, reform-based instruction, and classroom climate scales. For the majority of the scales, there was a student and a teacher version in the parent study and teachers mostly reported on their instructional practice in each of the courses that they teach. The lone exception was the reform instruction scale, for which teachers reported on their overall instructional practice – thus, there is neither a student report nor a classroom-level teacher report for this measure. In some cases, the items were adapted for student grade level, particularly for students in 5th versus 7th and 9th grades.).

All items were reviewed across student and teacher report and for each grade level and items were selected based on the extent to which they reflect the proposed dimensions of student-centered mathematics instruction. Because students and teachers can have different perspectives on instruction, not all items needed to include a parallel teacher and student item in order to be included in the student and teacher report scales. Items that aligned with the proposed conceptualization of student-centered instruction were pooled by dimension for the student and teacher report, separately. The following sections describe the constructs and measures from which items were reviewed for the quantitative study (see Tables 5-8).

3.4.2.2.1. The Classroom Assessment Scoring System for Secondary School Students

The Classroom Assessment Scoring System for secondary school students (CLASS-S) is an observational tool that provides an assessment of classroom quality in terms of emotional climate, classroom management, and instructional support domains (Pianta, Hamre, Haynes, Mintz, & La Paro, 2007). In the CLASS-S framework, each domain of classroom climate includes several dimensions, each of which measures several indicators based on observable behavioral markers (see Pianta, Hamre, & Mintz, 2012). In addition to the observational measure, there is also a validated student survey (Downer, Stuhlman, Schweig, Martinez & Ruzek 2015) and a teacher survey[10] of classroom climate that correspond with the observational measure.

Preliminary review of the CLASS-S student and teacher surveys suggests that at least 5 dimensions of the CLASS-S framework could include indicators that are aligned with the proposed conceptualization of student-centered instruction. Specifically, Teacher Sensitivity and Regard for Adolescent Perspectives in the Emotional Support Domain and Analysis and Inquiry, Content Understanding, and Instructional Learning Formats in the Instructional Support Domain align with various aspects of student-centered instruction. For example, the Emotional Support domain includes the dimension Teacher Sensitivity, which refers to the extent to which teachers exhibit "a timely responsiveness to the academic, social/emotional, behavioral developmental needs of individual students" (Pianta et al., 2012, pg. 17). A few specific indicators of teacher sensitivity could also be indicators with teachers' Flexible and Responsive Support of Student Understanding. Since the alignment between the CLASS-S and student-centered mathematics instruction occurs at the item and not the dimension level, each item and indicator in the aforementioned dimensions of the CLASS-S was reviewed for the extent of

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10 We are in the process of assessing and comparing the psychometric properties of the teacher report and comparing reliability and validity among teacher, student, and observer reports of classroom climate in the longitudinal parent study.

alignment with each dimension of student-centered mathematics instruction. CLASS-S items appear in Table 5.

3.4.2.2.2. Reform-oriented teaching practice

The Reform-Oriented Mathematics Teaching Practice measure was developed as part of the National Science Foundation's 2012 National Survey of Science and Mathematics Education Report (Banilower et al., 2013). A primary aim for developing the measure was to assess the extent to which mathematics teachers use pedagogical strategies that reflect current theories of learning, particularly the constructivist perspectives of learning underlying student-centered mathematics instruction (Banilower et al., 2013). The original survey has good reliability in a sample of 7,752 science and mathematics teachers from schools across the United States (Cronbach's $\alpha = .77$). Scale items appear in Table 8.

In line with student-centered mathematics instruction, the survey includes items that assess the extent to which teachers ask students to explain their thinking and to think deeply about the reasoning of their peers, both of which could also be indicators of teachers supporting Student Intellectual Authority and Responsibility for Learning. Items in the measure could also relate to the teachers' support of metacognition, which could be part of Relevant and Metacognitive Tasks. Like the CLASS-S, the items in the Reform-Oriented Teaching Practice survey were assessed at the item level for the extent to which they align with each dimension of student-centered instruction.

3.4.2.2.3. Autonomy supportive teaching practice

Measures for two types of autonomy support may include indicators of different aspects of student-centered mathematics instruction. “Fostering relevance” includes six items that assess teachers’ efforts to identify aspects of the learning process that are relevant to their interests, goals, and values (Assor et al., 2002). The measure has good reliability (Cronbach’s $\alpha = .81$) and is related to students’ emotions and cognitive and behavioral engagement in class (Assor et al., 2002). Items in the measure Fostering Relevance could assess indicators of relevant tasks, a key component of student-centered instruction. “Allowing criticism and encouraging independent thinking” includes six items that assess teacher practices that support students in expressing criticism in the learning content or task (Assor et al., 2002). The measure has good reliability (Cronbach’s $\alpha = .76$; Assor et al., 2002) and includes items that could be related to supporting Student Intellectual Authority and Responsibility for Learning. Autonomy support items appear on Table 6. As with the previous measures, the survey was reviewed at the item level.

3.4.2.2.4. Dialogic instruction

Finally, there is a measure of dialogic instruction in the data that assesses the extent to which mathematics teachers use social interaction to position students at the center of learning and to support students’ deep thinking about mathematical concepts. Developed based on the work of Stein et al. (e.g., 1996; Stein & Smith, 2011), the measure assesses indicators that could be aligned with each dimension of student-centered mathematics instruction in different ways (see Table 7).

3.4.2.3. Expert review of items

Once a preliminary list of potential indicators was complete, the list of items, along with how they group to assess the proposed dimensions of student-centered instruction, was reviewed by an expert in the learning sciences and student-centered instruction, an expert on mathematics instruction, and an expert on student engagement and motivation. Experts were asked to give feedback on: the proposed dimensions of student-centered mathematics instruction; whether the items described indicators of student-centered mathematics instruction, in general, and if they described indicators of each dimension, in particular; how items were worded; revisions to items based on their expert perspective of the indicator the items intend to measure.[11]

3.4.3. Quantitative study

3.4.3.1. Sample

I used student and teacher survey data from the study "Assessing Student Engagement in Math and Science in Middle School: Classroom, Family, and Peer Effects on Engagement" (NSF Grant Number 1315943) collected during the 2015-2016 academic year. The study includes data from 2,536 students in fifth (n= 851), seventh (n= 903), and ninth (n= 719) grades recruited from three school districts in Western Pennsylvania (none of which included the school within which the qualitative study was performed). Students in the sample (51.4% Caucasian, 29.9% African-American, 49.8% female, and 38% are eligible for free or reduced price lunch) are

11 While it was not possible to revise the items for this study, suggested revisions could indicate poor fit of an item to the dimension or that the item as written may not perform well (in general or with the other items) in the proposed scale. Experts' suggestions could inform revisions of items in future waves of the longitudinal study.

nested in 187 mathematics classrooms taught by 36 teachers. All students in the sample reported on their engagement in mathematics. However, approximately half of the students completed a "math version" of the survey that inquired into their mathematics coursework. Descriptive statistics are summarized in Table 4.

3.4.3.2. Procedures

All participants were invited to participate in the study using school-based recruitment during the Fall Semester of the 2015-2016 school year. Informational forms were sent home that described the study purpose and procedures and provided a place where parents and/or students could sign to opt out of the study. Students whose parents did not opt them out and who assented to study participation were provided with a computer-based survey that they completed during a full period of their math class. Research staff members were available to answer questions about survey items. Student surveys were anonymous aside from being linked to students' math classes. Mathematics teachers were invited to complete online surveys that asked them to report on their instructional practice in each of the classes that they teach. Specifically, mathematics teachers first reported what classes they teach, after which they reported on their instructional practice in each of those classes.

3.4.3.3. Measures

3.4.3.3.1. Student characteristics

Students' race, socioeconomic status, gender, and achievement level were used to assess measurement invariance by student characteristics. Students reported their race by selecting "all that apply" in response to 6 categories of race. For this analysis, I created a dichotomous variable indicating whether a student self-identified as Caucasian as a negative indicator of minority status (0). Biracial students were coded as minorities (1). Students' eligibility for reduced or free school lunches was reported by school districts and were used as a measure of students' socioeconomic status. Student gender was also student-reported and was coded dichotomously (female = 1). Students' overall grade point average (GPA) for the previous academic year (2014-2015) was reported by school districts and was used as a measure of a students' previous achievement.

3.4.3.3.2. Education outcomes

I assessed the predictive validity of the three dimensions of student-centered mathematics instruction on students' emotional, behavioral, cognitive, and social engagement in math class, students' mathematics course grades, and students' performance on the mathematics portion of the state standardized test. Students' mathematics course grades and performance on the Pennsylvania System of School Assessment (PSSA) for the current academic year (2015-2016) were reported by school districts and was used as the achievement outcome. For the purpose of the multi-group CFA, students will be broken into two groups: those performing in the top 25% of the sample (high achieving) and those performing in the lower 75% of the sample (average to low achieving).

Students' engagement in mathematics class was measured with the well-validated *Math and Science Engagement Scales* (Wang et al., 2016). The cognitive engagement scales included six items about students' use of deep learning strategies and self-regulated learning in science and mathematics (e.g., "I go through the work that I do for math class and make sure that it's right"). The behavioral engagement scales included seven items about students' involvement and investment in classroom activities in science and mathematics (e.g., "I put effort into math"). The emotional engagement scales included six items about students' positive and negative reactions to and value of classroom learning and activities in science and mathematics (e.g.: "I enjoy learning new things about math"). The social engagement scales included six items about students' willingness to value and consider others' perspectives and the quality of students' interactions in science and mathematics class (e.g., "I build on others' ideas"). All items were on a scale from 1 (*not at all like me*) to 5 (*very much like me*).

3.4.3.4. Analytic plan

3.4.3.4.1. Factor structure

In order to assess the psychometric properties of the student-centered instruction measure (RQ2a-c), I first used confirmatory factor analysis in M*plus* version 7.3 (Muthén & Muthén, 1998-2014). Specifically, I tested the conceptualization of student-centered instruction that was derived from synthesizing the *a priori* conceptualization and any revisions to that conceptualization based on the mixed method study of seventh grade mathematics teachers' implementation of it. I used five fit indices to evaluate the goodness of fit of the hypothesized model, including the chi-square statistic, the Comparative Fit Index (CFI > .90), the Tucker Lewis Coefficient (TLI > .95), the Root Mean Square Error of Approximation (RMSEA < .06), and the standardized root mean squared residual (SRMR < .08). The estimation method depended on whether the data was normally distributed, which was determined with a Shapiro-Wilks test. If the data was normally distributed, I used maximum likelihood; if not, I used the modified weighted least square estimation method for ordinal variables (WLSMV in Mplus; Muthén, du Toit, & Spisic, 1997) in order to avoid producing biased results when data is not normally distributed (Flora & Curran, 2004). I accounted for the clustered nature of the data by including students' mathematics class as a clustered variable in the analysis in M*plus* version 7.3 (Muthén & Muthén, 1998-2014).

3.4.3.4.2. Reliability

After determining the factor structure of the student and teacher reports, I assessed the reliability of each scale (RQ2b). Specifically, I tested the internal consistency of the teacher and student scales by running Cronbach's alpha on the student- and teacher-reported items for each dimension, which provided information about the extent to which the teacher and student reports consistently measure the dimensions of student-centered mathematics instruction. Generally, a scale that has a Cronbach's alpha of 0.80 to 0.79 is considered acceptable, 0.80 to 0.89 is considered good, and above 0.90 is considered excellent in terms of its internal consistency (DeVillis, 2012).

3.4.3.4.3. Measurement invariance

Finally, I assessed metric invariance by student and teacher characteristics (RQ2c) in order to examine whether the factor loadings are comparable across students and teachers with different characteristics. Metric invariance is a statistical property of a scale that assesses the extent to which the scale measures the construct comparably across individuals from different groups. In the current study, I examine whether students or teachers with different demographic or background characteristics have similar or different perspectives on student-centered mathematics instruction (Chen, 2007; Cheung & Rensvold, 2002). If there is not metric invariance across students and/or teachers with different characteristics, then that indicates that it would be difficult to interpret data from samples

To check for metric invariance, I conducted multi-group CFA with student self-reports by student race (minority vs. non-minority/Caucasian), socioeconomic status (eligibility for free or reduced price school lunch), gender (male vs. female), and achievement level (students in the upper quartile in vs. students performing in the lower three quartiles in math course grades). I assessed metric invariance for student self-report of each dimension of student-centered mathematics instruction. The difference between the CFI and RMSEA and a chi-square difference test was used to assess the degree of model fit. Specifically, a difference greater than .01 in the CFI and a difference greater than .015 in the RMSEA indicated that the student characteristic under consideration was related to meaningful differences in the measurement of the given dimension of student-centered mathematics instruction (Chen, 2007; Cheung & Rensvold, 2002).

3.4.3.4.4. Predictive validity

Predictive validity refers to the extent to which a scale is associated with other measures to which the scale is conceptually and empirically associated in other research (Cronbach & Meehl, 1955). In other words, in order for a scale to be considered valid, it must predict the types of outcomes with which it is associated in theory and in other research.

To examine the predictive validity of the student and teacher reports of the dimensions of student-centered mathematics instruction (RQ3), I conducted multiple regression analyses in M*plus* version 7.3 (Muthén & Muthén, 1998-2014) to investigate the extent to which students' and teachers' reports of student-centered mathematics instruction predicts students' behavioral, cognitive, emotional, and social engagement in mathematics, their math course grades, and their performance on standardized tests in mathematics (for students in grades 6-8 who completed the PSSA's). Specifically, I examined each dimension of student-centered instruction reported by students and teachers (i.e., relevant and cognitively challenging tasks; flexible and responsive support of student understanding; and supporting students' intellectual authority and responsibility for learning). Students' report of student-centered instruction (level 1) were nested in classrooms (level 2), which were also nested within teachers (level 3). Since teachers reported on their instructional practice in every class that they teach, teachers' reports of student-centered instruction appeared at the classroom-level (level 2).

The analyses was run in four stages. First, I used a null model –- essentially a one-way ANOVA without predictors—in which the outcome was allowed to vary at the student and classroom levels. Second, I examined the effects of student and teacher covariates. Student covariates included student gender (male vs. female), race (African-American versus Caucasian), socioeconomic status (full priced lunch versus free or reduced price lunch), and previous level of academic achievement (grade point average from the previous academic year). Teacher covariates included gender, race, years of teaching experience, and highest degree earned (bachelors or masters). Third, I ran separate hierarchical regressions to examine the extent to which student reports of student-centered instruction predict behavioral, cognitive, emotional, and social engagement and end-of-year course grades for mathematics. Separate models are preferable to a multivariate model that examines the effects on all outcomes in the same model because there is a high correlation between cognitive and behavioral engagement in mathematics. Finally, I ran separate models to examine the extent to which teacher reports of student-centered mathematics instruction predicted the same outcomes. In the full models (steps 3 and 4), all three dimensions of student-centered instruction were included. Student reports of student-centered mathematics instruction appear at the student level (Level 1). Teacher reports of their use of student-centered mathematics instruction in each of their classes appear at the classroom level (Level 2).

To interpret the results, I first used the output from the null models to: 1) estimate the intra-class correlations from the variance components at each level; and 2) assess the variance in student-centered instruction at the classroom-level. Of particular interest in the models were the standardized coefficients for the student and teacher reports of student-centered mathematics instruction that revealed the strength of the relationships between each report on the outcomes of interest, which described the extent to which student and teacher reports of student-centered instruction predicted the outcomes of interest when accounting for relevant student characteristics.

To improve interpretability, coefficients were reported in standardized form, and continuous predictors were centered around their grand-means. I addressed the clustering nature of the data by adjusting the mixed-effect linear model with a level-2 random effect for intercept variance, thereby achieving robust standard errors (Raudenbusch & Bryk, 2002).

3.4.3.4.5. Convergent validity of teacher and student reports

As a final validity check, I examined the congruence between teacher and student reports of student-centered mathematics instruction. Convergent validity is a type of construct validity that assesses if a scale is measuring what it is intended to measure by examining its correlation with a scale that measures a related construct (Campbell & Fisk, 1959). Conversely, divergent validity contributes to establishing construct validity by establishing that the scale is not associated with scales that measure dissimilar or opposing constructs (Campbell & Fisk, 1959). Ideally, I would assess convergence of the teacher and/or student scale with another student-centered instruction scale or with scales related to dimensions of student-centered instruction. Similarly, theoretically I could assess divergent validity by assessing the correlation between the student and/or teacher report of student-centered instruction with a validated scale of teacher-centered instruction. Unfortunately, there are no validated measures of student- or teacher-centered instruction that are publicly available and other measures of instruction or classroom climate that include components of student-centered instruction are too dissimilar (see the “3.4.3.5. Review of extant scales from the literature”). In this analysis, since I assume that teacher and student reports of student-centered instruction are comparable in the conceptualization and measurement the construct (in Chapters 2 and 3), I assessed the congruence between teacher and student reports. The test examines the validity of the assumption of congruence and is an exploratory assessment of convergent validity of the measures. However, whereas congruence between the teacher and student report can provide some evidence of convergent validity, the lack of congruence could mean multiple things, including differences in the indicators of each dimension for teachers and students or differences in the conceptualization itself.

To test convergent validity between student and teacher reports, I ran Pearson and Spearman correlations, which also served as a test of the correlation between the variables in the predictive validity tests. Pearson correlations assume a linear relationship between variables, whereas Spearman correlations are a non-parametric version of the Pearson correlation and can measure the relationship between rank-order data and/or capture non-linear associations. Given the relatively exploratory nature of the study, I used both. If the Spearman correlation is greater than the Pearson correlation, then this suggests a monotonic but non-linear relationship between the variables, which has implications for the hierarchical multiple linear regressions used in the predictive validity tests.

3.4.4. Missing data

There was a significant amount of missing student data. The lowest amount of missing data was among student reporting of their engagement in mathematics class (1.2% to 1.8%), likely because the student engagement items appear at the beginning of the student survey. Student demographic data – reported by school districts – also had relatively little missing data, ranging from 5.2% to 9.5%. One exception was students' previous GPA, 24% of which is missing. Student reporting of student-centered instruction, however, had a substantial amount of missing data. Climate and instructional items appeared later in the survey and exhibited 13.2% to 59.5% missing data. Little's Test revealed the data was not missing completely at random (chi-square = 441.724, df = 104, $p < .001$; Little, 1988).

The teacher data was missing significantly less data. Interestingly, there was no missing data in the teacher data among any variables except one. Teachers were missing 20% of their reports on Student Intellectual Authority and Responsibility for Learning. Little's Test on the teacher data revealed that the data is not missing completely at random ($\chi 2 = 11.882$, df = 3, $p < .05$; Little, 1988).

3.5. STUDY 1 RESULTS

3.5.1. Qualitative study results

The analyses consist of three phases designed to address the first research question "Based on the literature and existing measures of student-centered instruction and related constructs, what are the potential indicators and items of student-centered mathematics instruction?" (RQ1). First, I review the results from analyzing scales from the literature. Second, I outline the results of the item selection process. Finally, I describe the feedback from experts that contributed to the student-centered mathematics instruction scale that is examined in the quantitative study.

3.5.1.1. Review of extant scales from the literature

Indicators of the proposed dimensions of student-centered mathematics instruction appeared in three types of survey measures: sub-scales that are part of comprehensive classroom climate scales (Table 1), sub-scales in scales about specific types of instructional practice or methods (Table 2), and distinct measures of constructs related to the proposed theoretical frameworks (Table 3). In the classroom climate research, there were 6 measures that are publicly available that include 17 sub-scales with indicators that are conceptually aligned with student-centered mathematics instruction, including the Chicago Consortium's 5Essential Student Survey (2017), the Research Assessment Package for Schools (RAPS; IRRE, 1998), the School Success Profile (SSP; Bowen, 2005), the Individualized Classroom Environment Questionnaire (ICEQ; Fraser, 1990), the What is Happening in this Classroom measure (WIHIC, Chionh & Fraser, 1998), and the Classroom Assessment Scoring System for the secondary level (CLASS-S; Downer, Stuhlman, Schweig, Martinez, & Ruzek, 2015). There were also six instructionally-focused scales that included sub-scales and/or items that aligned with student-centered instruction, including the Personalized Learning Environment Questionnaire (PLQ) (Waldrip, Cox, Deed, Dorman, Edwards, Farrelly, et al., 2014), the Constructivist Classroom Learning Environment (CLES) (Taylor, Fraser, Fisher 1997), the Classroom Practices Teacher Survey (CPTS) (Nathanson, Sawyer, & Rimm-Kaufman, (2007), the Competency –Based Learning Survey (Ryan & Cox, 2016), the student and the teacher reports of the Patterns of Adaptive Learning Scales (PALS) (Midgley, Maehr, Hruda, Anderman, Anderman, Freeman et al., 2000), and the Classroom Practices Teacher Survey (CPTS, 2007). Finally, there were two measures that are aligned Relevant and Metacognitive Tasks (Reform Oriented Teaching Practice; Banilower, Smith, Weiss, Malzahn, Campbell, & Weis, 2013 and Autonomy Support – Fostering Understanding and Interest; Assor, Kaplan, & Roth, 2002) and two aligned with Student Intellectual Authority (Autonomy Support – Providing Choice and Autonomy Support – Allowing Criticism from Assor et al., 2002).

The literature search revealed several interesting and relevant findings regarding the extent to which and the ways in which student-centered mathematics instruction is measured. First, components of the dimensions of student-centered mathematics instruction appear in several existing classroom climate and instructional scales. However, the vast majority of measures (14/16) contain sub-scales that are content neutral and/or are written to apply to any subject. For example, some measures allow the survey administrator to fill in the target subject, while others simply refer to “my teacher.” Furthermore, the majority of the measures are student reports. The few exceptions are the CPS 5Essentials Student Survey, which includes general and subject-specific sub-scales (described in Table 2) and the Reform-Oriented Teacher

Practice scale, which is specifically a teacher report of mathematics instructional practice (described in Table 3). Finally, there are no measures on which student and teacher report on the same components of classroom climate or instructional practice related to student-centered mathematics instruction.

Interestingly and importantly, while there are several sub-scales in the classroom climate measures that are conceptually related to specific dimensions of student-centered instruction, the items within the sub-scales are aligned with multiple different dimensions. For example, the items within the Academic Work sub-scale of the Classroom Practices Teacher Survey (Nathanson et al., 2007) have items that align all three proposed dimensions of student-centered mathematics instruction. Several other sub-scales include items that measure indicators of multiple dimensions. In a few cases, all of the items of a scale fit with a single dimension. For example, the items from the Personal Relevance sub-scale of the Constructivist Classroom Learning Environment scale (CLES, Taylor et al., 1997) include indicators of Relevant and Metacognitive Tasks. However, it focuses specifically on relevance and does not describe the need for tasks to also require higher order thinking. Tasks that are relevant but not cognitively challenging are not aligned to adolescents' cognitive capacities and are not well suited for supporting independent inquiry or construction of knowledge. Taken together, the differentiation of item assignment within scales along with the fact that no single scale or sub-scale measures student-centered instruction or its components underscores the need and contribution of a well-validated scale.

In addition to confirming the value of developing a well-validated scale, the review of existing scales also confirmed the importance of evaluating extant scales in the parent study at the item level for the extent to which they measure indicators of each dimension. The results of reviewing the existing scales in the parent scale at the item level in order to develop the student-centered instruction scale is described below

3.5.1.2. Item selection

After reviewing the scales, 19 items from the student surveys and 22 roughly corresponding items from the teacher surveys described in Chapter 5 were included in the first draft of the measure. The descriptive statistics of each item appears in Table 9, along with how the items group according to dimension of student-centered instruction. The table also indicates from which scales each measure originates. Note that the student-centered instruction scale does not merely replicate and/or combine the related constructs. Instead, the items from the original scales separate and are re-constituted when conceptualized as elements of student-centered mathematics instruction.

3.5.1.3. Expert review

Experts reviewed the draft of items and gave feedback on 1) the proposed dimensions of student-centered mathematics instruction; 2) whether the items described are indicators of student-centered mathematics instruction, in general, and if the described indicators of each dimension, in particular; and 3) whether there were any issues with the wording or formatting of individual items, including offering specific suggestions about how to revise items based on their expert perspective of the indicator the items intend to measure.

3.5.1.3.1. Feedback on the proposed dimensions of student-centered mathematics instruction

The experts agreed that each of the proposed dimensions represented key components of student-centered mathematics instruction. Two of the experts asked why there was not a dimension or items within one or more dimensions relating explicitly to student collaboration or peer interactions. While there were items relating to teacher support of interactions among peers in the pilot study to the parent study, those items failed to perform well and were removed from the longitudinal study from which the items for the student-centered instructional scale were procured. The items in the pilot study inquired into the extent to which teachers supported peer interaction in class and the extent to which teachers supported students going to one another for assistance. The items showed low reliability in the pilot scale, and in preliminary analyses they were not predictive of student engagement or achievement in mathematics. As written, the items did not distinguish clearly between on-task and off-task social interaction in class. Thus, while student interaction is a key component of constructivist learning and relates to student academic motivation, more work may need to be done to develop and include items to specifically assess the extent to which instruction structures and supports productive academic collaboration and interaction. Items specific to pedagogical strategies that support productive discourse and collaboration between peers could be a good fit. Due to the inability to add items from the parent study for the dissertation, I discuss the importance and role of taking student collaboration into account in the measure and in conceptualizations of student-centered mathematics instruction in greater detail in the discussion chapter of the dissertation (Chapter 9).

One expert suggested that it might work well to separate meta-cognitive tasks and relevant tasks into separate domains or sub-domains under "Student-Centered Tasks." This is a good suggestion given that cognitive challenge or complexity and task relevance or personal interest drive engagement and learning through related but distinct mechanisms. Metacognitive tasks are hypothesized to have direct and indirect effects on learning. The constructivist perspective describes how the appropriate level of cognitive challenge directly impacts students' learning, and SDT perspectives of motivation explain how the developmental fit of cognitive challenge and its relation to the development of student competence and autonomy contribute to sustained, high quality engagement that leads to deep levels of understanding and learning. Task relevance, on the other hand, shapes learning by supporting students' engagement in learning tasks. However, since there are only six items in the student scale and seven in the teacher scale, I decided to keep the items in a single scale and to assess how well they load into a single factor in the confirmatory factor analysis.

Finally, an expert asked about the extent to which the indicators are specific to mathematics instruction and the extent to which these were general indicators of student-centered instruction. This is an interesting and important conceptual and empirical question. In the parent study, students are asked specifically about their mathematics courses in all of the items. However, the items could assess student-centered instructional practice in other content areas. Furthermore, additional items specific to mathematics pedagogy (for example, supporting productive peer interactions) could contribute to the measure being more subject-specific. The question does not change the structure or conceptualization of student-centered mathematics instruction. However, given the relevance and importance of the question to developing subject-specific measures, I frame this as an area for future research in the discussion of the dissertation (Chapter 9).

3.5.1.3.2. Feedback on items as indicators of the dimensions of student-centered instruction

Experts offered several suggestions on specific items. First, experts commented on two items that may be more valid when answered from the student perspective. First, experts suggested that the student report of “doing interesting activities” might have greater variation and be more reliable in the teacher report. Experts suggested that the teachers would likely consider their classes to be interesting and/or would be likely to report that they were and that they would be less likely to perceive or report on variation in interest. Furthermore, “interest” can apply to the student or the teacher (tasks are “interesting” from the student or teacher perspective), whereas task relevance in student-centered mathematics instruction refers specifically to the interest of students. Thus, the item “We do interesting activities in class” was dropped from the teacher report. Second, experts suggested that the student item “My math teacher stops to answer my questions” might be more valid than the comparable teacher item “I stop to answer students’ questions in class.” Teachers may be less likely to report that they do not stop to answer students’ questions in class due to social desirability bias (stopping to respond to students’ questions is likely considered a fundamental responsibility and role of a teacher). They may also be less aware of variation in the extent to which they address student questions across classes. As a result of this feedback, the teacher item was dropped.

Experts also pointed out that one item was not parallel in ways that may lead it to measure different things. The student item “I get to do things in math class other than listen to the teacher talk" is significantly different from the teacher item "Students engage in a variety of learning formats." The student item refers specifically to the teacher offering any activity other than lecturing, whereas the teacher item refers to task variety. Given that the student item was more representative of an indicator of student-centered-ness (i.e. offering less lecturing may be more indicative of a shift of classroom activity from the teacher to the student than task variety), the teacher item was dropped.

Experts suggested that one item may be less reliable and valid and may measure more of a teacher-centered approach when assessed from the teacher perspective. The teacher item, "Students get as much help from me as they need in order to understand what we are doing" could measure the tendency to devolve the level of cognitive challenge of a task by offering specific procedures or strategies on how to solve problems, indicative of a teacher-centered approach to scaffolding student work on open-ended or challenging tasks (Stein & Smith, 2011). The student item, "My math teacher keeps working with me until I understand what we are doing" is more aligned with teachers' scaffolding student work in ways that keep students at the center of the learning activity. Furthermore, it may be difficult for teachers to report on the extent to which their assistance is effective for students at the classroom level and students may be able to more accurately report on the extent to which teacher feedback is effective or helpful. Thus, the teacher item was dropped from the survey.

3.5.1.3.3. Suggested revisions of items

Experts also made several suggestions about ways to revise items to improve the range and quality of student responses. In some cases, the suggestions had consequences for whether items were included in the scale for the current study. Specifically, one expert pointed out that teachers' reports of items from reform instruction practice that are in a different format than the other items in the survey could affect how well it loads into the same factor as the other items. Instead of posing the indicator and having the frequency in the response scale (for example, "When I show my math teacher an answer, he/she asks me to explain how I got that answer"), the teacher items put the response scale in the item question ("How often do you: Ask students to explain and justify the methods they use for solving problems?"). I left the items in the set for the CFA and made a note to examine if there are any issues related to these items.

In other cases, expert feedback informed revisions of items for subsequent waves of data collection, which could shape future analyses of student-centered mathematics instruction. For example, one expert suggested that adding the word "often" to some of the items would improve the range of student responses (for example, "I *often* get to do things in math class other than listen to the teacher talk"). An expert also suggested that shifting the focus away from teacher behavior and towards allowances afforded in class, in general, might help students more accurately reflect on their classroom experience. For example, instead of asking students to consider, "My math teacher encourages me to consider different solutions and points of view," asking students "In class, I often have to consider different solutions and points of view" could include ways in which teachers may set up tasks that require students to consider others' points-of-view, which students may or may not attribute directly to the teacher. Also, in a couple of cases experts gave suggestions on how to re-word items that were dropped from the survey for this study. For example, an expert suggested that the teacher item "I stop to answer students' questions in class" could be revised to emphasize scaffolding and maintaining student responsibility. These suggestions did not change the survey for the current study. However, the suggestions were taken into consideration for revising items for future waves.

As a result of expert feedback, the original 19 items from the student report and 18 items from the teacher report were included in the confirmatory factor analysis examining the fit statistics for a three factor model of student-centered mathematics instruction proposed by the literature: 1) use of relevant and cognitively challenging tasks (6 student items; 5 teacher items); 2) flexible and responsive support of student understanding (7 student items; 8 teacher items); and 3) supporting students' intellectual authority and responsibility for learning (6 student items; 5 teacher items).

3.5.2. Quantitative study results

3.5.2.1. Psychometric properties of the student and teacher surveys

3.5.2.1.1. Dimensionality test with Confirmatory Factor Analysis (CFA) (RQ2a)

A two-level CFA with the three proposed factors was a good fit for the student report data (CFI = .911, TLI = .904, RMSEA = .043; see Table 11). Intra-class correlations supported that each student item is nested in their mathematics classrooms (.070 - .201; see Table 10). In the model, student responses were nested in their mathematics classrooms. All factor loadings were statistically significant and were all above .30 (Relevant and Metacognitive Tasks: M = .620, range = .538 - .727; Intellectual Authority and Responsibility for Learning: M = .580, range = .498 - .699; Flexible and Responsive Support of Student Understanding: M = .703, range = .619 - .764), supporting a 3-factor model of student-centered mathematics instruction. Standardized factor loadings appear in Table 12.

In order for the teacher model to have a satisfactory fit to the data, I made modifications to the type of model and to the items run in the CFA for the teacher scale. The two-level CFA with the teacher reports did not converge. The errors suggested that this could be due to a lack of variation in teachers' reports about how they teach in each of their classes. The standard deviation for the teacher items is lower than the student report and, more tellingly, the ICC's for the teacher items are very high (.681-.933; see Table 10).

To address the nested-ness of teacher reports of their instructional practice, I ran a 3-level CFA that adjusts the standard error. In this model, I also dropped the three reform-oriented teaching practice items because teachers reported them as an average across all of the classes that they teach (i.e. they were reported at the teacher-level, instead of at the classroom-level; ICC = 1.00, see Table 10). The three items were part of Flexible and Responsive Support of Student Understanding: "How often do you: Ask students to explain and justify the methods they use for solving problems?", "How often do you: Have students compare and contrast different methods that students in their class used to solve problems?", and "How often do you: Have students work in groups in class?" These were also items that experts suggested could be subject to respondent error due to item format. I also dropped the item about giving students feedback about their work. The item had two versions across fifth, seventh, and ninth grades: "I give students feedback on how to make their work better" for fifth graders and "I give students feedback about ways to learn more about what we are studying in class" for seventh and ninth grade students. In addition to having a low R-squared value in the CFA, the fifth grade version of the item could be described as teacher-centered. While teachers can give feedback on how to improve work in ways that are student-centered, this can also be an indicator of teachers positioning themselves as experts who provide students with procedures to use to solve problems.

Finally, the residuals of two sets of items were correlated to improve model fit. "Students in my classroom often come to me for help" was correlated with "I understand how students feel about what we are doing in class" within the dimension Flexible and Responsive Support of Student Understanding. Also, the residual for the item "When students construct their own ways of doing a problem, I have the students present the solution in their own words" was correlated with the residual for the item "Students are encouraged to express different points of view in class" within Student Intellectual Authority and Responsibility for Learning. In accordance with guidelines described by Brown (2015), there is a strong theoretical justification for correlating the residuals of these items. Each set of items describes processes that are related in classrooms and that are related in how they are hypothesized to shape adolescent education outcomes. "Students in my classroom often come to me for help" and "I understand how students feel about what we are doing in class" are both from the same teacher sensitivity scale and each represents a teacher's overall responsiveness to students' needs in class. While the items "When students construct their own ways of doing a problem, I have the students present the solution in their own words" and "Students are encouraged to express different points of view in class" come from different scales. They both express the teacher's effort to position students with intellectual authority and support student social interaction while learning in class.

After making these changes, a CFA that adjusted the standard errors for the nesting of teacher responses about their classrooms in teachers was a marginal fit to the data[12] (CFI =

.895, TLI = .867, RMSEA = .043; see Table 11).[13] All factor loadings for the teacher scale were statistically significant and were all above .30 (Relevant and Metacognitive Tasks: M = .707, range = .664 - .784; Student Intellectual Authority and Responsibility for Learning: M = .679, range = .517 - .768; Flexible and Responsive Support of Student Understanding: M = .537, range = .459 - .705), supporting a 3-factor model for teacher reports of student-centered mathematics instruction. Standardized factor loadings appear in Table 12.

3.5.2.2. Student and teacher scale reliability (RQ2b)

I concluded with a three-factor model of student and teacher reports of student-centered mathematics instruction. The student measure consisted of all 19 of the original proposed items: six items measuring indicators of Relevant and Metacognitive Tasks, six items measuring Student Intellectual Authority and Responsibility for Learning, and six items measuring Flexible and Responsive Support of Student Understanding. Reliability statistics are reported in Table 13. The Cronbach's alpha for each dimension was acceptable to good (Relevant and Metacognitive Tasks Cronbach's α = .785; Student Intellectual Authority and Responsibility for Learning Cronbach's α = .766; Flexible and Responsive Support of Student Understanding Cronbach's α = .89). The teacher report consisted of 14 of the original 22 proposed items. The Cronbach's alpha was good for Relevant and Metacognitive Tasks (.823) and acceptable Student Intellectual Authority and Responsibility for Learning (.766). The Cronbach's alpha for Flexible and Responsive Support of Student Understanding was questionable (.654).

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13 As a check, I ran the same 3-level CFA with teacher reports without accounting for the nesting of the data. The standard errors in the basic model were somewhat lower, but the model fit was substantially worse. Thus, I kept the model that adjusted for standard error for Study 1.

3.5.2.3. Metric invariance (RQ2c)

The multilevel three-factor model fit the student report data well, and measurement invariance held by student gender (male vs. female), race (African-American vs. Caucasian) and SES (free or reduced price lunch eligibility). The chi-square decreased from the baseline to the metric invariant model. However, the difference in the CFI is less than .01 and the difference in the RMSEA is less than .015 across models, indicating that none of these student characteristics relate to meaningful differences in the measure of student-centered mathematics instruction (Chen, 2007; Cheung & Rensvold, 2002; see Table 14).

In contrast, overall, the fit statistics for the teacher report of student-centered instruction indicate that there is not metric invariance across teacher characteristics. The chi-square difference was significantly higher in the measurement invariant model and, compared to the baseline models, the metric invariant models for teacher highest degree (bachelors vs. masters), teacher gender (male vs. female), and teacher years of experience (under or over 10 years) displayed a difference in the CFI greater than .01. The RMSEA does not change more than .015 across the baseline and metric invariant models. However, the other fit indices suggest that each of the teacher characteristics relates to meaningful differences in the measure of student-centered mathematics instruction (Chen, 2007; Cheung & Rensvold, 2002; see Table 15).

3.5.2.4. Validity evidence (RQ3)

3.5.2.4.1. Predictive validity of the student- and teacher-reports of student-centered mathematics instruction

The null models confirmed that each of the student outcomes (at Level 1) were nested in students' classrooms (at Level 2) (social engagement ICC = 0.131; emotional engagement ICC = 0.170; behavioral engagement ICC = 0.232; cognitive engagement ICC = 0.181; mathematics course grade: 0.371; see Table 4), supporting the use of a hierarchical model. Pearson and Spearman correlations between the predictors and outcomes appear in Tables 16 and 17. For ease of interpretation, the student and teacher results are reported in tables by outcome (Tables 18 - 23). In each table, Model 1 refers to the fully unconditional model. Model 2 includes only student and teacher covariates (at Levels 1 and 2, respectively). Model 3 includes student and teacher covariates and student report of the dimensions of student-centered mathematics instruction (at Level 1). Finally, Model 4 includes student and teacher covariates and teacher report of the dimensions of student-centered mathematics instruction (at Level 2).

3.5.2.4.1.1. Student report

Overall, student reports of student-centered instruction predicted their engagement and achievement in mathematics coursework when taking relevant student and teacher characteristics into account. However, some dimensions from student reports were more predictive of these outcomes than others. Student reports of teachers' implementation of Relevant and Metacognitive Tasks predicted all dimensions of student engagement (emotional engagement $\beta = .308$, $p < .001$; behavioral engagement $\beta = .100$; $p < .05$; cognitive engagement $\beta = .148$, $p < .001$; social engagement $\beta = .200$, $p < .05$).

Student report of Flexible and Responsive Support of Student Understanding was also associated with all dimensions of student engagement in mathematics class, as well as their mathematics course grades (emotional engagement $\beta = .381$; $p < .001$, behavioral engagement $\beta = .282$, $p < .001$; cognitive engagement $\beta = .271$, $p < .001$; social engagement $\beta = .110$, $p < .06$[14]; mathematics course grades $\beta = 2.699$, $p < .001$). Unlike the other dimensions of student-centered instruction, student report of Student Intellectual Authority and Responsibility for Learning did not predict student engagement in mathematics. However, it did predict their mathematics course grades ($\beta = 2.323$, $p < .05$). The student report did not predict state standardized test scores in mathematics, when taking student and teacher covariates into account.

3.5.2.4.1.2. Teacher report

Overall, teacher report of student-centered mathematics instruction followed a similar pattern as the student report in terms of which dimensions of student-centered mathematics instruction predicted which outcomes. However, interestingly, in some cases, teacher reports were negatively associated with student engagement and course grades. For example, similar to student report, teacher report of Relevant and Metacognitive Tasks in their classrooms was significantly predictive of all dimensions of student engagement. However, teacher report of Relevant and Metacognitive Tasks predicted declines in student engagement across the board (emotional engagement $\beta = -.310$, $p < .05$; behavioral engagement $\beta = -.230$, $p < .05$; cognitive engagement $\beta = -.260$, $p < .05$; social engagement $\beta = -.229$, $p < .05$). Teacher report of Student Intellectual Authority and Responsibility for Learning was also negatively associated with

[14] Given the exploratory nature of the study, I am including predictive associations with p < .06 in the text. They are indicated in tables with italics.

students' math course grades (β = -5.553, $p < .05$). In contrast, similar to the student report, teacher report of Flexible and Responsive Support of Student Understanding positively predicts student emotional engagement (β = .303, $p < .05$), behavioral engagement (β = .201, $p < .05$), and cognitive engagement (β = .258, $p < .05$) in mathematics. Teacher reports also did not predict state standardized test scores in mathematics, when taking student and teacher covariates into account.

3.5.2.4.2. Convergence and divergence between student and teacher reports (RQ4)

Correlations between student and teacher report of the dimensions of student-centered instruction are reported in Tables 16 and 17. For the most part, student and teacher reports of student-centered mathematics instruction were not significantly correlated with one another, with three exceptions. There is a significant, but weak correlation between teacher report of Relevant and Metacognitive Tasks and student report of Student Intellectual Authority and Responsibility for Learning (r = .148, rs = .177) and teacher report of Support and student report of Student Intellectual Authority and Responsibility for Learning (r = .114, rs = .162). There is also a significant but weak negative correlation between teacher report of Student Intellectual Authority and Responsibility for Learning and student report of Support (r = -.127, rs = -.119).

3.5.3. Summary

3.5.3.1. Conceptualization of student-centered mathematics instruction

This study contributes a much-needed conceptualization of student-centered mathematics instruction. Researchers and practitioners have remarked on the lack of clarity regarding exactly what education research and policy is referring to when it says that instruction is or should be "student-centered." This study contributes by synthesizing current theory and research to propose a conceptualization of student-centered mathematics instruction. Among the few conceptualizations that have been articulated in the literature (e.g. Lee & Hannafin, 2014), the perspective articulated here is unique in that it is multi-dimensional, subject specific, and aims to explicitly identify the motivational and learning components of student-centered mathematics instruction and the relation between them.

The multi-dimensional conceptualization used underlying theory to identify levers and mechanisms of student-centered instruction that are explored in other related areas of research. These areas of research were later tied back into the theoretical components to contribute to articulating the multiple dimensions or components of student-centered mathematics instruction. The result is a conceptualization that identifies potential areas of practice to target in implementing instructional reform and/or potential effects on students to study in the implementation of reform. For example, school administrators and educators could attend specifically to the challenge of supporting student intellectual authority while assessing and maintaining the appropriate level of challenge on those tasks. Research could study how the level of challenge of tasks relates to students taking responsibility for learning and exerting intellectual authority and how these interact to influence how students engage and learn in mathematics class. Implications for research and practice are discussed in more detail in the discussion in Chapter 5).

While the conceptualization is thorough, it is worth noting that it is not comprehensive. Some components of student-centered mathematics instruction were omitted from the conceptualization. In particular, some research described the use of engaging learning formats, like technology, manipulatives, and/or other interactive materials and the use of formative assessment as elements of student-centered instruction. They were omitted from the conceptualization in this study because they did not appear in the literature as core features of student-centered mathematics instruction. In other words, it was not clear that if either of these elements were *missing*, that the instruction is not student-centered. It was also not clear the ways in which these elements relate to the theoretical foundations for how and why student-centered instruction shapes engagement and learning, the focus of the conceptualization for this paper. However, it is possible – likely, even – that these elements are used by teachers to support the other dimensions of student-centered instruction. Thus, it is worth exploring the role of learning formats, technology, and assessments.

Finally, the conceptualization supports the view that student-centered instruction may shape youth outcomes in ways that are discipline specific (Staub & Stern, 2002; Wang et al., 2016). In other words, "discipline specificity" in this conceptualization focused on the ways in which students may experience student-centered instruction that shapes their engagement and achievement in mathematics. More work would need to be done to articulate the ways in which student-centered pedagogy is discipline specific – for example, from the perspective of teacher content knowledge, sequencing, and the like.

3.5.3.2. Measures of student-centered mathematics instruction

3.5.3.2.1. Student report

The study also makes a significant contribution of the first well-validated student report of student-centered mathematics instruction. Study 1 revealed that there is strong psychometric support and validity evidence for a student report of student-centered mathematics instruction. Experts agreed with the conceptualization and items for the student report of student-centered mathematics instruction. The proposed 3-factor model is a good fit to the student data, each of the dimensional scales were reliable, and there was evidence of measurement invariance across student demographic characteristics.

The student report was also predictive of mathematics outcomes, including their behavioral, cognitive, emotional, and social engagement (reported by students) and their mathematics course grades (reported by their teachers and school districts). Among the dimensions of student-centered instruction, student report of Metacognitive and Relevant Tasks is consistently predictive of student engagement in mathematics coursework (β ranges from .100 to .200). However, student report of teacher support of Intellectual Authority and Responsibility for Learning exhibits an even stronger predictive association with student outcomes, including mathematics course grades. Specifically, for every one-unit increase in students' report of Intellectual Authority and Responsibility for Learning, there is an increase in their behavioral engagement by .282, cognitive engagement by .282, social engagement by .110, and emotional engagement by .381 (on a 5-point scale), and an increase of 2.699 in their mathematics course grades (M = 76.45, SD = 15.51, range 27-100).

The fact that the student scale predicts the outcomes with which it is conceptually and empirically associated in prior research provides evidence that the scale is measuring what it is intended to measure, supporting the measure as an assessment of student perceptions of student-centered instruction. Furthermore, the fact that there was measurement invariance of the student report by student gender, race, and socioeconomic status means that the survey can be used with samples of adolescents who are diverse on these characteristics. Taken together, the sound psychometric properties of the measure paired with predictive validity and measurement invariance support the potential utility of the scale.

In addition to supporting the student-centered instruction scale, the psychometric support of the scale also contributes evidence support to the effectiveness of student-centered mathematics instruction on student engagement and achievement in mathematics. Each dimension in the student report predicted academic outcomes in mathematics for secondary school students, supporting the effectiveness of student-centered instructional practice and the value of using student report. Furthermore, in addition to consistently predicting mathematics outcomes, the student report of student-centered mathematics instruction consistently explained more variance in student outcomes at the classroom level than did the teacher report. Future research can use the measure to examine differential effectiveness for students of different characteristics (discussed in more detail in Chapter 9).

Third, the measure and validity evidence provide support for the conceptualization of student-centered mathematics instruction. This study provides a multi-dimensional conceptualization that articulates how student-centered instruction shapes outcomes for youth. A more in-depth and nuanced understanding of student-centered instruction can inform how teachers are supported in implementing student-centered instruction with their students.

3.5.3.2.2. Teacher report

While there is some evidence in support of the teacher report, overall, the teacher report of student-centered mathematics instruction had less psychometric and validity support in this study. Experts expressed concern over several items in the teacher scale, and several items were removed in order to attain a satisfactory fit of the 3-factor model to the teacher report data. Evidence suggested that measurement invariance did not hold across teacher gender, highest degree, and years of experience teaching. There was a considerable lack of alignment between the teacher and student reports, and the teacher reports predicted student outcomes in puzzling ways.

The relatively weak psychometric properties of the scale and unusual predictive associations with student outcomes indicate that the teacher scale may not be an accurate or reliable measure of student-centered instruction. The fact that the teacher scale does not predict student engagement or achievement in the way that the literature would suggest it should if it is measuring student-centered instruction suggests that the scale is not accurately measuring student-centered instruction from a teacher's perspective. Furthermore, the fact that the scale fails to demonstrate measurement invariance by teacher gender, race, or years or experience suggests that teachers may vary in their perceptions and/or reports of student-centered instructional practice. Taken together, this evidence fails to support the utility of the teacher report scale as it was tested here.

There are a number of factors that could have impacted the performance of the teacher scale that are worth noting. First, the sample size for validating the teacher scale was small. All told, 36 secondary school teachers reported on their instructional practice in the 146 classes that they teach. The number of teachers could be conceptualized as even smaller, since the teachers work in grade 5 – 9 in three very different school districts. Six of the teachers work in a high-performing, suburban, and predominately Caucasian community with very few students eligible for free or reduced priced lunch. Another 11 teachers work at a high-performing urban charter school system that serves predominately students who are African-American and low-income. Twenty-two of the teachers work at a relatively low-performing public urban school with students from a range of ethnic and racial backgrounds, mostly from middle to low-income families. Taken together, the schools contribute a large and diverse sample of fifth, seventh, and ninth grade students. However, the distribution of the teachers across these schools is skewed, and the overall number of teachers in each system is small, particularly in the high performing suburban school system. This could be important because student-centered instructional practice may vary in significant ways depending on the demographics, social dynamics, and instructional policies of each school system.

Second, another type of model could be a better fit for the data. Given the inability of the two-level model to converge and relatively subpar fit of the model that adjusted standard errors for the nesting of teacher reports of their instruction in each class within teachers, it could be valuable to go back and use exploratory factor analytic procedures to explore a different factor structure with the teacher data. In particular, exploratory factor analyses could examine whether a different type of model or factor structure is a better fit to the teacher data.

Another possibility is that the items in the teacher survey also may not adequately or accurately represent student-centered mathematics instruction from teachers' points-of-view. The majority of items in the teacher survey were originally developed for student scales. The CLASS-S scales, the autonomy support scales, and the dialogic instruction scale were developed as student reports and the items on these scales were adapted for the teacher report. Many times the adaptation consisted of rephrasing the item to be from the teacher perspective. For example, the item "My math teacher lets me know that if I do not agree with him/her, it is important that I express my disagreement" from the student report of Autonomy Support –

Allowing Criticism and Encouraging Independent Thinking was adapted for the teacher report as "I tell students that if they do not agree with me, it is important that they express their disagreement."

There are multiple potential problems with this approach for developing the teacher scale. First, the wording may not accurately represent how teachers support that indicator of autonomy support. Teachers may use a variety of strategies to "let students know" (wording from the student item) that expressing a point-of-view that dissents from their own apart from explicitly telling them that it is okay to disagree (wording from teacher item). Thus, the item wording may be worded to be parallel in a way that it is not comparable in terms of measuring the intended indicator of autonomy support. Similarly, it could be that the indicator itself may vary between teacher and student. Perhaps the teacher scale should focus on specific types of teacher behaviors or responses that support students' freedom to express criticism and that encourages independent thinking.

An exception to the limitation of having developed a teacher report from a student report scale is the Reform Oriented Teaching Practice scale. This scale was developed specifically for teachers to report on their own practice. The items were removed from this study because teachers reported them at the teacher level, whereas teachers reported on their instructional practices across each of the classes that they teach for the other indicators of student-centered mathematics instruction. Experts also pointed out that the response format could have been a source of respondent error due to differences in response format. In order to expand and develop the teacher report of student-centered mathematics instruction, these items were revised to have a comparable format to the other items. However, it would be helpful to get more feedback about the extent to which these items reflect indicators of student-centered mathematics instruction. To this end, the teacher report of student-centered instruction could be improved by starting from the ground up and working with teachers to identify the indicators of student-centered mathematics instruction and to develop the appropriate item wording for those indicators.

3.5.3.3. Opportunities and limitations of the student report

There are several ways in which the availability of a well-validated student survey of student-centered mathematics instruction could make meaningful contributions to education research, policy, and practice. In particular, the survey is valuable for research, practice, and policy related to student perceptions of instructional practice. The student survey is not an objective measure of instructional practice. Indeed, the study provides evidence that another informant may report on the same class in a very different way. However, as a valid and reliable measure of student-centered instruction *from students' perspectives,* there are several opportunities to use the measure to more deeply understand students' perceptions of instruction and relations to engagement, achievement, and student characteristics. For example, research explore variation in students' report predictive associations of student report with outcomes, and differential predictive associations based on student, classroom, teacher, or school characteristics (discussed in further detail in the discussion in Chapter 5). There are also several cautions about the limits of the student report. Because it is not an objective measure, the student report should not be used as an accurate or reliable measure of instructional practice for the purposes of assessing instructional quality, teacher evaluation, or as an objective measure of instructional practice. Until additional research can be done to determine the relationship between the student report and other measures of instructional quality or practice and how it relates to observer and other teacher reports, research with the measure should interpret findings as relating to student perspectives of instruction instead of as to the presence or absence of instruction in an objective sense.

3.5.3.4. Limitations and future directions

The study has several methodological and inferential limitations. First, self-report surveys of instructional practice provide a narrow perspective of the instruction that takes place in classrooms over time (see Rowan, Correnti, & Miller, 2002). Building on the student survey measure to develop a teacher log and/or observational study, student-centered instruction could provide more in-depth information about instructional practice and novel insight into how student-centered instruction unfolds in secondary mathematics classrooms over time.

The study is also limited due to its focus on only student-centered instruction. In practice, teachers often use a combination of student- and teacher-centered approaches (see Cohen, 1990; Smith, 2000). While student-centered mathematics instruction is often positioned as an instructional practice that is mutually exclusive with teacher-centered instruction, in practice the relationship between the two is likely more fluid and dynamic. Findings from this study should be interpreted as narrowly focused on developing a measure of student-centered instruction and investigating preliminary associations of student-centered mathematics instruction with mathematics outcomes. Future research should explore the complex relationship between student- and teacher-centered instruction and how the two interact in mathematics classrooms to shape student outcomes.

Finally, the generalizability of the study is limited due to the sample being collected from a metro area in a single geographic location. Future research could explore the psychometric properties of the scale and predictive validity in a sample of adolescents and teachers in different geographical and social contexts. Furthermore, while there has not been a study of differences in the implementation and effects of student-centered instruction across various cultural contexts, the availability of a well-validated student report measure could enable such a comparison.

4. STUDY 2: SEVENTH GRADERS' EXPERIENCE OF GETTING STUCK IN CLASSROOMS TRANSITIONING TO STUDENT-CENTERED MATHEMATICS INSTRUCTION

4.1. INTRODUCTION

Increasingly, American middle schools are changing the way they teach mathematics to their students. In place of a traditional teacher-centered approach that focuses on lecture and practicing mathematical procedures, many schools are calling on teachers to support students' engagement and achievement in mathematics by facilitating students' collaborative work on cognitively challenging tasks while shifting the cognitive effort, intellectual authority, and responsibility for learning from the teacher to the students (see Cohen & Ball, 1990; National Council of Teachers of Mathematics, 2014). However, while student-centered mathematics instruction is supported by motivation and learning theories (Piaget, 1960; Ryan & Deci, 2000; Vygotsky, 1978), we know very little about how students experience the transition from teacher- to student-centered mathematics instruction.

In particular, various aspects of student-centered instruction could be emotionally salient to adolescents who are accustomed to a more teacher-centered style of instruction. Learning how to solve more challenging problems in collaboration with peers in an environment in which students are responsible for their learning is a considerable shift from what is otherwise relatively routine academic work. For middle school students, this would be occurring at a time when they are developing their academic identity and working to meet their needs for competence, relatedness, and autonomy in school (see Eccles & Roeser, 2011). Indeed, while some research has found that student-centered instruction can increase adolescents' overall enjoyment of mathematics (for example Noyes, 2012), other research shows that it can be difficult for teachers implementing student-centered reforms to engage adolescents in cognitively challenging and open-ended tasks (Fulmer & Turner, 2014) and that adolescents can experience negative emotions when they are grappling with difficult problems (Newmann, Bryke, & Nagaoka, 2001; Spillane, 2001).

In particular, the experience of getting stuck while working on hard problems could have complicated effects on adolescents' attitudes towards mathematics. Experiencing negative emotions in math class and while doing math homework has been linked to declines in motivation and performance in math. However, studies tend to look at students' experiences of math class in general (see Pekrun, 2006), or their experience doing certain types of math work, such as homework (see Dettmers et al., 2011). Less research has been done on students' emotional experiences of specific phases of the process of math learning, such as getting stuck while working on math problems.

In this exploratory sequential mixed methods study, I investigate seventh graders' emotional experiences at a middle school that is in its first year of implementing student-centered instruction in its mathematics classes. The exploratory sequential mixed methods study design uses a qualitative study to explore a phenomenon that is not understood well or that is in need of deeper insight or clarification, which is then followed up by a quantitative study that examines relationships found in and built on the qualitative data (Creswell et al., 2003). Pursuant to this design, the current study integrates three phases of qualitative and quantitative data and analyses from teachers, observers, and students in order to generate new insight in the context of a holistic and robust assessment of students' experiences of student-centered mathematics instruction (Eisenhardt, 1989; Gay & Weaver, 2011). The mixed method study provides key insight into dynamic and transactional processes of adolescent development by exploring the range of emotions adolescents report that they experience when they get stuck working on math problems in their student-centered class and the complex relationship with their perceived competence in math over the course of the year.

4.2. LITERATURE REVIEW

4.2.1. Defining "stuck"

In this study, I define being stuck as students' inability to maintain productive cognitive engagement in learning tasks. Cognitive engagement refers to the level of mental investment, effort, and strategies that adolescents expend in class and on classwork. Productive cognitive engagement refers to cognitive engagement that effectively moves students towards learning goals, solving problems, or a deeper understanding. Unlike cognitive engagement, which refers to the in-the-moment quality of cognitive involvement in learning tasks, productive cognitive engagement refers to the cognitive investment, effort, and strategies that unfold throughout the learning process. In other words, from the constructivist perspective, whereas cognitive engagement is a key component of how children are active in constructing logical structures and systems of meaning that enable them to integrate new information with previous knowledge and experience (Piaget, 1960, 1972; Vygotsky, 1978), productive cognitive engagement refers to the cognitive involvement required for and involved in the successful creation of logical structures and systems of meaning.

Productive cognitive engagement is likely dynamic and non-linear as it unfolds throughout learning tasks. From an instructional and learning science perspective, the type of productive cognitive engagement that leads to deep levels of understanding is supported when students productively struggle on open-ended and challenging tasks. Cognitively challenging tasks provide students with opportunities to grapple with challenging mathematical concepts (Hiebert et al., 1996; Smith & Stein, 2011; Stein & Smith, 2011). Intellectual authority, responsibility for learning, and being flexible and responsive in supporting adolescents' understanding, support adolescents' motivation, engagement, and learning while working on challenging tasks (see Chapters 2 and 3 describing the tenants and theoretical foundations of student-centered mathematics instruction). Productive struggle occurs when students make progress towards solving a challenging problem by investing energy in coming up with one or multiple strategies. By maintaining productive cognitive engagement while productively struggling with challenging tasks, adolescents attain deep levels of learning by changing their conceptions or prior understanding through cognitive dissonance situated in social interaction (see Piaget, 1960; Vygotsky, 1978).

In contrast, students who are stuck are not productive in their cognitive engagement in the task and are not making any progress. For whatever reason, at the moment that they are stuck, they are not getting any traction to build momentum towards solving the problem. They may or may not be motivated or investing energy. Their worksheets might be blank or partially incomplete, they may indicate to the teacher that they do not know what to do next, and/or they may freeze or go blank when the teacher tries to "re-engage" them in the task. In the moment, they may be investing in cognitive engagement of one kind or another, but that engagement is not contributing to their progress or understanding of the problem.

Getting stuck is a well-known but unstudied phenomenon that is common to learning mathematics and may be even more common in student-centered mathematics classrooms that feature challenging tasks that students are responsible for solving. Getting stuck could be part of the cycle of productive struggle, an indicator of unproductive struggle, or could be a distinct cognitive and/or motivational phenomenon altogether. In and of itself, it is not understood very well. However, there is evidence that getting stuck may influence the emotions that adolescents experience in mathematics classes and may shape their sense of competence in mathematics over time.

4.2.2. Control-value theory of achievement emotions

The control-value theory (CVT) of achievement emotions provides a useful framework for thinking about how getting stuck could influence motivation, learning, and achievement in mathematics. Developed from the expectancy value theory of emotions (Pekrun et al., 2002), CVT provides an integrated framework for understanding the relationship between context, appraisals, emotions, and learning and achievement. Specifically, CVT explains that the achievement emotions that students experience in relation to their perceptions of their control (control appraisals) over the outcome of academic activities that are important to them (value appraisals) can influence the availability of students' cognitive resources, their interest and motivation, and the strategies they use during academic activities (Pekrun et al., 2007).

4.2.3. Achievement emotions

4.2.3.1. Description

Academic emotions are complex phenomena that involve the coordination of multiple psychological processes, including affect, cognitive processing, physiological responses, and motivation (Pekrun, 2012). Traditionally, emotions have been studied in psychology to understand psychopathology, psychological wellbeing, and relationships or attachment. Recently, researchers in education have begun exploring the role that emotions play in motivation, engagement, and achievement in school. Over the past decade, research has demonstrated that emotions are critical to students' productivity and success in school (Efklides & Volet, 2005; Linnenbrink, 2006; Schutz & Pekrun, 2007). While there has been an emphasis on test anxiety (e.g., Zeidner, 2007), research is expanding its focus to include a wider scope of emotional experiences and how they shape education outcomes. This research and the associated theoretical perspectives can help us understand how students experience getting stuck in their math classes and how it could shape their motivation and achievement in their mathematics coursework.

Generally, research on academic emotions has focused on achievement emotions. Achievement emotions refer specifically to the emotions that students experience in relation to their academic work, activities, or experiences. However, students can experience other types of emotions in the academic context. Social emotions relate to interactions and relationships with teachers and peers, epistemic emotions relate to cognitive states that occur when learning new information, and topic emotions relate to the specific nature of the academic content (see Pekrun, 2012). Together, achievement, social, epistemic, and topic emotions interact and overlap to shape adolescents' affective experiences of classroom learning.

Academic emotions have been described as having valence and energetic components, according to which emotions can be positive or negative and can have activating or deactivating effects (Feldman Barrett & Russell, 1998). Emotions that are positive and activating are associated with positive affect and energized activity – for example, enjoyment, pride, and hope. Positive emotions that are deactivating refer to the type of psychological well being associated with peaceful or satisfied withdrawal from activity, such as contentment, relaxation, and relief. Negative activating emotions include anger, anxiety, and shame. Negative deactivating emotions include boredom, sadness, and hopelessness (see Pekrun, 2012).

In addition to valence and activation, education emotions can vary in terms of whether they are focused on an activity that is happening in the moment or on an outcome that students are anticipating in the future (Pekrun, 2012). For example, students can experience achievement emotions in relation to the process of studying for an exam (activity achievement emotions, such as enjoyment or boredom) or in relation the prospect of success or failure on that exam (outcome achievement emotions, such as anxiety or excitement). Epistemic emotions tend to be activity focused, as they occur most often as a result of the cognitive processing that occurs during a task. For example, students can experience surprise, curiosity, anxiety, or confusion when the task that they are working on confronts or contradicts previously held beliefs or understanding (Kang et al., 2009). Social emotions can be either activity or outcome focused, as students experience emotions on the process of interacting with peers and teachers and in response to prospective feelings about future successes and failures in relationships in school.

4.2.3.2. Effects of achievement emotions

Emotions are ubiquitous in education settings and play a significant role in shaping adolescents' academic performance in a number of ways (Pekrun, Goetz, Titz, & Perry, 2002a). First, emotions can distract students from focusing on their academic work by becoming the object of students' attention (Ellis & Ashbrook, 1988), which can have detrimental effects on their academic performance (Meinhardt & Pekrun, 2003). Second, emotions are a significant part of and influence on students' motivation to learn. Students' enjoyment, curiosity, boredom, and frustration during a learning activity are part of their emotional engagement in learning, a key predictor of their achievement in mathematics (see Wang et al., 2016). Third, emotions shape the learning strategies that students use during academic tasks and in response to setbacks or challenges. The emotions that students experience while working on a task can facilitate or interfere with their cognitive flexibility, creativity, and ability to reason or think critically about the work that they are doing (see Pekrun, 2012). Lastly, emotions can influence students' self-regulatory strategies, like goal setting, self-monitoring, self-directed effort, and purposeful implementation of various learning strategies.

The relationship between emotional valence and effects on education outcomes in mathematics is complex. Positive and negative emotions can evoke an adaptive or maladaptive response, depending on the emotion, the level of activation it entails, the demands of the context or activity, and attribution process it triggers in students. For example, positive emotions can support self-regulation, goal setting, and problem-solving (Clore & Huntsinger, 2009; Pekrun et al., 2002b), but positive emotions can also contribute to an unrealistic assessment of one's ability or make students less likely to think deeply about their academic work (Pekrun et al., 2002b).

The effects of negative emotions may be even more complex and is less studied. Research on negative activating emotions has focused on anxiety. We know much less about other activating emotions like anger, shame, or confusion. Evidence and theory suggests that anger, shame, and confusion can distract students from focusing on learning tasks. Students' feelings of shame have been linked to low levels of effort and academic performance (Pekrun et al., 2004). However, if the cognitive demand of the task is low, then students' fear of failure (shame) can support the use of superficial strategies that may contribute to their success on those tasks (see Pekrun, 2012). Similarly, overall anger is negatively associated with engagement, motivation, and self-regulation of learning (Assor, Kaplan, Kanat-Maymon, & Roth, 2005; Pekrun, Goetz, Franzel, Barchfeld, & Perry, 2011). However, there is evidence that anger can be translated into motivation when students have high expectations for success (Lane, Whyte, Terry, & Nevill, 2005). Negative deactivating emotions – boredom, helplessness, and hopelessness – are considered to be detrimental. Boredom, helplessness, and hopelessness have each been linked to reduced motivation and performance (Pekrun, Goetz, Daniels, Stupinsky, & Perry, 2010; Pekrun et al., 2002a; Pekrun, Elliot, & Maier, 2009).

4.2.4. Achievement emotions of getting stuck

Research in academic emotions has focused mostly on the academic activities of homework and studying and the academic outcomes related to prospective feelings about testing or assessment (see Pekrun et al., 2007). Getting stuck may fall somewhere in between or across being activity or object focused. On the one hand, getting stuck occurs during learning activities. On the other hand, getting stuck is itself an outcome that students can anticipate – a failure to know what to do to solve a problem that is consequential to your grades, sense of competence, and ability to meet internal and external expectations and goals for success. In this way, getting stuck may be either activity- or outcome-focused or could cross both; it is a phenomenon associated with the learning process as it unfolds in real time and as students anticipate it will unfold in the future.

Research supports that getting stuck is likely an emotionally salient experience for adolescents. While students' experiences of getting stuck have not yet been studied in depth, research on students' inability to successfully solve mathematics problems has found that students experience a wide range of emotions that influence their motivation, strategy use, and self-regulation (Pekrun et al., 2004). Also, research on the implementation of student-centered mathematics instruction has found that students often respond to productive struggle with negative emotions in class (Newmann et al., 1992; Spillane, 2001).

Despite the fact that student-centered instruction features cognitively challenging tasks, we know very little about how students experience that kind of academic work. From the perspective of learning theory, raising the cognitive demand of students' academic work provides opportunities for students to attain a deep understanding of mathematics (Stein & Smith, 2011; Resnick et al., 2008). From a motivational perspective, providing adolescents with relevant and challenging tasks can increase their motivation, engagement, and learning by supporting their psychological needs with tasks that are also aligned with their cognitive skills and increasing inclination towards social interaction (see Eccles & Roeser, 2009; Deci & Ryan, 1987). From the perspective of academic emotions, cognitively challenging tasks can be cognitively activating in ways that support motivation, engagement and learning (Pekrun, 2012).

However, if the demands of a task are such that adolescents do not feel that they are likely to succeed, then students can experience emotions that thwart their motivation by undermining their sense of control and competence (Pekrun, 2006). A significant amount of work has been done in the learning sciences and curriculum and instruction about the appropriate level of challenge of a task (e.g., Smith & Stein, 2011; Stein et al., 2008). This research has conceptualized "appropriate level of challenge" as an objective quality of a task, referring to a level of difficulty that is outside of students' current level of ability but it is within reach. Similarly, research in academic emotions highlights the potential for cognitively challenging tasks to increase students' perceptions of control and sense of competence in mathematics (Pekrun, 2012). However, students' perceptions of their ability to succeed in solving a challenging task is critically important to the emotions that they experience, the strategies students use, and the impact on their motivation and attitudes about mathematics over time (Craig et al., 2008; Pekrun, 2006, 2012).

In particular, emotions experienced when stuck could shape students' sense of competence in mathematics. In the control-value theory of emotions, competence beliefs are considered an antecedent to affective experiences or emotions (see Pekrun, 2012). In socio-cognitive theory, however, competence beliefs are related to self-efficacy, which is strongly influenced by students' emotional experiences. For example, experiencing anxiety is related to low self-efficacy because students interpret the anxiety as indicating that they will not be successful (Bandura, 1997). In the United States, secondary school students who experience math anxiety feel less capable of solving math problems (e.g., Pajares & Urdan, 1996). Both perceived competence and self-efficacy are strongly associated with academic motivation and achievement in mathematics (e.g., Britner & Pajares, 2006; Lau & Roeser, 2002), and a recent factor analysis found the two constructs to be related (Hughes, Galbraith, & White, 2011). Thus, it could be valuable to study how getting stuck is experienced by students and how those experiences influence their sense of control over time.

4.2.5. Student demographic characteristics

Students' emotional experiences of getting stuck could vary by students' race, socioeconomic status, gender, and level of achievement in mathematics. Race and gender have been directly and indirectly associated with negative emotional processes in relation to mathematics coursework in previous research. Anxiety related to the potential for activating stereotype threat has been associated with declines in engagement in mathematics among African-American students (Aronson, Fried, & Good, 2002), and experiencing stereotype threat can trigger emotions that are detrimental to learning (Mangels et al., 2012). Female students have experienced significant math-related anxiety (e.g., Lau & Roeser, 2002) and are less interested in math than their male peers (Frenzel et al., 2010; Hyde et al., 1990). Low-income students' emotional experiences have not been studied. However, their decline in mathematics engagement is well documented (National Center for Education Statistics, 2015; U.S. Department of Education, 2015), and CVT posits that emotional experiences related to control and value appraisals could play a role (Pekrun et al., 2002). Finally, students' level of achievement in mathematics could be related to their emotional experience of getting stuck in complex ways. On the one hand, getting stuck could contribute to negative achievement emotions in low-achieving students because it could trigger appraisals of low control and/or value of mathematics coursework. On the other hand, high-achieving students have developed strategies for succeeding in learning contexts that are traditionally teacher-centered, within which high-achieving students experience continuous flow of productive cognitive engagement in relatively straightforward tasks. Getting stuck while working on cognitively challenging and open-ended tasks could confront high-achieving students' appraisals of control, which could trigger negative achievement emotions.

4.2.6. Achievement emotions in algebra coursework

It could be particularly valuable to study adolescents' emotional experiences of the transition to student-centered instruction in algebra coursework. Fluency in algebraic concepts and procedures are increasingly considered skills that are important for participating productively in life outside of school (Evan et al., 2006; U.S. Department of Labor, 2007). The mathematical knowledge and skills taught in required secondary mathematics coursework are important for participating productively in life outside of school (Evan et al., 2006; U.S. Department of Labor, 2007). Furthermore, algebra coursework is a gateway to the advanced courses in mathematics that contribute to high school graduation rates, college readiness, and college completion (Evan et al., 2006), which have become increasingly important for attaining well-paying jobs (Achieve, 2006; Carnevale & Desroches, 2003).

Specifically, adolescents underachieve in algebra coursework and lack the algebraic skills needed to succeed in postsecondary education (Baldi et al., 2007; Evan et al., 2006; U.S. Department of Education, 2007, 2015). To be successful in mathematics coursework and to be eligible for STEM careers, American adolescents need to develop the cognitive skills needed to solve challenging mathematics problems (e.g. Common Core, 2015; National Council of Teachers of Mathematics, 2014; Resnick et al., 2008). This is particularly critical beginning with algebra coursework, which requires a range of complex skills, like conceptual understanding, strategic competence, and adaptive reasoning (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; US Department of Education, 2007, 2015). Unfortunately, algebra coursework begins in earnest after the transition to middle school, at which point adolescents experience a significant decline in their engagement in mathematics coursework (Martin et al., 2014; Wigfield et al., 2006).

4.2.7. Contribution and gaps in the literature

Studying adolescents' experience getting stuck in mathematics could contribute to multiple gaps in the current literature on students' academic emotions and how student-centered instructional reforms shape adolescents' outcomes in mathematics. First, studying students' emotional experiences when they are stuck can help us better understand the range of academic emotions that students can experience in mathematics coursework. Besides anxiety, there has not been as much research about negative emotions (e.g., Pekrun & Frese, 1992; Schutz & Lanehart, 2002), particularly in a real-life classroom setting (Pekrun, 2012). Getting stuck while working on problems in mathematics class is likely to be associated with a variety of emotions, in part because it entails multiple objects of focus and can be both personally and socially salient to students. For example, students who are stuck while trying to solve a challenging mathematics problem could experience an epistemic emotion, such as frustration or confusion, while they are in the process of trying to figure out how to solve the problem. Alternatively, they may experience the achievement emotion of anxiety if they are focused on the prospect of not being able to solve the problem. Or, they could experience the social emotion of shame if they are concerned about the teacher misinterpreting or having an otherwise negative response to their lack of progress.

Studying students' experiences of getting stuck could also help us better understand key components of the learning process that are salient for student motivation and achievement. Currently, educational psychologists tend to focus on structural differences in learning tasks, such as focusing on academic work and dividing it into homework, studying, and testing (see Pekrun et al., 2011). It could be that the differences between these tasks are relatively superficial and that students' experiences and outcomes in mathematics are shaped by more fundamental and emotionally salient components of the learning process. Getting stuck, for example, could be a fundamental component of the learning process and is likely to be emotionally salient to youth and their engagement and achievement in mathematics.

Finally, studying students in classrooms who are in the process of undergoing instructional reform to student-centered instruction could help us understand how these instructional policies impact students' experiences in mathematics class and motivational beliefs towards mathematics coursework. Education research has studied the effects of cognitively challenging tasks, supporting students' autonomy, flexible and responsive support of student understanding, and teachers espousing student-centered learning goals on student achievement in mathematics (see Chapters 1-3). However, we have not yet studied how students' experiences in the context of these teaching practices shape outcomes in mathematics.

4.3. AIMS AND RESEARCH QUESTIONS

The study aims to describe seventh grade adolescents' experiences of their mathematics teachers implementing a reform to student-centered instruction. In particular, the study examined the emotional experience of getting stuck in their algebra coursework and how the frequency and nature of these emotions varied by student characteristics and how students' emotional experiences influence their self-competence in math. This mixed method case study examined the reported experiences and perceptions of 214 seventh graders at a middle school that was implementing reform-based, student-centered instruction in their math classrooms. The specific research questions of the study included:

4.3.1. Qualitative study

1. How do seventh grade students describe what it is like when they are stuck on math problems?

4.4.2. Quantitative study

2. What are the relationships between students' race, gender, socioeconomic status, and level of achievement in mathematics with how often they get stuck and the types of emotions they report about getting stuck in math?
3. Does how often a student gets stuck relate to the types of emotions they report experiencing when they are stuck?
4. How do students' experiences being stuck relate to their sense of competence in math at the end of the year?

4.4. METHODS

4.4.1. Mixed methods

The study employed a mixed method sequential exploratory design (Creswell et al., 2003). The first phase was a qualitative study of students' emotional experience of getting stuck. The second phase was a quantitative study that described the prevalence and associations of students' experiences with demographic characteristics and tested the predictive association of emotional experiences getting stuck with changes in competence from the fall (November) to the end of the school year (June) (Johnson & Onweugbuzie, 2004; Onweugbuzie & Leech, 2006).

The study included multiple aims and components that were well served by a mixed method approach (from Yoshikawa, Weisner, Kalil, & Way, 2007). First, the study aimed to understand a complex process that integrates adolescents' psychological experiences and their socialization and development in schools (Yoshikawa et al, 2007). Understanding the nature and importance of adolescents' emotional experiences getting stuck in mathematics coursework required accurately capturing or describing their experiences, assessing the prevalence of those experiences, and examining the predictive association of those experiences with important outcomes (Yoshikawa et al, 2007). By combining qualitative and quantitative analyses, I was able to elaborate on the nature of getting stuck (Greene, Caracelli, & Graham, 1989), while also enhancing the understanding of its significance to adolescent outcomes (Collins et al, 2006).

Mixed methods can also help to identify the ways in which adolescents' emotional experiences of getting stuck could be a reciprocal and dynamic process, wherein features of student-centered instruction shape adolescents' emotional experiences of getting stuck, and those experiences influence their engagement and teacher response. Mixed methods informed these processes by identifying potential reciprocal relations between instruction and students' emotional experiences (Yoshikawa et al, 2007).

In this way, the mixed method analysis can identify and explore a potential mechanism that influences the effectiveness of student-centered instruction in supporting engagement and achievement in mathematics. By combining the insight provided by the qualitative study with quantitative descriptive, comparative, and predictive analyses, the study findings can add to our understanding of learning emotions and processes shaping the effectiveness of student-centered instruction and generate new ideas for future research (Newman, Ridenour, Newman, & DeMarco, 2003).

The sequential exploratory design also enabled me to capture a broad range of emotional experiences. First, by starting with the qualitative methods, I captured the range of students' emotional experiences, which helped to prevent me from glossing over emotions that are not experienced often. The subsequent quantitative analysis looked for patterns of association with student characteristics, even among emotions that were reported in relatively low frequency. For some students, these emotional experiences may have a particularly strong effect on their outcomes. Math education policies are interested in mechanisms that keep minority and low-income students from doing well in math, and these students can make up a small proportion of the student population in schools (Greene et al., 1989).

4.4.2. The study contexts and participants

4.4.2.1. The school

The study took place at Lakeside Middle School, a suburban school serving approximately 800 seventh and eighth grade students in Western Pennsylvania. The school is organized into teams of teachers and students. Each team included one teacher in each content area who collaborated to educate and track approximately 125 students. The school was nationally recognized for students attaining high levels of math achievement with teachers using direct instruction. During the 2013-2014 school year, however, the school initiated a significant change to the way they teach mathematics. First, the district implemented a school-wide instructional program to make mathematics instruction more student-centered. The program, called Lead-to-Learn, worked with teachers and administrators to develop “meta-cognitive classrooms,” wherein students were positioned to take the lead in engaging in relevant and cognitively challenging tasks by working while interacting with their teacher and their peers. To support the instructional reform, the mathematics department in the middle school started using Connected Math Project (CMP) curriculum, a problem-based approach that promoted inquiry-based teaching and learning in math classrooms. As part of these initiatives, the teachers were observed and coached by Lead-to-Learn trainers and met weekly to discuss and review the implementation of CMP-type activities and strategies.

The school was selected because it is a setting where there were minimal barriers to collecting the necessary data (Yin, 2014), and the process under study was readily observable (Eisenhardt, 1989; Miles, Huberman, & Saldaña, 2013). The study was approved by the Institutional Review Board at the University of Pittsburgh and by the school board and administration of the participating district.

4.4.2.2. The students

Approximately half of the seventh grade students (n=214) participated in the study. These students were selected because they were the students of two teachers who participated in a case study on the implementation of student-centered instructional reform. The students were in two different cohort teams at the school, meaning that they had two different mathematics teachers and did not share any other coursework together. Among the sample, 11.13% were minorities, 54.38% were female, 7.55% were eligible for free or reduced price lunch, and 25.23% were enrolled in honors-level algebra classes.

The students were in eight classrooms taught by the two teachers taking the lead on implementing the instructional interventions in the school. To review, “Mrs. Miller” was a former engineer, had over 10 years of experience teaching mathematics, was finishing her doctoral studies in mathematics curriculum and instruction, and was considered an “expert” in student-centered mathematics instruction. Her colleague, Mr. Smith, had 17 years of experience using predominately teacher-centered mathematics instruction. Along with Mrs. Miller, he agreed to focus on supporting its implementation in the mathematics department at the school. Mr. Smith reported that he had wanted to try changing the way he teaches to include more student-centered practices for a while but hesitated to do so on his own.

The study focused on the students in each of the teacher’s honors class and three of their regular pre-Algebra classes. The students in Mr. Smith’s remedial class were omitted because he co-teaches the course over two class periods, which changed the dynamic in ways that could be misconstrued as being related to the ability-level grouping. The students from Mrs. Miller’s fourth regular pre-Algebra class were omitted because that section was the focus of her dissertation, which could have led to changes in the way she was teaching the class, the frequency or ways in which students were stuck, or students' experiences getting stuck in ways that were not related to her use of student-centered mathematics instruction broadly.

4.4.3. Procedures

All participants were invited to participate in the study using school-based recruitment during the Fall Semester of the 2014-2015 school year. With the teachers' help, the research team distributed letters about the project to students in their math classes. Students whose parents did not opt their children out of participation and who agreed to participate in the study completed surveys during their math class in the fall (early October), in the spring (early May), and at the end of the academic year (June). To protect student privacy and in order to support students' openness and honesty in their reporting, the survey was distributed in large manila envelopes, and students were instructed to return their survey with the envelopes sealed. Students were informed – verbally during survey administration and in writing on the survey – that the research team was interested in understanding their experiences and that their responses would be confidential. Student surveys were anonymous and linked to their math classes. Students were compensated with a small gift for their work on the survey. The study was approved by the Institutional Review Board at the University of Pittsburgh and the school board and school administration of the participating district.

4.4.4. Qualitative study

4.4.4.1. Check for student-centered instruction

Since the study examines student experiences of getting stuck in the context of a school-wide reform to student-centered mathematics instruction, I first examined the extent to which each teacher's classes exhibited activity formats that were consistent with implementing the reform. In order to capture each teacher's instructional practice throughout the school year, I videotaped four full days of each teacher's instruction throughout the academic year, specifically, one full day of instruction in October, January, March, and May.

In order to check for activity formats consistent with student-centered instruction, I used a coding scheme based on Schoenfeld's (2013) The Teaching for Robust Understanding of Mathematics (TRU Math) coding scheme. The codes included: whole class activities (including teacher exposition and whole class discussion), small group work, student presentations, and individual work. The instructional reforms would have students spending a significant amount of time doing group work and student presentations.

Two undergraduate students coded the first forty minutes of each lesson (out of a 42-minute period) in the Interact Video Coding software by Mangold International, Inc. Inter-rater reliability was assessed by having 30% of the classroom observations co-coded and by calculating Cohen's Kappa in the coding software (Fleiss, 1981; Landis & Koch, 1977). The activity formats for each teacher were examined by calculating the amount of time spent in each activity type.

4.4.4.2. Data on students' stuck emotional experience

On a survey administered to students in early May, students were asked to respond to open-ended questions about their experiences when they get stuck solving math problems. Due to the exploratory and novel nature of the study, open-ended prompts needed to be developed. With the assistance of an expert in mathematics pedagogy and an expert in motivational and developmental psychology at the University of Pittsburgh, I crafted a list of brief open-ended questions to which students would report on their experiences getting stuck. Once a preliminary draft of the open-ended items was complete, I pilot tested the items in a semi-structured interview format with three female, Caucasian seventh grade students; one male, Caucasian eighth grade student; and one male, Indian ninth grade student.

To help students recall their experiences of being stuck in their math class and to distinguish being stuck from when they were making progress on hard math problems, students were first asked, “When was the last time you worked on a hard math problem?” “When you are given a hard math problem in class, what do you do?” After responding to these questions, students then responded to prompts about their experiences getting stuck specifically, including: “What do you do when you are completely stuck – when you are not sure what to do next?” and “How does it feel when you are stuck? What goes through your mind, if anything?” The items were purposefully written to illicit student reports about their experiences, broadly, so as not to lead students to report any particular sort of emotional experiences. Students’ open-ended survey responses were collected and transcribed into an Excel document.

4.4.4.3. Analytic plan

4.4.4.3.1. Students’ emotional experiences when they are stuck

In order to describe students’ reports about their experiences (RQ1), I used an inductive and deductive qualitative analysis that used multiple rounds of analysis with *a priori* and grounded emotion categories (described in more detail below). The analysis included a systematic review of the excerpts followed by two rounds of coding. The study investigator and two psychology undergraduate students reviewed and coded the data.

In the first round of coding, two undergraduate psychology students coded excerpts with the discrete emotions reported in the Achievement Emotions Questionnaire (AEQ; Pekrun, Goetz, & Perry, 2005). Table 24 displays the emotions described in the achievement emotions framework, including enjoyment, hope, pride, relief, anger, anxiety, shame, hopelessness, and boredom. In order to be exhaustive, we used each of these emotions as codes in Round 1 coding. Given the exploratory nature of understanding students' emotional experiences of being stuck, we also included an "other" code that identified emotional experiences that did not clearly fit into any of the Achievement Emotions categories. Excerpts coded as "other" were evaluated for themes. In order to assist in thematic analysis (for example, by identifying the co-occurrence of codes), students' whole responses were the unit of analysis, and raters applied all codes that were applicable to each student's response.

Then, to develop the final coding scheme for understanding students' emotional experiences of getting stuck, I inductively identified themes not captured by the other codes by reviewing the codes that were applied from the original framework (Table 24), by evaluating codes that commonly co-occur in the same excerpt and that are conceptually related, and by evaluating the types of emotional experiences that are in the "other" bin (Saldana, 2011; Miles & Huberman, 1994; Strauss & Corbin, 1998). Since the first round of coding resulted in changes to the coding scheme, the student responses were re-coded to investigate the *a priori* and emergent themes from the first round of coding. The results of the thematic analysis and resultant Round 2 codes are described in the study results.

4.4.5. Quantitative study

4.4.5.1. Data

4.4.5.1.1. Emotional experiences being stuck

Qualitative codes of students' emotional experiences getting stuck were transformed into dichotomous variables, one for each emotion category.

4.4.5.1.2. Self-competence in math

Perceived competence was assessed with items from a shortened version of the Attitudes Towards Mathematics Inventory (revised by Lim & Chapman 2013) in early October and in June. Students' self-competence in math was assessed with 5 items that measured students' confidence in math class and when studying math (for example "I feel a sense of insecurity when attempting mathematics" and "I am always confused in my math class"). Students responded on a 4-point Likert-type scale ranging from *strongly disagree* (1) to *strongly agree* (4). The self-competence scale demonstrated good reliability in the fall and spring (see Table 27).

4.4.5.1.3. Frequency of getting stuck

In early May, students were asked to report on how often they got stuck in math class. In order to anchor student responses to a similar time frame, students were asked to report how often they got stuck "in the last couple of weeks." Students responded to this question after answering open-ended questions about their experiences being stuck (previously described). Students responded on a 4-point scale ranging from almost never (1) to very often (4) (see Table 27 for descriptive statistics).

4.4.5.1.4. Student and classroom covariates

At the end of the school year (in June), students completed a demographic survey on which they reported their gender, ethnicity, free or reduced lunch status, and previous math grade. Students indicated with which racial or ethnic groups they identified, including: White or European-American, Black or African-American, Hawaiian Native or Pacific Islander, Asian or Asian-American, American Indian or Alaskan Native, Hispanic or Latino/a. Given the small sample size and small proportion of students who identified as a minority (around 11%), students who selected any minority category were coded as minority (1) – students who indicated only White or European-American were coded as non-minority (0). Student math grades were reported on a 10-point scale ranging from F (1) to A (10). In order to check the accuracy of student self-reports of their end-of-year course grades, the school district provided classroom-level averages for these variables. The school-reported classroom-level averages did not differ significantly from the averages generated by student reports. Variables were created in the data to indicate whether students were enrolled in general or honors-level algebra courses and who taught their math class.

4.4.5.2. Analytic plan

In order to examine the relationship between student characteristics and frequency of getting stuck (RQ2 and RQ3), I ran Analysis of Variance to test for mean level differences in how often students' got stuck with student characteristics, including gender, minority status, free lunch eligibility, and whether they were enrolled in an honors course. I used chi-square tests to examine the relationship between these student characteristics and the types of emotions they report about getting stuck in math. The chi-square tests examined the relationship between each student characteristic and each emotion category. I also used Analysis of Variance to test for whether the type of emotions students report when stuck was associated with how often a student reported being stuck.

Finally, I used multiple regression models in Stata 14.0 (StataCorp, 2015) in order to examine how student experiences when stuck influenced their attitudes toward math at the end of their seventh grade year (RQ4). I used model building to examine three sequential models. The first model tested for the effects of student covariates on students' self-competence at the end of the year, including students' level of self-competence in math in the fall and their gender, minority status, free or reduced-price lunch eligibility, and previous math grade. Due to the fact that the sample size was too small to address the clustering nature of the data, I controlled for teacher, class period, and whether the class was a general or honors' level algebra course in the analyses (Laird and Ware, 1982; Raudenbusch & Bryk, 2002). In Model 2, I added students' report of their emotions when they were stuck to the predictors used in Model 1. In Model 3, I included how often students were stuck as a predictor, to test if frequency of getting stuck predicted self-competence and if other predictors that were significant fell below statistical significance when frequency of getting stuck was taken into consideration.

4.4.5.3. Missing data

Overall, the response rate from students was good. The missing data for any single variable averaged .5-6.0%. Missing data analyses revealed primarily arbitrary patterns of missing data and Little's Test reveal that data appear to be missing completely at random (MCAR; Little, 1988). Since data are considered MCAR and the assumption of normality of residuals for regressions is satisfied, full information maximum likelihood for missing values was applied in M*plus* version 7.3 (Muthén & Muthén, 1998-2014). To improve interpretability, the coefficients are reported in standardized form.

4.5. RESULTS

4.5.1. Qualitative study

4.5.1.1. Instructional format

Since the study examined student experiences of getting stuck in the context of a school-wide reform to student-centered mathematics instruction, I first investigated the extent to which each teacher's classes exhibited activity formats that were consistent with implementing the reform. Kappa scores revealed substantial or good to excellent agreement between raters for each category of codes: structure .80; struggle .69; and affect .77 (Fleiss, 1981; Landis & Koch, 1977).

Coding revealed that the overall instructional formats of both teachers were comparable and were in line with a student-centered instructional approach. Both teachers spent approximately two-thirds of class time having students work in groups and one-third of the time leading the whole class (see Figures 10 and 11). Video analysis revealed that the whole class format was predominately used for launching students into group work at the beginning of class and for whole group discussion, predominately at the end of class, although occasionally when the teacher wanted to bring everyone together to clarify or discuss a particular issue or idea arising in group work. Teacher B had students participate in more student presentations (12% of the overall time, compared to 2% for Teacher A).

Each teacher also used a similar distribution of formats for their general pre-algebra classes and changed the format for their honors classes, although Teacher B displayed somewhat more variability across general-level class periods (see Figures 12 and 13). Teacher A had the honors class spend more time in group work and Teacher B had the honors class spend more time in student presentations. This suggests that students experienced getting stuck in largely collaborative learning environments where they were mostly working with their peers in small groups. It also suggests that students in honors classes may be in classrooms characterized by activity formats that enhance student accountability and intellectual authority in solving math problems, as student presentations and group work is intended to do (see Michaels et al., 2008; Stein & Smith, 2008).

4.5.1.2. Final coding scheme and emotion categories for students' experiences getting stuck

The first step in understanding students' emotional experiences was to determine if the Achievement Emotions described by Pekrun and colleagues (Pekrun et al., 2011) were accurate and sufficient to describe students' emotional experiences when they get stuck working on math problems. Review of Round 1 coding revealed that the negative learning emotions captured a significant portion of students' experiences. The following paragraphs describe the changes that were made to the coding scheme in order to capture students' emotions when they were stuck. The original coding scheme from Round 1 is outlined in Table 24. The revised coding scheme appears in Table 25.

First, students described physical symptoms when they are stuck, such as feeling as though they have a headache or sweaty palms. Headaches and perspiration can be somatic experiences that are associated with anxiety or worry. Indeed, students' descriptions of physical complaints often accompanied descriptions of feeling worried, anxious, or tense. In addition to describing physical symptoms, students also described feeling "stressed" in relation to feeling anxious and worried. Thus, in Round 2, we revised the code "anxiety" to include anxiety, stress, and students' reports of somatic symptoms.

Second, students often expressed anger and frustration together. In the original framework (Table 24), anger and frustration are both listed as activating emotions with an activity focus. Anger also appears as an activating emotion with an outcome focus. However, in our coding anger and frustration co-occurred in almost 40% of student excerpts that were coded with either code. Given the amount of overlap in the data and given the conceptual association of the two types of emotion in the original framework, we combined them in the Round 2 coding scheme (Table 25). Since it is not clear conceptually or in student reports if students experience or perceive being stuck as an activity focus (i.e. as it relates to the work they are doing in class in the moment) or outcome focus (i.e. as it relates to successfully completing the problem), anger and frustration appear in both the activity and outcome focus domains in the revised coding scheme.

Students expressed negative emotions that are not part of the original set of achievement emotions. First, students reported feeling confusion when they were stuck. Given the fact that confusion is an emotion with cognitive components in relation to an activity, it is categorized as an activity focus emotion. However, since students could find confusion as either activating or deactivating, it is listed under both. Students also described a non-descript negative feeling, such as feeling “bad” or “not good.” Therefore, a code was created to capture students’ non-descript negative feelings.

Students also reported some positive emotions, including feeling enjoyment, excitement, and a positive sense of feeling challenged. However, the positive emotions did not occur at a high enough frequency to distinguish them meaningfully in accordance with Pekrun’s framework or otherwise. Thus, a “positive” code was created to capture any positive experience of getting stuck.

Finally, an “other” category was maintained to capture student reports of emotionally salient experiences that are not captured by the revised coding categories.

The final coding scheme maintained several of the achievement emotions, with some modifications and additions. The final emotion codes appear in Table 25. Emotion categories that appear in parents in the table describe potentially important emotions that students described when they are stuck. These emotions did not occur at a sufficiently high frequency to code and include in the quantitative analyses. However, their occurrence and potential significance are discussed briefly in the following sections and in more detail in the summary of the chapter.

The same two undergraduate psychology students who coded student reports in Round 1 used the revised coding scheme to re-code the data in Round 2. In round two, raters co-rated 25% of the transcripts and achieved an average of 89% agreement across codes (Fleiss, 1981; Landis & Koch, 1977; % agreement for each code outlined in Table 27). Table 26 and 27 outline the emotion categories that were coded and the frequency with which each code was assigned in the data. Table 26 shows examples of each emotion code from student reports. The frequency describes the percent of students in the sample who expressed a given emotional experience when he or she was stuck while working on math problems. Note that because excerpts were coded with all emotions that apply to the excerpt, the percentage reflects the percent of students whose report included that emotion and, thus, when summed, the percentage exceeds 100.

4.5.1.3. Student emotional experience of being stuck (RQ1)

Over half of the students reported negative emotional experiences of being stuck while working on math problems (see Table 26). Students often reported on how their negative experiences made them reflect on their ability, for example, "I can get frustrated sometimes, and I just wish I was smart enough to understand" and, "When I'm stuck… I feel like I have failed." For some, being stuck was meaningful to them socially. In addition to reporting feeling "embarrassed," students described feeling "lonely" and "alone" when they were stuck (for example "I feel alone and like I'm the only one that does not know it"). Lastly, numerous students reported a somewhat non-descript sense of feeling "bad." It is not clear if these students felt anger, frustration, anxiety, stress, or shame, and they were unable to specify the feeling. These excerpts tended to be brief (one or two word responses), and the students articulated their negative affect as feeling "bad," "weird," "not good," or "terrible."

In terms of emotion categories, one out of five students in the sample reported feeling anger and frustration when they were stuck. Some students focused on their anger and frustration while others described anger and frustration occurring along with other emotions. The most common feeling described with anger and frustration was anxiety or stress. For example, a student reported, "It's frustrating that I can't solve it, I get mad and stressed." Expressions of anger were often accompanied by attributions and a desire or need to solve the problem. For example, students reported, “It is a huge challenge and it can be very frustrating and I just want to be able to solve it,” “I get angry sometimes because I've tried so many ways,” and “I can get frustrated sometimes and I just wish I was smart enough to understand.”

In general, approximately one out of ten students expressed feeling anxious or stress when they are stuck. Students reported feeling anxious, nervous, or panicky (“It feels nerve racking and straining”; “I panic when I get stuck”); fearful (“I feel scared to get it wrong”); stressed (“It feels very stressful because I don’t know if I'm doing it right or not”); and somatic experiences (“I get sweaty and over think”). One in ten students also reported a non-descript sense of feeling “bad,” which included a sense of uneasiness (“It feels weird”), feeling upset (“I feel upset”), and feeling “not good.”

In our sample, shame and helplessness did not occur very often and tended to co-occur. For these reasons, we combined the shame and hopeless coded excerpts into a parent code. When combined, 3% of students reported feeling helplessness and shame. Examples include: “What I think is that ‘it's impossible I will never get it,’” “…I feel like I fell into an abyss,” and “I feel disappointed and embarrassed.”

One out of ten students also reported a non-descript negative feeling when stuck. Their language did not describe what we would consider a discrete emotion. Students said things like, “I don’t feel good. It feels weird,” and “It feels bad.”

Approximately 13% of students expressed feeling a sense of confusion when stuck. Excerpts include, “I feel really confused,” and “I keep wondering ‘how do I solve this problem?’”

A few students also expressed positive experiences of being stuck. While there were not enough excerpts to make distinct conceptual categories for positive emotions, most students who reported positive emotions shared the theme of being motivated by challenge. For these students, they perceived being stuck as a challenge that they were motivated and excited to overcome. One student reported, “I like facing a challenge. Whenever I am stuck, I like working that much harder to get the answer.” Another student said, “I like problems even better when I'm stuck.” And yet another student described how feeling negative feelings like frustration might be linked to positive feelings when they find a path forward: “It feels really good. I ask a friend. I get a little frustrated but then I think different ways to find the answer.”

In some cases, students described feeling “challenged” as a negative experience, such as feeling that the work is “too hard.” In other cases, students described feeling “challenged” without qualifying if the experience was positive or negative. In these cases, “challenge” was coded in the “other” category maintained in Round 2 coding. The “other” category also included student reports that being stuck felt “weird” and “hard.”

4.5.2. Quantitative study

4.5.2.1. How student characteristics relate to frequency and type of emotional experience when stuck (RQ2)

In order to explore the relationships between students’ demographic characteristics and ability level with how often they get stuck and the types of emotions they report about getting stuck in math (RQ2), qualitative codes of students’ emotional experiences getting stuck will be transformed into dichotomous variables, one for each emotion category: none/neutral, anger/frustration, anxiety/stress, helplessness/shame, bad/non-descript, confusion, and positive (see Tables 26 and 27 for descriptive statistics and examples, respectively).

Similarly, there were not many relationships between the kind of emotions students described when they were stuck with students' demographic and ability-level characteristics (see Table 28). Students' free lunch eligibility and whether students were in an honors or general-ability class was not significantly associated with any of the categories of emotions students described when they were stuck. However, female students were more likely than male students to report positive feelings when they were stuck, $\chi 2$ (1, N = 180) = 3.36, p = .06. Also, minority students were significantly more likely to feel helplessness or shame and to feel anger or frustration than Caucasian students, $\chi 2$ (1, N = 180) = 6.29, p = .01.

4.5.2.2. The relationship between the frequency getting stuck and the type of emotional experience when stuck (RQ3)

Overall, there were few mean level differences in how frequently students reported being stuck in math by student characteristics. There were no mean level differences in how often students got stuck while working on math between students of different genders, free lunch eligibility, and between students in honors and non-honors math classes (see Table 29). In contrast, minority students reported being stuck slightly more often than Caucasian students, $F(1, 179)$ = 1.37, p = .055.[15]

For the most part, students' reports of emotions experienced when they are stuck are not related to how often they get stuck in math. Even students who did not report any emotions

15 Due to the small sample size and exploratory nature of the study, we display significance at three levels: *p<.07, **p<.05, and ***p<.001. This enables us to highlight associations that are promising for future research in larger scale quantitative studies or in more in-depth qualitative or mixed methods investigations.

when they were stuck did not significantly differ from students who did in how often they got stuck, $F(1, 179) = 1.15$, $p = .63$. One exception is hopelessness and shame. Students who reported experiencing hopelessness and shame reported significantly higher frequency of being stuck than students who did not, compared to students who did not report feeling hopelessness or shame, $F(1, 179) = 4.82$, $p = .02$.

4.5.2.3. Emotional experiences when stuck and competence in mathematics (RQ4)

For all models, students' self-competence in the fall was the strongest predictor of their self-competence in math at the end of the year ($\beta = .295$ to .484, $p < .001$; see Table 30). Students' eligibility for free lunch and students' enrollment in honors coursework also consistently negatively predicted their self-competence in math at the end of the year (free lunch $\beta = -.127$ to -.242, $p < .001$; honors $\beta = -.101$ to -.169, $p < .05$).

Apart from these similarities, the models display important differences. In Model 2, when students' emotional experiences of being stuck were added into the model, students' experiences of anger and frustration when stuck while working on math problems predicted a significant increase in self-competence in math at the end of the year ($\beta = .146$, $p < .001$). However, when I took into consideration how often students got stuck in math class, the effect of anger and frustration dropped to non-significance ($\beta = .100$, $p > .05$), and the frequency of getting stuck became the strongest predictor of self-competence in math at the end of the year ($\beta = .561$, $p < .001$).

4.5.3. Summary

While exploratory, the study presents strong evidence that getting stuck is a prevalent and emotionally salient experience for middle school students in classrooms whose teachers are implementing student-centered mathematics instruction. Thus far, research on achievement emotions has focused on how experiences during specific types of academic activities, such as doing homework, studying, or taking an exam, can trigger cost value appraisals that evoke emotional responses (Pekrun et al, 2007). This study reveals that getting stuck could be part of the learning process that cuts across different types of academic activities and that could evoke an emotional response by challenging students' sense of control and value of mathematics (Pekrun et al., 2007).

4.5.3.1. Emotional salience for adolescents

Getting stuck while working on mathematics problems may be particularly emotionally salient for adolescents. Throughout middle school, adolescents strive to shape their identities and form meaningful relationships with teachers and peers while seeking to have their psychological needs met in school (Eccles & Roeser, 2009). The student-centered classrooms in this study emphasized group work and students' intellectual authority and responsibility for learning (both key components of the type of student-centered instruction adopted by the school). While this framework afforded rich opportunities for developing a sense of autonomy, students' sense of relatedness and competence could have been hindered by the public nature of getting stuck while working with peers. Their sense of competence in mathematics and their sense of relatedness with their teachers could also have been undermined if they experienced getting stuck as an indication of low ability or if they felt that their teacher assigned work that was too challenging or was not offering sufficient support for them to be successful. In these ways, adolescents may be more sensitive to perceiving that they have a limited ability to control how successful they are in mathematics class. Furthermore, research documenting declines in engagement in mathematics starting in middle school suggest that young adolescents' sense of value of mathematics may be more malleable and subject to decline, both of which could contribute to adolescents have a strong and effectual emotional response to getting stuck.

Indeed, the majority of seventh grade students in the study responded to an open-ended question about what it was like when they were stuck by reporting negative emotions. Moreover, while student responses were brief, they often included vivid descriptions of their experiences, and at times used strong emotional language. For example, some students reported that when they are stuck they feel as though they "fell into an abyss," suggesting a relatively extreme sense of hopelessness or helplessness. Other students described intense feelings of anxiety or stress, including feeling "panicked," "nerve-wracked," and "strained." Students even described physical responses to being stuck, known to be somatic symptoms related to anxiety, including getting "sweaty" and having "big headaches."

4.5.3.2. Supporting adolescents when they are stuck

Understanding the emotions that adolescents experience when they are stuck is key for supporting adolescents' success in student-centered mathematics classroom. Student-centered instruction aims to support students' deep understanding of mathematics by supporting students' productive engagement in challenging and meaningful tasks, tasks that are within and perhaps even on the edge of students' zone of proximal development (ZPD described in Chapter 3; Vygotsky, 1978). In this way, students are likely to get stuck while working on mathematical tasks in student-centered classrooms by design. The task for students is to engage in productive struggle and to persist when they get stuck. Depending on the type of emotion, the level of activation it entails, the demands of the context or activity, and attribution process it triggers in students (Pekrun et al., 2007), achievement emotions can support or undermine students' persistence when they are stuck.

In regards to students' report of strong negative emotions, it is difficult to say how the intensity of the emotion would influence their ability to stay engaged and work through being stuck. Feeling as though you are failing and other feelings of shame, hopelessness, and helplessness have been associated with declines in engagement and achievement in previous research (Pekrun et al., 2010; Pekrun et al., 2002a; Pekrun, Elliot, et al., 2009), especially when working on challenging tasks (Pekrun et al, 2004). However, experiencing anxiety, anger, and frustration while working on academic tasks has been linked with both adaptive and maladaptive responses (Pekrun et al., 2007). Anxiety, anger, or frustration could have similarly mixed effects on student engagement and learning when stuck. On the one hand, some anxiety, anger, or frustration, could fuel students' motivation to persist. However, extreme levels of anxiety and/or anxiety accompanied with significant somatic symptoms could interfere with students' ability to focus, identify, and implement effective learning strategies.

Regarding anger or frustration, the study contributes to growing evidence that experiencing anger or frustration can be part of a productive response to academic challenge. In the study, students who experienced anger or frustration when stuck experienced an increase in their sense of competence in mathematics over the course of the school year, even when controlling for demographic characteristics, classroom characteristics, and their level of achievement in mathematics. Previous research has documented that anger can fuel motivation to overcome challenges when students have a high expectations for success (Lane et al., 2005).

In the case of getting stuck, students' feelings of anger or frustration may contribute directly and indirectly to their sense of competence. Indirectly, anger or frustration may fuel the type of persistent effort that helps them overcome challenge, which improves student competence. More directly, students might experience anger or frustration because they feel a need or desire to resolve the ambiguity or lack of momentum experienced when they are stuck. In this case the anger or frustration is part of a determined drive to overcome being stuck. When successful, students not only overcome being stuck, they also can attribute the success to their own sense of drive or perseverance, which might contribute to them feeling competent not just in relation to their mathematical knowledge or skills, but also competent to overcome challenge more generally.

4.5.3.3. Cognitive achievement emotions

The study also contributes to evidence that achievement emotions might include experiences that are often considered to be primarily cognitive. Specifically, in this study students described feeling confused and challenged when they are stuck. Recently, emotion researchers have started to investigate confusion as an emotion that students experience during academic activities (e.g. D'Mello, Lehman, Pekrun & Graesser, 2014; Pekrun & Stephens, 2010; Silvia, 2010). Defined by D'Mello and colleagues (2014) as a "state of cognitive disequilibrium that is triggered by contradictions, conflicts, anomalies, erroneous information, and other discrepant events" (pg. 153), confusion is described as being potentially beneficial if it arises, is managed, and is resolved productively. In this study, it was not clear if students' experience of confusion was productive or unproductive, or activating or deactivating. Interestingly, in some cases, student excerpts indicated that confusion is the emotional experience of "stuckness" (for example "the confusion of being stuck" or a feeling of one's "mind going blank and not knowing what to do next"). More research is needed to explore confusion in the context of being stuck while working on challenging problems, its relationship to other achievement emotions (whether feeling confused trigger other activating or deactivating emotions), and how it shapes student engagement in academic tasks.

Several students also reported that they feel "challenged" when they are stuck. Student excerpts were too brief to understand what students mean when they say they "feel challenged" or to conceptualize "challenge" as an affective experience or academic emotion. Indeed, it was difficult to ascertain if students meant that feeling challenged was positive or negative. Building on the definition of confusion, it seems reasonable to explore whether challenge might refer to a state of cognitive exertion that is triggered by open-ended, complex, or ambiguous tasks. Given the foundational nature of challenge to student-centered mathematics instruction and given efforts to expand achievement emotions to include cognitive-affective states students experience when learning, it might be worth investigating the affective components and consequences of feeling challenged.

The study also revealed that students might experience types of negative and deactivating emotions that undermine their engagement and achievement that are either described or experienced as general or non-descript, in a way that has theoretical and methodological implications. Research in achievement emotions asks students to report on the presence or absence of or the extent to which they experience specific discrete emotions. Approximately ten percent of students in the study described their emotions in a way that does not neatly fit into a specific type of emotion. Instead, they described a "bad" or "weird" feeling when stuck. It is not clear if these students were unable to identify the specific emotion they experienced or if their emotional experience is accurately described as a non-descript "bad" feeling. Methodologically, it is not clear whether students who have a non-descript "bad" feeling would be able to accurately identify a specific emotion if offered a range of choices, if they would endorse a specific negative emotion in hopes of capturing the non-descript bad feeling in some way, or if they would not endorse any discrete emotions since they do not seem like an accurate portrayal of the non-descript "bad" feeling. Theoretically, it is interesting to think about the extent to which a non-descript "bad" feeling is a valid type of feeling that is distinct from specific discrete negative emotions, with distinct causes and effects.

4.5.3.4. Achievement emotions profile of being stuck

The study also suggests that while getting stuck may be a common experience of the learning process that cuts across different types of learning activities (homework, studying, or taking tests), it is also a distinct and unique phenomenon. A particularly unique feature of getting stuck is that it describes a state of irresolution. The revised coding categories based on Round 1 coding (Table 25) reveal that students did not report any deactivating positive emotions. Deactivating positive emotions like relaxation, relief, and contentment are common during learning activities. However, the nature of being stuck is such that students cannot experience these emotions. In fact, it is possible that part of what makes being stuck emotionally salient is the suspension of the satisfaction, relief, or contentment that comes from solving a problem. Negative emotions that are associated with an outcome are also suspended, including disappointment and sadness, although previous relief or disappointment could play a significant role in the control value appraisals that shape students' emotions when they are stuck.

4.5.3.5. Student characteristics

Finally, the study also suggests that student characteristics may influence the way that students experience being stuck in mathematics class. Specifically, minority students might experience being stuck differently in important ways. Minority students were more likely to report feeling helplessness or shame and anger or frustration when stuck. Furthermore, minority students also reported getting stuck more often than their Caucasian peers. Given the negative effects of hopelessness and shame on student effort and achievement, it is important to understand if minority students are more likely to report this emotion because of attributions or experiences related to their minority status, or because they are stuck more often than their Caucasian peers.

4.5.3.6. Limitations and future directions

The study has several limitations. First, the sample was small and specific to a single school. The sample also consisted of a small sample of students who are relatively homogenous in terms of their racial, ethnic, and socioeconomic backgrounds. Thus, the results are preliminary, and more research should be done to study the prevalence and importance of getting stuck among a larger sample. Specifically, it would be important to study how student characteristics relate to stuck prevalence and effects. Second, the study examined getting stuck in the context of reform to student-centered mathematics instruction. While classroom observations assessed the alignment of activity formats with a student-centered approach, there was no assessment of student-centered instruction. Future research interested specifically in understanding the experience of getting stuck in student-centered instructional environments should include a robust measure of student-centered mathematics instruction. In particular, it would be helpful to assess the quality and level of cognitive challenge of the tasks and instructional support. The study assumed that student reports of getting stuck represent the kind of getting stuck that occurs when tasks are at the appropriate level of challenge and with appropriate scaffolding. It is likely that there are different levels or ways in which students can get stuck. Similarly, it would be helpful to assess specifically how students respond when they are stuck, attributions students make when they are stuck, and the relation of responses and attributions to the length of time that students spend not knowing how to move forward. Furthermore, the measure of getting stuck was cross-sectional, retrospective, and activity neutral. Students' experiences of getting stuck may vary over the course of the year and depend on whether they are stuck working on problems in class, working on homework, or taking a test. Future research could apply in-depth longitudinal methods like a daily diary study to get more detailed information from students.

5. DISCUSSION AND IMPLICATIONS

Instructional reforms are changing how adolescents are taught and learn mathematics. Yet, we have yet to fully understand how student-centered instruction unfolds in classrooms and how it shapes student outcomes for youth. In 1990, Cohen and Ball declared that "it is time for education researchers…to 'open up the black box' of schooling and look more closely at the kinds of instruction occurring in schools that adapt innovative programs" (pg. 334). Almost three decades later, these dissertation studies make significant contributions to "opening the black box" by articulating a multi-dimensional conceptualization of student-centered mathematics instruction based on theory and research, developing a valid and reliable student report measure, providing evidence of the effects of student-centered mathematics instruction, and by revealing that students' experiences getting stuck may be particularly salient to their engagement, achievement, and motivational trajectories in student-centered classrooms.

5.1. TEACHER SUPPORT

Taken together, the dissertation studies suggest that we could benefit from better understanding how to support adolescents' perceptions of and engagement in open-ended and challenging tasks. Study 1 found that both task quality and teacher support predicted the quality of students' involvement in mathematics. Specifically, students' report of their teachers' implementation of relevant and metacognitive tasks and of their teachers' flexible and responsive support predicted their emotional, behavioral, cognitive, and social engagement. Along similar lines, Study 2 revealed that students who are adapting to student-centered mathematics instruction experience a range of emotions when they get stuck while working on challenging tasks, which could jeopardize the quality of their involvement and success in the task. These findings highlight the importance of identifying what type of support is appropriate and effective for adolescent engagement and achievement in student-centered mathematics.

There is a long line of research based in humanist and person-centered perspectives (Rogers, 1961) and attachment theory (Ainsworth & Bowlby, 1991) that emphasizes the importance of warm, caring, and accepting social interactions and relationships in child development. This perspective has informed a wide range of educational reforms that aim to improve engagement and achievement in school, particularly among low-income and minority youth (e.g. Nathanson et al., 2007). While there is an abundance of evidence supporting the importance of warm and caring interactions and relationships to adjustment, wellbeing, and achievement in school, the evidence is mixed as to their role specifically in supporting adolescents' persistent engagement in challenging academic tasks. Some studies have found that having support in the context of cognitive challenging tasks and high expectations for student thinking is associated with higher levels of academic achievement in adolescents (e.g. Lee & Smith, 1999; Shouse, 1996). However, other studies have found that teacher warmth and caring is negatively related to academic achievement among youth when academic challenge is taken into consideration (e.g. Phillips, 1997).

Warmth or caring could be associated with declines in students' efforts to work on cognitively challenging tasks for a number of reasons. Teachers could respond with warmth and caring as a way of supporting students when they sense that the task might be too challenging for them. Research has demonstrated that students can be difficult to engage in challenging open-ended tasks (Fulmer & Turner, 2014), that students can express negative emotions when they are struggling with difficult tasks (Newmann et al., 1992; Spillane, 2001), and that teachers can respond by devolving the level of challenge of the task (Smith & Stein, 2011) or implementing a more teacher-centered strategies (Fulmer & Turner, 2014). Devolving the level of challenge can implicitly communicate low expectations for success, which can undermine student motivation and engagement on academic tasks (Wigfield & Eccles, 2000).

Making the task easier to solve and transitioning the intellectual authority and responsibility for learning back towards the teacher can also evoke emotions that, while positive, undermine student engagement. Students' experience of relief or contentment can lead students to withdraw from academic activity. Furthermore, students' experience of positive achievement emotions can interfere with how deeply they think about their academic work (Pekrun et al., 2002b). Emotions that have deactivating effects on student engagement could be especially detrimental when students are stuck – a time when they may need to sustain high levels of engagement in order to be successful. In the study, students who reported feeling angry when they were stuck experienced an increase in their sense of competence in mathematics over the course of seventh grade.

In this study, "teacher support" does not refer explicitly to the extent to which teachers are warm or caring in their interactions with students. Instead, the type of teacher support operationalized in the measure of student-centered mathematics instruction refers specifically to teachers' persistent monitoring and scaffolding of students' understanding in ways that maintain students' intellectual authority and an appropriate level of cognitive challenge of the task. This type of "support" emphasizes teachers' ability and willingness to notice and respond to variation in adolescents' developmental skills, background knowledge and skills, learning preferences, personal interests and goals, and cultural, social, and historical context that shape adolescents' experiences in mathematics.

While the affective quality of teacher interactions is not conceptualized as a component of student-centered mathematics instruction, the role of teachers' unconditional regard of students is implied as a fundamental component. Carl Rogers (1959) defines unconditional regard as the complete and total acceptance of a human being. Items in each of the domains include indicators of teachers' recognition and acceptance of students as individual learners. Making tasks relevant to students implies respecting their experiences, interests, values, and goals. Supporting students as having intellectual authority in class requires teachers to respect students' ideas and ability to reason and construct their own understanding. Working with students to succeed on cognitively challenging tasks implies that the teacher recognizes students' ability to work through the problem. In these cases, unconditional regard is more specific than acceptance of students as people – it refers specifically to unconditional regard of students *as thinkers and learners.*

Furthermore, unlike teacher warmth of caring, teachers' unconditional regard of students as learners communicates high expectations for success and does not aim to directly address or remediate the emotions that students experience during academic tasks. Instead, unconditional regard of students as learners could help students experience or make sense of what it feels like to work on challenging problems or to get stuck in productive ways – by focusing on student mastery through incremental progress and by normalizing the fact that learners have different levels and types of understanding of mathematical concepts and skills. Thus, while it is not explicitly articulated as a component of student-centered instruction, it is possible that teachers' unconditional regard of students as learners describes a quality of teacher interactions that is an important – or even necessary – component to student-centered mathematics instruction.

Nevertheless, it is important to recognize that teachers' affective responses to students when implementing student-centered instruction could affect student engagement and achievement in important ways. Teacher negativity or criticism could fuel unproductive attributions of getting stuck or failure and could influence the initiation or persistence of unproductive negative learning emotions. Teacher affect could also affect the extent to which students genuinely see themselves as intellectual authorities who can construct their own understanding of mathematics or who can contribute to the co-construction of understanding among peers. Intellectual authority means thinking freely and being able and willing to take risks, which could be hindered by teacher negativity or criticism. In this way, teacher affect could be an important mechanism in the effectiveness of student-centered instruction.

5.2. EXPLORING "STUCKNESS" TO DEVELOP OUR UNDERSTANDING OF STUDENT-CENTERED INSTRUCTION

Studying student "stuckness" – the phenomenon of students getting stuck while working on academic tasks – could inform the implementation of student-centered mathematics instruction and deepen our theoretical understanding of engagement and achievement in mathematics. A central component of student-centered instruction is supporting students' productive engagement in tasks that are within students' zone of proximal development (ZPD described in Chapter 3; Vygotsky, 1978). It is not clear if getting stuck is an indication that a task is too difficult, or if getting stuck is an indicator that tasks are sufficiently challenging. Future research can explore the ways in which students get stuck and how types of stuckness relate to appropriate levels of difficulty. Understanding different ways of getting stuck can also inform how teachers intervene to support student engagement.

More fundamentally, it would be helpful to know when students identify themselves as stuck. Students' sense that they are stuck could be at least partly a matter of their perception. Some students who are stalled or not making progress on a problem might just think that they're working on it and not describe those times as being "stuck" or unable to make progress. Other students may identify brief moments of considering how to move forward being "stuck." Furthermore, students who are disengaged from school or mathematics specifically may feel stuck as a result of a lack of cognitive and behavioral engagement in tasks; they may not invest the necessary energy to get started in the first place. For these reasons, additional research can explore what students think it means to be stuck, thresholds for feeling "stuck," and how these thresholds relate to student emotions, attributions, and responses.

Once the characteristics of stuckness are operationalized in more detail, it could also be important to understand the effects of stuck frequency. In Study 2, frequency of getting stuck was related to feelings of hopelessness and shame. This suggests that the amount of time that students spend stuck could be related to their attribution processes and overall enjoyment of and motivation in mathematics coursework. Students who are often stuck may feel that they are not good at math or that the teacher makes the coursework too challenging for them. Students who are used to teacher-centered mathematics instruction can resist challenge and interpret the work as unfairly challenging or interpret the positioning of intellectual authority and responsibility for work as receiving insufficient support from their teacher (Fulmer & Turner, 2014).

Finally, from a theoretical perspective, studying student stuckness could contribute to our understanding of the construction of knowledge and experiences of self-determination in mathematics coursework. Getting stuck is likely a normal – and perhaps beneficial – component of the learning process prescribed by constructivist perspectives. Future research could examine getting stuck from a learning science perspective to understand how "stalls" in momentum relate to learning processes like cognitive assimilation and conceptual change and to identify what types of learning strategies support student progress when stuck.

5.3. STUDENT CHARACTERISTICS

The dissertation studies also provided some information about the role that student characteristics could play in the effectiveness of student-centered mathematics instruction. The availability of a validated measure of student-centered mathematics instruction makes it possible to follow-up on preliminary findings described below to explore differential effects of student-centered mathematics instruction.

5.3.1. Race

The two dissertation studies provided different types of information about the ways in African American students may experience and respond to student-centered mathematics instruction. The hierarchical linear models in Study 1 demonstrated that African American students have significantly lower course grades in mathematics than their peers (see Tables 18-23). This finding underscores one of the primary purposes of instructional reform, which is to improve mathematics achievement specifically among minority youth. However, while Study 1 examined student-centered instructional practices used by teachers in a range of schools, it did not study student outcomes in student-centered mathematics classrooms.

Study 2, however, revealed that getting stuck could be a complex and consequential experience for minority youth. Minority students reported getting stuck significantly more often than their peers *and* reported feeling helplessness or shame and anger or frustration when stuck. In addition to potentially influencing attributions (suggested in the summary of Chapter 8), the experience of getting stuck could activate stereotype threat among minority students, which has been associated with experiencing emotions that are detrimental to learning in other research (Mangels, Good, Whiteman, Maniscalco, & Dweck, 2012). In this way, getting stuck could be a landmine for minority students, especially if the stuckness is not successfully resolved. On the other hand, getting stuck could provide an opportunity for minority youth to overcome stereotype threat if appropriate and sufficient supports are available. Minority students who experience a positive academic self-image (Croizet et al., 2010; Van Loo & Rydell, 2013) and believe that all students can improve their ability through effort (Boaler, 2013) experience less stereotype threat and increased engagement in academic coursework. Overcoming stuckness could instill a sense of competence in mathematics that makes minority youth resilient to fear of stereotype threat and/or setbacks or failure in mathematics coursework.

5.3.2. Socioeconomic status

The dissertation studies revealed that the students in our sample from low socioeconomic backgrounds may be particularly at risk for underachievement in mathematics. The hierarchical linear models in Study 1 (Tables 18-23) showed that being eligible for free or reduced-price lunch is associated with lower behavioral engagement in mathematics coursework and a significant lower mathematics course grade. Furthermore, low-income students in Study 2 had significantly lower self-competence in mathematics. In both studies, the effects of socioeconomic status were consistent across models, after accounting for student and teacher covariates and reports of instruction in Study 1 and after accounting for student and classroom characteristics and emotional experiences of getting stuck in Study 2. These findings conform to national trends demonstrating that low-income students disproportionately disengage from and underachieve in their algebra coursework (National Center for Education Statistics, 2015; U.S. Department of Education, 2015). More research is needed to know if there are differential experiences or effects of student-centered instruction with low-income students.

5.3.3. Gender

Study 1 replicated well-documented findings in education research: female secondary students attain higher grades but are less emotionally engaged in mathematics than their male peers (see Tables 18-23). Thus, the female secondary students in the sample were equally or even more qualified or prepared as their male peers to take advanced mathematics and pursue math-intensive STEM careers. However, the fact that they were less interested in mathematics and experience less enjoyment of their mathematics coursework suggests that they may choose not to (Clewell & Campbell, 2002; Wang, Eccles, & Kenny, 2013).

Previous research suggests that girls could benefit from student-centered mathematics instruction because it features relevant and meaningful tasks (Baker & Leary, 1995; Geist & King, 2008; Burkam, Lee, & Smerdon, 1997) and social interaction (Gilligan, 1982; Zohar, 2006). Interestingly, Study 2 suggests that female students may also benefit from academic challenge. In Study 2 female students were significantly more likely to report *positive* emotions when they were stuck. There were too few positive emotions in the study to explore what types of positive emotions females reported and how they differ from their male peers. However, it is interesting to consider female students' overall positive emotional experience of being stuck, given the fact that girls are prone to experiencing more math-related anxiety (e.g., Lau & Roeser, 2002) and tend to report being less interested in mathematics than their male peers (Frenzel, Goetz, Pekrun, & Watt, 2010; Hyde, Fennema, Ryan, Frost & Hopp, 1990). While preliminary, it would be interesting to explore whether girls' experience of academic challenge could be a gateway to mathematics interest and/or if positive emotions or experiences getting stuck could buffer them against mathematics anxiety.

5.4. IMPLEMENTATION

The availability of a validated measure of student-centered mathematics instruction will also enable us to study differential implementation of student-centered instruction by teachers. In mathematics education policy and research there is often an assumption that instructional practice is stable – that the quality of mathematics instruction is comparable across the classes that teachers teach. This assumption is critical to the many efforts of policy makers and educators to improve excellence and equity in mathematics education by reforming mathematics instruction (National Council of Teachers of Mathematics, 2014). In school districts, instructional policies are often implemented and evaluated by observation and at the teacher level. For example, school administrators can implement state-mandated teacher evaluation systems by assessing the quality of a teacher's practice from observing as few as one of his or her class periods (see Danielson, 2012). Similarly, education research tends to investigate between-teacher differences in overall instructional practice or to focus on instructional practice in specific classes (see Ball & Rowan, 2004).

Focusing on between-teacher differences or on instructional practice in one class could obscure potentially significant within-teacher variation in instructional practice (Meyer, 1999). One of the few studies that has examined within-teacher variation of instructional practice revealed that variation within teachers could exceed the amount of variation between teachers in the same school (Raudenbush, Rowan, & Cheong, 1993). The student report survey developed in Study 1 could make it possible study within-teacher variation in instructional practice.

In particular, in the context of aiming to address inequities in mathematics education, it is important to consider that the racial and socioeconomic characteristics of students in classrooms could influence the extent to which teachers use student-centered practices in complex ways. Research shows that teachers adapt their instructional strategies and the amount of individualized support they offer in response to students' academic skills (Kiuru, Nurmi, Leskinen, Torppa, Poikkeus, Lerkkanen et al., 2015) and classroom behavior (Nurmi & Kiuru, 2015). Mathematics teachers, in particular, are more likely to use teacher-centered practices when they perceive that students are not working hard, are off task (Kiuru et al., 2015; Nurmi & Kiuru, 2015), or resist working on challenging tasks (Fulmer & Turner, 2014).

Mathematics teachers' perceptions of student ability and effort could disproportionately affect how they teach to classes with predominately minority or low-income students. Teachers are more likely to perceive minorities as having less mathematics ability in schools with a high concentration of minorities (e.g. Flores, 2007) and minority students who are motivated to do well in mathematics may, in turn, opt out of fully participating in student-centered classrooms in order to avoid invoking a negative performance stereotype (Sackett, Hardison, & Cullen, 2004; Steele & Aronson, 1998). Students from low-income families may also be less likely to participate in mathematics class in ways that engender more student-centered practices since they are not as well socialized as higher income peers in how to communicate in ways that are rewarded in school (e.g. Hart & Risley 1995).

Implementing less student-centered mathematics instruction in classrooms with a higher concentration of minority or low-income students could contribute to significant differences in mathematics outcomes based on race and socioeconomic status. A few studies suggest that student-centered instructional practices can increase adolescents' enjoyment (Noyes, 2012) and understanding of mathematics (Saraghi & Napitulu, 2015). Overall, however, very little research has been done on the effects of student-centered instruction. Examining whether there is differential implementation or a differential effect on student outcomes based on race or socioeconomic status could help us understand how to better support – or modify –instructional reforms meant to remedy gaps in engagement and achievement for minority and low-income (U.S. Department of Education, 2015).

5.5. THEORETICAL INTEGRATION

Finally, the dissertation studies provide preliminary support for integrating constructivist theories of learning and self-determination theories of motivation. Taken together, the research questions for Study 1 support the pathways proposed in Figures 6 and 7, particularly when using the student reports. Student reports of student-centered instruction predicted students' cognitive, behavioral, emotional, and social engagement in mathematics (illustrated in blue in Figure 7) *and* their mathematics course grades (illustrated in yellow in Figure 7). This confirms that there is a relationship between student reports of student-centered mathematics instruction and engagement and achievement in mathematics. More research and conceptual analysis are needed to explore other components of the proposed model. In particular, path analyses could explore whether engagement partially or fully mediates the association between student-centered mathematics instruction and student achievement.

A new set of research questions and analyses is required, however, in order to inform and validate the integration of constructivist and self-determination perspectives. First, students' social engagement could be modeled to determine if and how students' social engagement is related in class, and how much that relationship is associated with student achievement through the construction or co-construction of knowledge. It is not clear, however, if the conceptualization of social engagement used in Study 1 is a good fit for the integrated model. The measure conceptualizes social engagement as consisting of multi-dimensional components that were developed to capture individual student's social engagement. The measure includes behavioral items (e.g. "When working with others in math, I don't share my ideas" and "I work with classmates to come up with ways to solve problems in math class"), cognitive items (e.g. "I try to understand other people's ideas in math class" and "I build on others' ideas in math"), and emotional items (e.g. "I don't care about other people's ideas in math" and "I don't like working with my classmates in math"). The scale could be revised to capture indicators of the type of social engagement that contributes specifically to construction and co-construction of understanding. Alternatively, student report of the indicators in the extant measure could use latent class analyses to examine patterns of social engagement (patterns among the six items) in classes that are high and low in student-centered instruction, which could help us start to understand what types of indicators and collections of indicators of social engagement are brought the fore in student-centered mathematics classrooms.

5.6. FUTURE DIRECTIONS

Developing a student report of student-centered mathematics instruction will enable me to continue to investigate its implementation and effects. Specifically, I will further investigate the relationship between instructional practice and student characteristics with in-depth, mixed method, and longitudinal studies of mathematics teachers' instructional practice and students' experience of and participation in mathematics coursework. I will look more closely at how student characteristics shape instruction, the role of teacher characteristics in predicting within-teacher variation in instructional practice, and how student-centered mathematics instruction shapes the nature of teacher's work, all of which could provide key insights into the nature of mathematics teaching and learning. In future research, I will also extend the study of the effects of student-centered mathematics instruction in order to provide critically needed insight and evidence into how student-centered instruction shapes mathematics outcomes for youth. Specifically, I intend to examine what aspects of learning climates support the effectiveness of student-centered mathematics instruction for minority and low-income youth. The aim of this work is to develop an understanding of student-centered instructional practice in mathematics education policy and practice that effectively supports mathematics teachers and mathematics outcomes for minority and low-income youth.

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Table 1

Classroom and school climate measures that include indicators of the dimensions of student-centered mathematics instruction

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
2017 CPS 5Essentials Student Survey	https://www.uchicagoimpact.org/sites/default/files/2017%20CPS%205Essentials%20Student%20Survey.pdf	S	Gen	Academic Press	0.89	In my [TARGET] class, my teacher:			
						Expects everyone to work hard.			✓
						Expects me to do my best all the time.			✓
						Wants us to become better thinkers, not just memorize things.	✓		
						In my [TARGET] class how often:			
						Are you challenged?	✓		
						Do you have to work hard to do well?	✓		
						Does the teacher ask difficult	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						questions on tests?			
						Does the teacher ask difficult questions in class?	✓		
		S	Gen	Classroom rigor	0.83	My teacher:			
						Often connects what I am learning to life outside of the classroom.	✓		
						Encourages students to share their ideas about what we are studying in class.		✓	
						Often requires me to explain my answers.		✓	
						Encourages us to consider different solutions or points of view.	✓		
						Doesn't let students give up when the work gets hard.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
		S	Math	Math instruction	0.87	In your MATH class, how often do you do the following:			
						Apply math to situations in life outside of school.	✓		
						Discuss possible solutions to problems with other students.	✓		
						Explain how you solved a problem to the class.		✓	
						Write a few sentences to explain how you solved a math problem.		✓	
						Write a math problem for other students to solve.		✓	
Research Assessment Package for Schools (RAPS)	Institute for Research and Reform in Education, Inc.,1998	S	Gen	Teacher autonomy support	0.77	My teacher doesn't explain why we have to learn certain things in school. [R]	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						My teacher thinks what I say is important.		✓	
						My teacher interrupts me when I have something to say. [R]		✓	
						My teacher tries to control everything I do. [R]		✓	
School Success Profile (SSP)	Bowen, 2005; https://www.schoolsuccessonline.com/documents/ssp.pdf	S	Gen	Teacher support	0.74 - 0.90	Indicate your level of agreement with each of the following statements about your teachers at school:			
						Listen to what I have to say.		✓	✓
						Show me respect.			✓
				Academic relevancy	0.74 - 0.90	Indicate your level of agreement with each of the following statements about your teachers at school:			

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						Ask me about my interest in future jobs and careers.	✓		
						Help me relate what I am learning in the classroom to the real world.	✓		
						Help me see the value of what I am learning in the classroom to the real world.	✓		
						Help me see the value of what I am learning in the classroom.	✓		
						Help me relate what I am learning in the classroom to my own experiences and interests.	✓		
						Explain the importance of assignments to my	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						learning.			
						Help me relate what I am learning in the classroom to potential jobs and careers.	✓		
						Assign work that connects what I am learning in the classroom to future jobs and careers.	✓		
						Encourage me to think about my future as an adult.	✓		
				Academic rigor	0.74 - 0.90	Indicate your level of agreement with each of the following statements about your teachers at school:			
						Expect me to do my best.			✓
						Assign work that makes me think.	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						Ask questions that make me think.	✓		
						Assign work that challenges me.	✓		
Individualized Classroom Environment Questionnaire (ICEQ)	Fraser, 1990	S	Gen	Personalization	0.79	The teacher talks with each student.			✓
						The teacher takes a personal interest in each student.			✓
						The teacher remains the front of the class rather than moving-about and talking with students. [R]		✓	
						Students' ideas and suggestions are used during classroom discussion.		✓	
						The teacher tries to find out what each student wants to learn about.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
				Participation	0.70	Students discuss their work in class.		✓	
						The' teacher talks rather than listens. [R]			✓
						Students give their opinions during discussions.		✓	
						The teacher lectures without students asking or answering questions. [R]	✓	✓	
						Students explain the meaning of statements, diagrams and graphs.		✓	
						Students sit and listen to the teacher. [R]	✓	✓	
				Independence	0.68	The teacher decides where students sit. [R]		✓	
						Students choose their partners for group work. [R]		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						Students are told exactly how to do their work. [R]		✓	
						The teacher decides which students should work together. [R]		✓	
				Investigation	0.71	Students find out the answers to questions from textbooks rather than from investigations. [R]		✓	
						Students carry out investigations to test ideas.		✓	
						Students find out the answers to questions and problems from the teacher rather than from investigations. [R]		✓	
						Students are asked to think about the evidence behind statements.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						Students carry out investigations to answer questions coming from class discussions.		✓	
						Students carry out investigations to answer questions that puzzle them.		✓	
						Investigations are used to answer the teacher's questions.		✓	
				Differentiation	0.76	The teacher goes out of his way to help each student.			✓
						All students in the class do the same work at the same time. [R]			✓
						Different students do different work.			✓
						The teacher helps each student who is having trouble with his or her work.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						Students who work faster than others move on to the next topic.			✓
What is happening in this classroom (WIHIC)	Chionh & Fraser, 1998	S	Gen	Teacher support	0.88	The teacher goes out of his/her way to help me.			✓
						The teacher helps me when I have trouble with the work.			✓
						The teacher's questions help me understand.			✓
				Involvement	0.84	I discuss ideas in class.		✓	
						I give my opinions during class discussions.		✓	
						The teacher asks me questions.		✓	
						My ideas and suggestions are used during discussion.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						I explain my ideas to other students.		✓	
						Students discuss with me how to go about solving problems.		✓	
						I am asked to explain how I solve problems.		✓	
				Investigation	0.88	I am asked to think about evidence for statements.		✓	
						I explain the meaning of statements, diagrams, and graphs.		✓	
Classroom Assessment Scoring System – Secondary Level (CLASS-S)	Downer, Stuhlman, Schweig, Martinez & Ruzek 2015	S	Gen	Emotional Support Domain	0.83	My teacher encourages me to share my ideas in class.		✓	
						My teacher stops to answer my questions.			✓
						My teacher helps me when I need help.			✓
						I feel comfortable in this class.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
				Instructional Support Domain	0.86	I use all kinds of interesting materials in this class.	✓		
						I get to do a lot in this class, not just listen to my teacher talk	✓		
						My teacher explains it in a new way if I say that I don't understand something.			✓
						My teacher connects learning to what I already know.	✓		
						My teacher provides challenging work in this class.	✓		
						I speak up to share my ideas about class work.		✓	
						My teacher helps me to solve problems myself.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Task	SCI Auth	SCI Sup
						My teacher asks me to think about what I have learned at the end of activities.	✓		
						My teacher suggests ways to make my work better.			✓
						My teacher keeps working with me until I understand what we are doing.			✓

Table 2

Instruction or classroom learning environment measures that include indicators of the dimensions of student-centered mathematics instruction

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
Personalized Learning Environment Questionnaire (PLQ)	Waldrip, Cox, Deed, Dorman, Edwards, Farrelly, Keeffe, Lovejoy, Mow, Prain, sellings, Yager 2014	S	Gen	Learning environment – Teacher support	0.86	Teachers in my school help me.			✓
						Teachers help me when I have trouble with my work.			✓
				Learning environment – shared control	0.85	I review with the teacher how well I am learning.		✓	✓
						I plan with the teacher which activities are best for me.		✓	✓
						I plan with the teacher how much time I spend on learning activities.		✓	✓
				Learning environment – student negotiation	0.83	In class, I explain my understandings to other students.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						In class, I ask other students to explain their thoughts about their work.		✓	
						In class, other students explain their ideas to me about their work.		✓	
				Learning environment – Personal relevance	0.81	I am asked to apply my learning to real life situations.	✓		
						My assessment tasks are useful to everyday events.	✓		
						Assessment tasks are connected to what I do outside of school.	✓		
Constructivist Classroom Learning Environment (CLES)	Taylor, Fraser, Fisher 1997	S	Math	Personal relevance	0.81	I learn about the world outside of school.	✓		
						It's OK to ask the teacher "why do we have to learn this?	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						New learning starts with problems about the world outside of school.	✓		
						The activities are among the most interesting at this school.	✓		
						I learn how mathematics can be part of my out-of-school life.	✓		
						The activities make me interested in mathematics.	✓		
						I get a better understanding of the world outside of school.	✓		
						I learn interesting things about the world outside of school.	✓		
						What I learn has nothing to do with my out-of-school life. [R]	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						What I learn has nothing to do with the world outside of school. [R]	✓		
				Shared control	0.85	I help the teacher to plan what I'm going to learn.		✓	✓
						I help the teacher decide how well my learning is going.		✓	
						I have a say in deciding the rules for classroom discussion.		✓	
						I have a say in deciding how much time I spend on an activity.		✓	
				Critical voice	0.79	I feel free to question the way I'm being taught.		✓	
						It's OK to complain about activities that are confusing.		✓	
						It's OK to complain about anything that stops me from learning.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						I'm free to express my opinion.		✓	
						I feel unable to complain about anything.		✓	
				Student negotiation	0.68	I get the chance to talk to other students.		✓	
						I talk with other students about how to solve problems.		✓	
						I try to make sense of other students' ideas.	✓		
						I ask other students to explain their ideas.		✓	
						Other students ask me to explain my ideas.		✓	
Classroom Practices Teacher Survey (CPTS)	Nathanson, Sawyer, & Rimm-Kaufman, (2007)	T	Gen	Academic Work	0.82	I provide students with opportunities to work in whole group, small group, partner and individual work activities.	✓		

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						Students have regular and predictable opportunities to share their work with other students.		✓	
						I carefully consider my students' developmental needs when choosing lessons and materials.			✓
						I provide students a set of choices about what kind of work to do, how to do the work, or both.		✓	
						When my students are working on activities of their own choosing, I have structures in place that assist them in planning their activity.			✓
						When my students are working on activities of their own choosing, I have structures in place that assist them in reflecting on their work.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
Competency –Based Learning Survey	Ryan & Cox, 2016; https://ies.ed.gov/ncee/edlabs/regions/northeast/pdfs/REL_2016165.pdf	S	Gen	Progression through demonstration of mastery		In courses at my school, students must show their learning on each competency in more than one way. For example, students must show that they have mastered the competency on more than one assignment, assessment, or exam.	✓		
						In courses at my school, students are able to choose how they want to show what they have learned from several different options. For example, options such as taking a test, writing a paper, completing a project, etc.		✓	
						Students at my school are able to progress at their own individual pace in courses.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						I am able to move on to the next competency when I am ready, even if other students in the course are not ready.			✓
						I understand how the competencies in my courses will help me in the future.	✓		
						My teachers give me a rubric so that I know how I am progressing on each competency.	✓		
				Personalization		My teachers notice if I need extra help.			✓
						My teachers or a counselor/advisor discussed how I am doing on each competency with me.			✓
						My teachers teach the material in several different ways in order to help students learn.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						I have had opportunities to choose how to show my teachers what I have learned.		✓	
				Development of skills and dispositions		When I have trouble learning something new, my teachers give me advice and strategies that help me to keep trying.			✓
						My teachers notice when I take extra time and effort on something that is difficult for me.			✓
						If I get a low score on an assessment, my teachers help me figure out how I can still do well in the class.			✓
						Teachers encourage students to take responsibility for their work.		✓	

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
				Flexible assessment		I have created drawings or models to show what I have learned.	✓		
						I have given a presentation to show what I have learned.	✓		
						I have completed a project at school to show what I have learned.	✓		
Patterns of Adaptive Learning Scales (PALS) Student report	Midgley, Maehr, Hruda, Anderman, Anderman, Freeman et al., 2000 http://www.umich.edu/~pals/PALS%202000_V12Word97.pdf	S	Gen	Academic Press	0.79	When I've figured out how to do a problem, my teacher gives me more challenging problems to think about.	✓		✓
						My teacher presses me to do thoughtful work.	✓		✓
						My teacher asks me to explain how I get my answers.		✓	
						When I'm working out a problem, my teacher tells me to keep thinking until I really understand.	✓		✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						My teacher doesn't let me do just easy work, but makes me think.	✓		
						My teacher makes sure that the work I do really makes me think.	✓		
						My teacher accepts nothing less than my full effort.			✓
Patterns of Adaptive Learning Scales (PALS) Teacher Report	Midgley, Maehr, Hruda, Anderman, Anderman, Freeman et al., 2000 http://www.umich.edu/~pals/PALS%202000_V12Word97.pdf	T	Gen	Mastery approaches	0.69	I make a special effort to recognize students' individual progress, even if they are below grade level.			✓
						During class, I often provide several different activities so that students can choose among them.	✓	✓	
						I give a wide range of assignments, matched to students' needs and skill level.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
Classroom Practices Teacher Survey (CPTS)	Nathanson, L., Sawyer, B., & Rimm-Kaufman, S.E. (2007)	T	Gen	Academic Work	0.82	I provide students with opportunities to work in whole group, small group, partner and individual work activities.	✓		
						Students have regular and predictable opportunities to share their work with other students.		✓	
						I carefully consider my students' developmental needs when choosing lessons and materials.			✓
						I provide students a set of choices about what kind of work to do, how to do the work, or both.		✓	
						When my students are working on activities of their own choosing, I have structures in place that assist them in planning their activity.			✓

Measure	Availability	T/S	Math /Gen	Sub-Scale	Cronbach's α	Items	SCI Tasks	SCI Auth	SCI Sup
						When my students are working on activities of their own choosing, I have structures in place that assist them in reflecting on their work.			✓

Table 3.
Classroom and school climate measures that include indicators of the dimensions of student-centered mathematics instruction

SCI Dimension	Scale	Availability	Sub-scale	What it measures	T/S	Math/ Gen	Items	Cronbach's α	Effects
Relevant and metacognitive tasks	Reform Oriented Teaching Practice	Banilower, E. R., Smith, P. S., Weiss, I. R., Malzahn, K. A., Campbell, K. M., & Weis, A. M. (2013)		Assesses the extent to which teachers use mathematics instruction that mirrors the current understanding of learning	T	Math	How often do you: Have students consider multiple representations in solving a problem (e.g., numbers, tables, graphs, pictures. Have students explain and justify their method for solving a problem. Have students compare and contrast different methods for solving a problem.	4 items: 0.77	More commonly used in mathematics classes consisting mainly of high achievers and is less common in high school. Associated with improved state standardized test scores.

SCI Dimension	Scale	Availability	Sub-scale	What it measures	T/S	Math/ Gen	Items	Cronbach's α	Effects
							Have students present their solution strategies to the rest of the class.		
	Autonomy Support	Assor, Kaplan, Roth 2002	Fostering Understanding and Interest (aka Fostering Relevance)	Measures teachers attempts to help students experience the learning process as relevant to and supportive of their self-determined interests, goals and values	S	Gen	My teacher listens to my opinions and ideas. My teacher respects students who tell her what they really think and are not ingratiating. My teacher tells us that if we do not agree with her – it is important that we would express our disagreement.	6 items: 0.81	Predicts positive feelings and behavioral and cognitive engagement in 6th and 8th grade students

SCI Dimension	Scale	Availability	Sub-scale	What it measures	T/S	Math/ Gen	Items	Cronbach's α	Effects
							My teacher is willing to listen to students' complaints regarding her. My teacher allows me to decide about things by myself. My teacher allows us to talk about things that we find.		
Student intellectual authority and responsibility for learning	Autonomy support	Assor, Kaplan, Roth 2002	Providing choice	Examines teachers use of instructional practices that enable students to choose tasks that they perceive as consistent with their goals and interests.	S	Gen	When I am doing something that interests me, my teacher gives me enough time to finish it. My teacher allows me to choose how to do my work in the classroom.	6 items: 0.75	No effects on 6th to 8th graders positive feelings, negative feelings, or behavioral and cog engagement

SCI Dimension	Scale	Availability	Sub-scale	What it measures	T/S	Math/ Gen	Items	Cronbach's α	Effects
							My teacher asks us which topics we would like to study more and which we prefer to study less. My teacher asks us if there are things we would like to change in the way we study. My teacher allows me to choose to study topics that interest me. When my teacher gives us an assignment she allows us to choose which questions to answer.		

SCI Dimension	Scale	Availability	Sub-scale	What it measures	T/S	Math/ Gen	Items	Cronbach's α	Effects
							My teacher encourages me to work in my own way.		
			Allowing criticism	Measures teacher behaviours that are assumed to evoke feelings of interest because the expression of dissatisfaction by students might cause teachers to make learning tasks more interesting	S	Gen	My teacher listens to my opinions and ideas. My teacher tells us that if we do not agree with her, it is important that we would express our disagreement. My teacher is willing to listen to students' complaints regarding her. My teacher respects students who tell her what they really think and are not ingratiating.	6 items: 0.76	Predicts positive feelings in 6^{th} and 8^{th} graders

SCI Dimension	Scale	Availability	Sub-scale	What it measures	T/S	Math/ Gen	Items	Cronbach's α	Effects
							My teacher allows me to decide about things by myself.		

Table 4

Predictors and outcomes of student and teacher reports of student-centered mathematics instruction collected in the fall of 2015

	Reported By	N	Mean	SD	Range From	Range To	Percent	Cronbach's α	ICC
Student Characteristics									
Gender (Female)	School District	2,172					49.72		
Race/Ethnicity									
Caucasian	School District	1,045					48.11		
African American	School District	707					32.55		
Asian	School District	26					1.20		
Native American	School District	11					0.51		
Hispanic	School District	13					0.60		
Bi- or multi-racial	School District	273					12.56		
Grade									
5th	School District	682					31.47		
7th	School District	825					38.07		
9th	School District	608					28.06		
Free or reduced lunch	School District	840					37.51		
Previous achievement									
14-15 Grade point average	School District	2,503	3.16	0.767	0	4.05			
Teacher Characteristics									
Gender (Female)	Teacher	36					70.60		
Race/Ethnicity									
Caucasian	Teacher	36					93.84		
African American	Teacher	36					6.16		
Asian	Teacher	36					2.05		
Native American	Teacher	36					0		
Hispanic	Teacher	36					0		
Bi- or multi-racial	Teacher	36					0		
Years Experience	Teacher	36	11.54	8.06	2	34			
Highest Degree									
Bachelors	Teacher	36					23.50		
Masters	Teacher	36					76.50		
Student Outcomes									
Current achievement									

15-16 Math grade	School District	2,199	76.45	15.51	27	100		0.371
15-16 PSSA Math Score	School District	1,495	978.99	125.98	730	2112		0.260
Mathematics Engagement								
Emotional Engagement	Student	1,838	3.60	1.04	1	5	0.90	0.170
Behavioral Engagement	Student	1,838	4.03	0.73	1	5	0.82	0.232
Cognitive Engagement	Student	1,836	3.77	0.74	1	5	0.74	0.181
Social Engagement	Student	1,836	3.65	0.77	1	5	0.75	0.131

Note: The intra-class correlation for each outcome variable in the null models (predicting the outcome while nesting the outcome in mathematics classrooms).

Table 5

CLASS-S student and teacher items

CLASS		Student Survey	Teacher Survey
Domain	Dimension	Item	Item
Emotional Support	Positive Climate	5th: My math teacher likes me // 7th & 9th: My math teacher respects me.	5th: I enjoy teaching my class. // 7th & 9th: Respecting students is a high priority in my classroom.
		My math teacher says nice things to me.	5th: I give students positive feedback in class. // 7th & 9th: Students in my classroom get positive feedback and support.
	Teacher Sensitivity	My math teacher helps me when I need help.	Students in my classroom often come to me for help.
		I feel comfortable in math class.	Students feel comfortable in my class.
		5th: My math teacher understands my feelings. // 7th & 9th: My math teacher understands how I feel about things in class.	5th: I understand how students are feeling in class. // 7th & 9th: I understand how students feel about what we are doing in class.
	Regard for Adolescent Perspectives	My math / science teacher encourages me to share my ideas in class.	Students have opportunities to share their ideas in my class.
		5th: My math teacher lets me know that it's okay to have different opinions in class. // 7th & 9th: My math teacher lets me know that if I do not agree with him/her, it is important r, it is important that I express my disagreement.	I tell students that if they do not agree with me, it is important that they express their disagreement.
		My math teacher stops to answer my questions.	I stop to answer students' questions in class.

CLASS		Student Survey	Teacher Survey
Domain	Dimension	Item	Item
Classroom Organization	Negative Climate	My math teacher gets annoyed with me.	I feel irritated when I am teaching.
	Behavior Management	5th: My math teacher knows if I am on task in class. // 7th & 9th: My math teacher knows if I am on task in class.	5th: I monitor students' behavior in class. // 7th & 9th: I know if students are on task.
		5th: I act the way my math teacher wants me to. // 7th & 9th: I am respectful to my math teacher.	5th: Students follow classroom rules and routines. // 7th & 9th: Students are respectful.
		If students are acting up in math / science class, my teacher will do something about it.	I respond to students who are acting up in class.
	Productivity	I stay busy in math / science class	Students stay busy in this class.
		5th: My math teacher has everything ready for me to start my work. // 7th & 9th: My math / science teacher is prepared for class.	Materials and activities are ready for students when students come in to class.
Instructional Support	Instructional Learning Formats	I do all kinds of interesting activities in math / science class.	We do interesting activities in class.
		I get to do things in math /science class other than listen to the teacher talk.	Students engage in a variety of activities and learning formats in my class.
	Content Understanding	My math / science teacher explains it in a new way if I say that I don't understand something.	I explain ideas in a new way if a student doesn't understand.
		My math/ science teacher connects what I am learning to what I already know.	I connect what we are learning to what students already know.
		My math / science teacher provides challenging work in math /science class.	Students engage in challenging work in this class.

CLASS		Student Survey	Teacher Survey
Domain	Dimension	Item	Item
		I have to think hard in order to solve the kinds of problems that we work on in	Students have to think hard in order to solve the problems I give them in class.
	Analysis and Inquiry	5th: My math teacher helps me to solve problems myself. // 7th & 9th: My math / science teacher encourages me to solve problems on my own.	5th: I encourage students to come up with their own ways to solve problems. // 7th & 9th: I encourage students to solve problems on their own.
		My math / science teacher asks me to think about what I have learned at the end of activity.	We think about what we have learned at the end of activities.
		5th: My math teacher encourages me to consider different ways to solve problems. // 7th & 9th: My math / science teacher encourages me to consider different solutions and points of view.	Students are given opportunities to consider different solutions or points of view.
	Quality of Feedback	5th: My math teacher suggests ways to make my work better. // 7th & 9th: My math / science teacher suggests ways that I can learn more.	5th: I suggest ways students can make their work better. // 7th & 9th: I give students feedback about ways to learn more about what we are studying in class.
		My math /science teacher keeps working with me until I understand what we are doing.	Students get as much help from me as they need in order to understand what we are doing.
		My math /science teacher gives clear instructions for how to do well in math / science class.	I give students instructions on what they need to do in order to do well in class

Table 6

Autonomy support student and teacher items from Assor, Kaplan, & Roth (2002)

Autonomy Support	Student Survey Items	Teacher Survey Items
Allowing criticism and encouraging independent thinking	1. My math teacher lets me know that if I do not agree with him/her, it is important that I express my disagreement.	1. I tell students that if they do not agree with me, it is important that they express their disagreement.
	2. My math teacher encourages me to solve problems on my own.	2. I show students how to solve problems by themselves.
		3. I listen to students' opinions and ideas.
Fostering relevance	1. My math teacher applies the subject to problems and situations in life outside of school.	1. I apply what we are learning in class to life outside of school.
	2. My math teacher explains why it is important to study certain subjects in math class.	2. We talk about why it is important to know what we are learning in class.

Table 7

Dialogic instruction student and teacher item from Stein & Smith, 2011

Student Survey Items		Teacher Survey Items	
1.	When other students present their work on math problems, I get to see a lot of different ways to solve the same problem.	1.	When students construct their own ways of doing a problem, I have the students themselves share their approaches with the rest of the class using their own ways of expressing themselves.
2.	After we have worked on a really difficult problem, my math teacher allows us to watch how other students solved it.		
3.	7th & 9th only: When I show my math teacher an answer, he/she asks me to explain how I got that answer.	2.	*Matching teacher item in reform-oriented instruction*
4.	7th &9th only: After we have worked on a really difficult problem, my math teacher allows us to watch how other students solved it.	3.	When students construct their own ways of doing a problem, I have the students themselves share their approaches with the rest of the class using their own ways of expressing themselves.
		4.	I listen to students' opinions and ideas.

Table 8

Reform oriented teaching practice teacher items, reported by teachers at the classroom level, from Banilower, Smith, Weiss, Malzahn, Campbell, & Weis (2013)

In your mathematics classes, how often do you:
Explain mathematical ideas to the whole class
Engage the whole class in discussions
Have students work in small groups
Have students attend presentations by guest speakers focused on mathematics in the workplace
Have students consider multiple representations in solving a problem (for example: numbers, tables, graphs, pictures)
Have students explain and justify their method for solving a problem
Have students compare and contrast different methods for solving a problem

Table 9

Indicators for student and teacher report of the three dimensions of student-centered mathematics instruction (the three dimensions summed for overall student and teacher report of student-centered instruction) collected in the fall of 2015

Dimension	Scale		Student Items	N Students	Mean	SD		Teacher Items	N Classes	Mean	SD
Relevant and metacognitive tasks	CLASS Instructional Support	1.	I get to do all kinds of interesting things in math class.	1,656	3.31	1.30	1.	We do interesting activities in class.	146	3.52	0.82
		2.	I get to do things in math class other than listen to the teacher talk.	1,656	3.21	1.35	2.	Students engage in a variety of activities and learning formats in my class.	146	3.77	0.87
	CLASS Analysis and Inquiry	3.	My math teacher asks me to think about what I have learned at the end of activities.	1,656	3.37	1.27	3.	We think about what we have learned at the end of activities.	146	3.87	0.79
		4.	My math teacher encourages me to consider different solutions and points of view.	1,656	3.84	1.11	4.	Students are given opportunities to consider different solutions or points of view.	146	3.87	0.79
	CLASS Content Understanding and Autonomy Support – Fostering Relevance	5.	My math teacher connects what I am learning to what I already know.	1,656	3.77	1.09	5.	I connect what we are learning to what students already know.	146	4.54	0.53
		6.	My math teacher applies the subject to problems and situations in life outside of school.	1,154	3.40	1.24	6.	I apply what we are learning in class to life outside of school.	146	3.91	0.80
							7.	We talk about why it is important to know what we are learning	146	3.95	0.71

Dimension	Scale		Student Items	N Students	Mean	SD		Teacher Items	N Classes	Mean	SD
Student intellectual authority and responsibility for learning	CLASS Analysis and Inquiry	1.	5th: My math teacher helps me to solve problems myself. // 7th & 9th: My math teacher encourages me to solve problems on my own	1,656	4.08	1.06	1.	5th: I encourage students to come up with their own ways to solve problems. // 7th & 9th: I encourage students to solve problems on their own.	146	4.30	0.73
	CLASS Regard for Adolescent Perspectives And Autonomy Support – Allowing Criticism and Encouraging Independent Thinking	2.	My math teacher encourages me to share my ideas in class.	1,643	3.53	1.23	2.	Students have opportunities to share their ideas in my class.	146	4.32	0.69
		3.	5th: My math teacher lets me know that it's okay to have different opinions in class. // 7th & 9th: My math teacher lets me know that if I do not agree with him/her, it is important that I express my disagreement.	1,643	3.63	1.27	3.	I tell students that if they do not agree with me, it is important that they express their disagreement.	146	3.66	1.03
	NSF Reform Mathematics Instruction	4.	When I show my math teacher an answer, he/she asks me to explain how I got that answer.	778	3.91	1.11	4.	How often do you: Ask students to explain and justify the methods they use for solving problems.	146	4.25	0.61
		5.	My math teacher has us compare and contrast different methods that students used to solve problems in class.	778	3.50	1.15	5.	How often do you: Have students compare and contrast different methods that students in their class used to solve problems.	146	3.55	0.81
	Dialogic Instruction	6.	When we work on a difficult problem, my math teacher asks me to share	778	3.30	1.23	6.	Students are encouraged to express different points of	146	4.19	0.68

Dimension	Scale		Student Items	N Students	Mean	SD		Teacher Items	N Classes	Mean	SD
			how I solved the problem with the rest of the class.					view in class.			
	NSF Reform Mathematics Instruction						7.	How often do you: Have students work in groups in class.	146	3.63	0.80
	Dialogic Instruction						8.	When students construct their own ways of doing a problem, I have the students present the solution in their own words.	146	3.72	0.85
Flexible and responsive support of student understanding	CLASS Regard for Adolescent Perspectives	1.	My math teacher stops to answer my questions.	1,643	3.80	1.23	1.	I stop to answer students' questions in class.	146	4.88	0.33
	CLASS Quality of Feedback	2.	My math teacher suggests ways that I can learn more.	1,643	3.78	1.16	2.	5th: I give students feedback on how to make their work better; 7th & 9th: I give students feedback about ways to learn more about what we are studying in class.	146	4.12	0.76
		3.	My math teacher keeps working with me until I understand what we are doing.	1,643	3.84	1.21	3.	Students get as much help from me as they need in order to understand what we are doing.	146	4.36	0.54
	CLASS Content Understanding	4.	My math teacher explains it in a new way if I say that I don't understand something.	1,656	3.88	1.18	4.	I explain ideas in a new way if a student doesn't understand.	146	4.54	0.53

Dimension	Scale		Student Items	N Students	Mean	SD		Teacher Items	N Classes	Mean	SD
	CLASS Teacher Sensitivity	5.	My math teacher helps me when I need help.	1,643	4.24	1.05	5.	Students in my classroom often come to me for help.	146	3.77	0.96
		6.	5th: My math teacher understands my feelings. // 7th & 9th: My math teacher understands how I feel about things in class.	1,643	3.47	1.30	6.	5th: I understand how students are feeling in class. // 7th & 9th: I understand how students feel about what we are doing in class.	146	3.92	0.56
	Autonomy Support – Allowing Criticism and Encouraging Independent Thinking and Dialogic Instruction						7.	I listen to students' ideas and opinions	146	4.36	0.62
	CLASS Positive Climate	7.	5th: My math teacher likes me // 7th & 9th: My math teacher respects me.	1,642	4.28	1.07					

Table 10

Intra-class correlation for student and teacher items of student-centered mathematics instruction

Dimension	Student Items	ICC	Teacher Items	ICC
Relevant and metacognitive tasks	I get to do all kinds of interesting things in math class.	0.201		
	I get to do things in math class other than listen to the teacher talk.	0.116		
	My math teacher asks me to think about what I have learned at the end of activities.	0.079	We think about what we have learned at the end of activities.	0.759
	My math teacher encourages me to consider different solutions and points of view.	0.134	Students are given opportunities to consider different solutions or points of view.	0.843
	My math teacher connects what I am learning to what I already know.	0.105	I connect what we are learning to what students already know.	0.917
	My math teacher applies the subject to problems and situations in life outside of school.	0.111	I apply what we are learning in class to life outside of school.	0.933
			We talk about why it is important to know what we are learning in class.	0.862
Student intellectual authority and responsibility for learning	5th: My math teacher helps me to solve problems myself. // 7th & 9th: My math teacher encourages me to solve problems on my own	0.093	5th: I encourage students to come up with their own ways to solve problems. // 7th & 9th: I encourage students to solve problems on their own.	0.742
	My math teacher encourages me to share my ideas in class.	0.125	Students have opportunities to share their ideas in my class.	0.865
	5th: My math teacher lets me know that it's okay to have different opinions in class. // 7th & 9th: My math teacher lets me know that if I do not agree with him/her, it is important that I express my disagreement.	0.110	I tell students that if they do not agree with me, it is important that they express their disagreement.	0.776
	When I show my math teacher an answer, he/she asks me to explain how I got that answer.	0.99	How often do you: Ask students to explain and justify the methods they use for solving problems.	1.000
	My math teacher has us compare and contrast different methods that students used to solve problems in class.	0.086	How often do you: Have students compare and contrast different methods that students in their class used to solve problems.	1.000
	When we work on a difficult problem, my math teacher asks me to share how I solved the problem with the rest of the class.	0.114	Students are encouraged to express different points of view in class.	0.858
			How often do you: Have students work in groups in class	1.000
			When students construct their own ways of doing a problem, I	0.906

Dimension	Student Items	ICC	Teacher Items	ICC
			have the students present the solution in their own words.	
Flexible and responsive support of student understanding	My math teacher stops to answer my questions.	0.150		
	My math teacher suggests ways that I can learn more.	0.07		
	My math teacher keeps working with me until I understand what we are doing.	0.121	Students get as much help from me as they need in order to understand what we are doing.	0.867
	My math teacher explains it in a new way if I say that I don't understand something.	0.166	I explain ideas in a new way if a student doesn't understand.	0.662
	My math teacher helps me when I need help.	0.189	Students in my classroom often come to me for help.	0.681
	5th: My math teacher understands my feelings. // 7th & 9th: My math teacher understands how I feel about things in class.	0.159	5th: I understand how students are feeling in class. // 7th & 9th: I understand how students feel about what we are doing in class.	0.750
	5th: My math teacher likes me // 7th & 9th: My math teacher respects me.	0.191	I listen to students' ideas and opinions	0.948

Table 11

Fit statistics of models for student-report and teacher-report student-centered mathematics scales

	Model	*df*	χ^2	CFI	TLI	RMSEA
Student	1^{st} Order, Multi-Level CFA	317	1277.28***	.911	.904	.043
Teacher	1^{st} order CFA, controlling for nesting	72	95.208*	.895	.867	.049

Note. *** $p < .001$; Comparative Fit Index (CFI), Tucker Lewis Index (TLI), Root Mean Square Error of Approximation

Table 12

Factor loadings for student and teacher report of student-centered mathematics Instruction

Student Items	SCI Task	SCI Authority	SCI Support	Teacher Items	SCI Task	SCI Authority	SD Support
1. I get to do all kinds of interesting things in math class.	.642						
2. I get to do things in math class other than listen to the teacher talk.	.471						
3. My math teacher asks me to think about what I have learned at the end of activities.	.634			1. We think about what we have learned at the end of activities.	.676		
4. My math teacher encourages me to consider different solutions and points of view.	.710			2. Students are given opportunities to consider different solutions or points of view.	.679		
5. My math teacher connects what I am learning to what I already know.	.727			3. I connect what we are learning to what students already know.	.664		
6. My math teacher applies the subject to problems and situations in life outside of school.	.538			4. I apply what we are learning in class to life outside of school.	.784		
				5. We talk about why it is important to know what we are learning in class.	.732		
1. 5th: My math teacher helps me to solve problems myself. 7th & 9th: My math teacher encourages me to solve problems on my own		.580		1. 5th: I encourage students to come up with their own ways to solve problems. 7th & 9th: I encourage students to solve problems on their own.		.517	
2. My math teacher encourages me to share my ideas in class.		.699		2. Students have opportunities to share their ideas in my class.		.696	
3. 5th: My math teacher lets me know that it's okay to have		.675		3. I tell students that if they do not agree with me, it is important		.761	

	Student Items	SCI Task	SCI Authority	SCI Support		Teacher Items	SCI Task	SCI Authority	SD Support
	different opinions in class. 7th & 9th: My math teacher lets me know that if I do not agree with him/her, it is important that I express my disagreement.					that they express their disagreement.			
4.	When I show my math teacher an answer, he/she asks me to explain how I got that answer.		.504						
5.	My math teacher has us compare and contrast different methods that students used to solve problems in class.		.526						
6.	When we work on a difficult problem, my math teacher asks me to share how I solved the problem with the rest of the class.		.498		4.	Students are encouraged to express different points of view in class.		.655	
7.	We do group work in math class.		.697						
					5.	When students construct their own ways of doing a problem, I have the students present the solution in their own words.		.768	
1.	My math teacher stops to answer my questions.			.619					
2.	My math teacher suggests ways that I can learn more.			.745					
3.	My math teacher explains it in a new way if I say that I don't understand something.			.764	1.	I explain ideas in a new way if a student doesn't understand.			.705
4.	My math teacher keeps working with me until I understand what we are doing.			.762					
5.	My math teacher helps me when			.675	2.	Students in my classroom often			.474

	Student Items	SCI Task	SCI Authority	SCI Support		Teacher Items	SCI Task	SCI Authority	SD Support
	I need help.					come to me for help.			
6.	5th: My math teacher understands my feelings. 7th & 9th: My math teacher understands how I feel about things in class.			.662	3.	5th: I understand how students are feeling in class. 7th & 9th: I understand how students feel about what we are doing in class.			.509
7.	5th: My math teacher likes me // 7th & 9th: My math teacher respects me.			.724					
					4.	I listen to students' ideas and opinions			.459

Note. All factor loadings were significant at $p < .05$

Table 13

Scale reliability for the student and teacher report of student-centered mathematics instruction

Dimension	Student				Teacher			
	N	Mean	SD	Cronbach's α	N	Mean	SD	Cronbach's α
Relevant and metacognitive tasks	1132	3.52	0.85	.785	146	3.87	0.79	.823
Student intellectual authority and responsibility for learning	756	3.66	0.79	.766	146	4.04	0.60	.803
Flexible and responsive support of student understanding	1621	3.9	0.89	.873	146	4.15	0.48	.654

Table 14

Test of measurement invariance for the student-report student-centered mathematics instruction scale

Math	Model	df	χ2	CFI	TLI	RMSEA (90% CI)	Δχ2	Δdf
Gender (male vs. female)	1. Baseline Model	314	827.60***	.917	.910	.063 (.058, .069)		
	2. Metric invariance	333	844.28***	.918	.914	.062 (.057, .067)	16.68	19
Race (European American vs. African American)	1. Baseline Model	314	1347.86***	.903	.894	.066 (.062, .070)		
	2. Metric invariance	333	1427.53***	.897	.895	.066 (.062, .069)	220.33***	19
SES (regular vs. free/reduced lunch)	1. Baseline Model	314	1386.30***	.900	.892	.066 (.062, .069)		
	2. Metric invariance	333	1480.59***	.893	.891	.066 (.063, .069)	94.29***	19

Note. * $p < .05$; ** $p < .01$; *** $p < .001$. -- Indicates that the more constrained model provides better fit than the less constrained model and thus difference test cannot be performed; Comparative Fit Index (CFI), Tucker Lewis Index (TLI), Root Mean Square Error of Approximation (RMSEA).

Table 15

Test of measurement invariance for the teacher-report student-centered mathematics instruction scale

Math	Model	df	χ2	CFI	TLI	RMSEA (90% CI)	Δχ2	Δdf
Highest degree (bachelors vs. masters)	1. Baseline Model	160	777.97***	.529	.465	.239 (.223, .256)		
	2. Metric invariance	173	830.11***	.500	.474	.237 (.221, .251)	52.14***	13
Teacher gender (male vs. female)	1. Baseline Model	160	863.13***	.506	.438	.255 (.239, .272)		
	2. Metric invariance	173	916.12***	.478	.451	.252 (.236, .268)	52.99***	13
Years experience (under 10 years vs. over 10 years)	1. Baseline Model	160	826.81**	.506	.438	.248 (.232, .265)		
	2. Metric invariance	173	920.26**	.446	.417	.253 (.237, .269)	93.45***	13

Note. * $p < .05$; ** $p < .01$; *** $p < .001$. -- Indicates that the more constrained model provides better fit than the less constrained model and thus difference test cannot be performed; Comparative Fit Index (CFI), Tucker Lewis Index (TLI), Root Mean Square Error of Approximation (RMSEA).

Table 16

Pearson correlations between model student and teacher predictors and outcomes

	1	2	3	4	5	6	7	8	9	10
1. Student SCI Task										
2. Student SCI Authority	.716**									
3. Student SCI Support	.706**	.652**								
4. Teacher SCI Task	.042	.148**	-.047							
5. Teacher SCI Authority	-.024	.035	-.127**	.711**						
6. Teacher SCI Support	.015	.114*	-.005	.606**	.530**					
7. Cognitive Engagement	.486**	.467**	.496**	-.012	-.036	.083*				
8. Behavioral Engagement	.447**	.405**	.497**	-.038	-.072*	.070*	.746**			
9. Emotional Engagement	.536**	.439**	.490**	.038	.042	.097**	.621**	.619**		
10. Social Engagement	.410**	.416**	.353**	-.007	.031	.039	.534**	.436**	.399**	
11. Math Course Grades	.257**	.280**	.305**	-.052	-.009	-.009	.345**	.441**	.315**	.188**

Table 17

Spearman correlations between model student and teacher predictors and outcomes

	1	2	3	4	5	6	7	8	9	10
1. Student SCI Task										
2. Student SCI Authority	.688**									
3. Student SCI Support	.694**	.640**								
4. Teacher SCI Task	.058	.177**	-.004							
5. Teacher SCI Authority	-.044	.012	-.119**	.695**						
6. Teacher SCI Support	.032	.162*	-.014	.593**	.528**					
7. Cognitive Engagement	.486**	.457**	.510**	-.006	-.043	.085*				
8. Behavioral Engagement	.436**	.393**	.501**	-.030	-.070*	.075*	.746**			
9. Emotional Engagement	.515**	.412**	.477**	.051	.033	.110**	.633**	.630**		
10. Social Engagement	.420**	.423**	.387**	-.005	.025	.037	.537**	.443**	.414**	
11. Math Course Grades	.261**	.264**	.317**	-.021	-.128	-.020	.387**	.476**	.350**	.213**

Table 18

Hierarchical linear models predicting emotional engagement in mathematics with student and teacher reports of student-centered mathematics instruction

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Intercept	3.546**	(0.043)	3.245**	(0.237)	3.081**	(0.212)	3.128**	(0.234)
Fixed Effects								
Level 1: Student								
Student Covariates								
Male			0.219**	(0.045)	0.220**	(0.040)	0.221**	(0.045)
African American								
Free/red lunch								
Previous GPA			0.252**	(0.049)	0.181**	(0.047)	0.248**	(0.048)
Student Report Classroom Climate								
Tasks					0.308**	(0.044)		
Student Authority								
Support					0.381**	(0.038)		
Level 2: Classroom								
Teacher Covariates								
Male			-0.267*	(0.095)			-0.260*	(0.091)
African American			0.510**	(0.143)	0.441**	(0.113)		
Years experience							-0.011*	(0.005
Highest degree			0.331*	(0.108)	0.280*	(0.099)	0.388**	(0.100)
Teacher Report Classroom Climate								
Tasks							-0.310*	(0.093)
Student Authority								
Support							0.303*	(0.116)
Random Effects								
Between Level Variance, u_0	0.184**	(0.013)	0.113**	(0.025)	0.050**	(0.013)	0.091*	(0.029)
Within Level	0.900**	(0.019)	0.875**	(0.036)	0.676**	(0.032)	0.873**	(0.036)

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Variance, $_r$								
AIC	5172.38		19660.92		28031.87		20156.46	
BIC	5188.91		19818.16		28241.52		20366.11	
Pseudo R-Squared								
Level 1			.028		.249		.030	
Level 2			.386		.728		.505	

*p<.05, **p<.00

Table 19

Hierarchical linear models predicting behavioral engagement in mathematics with student and teacher reports of student-centered mathematics instruction

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Intercept	3.965**	(0.033)	3.924**	(0.154)	3.883**	(0.120)	3.838**	(0.165)
Fixed Effects								
Level 1: Student								
Student Covariates								
Male								
African American								
Free/red lunch			-0.131*	(0.044)	-0.090*	(0.036)	-0.122*	(0.043)
Previous GPA			0.312**	(0.040)	0.271**	(0.037)	0.309**	(0.040)
Student Report Classroom Climate								
Tasks					0.100*	(0.041)		
Student Authority								
Support					0.282**	(0.033)		
Level 2: Classroom								
Teacher Covariates								
Male			-0.210*	(0.078)			-0.198*	(0.070)
African American			0.163*	(0.079)				
Years experience			0.007*	(0.004)	0.011*	(0.003)		
Highest degree			0.266**	(0.064)	0.224**	(0.061)	0.297**	(0.069)
Teacher Report Classroom Climate								
Tasks							-0.230*	(0.072)
Student Authority								
Support							0.201*	(0.086)
Random Effects								
Between Level Variance, u_0	0.124**	(0.016)	0.039*	(0.012)	0.018*	(0.008)	0.029*	(0.010)

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Within Level Variance, r	0.411**	(0.022)	0.391**	(0.021)	0.320**	(0.018)	0.390**	(0.021)
AIC	3840.57		18256.76		26727.09		18750.16	
BIC	3857.15		18414.00		26936.74		18959.81	
Pseudo R-Squared								
Level 1			.049		.049		.051	
Level 2			.685		.855		.847	

*p<.05, **p<.001

Table 20

Hierarchical linear models predicting cognitive engagement in mathematics with student and teacher reports of student-centered mathematics instruction

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Fixed Effects	3.719**	(0.031)	3.790**	(0.196)	3.685**	(0.166)	3.671**	(0.212)
Level 1: Student								
Student Covariates								
Male								
African American								
Free/red lunch			-0.140*	((.048)	-0.096*	(0.041)	-0.129*	(0.049)
Previous GPA			0.239**	(0.038)	0.189**	(0.035)	0.233**	(0.037)
Student Report Classroom Climate								
Tasks					0.148**	(0.038)		
Student Authority								
Support					0.271**	(0.028)		
Level 2: Classroom								
Teacher Covariates								
Male			-0.225*	(0.081)			-0.192*	(0.075)
African American			0.226*	(0.097)	0.241*	(0.086)		
Years experience					*0.008**	*(0.004)*		
Highest degree			0.194*	(0.083)			0.222**	(0.087)
Teacher Report Classroom Climate								
Tasks							-0.260*	(0.085)
Student Authority								
Support							0.258*	(0.093)
Random Effects								
Between Level Variance, u_0	0.098**	(0.013)	0.047**	(0.012)	0.026*	(0.009)	0.032*	(0.011)
Within Level	0.445**	(0.019)	0.432**	(0.019)	0.343**	(0.016)	0.431**	(0.019)

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Variance, $_r$								
AIC	3888.76		18365.02		26791.38		18857.03	
BIC	3905.29		18522.26		27001.08		19066.69	
Pseudo R-Squared								
Level 1			.029		.229		.031	
Level 2			.520		.735		.673	

*p<.05, **p<.00

Table 21

Hierarchical linear models predicting social engagement in mathematics with student and teacher reports of student-centered mathematics instruction

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Intercept	3.603**	(0.029)	3.451**	(0.204)	3.431**	(0.167)	3.382**	(0.218)
Fixed Effects								
Level 1: Student								
Student Covariates								
Male								
African American								
Free/red lunch								
Previous GPA							0.142**	(0.037)
Student Report Classroom Climate								
Tasks					0.200*	(0.076)		
Student Authority					*0.148**	(0.077)		
Support					*0.110**	(0.057)		
Level 2: Classroom								
Teacher Covariates								
Male								
African American								
Years experience								
Highest degree								
Teacher Report Classroom Climate								
Tasks							-0.229*	(0.092)
Student Authority								
Support								
Random Effects								
Between Level Variance, u_0	0.077**	(0.015)	0.050*	(0.016)	0.036*	(0.013)	0.043*	(0.014)

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Within Level Variance, $_r$	0.515**	(0.019)	0.528**	(0.025)	0.457**	(0.028)	0.511**	(0.019)
AIC	4119.85		8365.48		12281.54		19141.26	
BIC	4136.38		8461.46		12422.99		19350.92	
Pseudo R-Squared								
Level 1			.025		.113		.008	
Level 2			.351		.532		.442	

*p<.05, **p<.00

Table 22

Hierarchical linear models predicting mathematics course grades with student and teacher reports of student-centered mathematics instruction

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Intercept	77.878**	(0.811)	75.721**	(6.405)	75.515**	(6.163)	78.051**	(6.009)
Fixed Effects								
Level 1: Student								
Student Covariates								
Male			*-1.022**	*(0.530)*			-1.057*	(0.531)
African American			-1.979*	(0.958)	-2.080*	(0.947)	-1.889*	(0.938)
Free/red lunch			-4.645**	(0.695)	-4.406**	(0.695)	-4.618**	(0.689)
Previous GPA			9.066**	(1.122)	8.880**	(1.099)	9.021**	(1.139)
Student Report Classroom Climate								
Tasks								
Student Authority					2.323*	(0.985)		
Support					2.699**	(0.757)		
Level 2: Classroom								
Teacher Covariates								
Male								
African American								
Years experience								
Highest degree								
Teacher Report Classroom Climate								
Tasks								
Student Authority							-5.553*	(2.258)
Support								
Random Effects								
Between Level Variance, u_0	92.292**	(13.064)	51.360**	(10.623)	48.529****	(9.965)	41.736**	(8.991)

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Within Level Variance, $_r$	156.045**	(12.916)	128.588**	(11.002)	120.009**	(9.484)	128.532**	(10.995)
AIC	18599.08		32742.28		41494.31		33241.01	
BIC	18616.32		32899.16		41703.90		33450.18	
Pseudo R-Squared								
Level 1			.176		.231		.176	
Level 2			.444		.474		.548	

*p<.05, **p<..001

Table 23

Hierarchical linear models predicting State Standardized Test in Mathematics with Student and Teacher Reports of Student-Centered Mathematics Instruction

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Intercept	955.033**	(11.351)	1038.087**	(65.966)	1032.025**	(65.235)	1040.973**	(60.068)
Fixed Effects								
Level 1: Student								
Student Covariates								
Male								
African American			-58.024**	(14.360)	-57.176**	(13.936)	-57.518**	(14.699)
Free/red lunch			-43.424**	(10.788)	-42.126**	(10.833)	-43.489**	(10.760)
Previous GPA			83.125**	(17.717)	83.688**	(18.362)	82.375**	(17.746)
Student Report Classroom Climate								
Tasks								
Student Authority								
Support								
Level 2: Classroom								
Teacher Covariates								
Male								
African American			55.972**	(16.873)	55.840**	(16.619)		
Years experience			3.267*	(1.371)	3.174*	(1.369)	3.475*	(1.454)
Highest degree								
Teacher Report Classroom Climate								
Tasks								
Student Authority								
Support								
Random Effects								
Between Level Variance, u_0	9801.235**	(19990.044)	3317.365**	(1293.353)	3349.30**	(1356.31)	3189.945**	(1239.28)

	Model 1: Fully Unconditional Model		Model 2: Covariates Only		Model 3: Student Report		Model 4: Teacher Report	
	β	(S.E.)	β	(S.E.)	β	(S.E.)	β	(S.E.)
Within Level Variance, r	27161.84**	(5808.066)	24700.03**	(6159.211)	24099.78**	(9.484)	24708.611**	(6153.61)
AIC	18508.65		32905.86		41494.31		33407.82	
BIC	18524.39		33062.74		41905.46		33617.00	
Pseudo R-Squared								
Level 1			0.091		0.113		0.090	
Level 2			0.662		0.658		0.675	

*p<.05, **p<..001

Table 24

Achievement emotions categories as outlined by Pekrun and colleagues (2007), used for round 1 coding of student emotions when stuck

Object Focus	Positive		Negative	
	Activating	Deactivating	Activating	Deactivating
Activity Focus	- Enjoyment	- Relaxation	- Anger - Frustration	- Boredom
Outcome Focus	- Joy - Hope - Pride - Gratitude	- Contentment - Relief	- Anxiety - Shame - Anger	- Sadness - Disappointment - Hopelessness

Table 25

Achievement emotion categories, revised for round 2 coding of student emotions when stuck

Object Focus	Positive		Negative	
	Activating	Deactivating	Activating	Deactivating
Activity Focus	- Positive (Enjoyment, Excitement)		- Anger or Frustration	- Bad
			- Confusion	- Confusion
Outcome Focus	(Positive Challenge)		- Anger or Frustration	- Helpless/shameful
			- Anxiety, stress, somatic complaints	(Negative challenge)

Table 26

Examples and frequency of students' emotional experiences when they are stuck working on math problems

Code	Examples	Percent
Anger or frustration	"It is a huge challenge and it can be very frustrating and I just want to be able to solve it." "It's frustrating that I can't solve it. I get mad and stressed."* "I get angry sometimes because I've tried so many ways" "I can get frustrated sometimes and I just wish I was smart enough to understand."	19.58
Anxiety or stress	"Terrible when I'm stuck… I feel scared to get it wrong." "It feels stressful and useless if I don't know the answer." "It feels nerve racking and straining." "I panic when I get stuck." "I get sweaty and over think." "It feels very stressful because I don't know if I'm doing it right or not."	10.58
Helplessness or shame	"What I think is that 'it's impossible I will never get it." "When I'm stuck, I first sit and think slowly and then I'll ask for help… even though I feel like I fell into an abyss." "I feel alone and like I'm the only one that does not know it." "I feel embarrassed." "When I'm stuck… I feel like I have failed."	3.17
Bad (non-descript)	"I don't feel good. It feels weird" "I feel upset"	10.05

Code	Examples	Percent
	"It feels bad."	
Confusion	"I feel really confused" "I keep wondering 'how do I solve this problem?'" "The confusion of being stuck"	13.01
Positive	"It feels really good. I ask a friend. (I get a little frustrated) but then I think different ways to find the answer." "It feels great, I love it. Nothing goes through my mind, except solve it." "I like problems even better when I'm stuck." "I like facing a challenge. Whenever I am stuck, I like working that much harder to get the answer."	3.17
Other	"I feel weird" "I feel challenged" (valence unclear) "It feels hard" (valence unclear) "My mind goes blank"	6.00

Table 27

Descriptive statistics of variables for study 2 (Students' emotions when stuck in mathematics class, n=214)

	Percent	***M***	**SD**	**Cronbach's α**	**%Agreement**
Self-Competence in Math					
November		3.197	.645	.868	
May		3.194	.631	.864	
Frequency of getting stuck		1.445	.870		
Experience of being stuck					
None/neutral	41.27%				90%
Anger/frustration	19.58%				88%
Anxiety/stress	10.58%				87%
Helplessness/shame	3.17%				89%
Bad (non-descript)	10.05%				87%
Confusion	13.01%				82%
Positive	3.17%				92%
Other	6.00%				n/a
Demographics					
Free or reduced lunch	8.41%				
Female	56.71%				
Minority	11.13%				
Previous math grade		8.872	1.612		
Honors	25.70%				

Table 28

Differences in students' emotions being stuck by student demographics

	Stress/ Anxiety		Helplessness/ Shame		Bad/non-descript		Anger/ Frustration		Positive	
	%	X^2	%	X^2	%	X^2	%	X^2	%	X^2
Gender										
Female	9.8	.02	4.9	1.30	10.4	.01	7.7	1.28	4.9	3.36*
Male	10.4		1.5		11.0		20.7		0	
Minority										
Minority	0	1.61	15.4	6.29**	23.1	2.22	28.3	3.61*	0	.40
Caucasian	11.1		2.2		9.6		16.0		3.0	
Free lunch										
Eligible	25.0	2.05	0	.30	12.5	.03	46.1	.14	0	.63
Not eligible	9.3		3.6		10.7		40.7		2.9	
Course Level										
Honors	10.7	.02	5.7	1.36	9.4	1.75	0		5.7	.23
General	11.9		2.2		12.2		19.3	1.89	1.5	

*p<.07, **p<.05, ***p<.001

Table 29

Mean differences in the frequency of being stuck when working on math problems by demographics and emotional experiences of being stuck

	M	SD	F-Statistic
Gender	1.52	.89	.64
Female			
Male	1.40	.84	
Minority			
Minority	1.71	1.14	1.37*
Caucasian	1.43	.83	
Free lunch eligibility			
Eligible	2.00	1.00	3.55
Not eligible	1.44	.86	
Math ability placement			
Honors	1.42	.82	.01
General	1.43	.89	
No emotions reported			
Yes	1.48	.85	1.15
No	1.34	.86	
Anxiety/Stress			
Yes	1.7	.87	2.42
No	1.39	.86	
Hopeless/Shame			
Yes	2.17	.41	4.82**
No	1.39	.86	
Bad/non-descript			
Yes	1.68	1.00	2.03
No	1.38	.84	
Anger/Frustration			
Yes	1.26	.78	1.57
No	1.46	.87	
Confusion			
Yes	1.34	.81	1.31
No	1.20	.79	
Positive			
Yes	1.2	.84	.34
No	1.43	.86	

*p<.07, **p<.05, ***p<.001

Table 30

Predicting students' self-competence in math at the end of 7th Grade

	Self-Competence in Math (June)		
	Model 1	Model 2	Model 3
Demographics			
Minority	.070	.119	.082
Female	.073	.047	.071
Free or reduced lunch	-.242**	-.222**	-.127*
Previous math grade	.090	.055	-.041
Previous Attitude			
Self-competence in math (October)	.481***	.480***	.295***
Classroom			
Teacher	.044	.038	.033
Honors	-.153**	-.169	-.101*
Stuck Emotions			
Anger/Frustration		.146**	.100
Stress/Anxiety		-.037	-.013
Helplessness/Shame		-.063	.018
Bad		-.027	-.014
Confusion		-.039	-.011
Positivity		.077	.066
Stuck Frequency			-.561***
R^2	.286	.342	.554

*p<.07, **p<.05, ***p<.001

Figure 1

Vygotsky's portrayal of the zone of proximal development (1976)

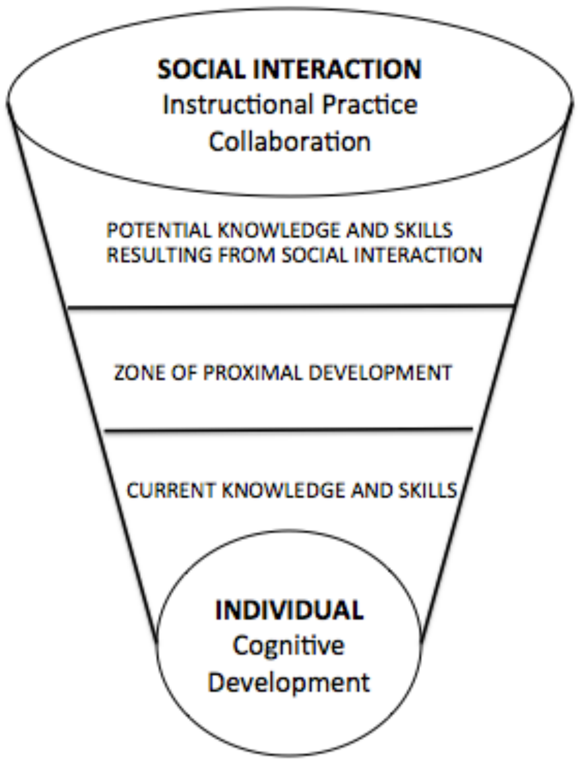

Figure 2

Three innate psychological needs that comprise the self-determination theory of student motivation (Deci & Ryan, 2000)

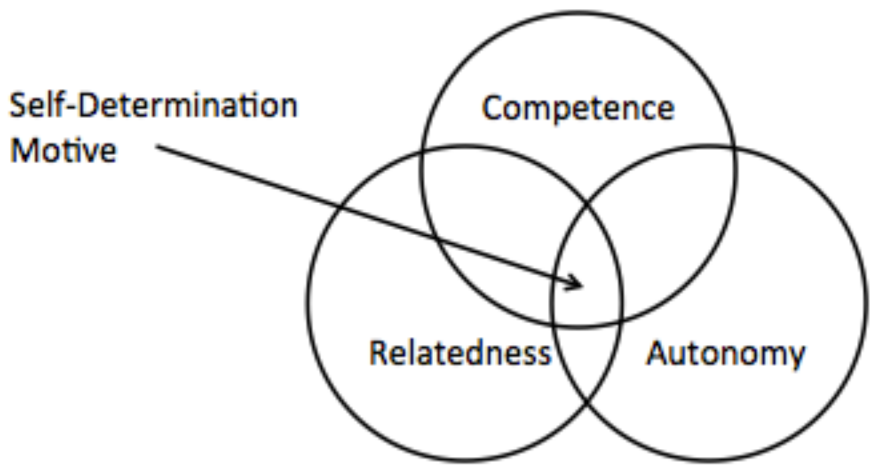

Figure 3a

Model illustrating how mathematics instruction, as a feature of context, influences engagement and achievement through meeting adolescents' psychological needs, adapted from a pictorial representation of the self-determination model of motivational development printed in Skinner and Chi (2012)

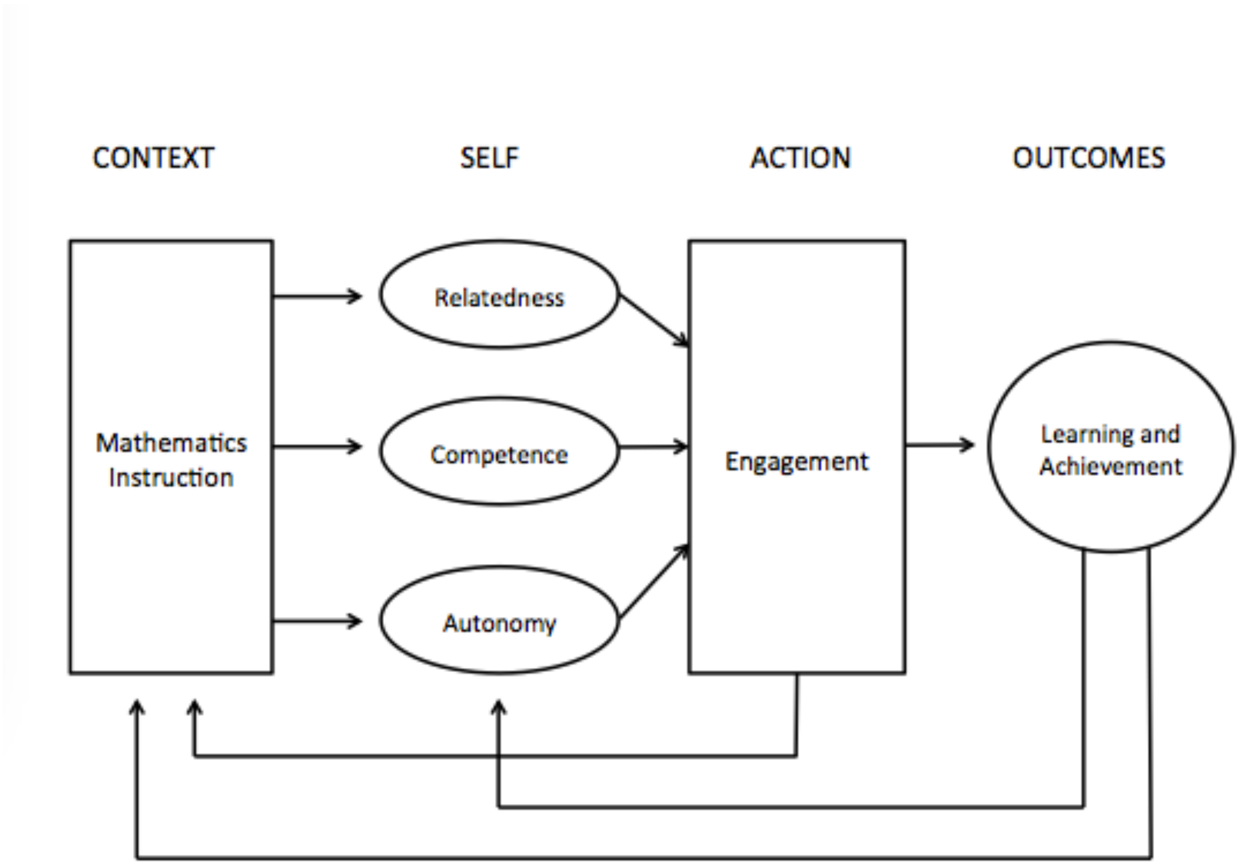

Figure 3b

Further adapted figure of a pictorial representation of the self-determination model of motivational development (Skinner & Chi, 2012) illustrating the ways in which components of student-centered mathematics instruction meet students psychological needs in ways that shape learning outcomes by influencing their engagement in mathematics tasks and coursework.

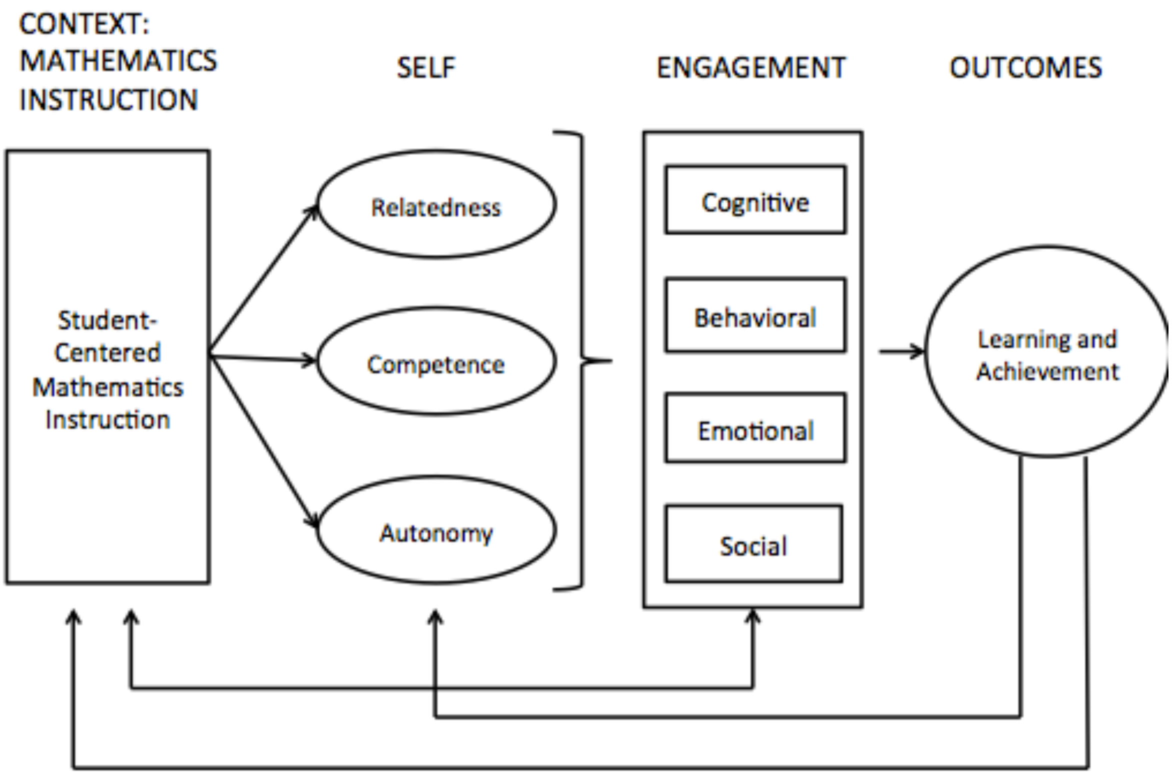

Figure 4

Hypothesized direct and indirect effects of student-centered mathematics instruction on engagement

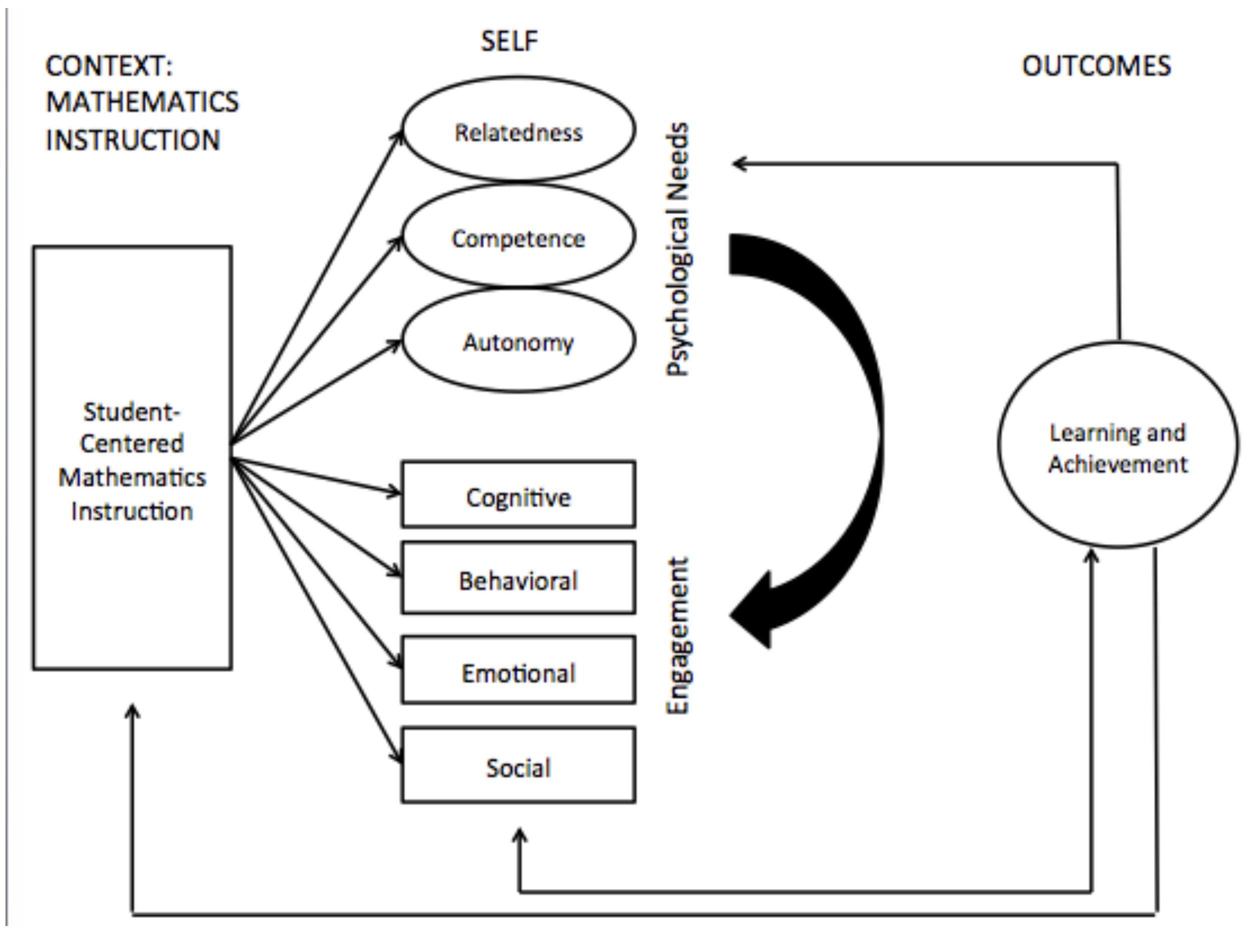

Figure 5

How the components of SCI relate to constructivist theories of learning and Self-Determination Theory of motivation

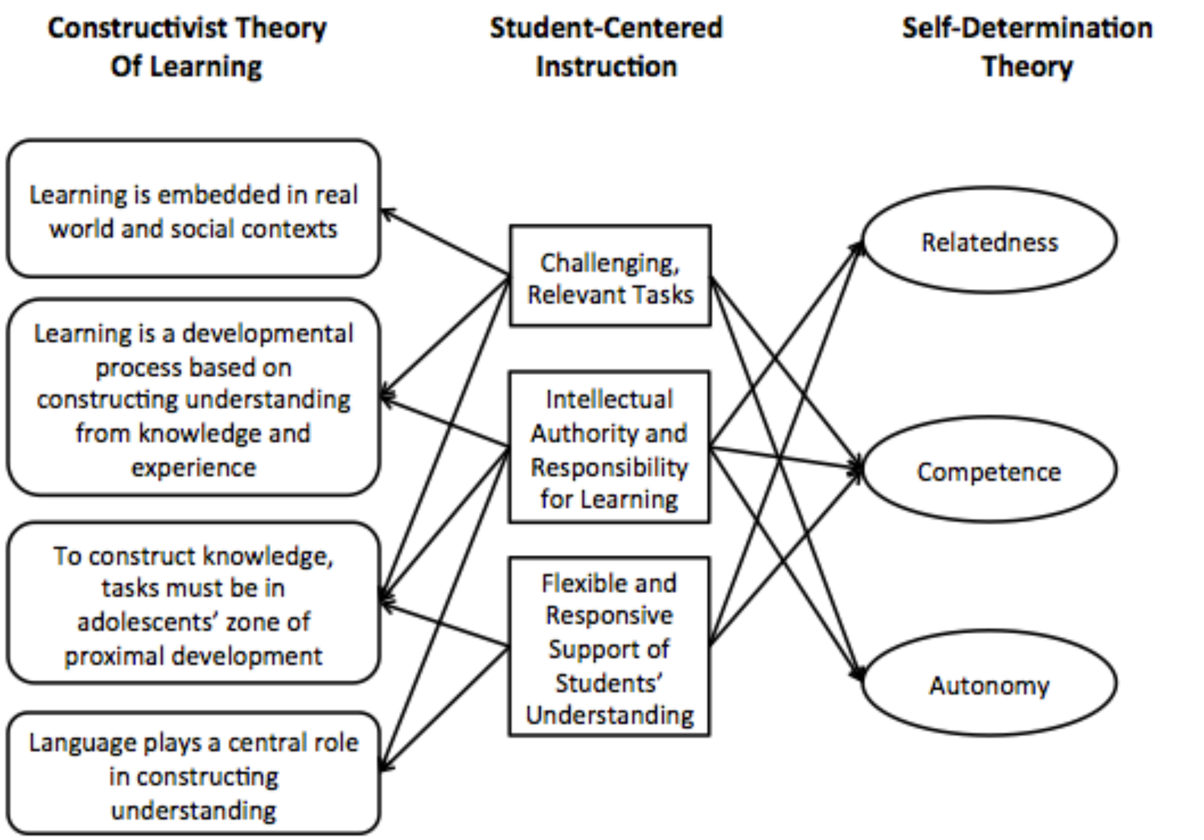

Figure 6

Integrating constructivist theories of learning with Self-Determination of motivation to understand the effects of student-centered mathematics instruction on student engagement and mathematics

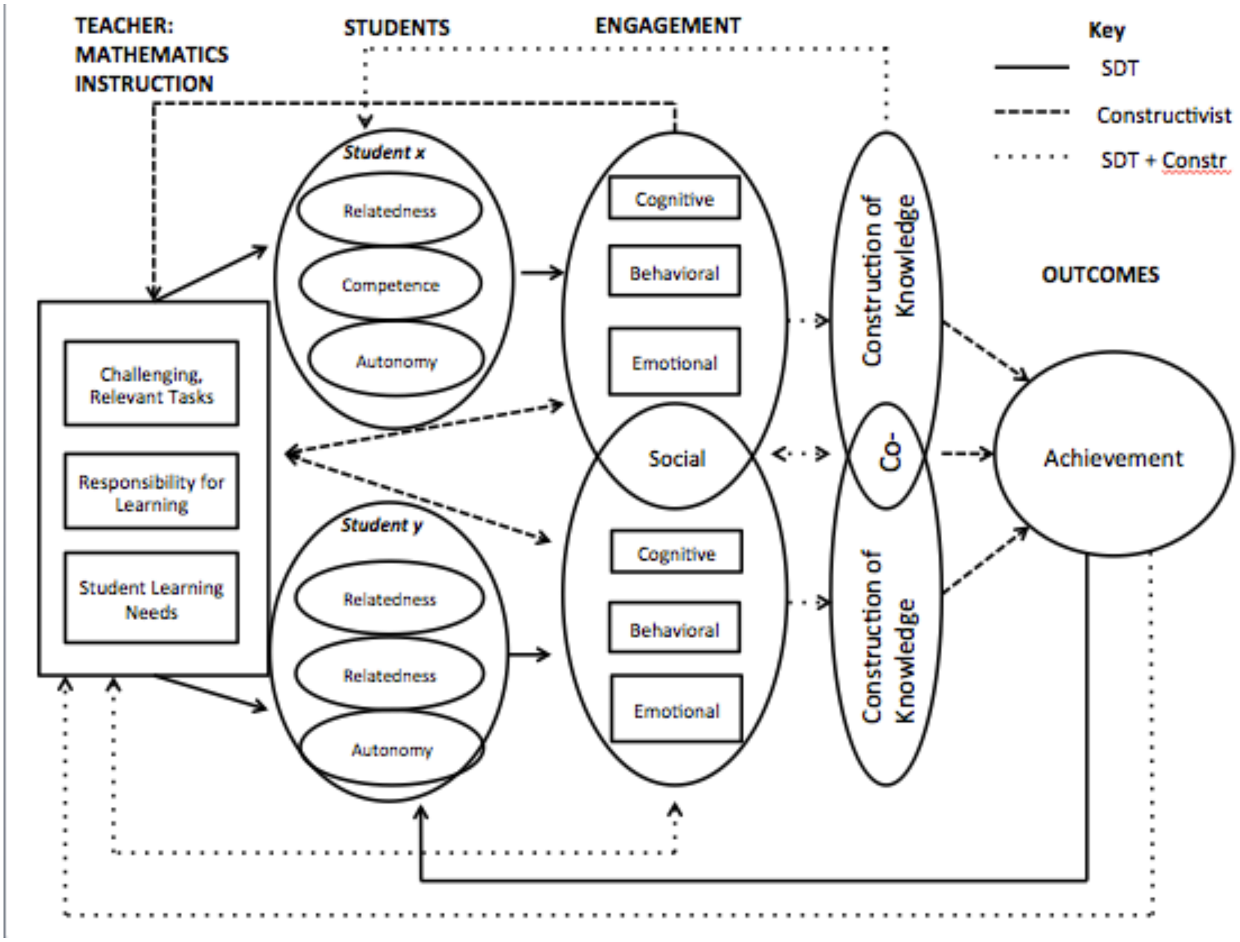

Figure 7

Figure 6 revised to illustrate research questions examining predictive associations of student-centered mathematics instruction with engagement and achievement

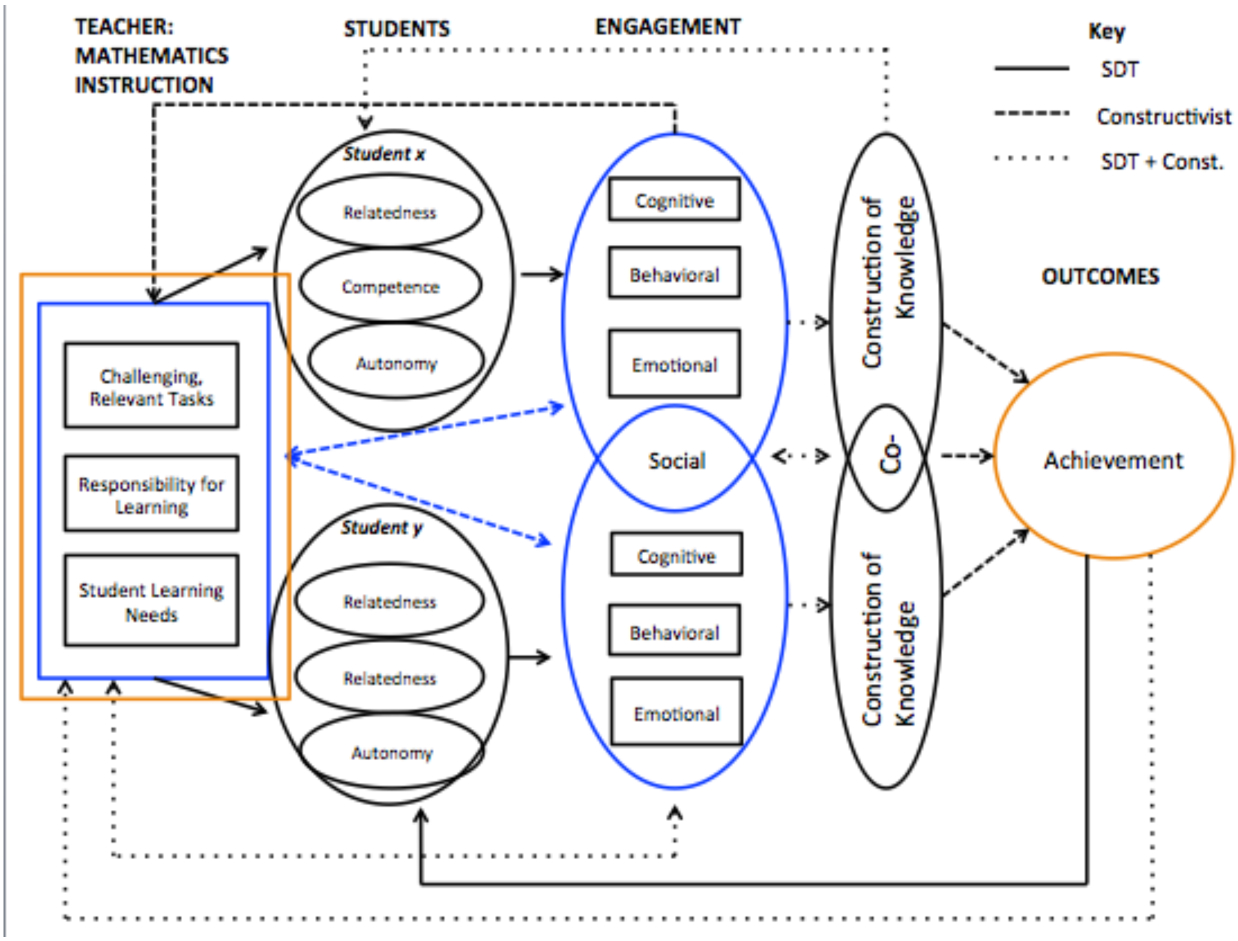

Figure 8

Overview of control value theory of emotions (Pekrun et al., 2007)

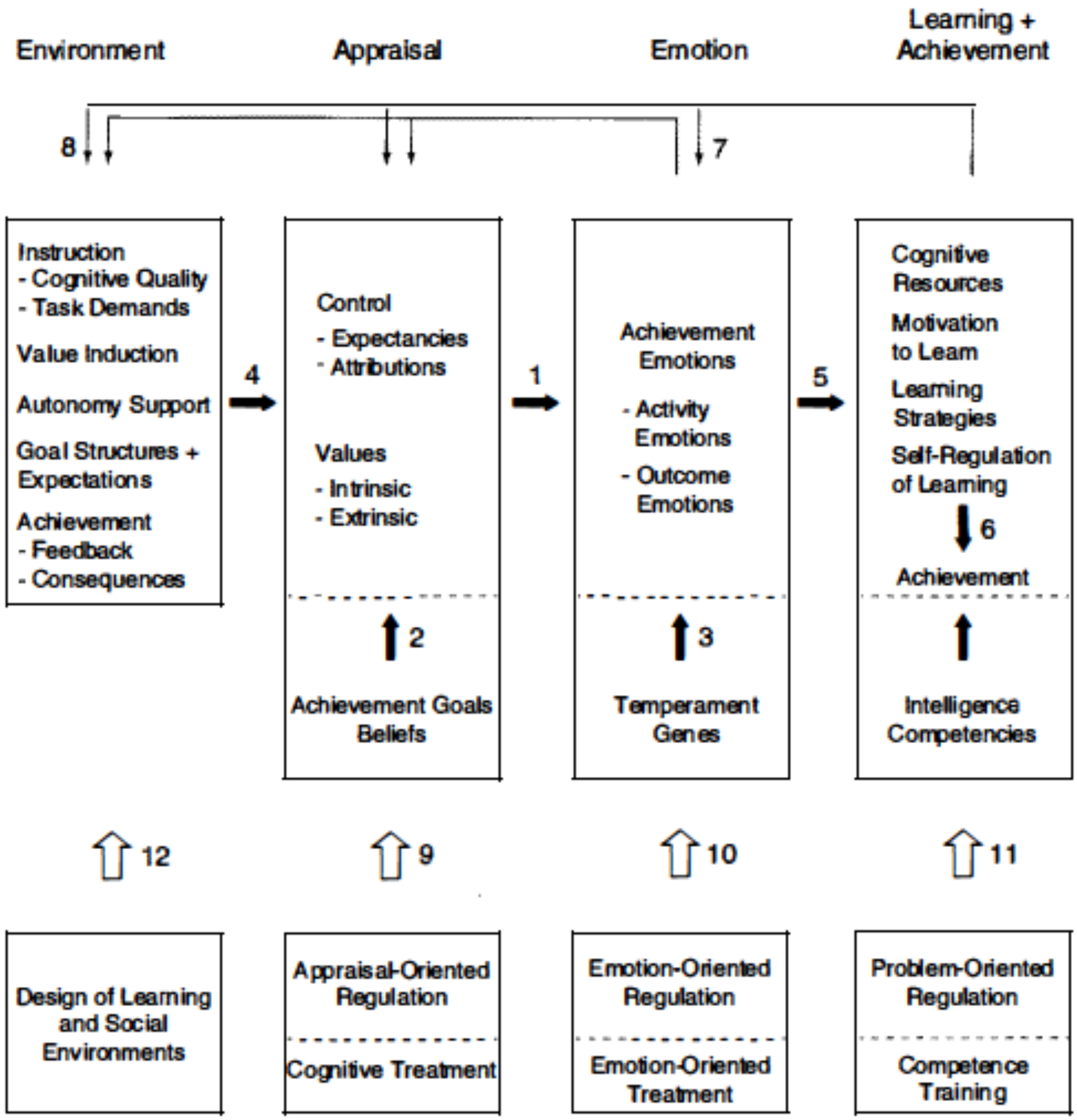

Figure 9

Overview of control and value appraisals of Control Value Theory of Achievement Emotions (Pekrun 2006)

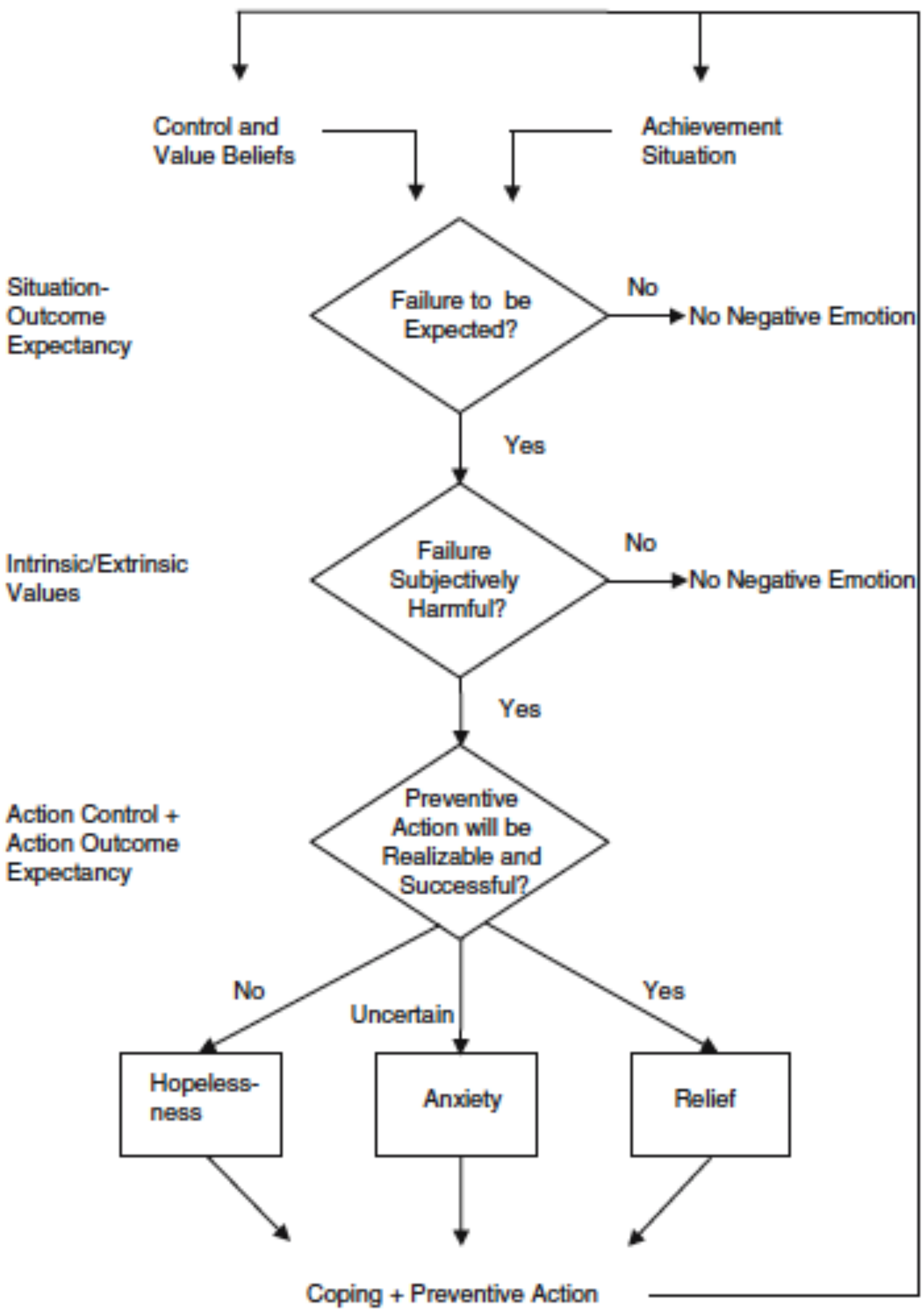

Figure 10

Average distribution of student-centered instruction activity format for Mr. Smith

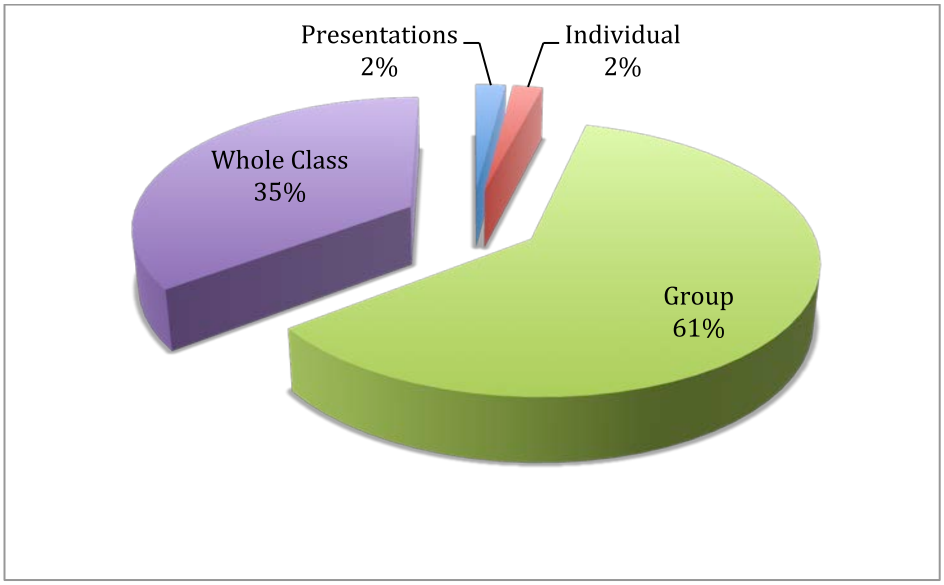

Figure 11

Average distribution of student-centered instruction activity format for Mrs. Jones

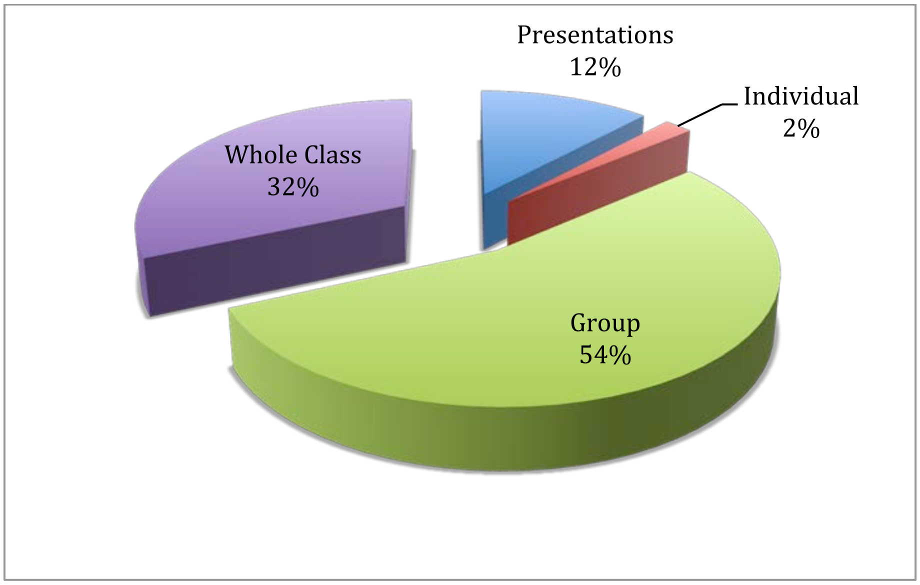

Figure 12

Average time spent in each activity type for Mr. Smith by class period

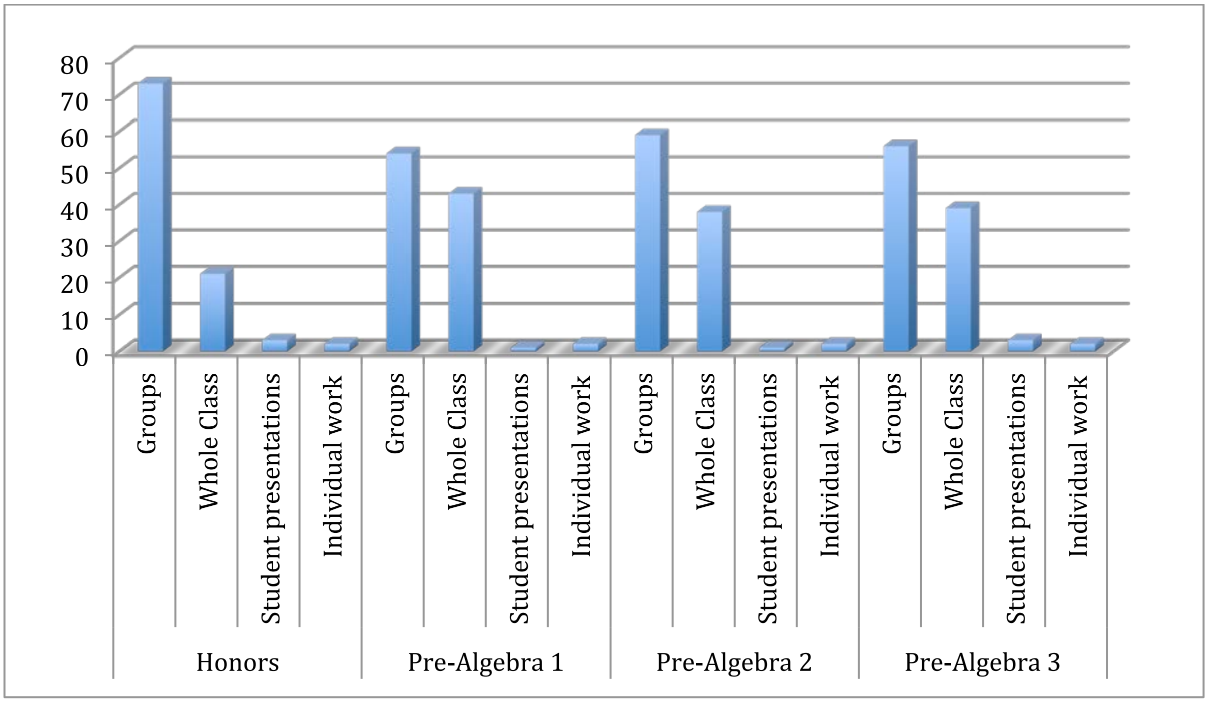

Figure 13

Average time spent in each activity type for Mrs. Jones by class period