Topological description of periodic structures of an asymmetric nonlinear oscillator

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Summary. The bifurcation structure of periodic solutions of a harmonically driven asymmetric nonlinear oscillator (Rayleigh–Plesset equation, describing bubble dynamics) is examined. The control parameters were the amplitude and frequency of the driving with frequency values higher than the subharmonic resonance frequency of the system. In the investigated parameter region, the endoskeleton of the bifurcation structure, composed by solutions with low periodicities, can be described by an asymmetric Farey-ordering tree. To each periodic domain, a sub-structure can be associated created by period-n tupling processes, whose topology are governed by a two-sided symmetric Farey tree. Higher order sub-structures apparently exhibit self-similar features.

Topology of the bifurcation structure

The extensive study of harmonically driven nonlinear oscillators has revealed several topological universalities in the last few decades with respect to a single control parameter. For instance, the standard Feigenbaum period doubling cascades or the alteration of periodic and chaotic windows via crises [1]. The topological description in two or more dimensional parameter space, however, is less elaborated. The present investigation intends to extend our knowledge on bi-parametric bifurcation structure in the amplitude-frequency parameter plane of the driving via thorough numerical analysis.

The mathematical model describing the radial oscillation of a single spherical gas bubble in water (Rayleigh–Plesset equation [2]) can be written as

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho_L}\left(p_G + p_V - \frac{2\sigma}{R} - 4\mu_L\frac{\dot{R}}{R} - P_\infty - p_A\sin(\omega t)\right), \qquad (1)$$

where $R(t)$ is the time dependent bubble radius, $\rho_L = 997.1\,\mathrm{kg/m^3}$ is the liquid density, p_G is the gas pressure inside the bubble following a simple polytropic state of change, $p_V = 3166.8\,\mathrm{Pa}$ is the vapour pressure, $\sigma = 0.072\,\mathrm{N/m}$ is the surface tension, $\mu_L = 8.902^{-4}\,\mathrm{Pa\,s}$ is the viscosity, $P_\infty = 5458\,\mathrm{Pa}$ is the ambient pressure, p_A is the amplitude and ω is the frequency of the driving.

Figure 1 shows the bifurcation structure of the periodic solutions as a function of the pressure amplitude p_A at frequency approximately 3 times the linear eigenfrequency. The bold fractions denote the winding numbers [3] of the saddle-node bifurcations of the corresponding periodic windows. The main periodic structure is governed by a winding number sequence $1/2$, $1/3$, ..., $1/9$ with an increment of $0/1$ constituting an asymmetric Farey tree. The sub-structures associated to each main periodic windows, however, form a double sided Farey tree. For instance, in case of the periodic domain of order $1/3$ (see the left hand side of Fig. 1): $2/7$, $3/10$, ... $1/3$ ..., $5/18$, $4/15$, $3/12$, $2/9$. The increment is exactly $1/3$ from both sides. According to more detailed simulations, the higher order sub-structures show the same double-sided topological behaviour.

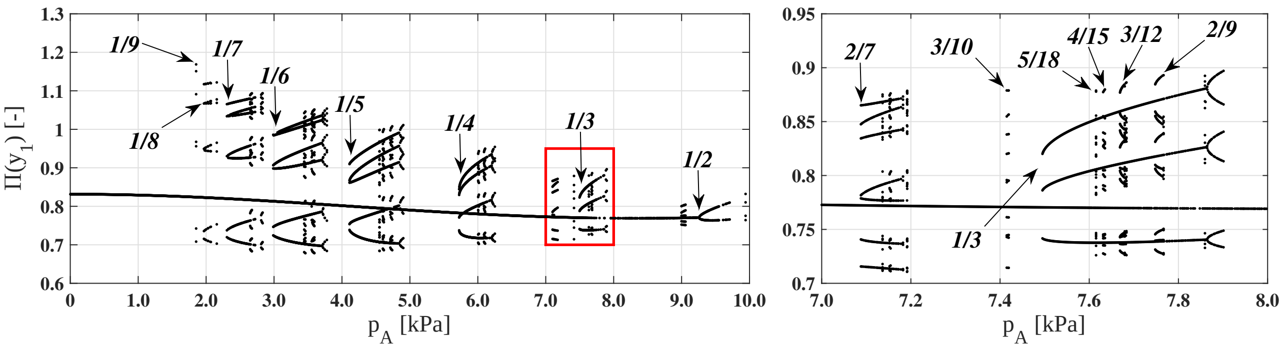

Figure 1: First component of the Poincaré section of the dimensionless bubble radius as a function of the pressure amplitude.

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