

A THEORETICAL APPROACH TO ESTIMATE THE TIME LAG OF BUILDING ENVELOPES

Assoc. Prof. Dr. Akos Lakatos¹

¹ University of Debrecen Faculty of Engineering, **Hungary**

ABSTRACT

The investigations of the building elements from the point of view of energy engineering became very important since the last 50 years. Insulation of the building envelope is one of the keys to reduce the heating energy loss. The mainly used thermal insulating materials are Expanded Polystyrene (EPS), Extruded Polystyrene (XPS) and mineral wool materials, moreover the nano-sized insulators (e.g.: aerogel, vacuum insulations etc.) are requiring spaces for themselves also. Aerogels are nanoporous lightweight materials that were discovered more than 70 years ago. Nowadays their applications are truly widespread. In this article measurement results and theoretical backgrounds are presented. Thermal parameters of building envelope, such as heat storage capacity, heat-loss coefficient, time constant, time lag, heat transfer coefficient etc. are very useful during the analysis of thermal behavior of buildings. This paper describes a methodology developed to calculate the time lag based on our previous measurement results. The thermal diffusion and time lag calculations are based on the Onsager-equations and atomic diffusion rules.

Keywords: Time lag, building structures, error function, aerogel

INTRODUCTION

In the European Union buildings account for a 20–40% of the total final energy consumption [1]. Because no one of the member states is independent from energy point of view, in the building sector the main goals are the increase of energy efficiency and utilization of renewable energy sources. This is part of the 20-20-20 EU target. To fulfil the fixed goals several Directives were prepared. One of the Directives is the 2002/91/EC Directive dealing with energy performance of buildings. [2] This Directive in 2010 was revised and adopted as 2010/31/EU Directive [3]. Buildings are also responsible for CO₂ emissions with a consequential impact on global warming and changing the green-house effect. The building envelope is the physical shell that separates the interior of the building from the outdoor environment. The purpose of the envelope of a building is to act as a passive climate modifier to help in maintaining an indoor environment more suitable for habitation than the outdoors. [4-7] At first, thermal conductivity measurement results of individual insulation materials were achieved by a Holometrix type Heat Flow Meter. Moreover, in this paper a method to calculate the time lag of different building structures is presented. Two different walls were investigated: an ordinary 30 cm thick brick wall with (2 cm thick) plaster at both sides, and the same wall covered with 1,3 cm thick aerogel insulation. The time delay ability due to the thermal mass is known as the time lag.

THEORY AND CALCULATIONS

The diffusion of atoms and time lag

In order to fully understand flux, diffusion must first be defined. Diffusion is caused by random molecular motion that leads to complete mixing. It follows then that flux can be described as “the rate per unit area at which mass moves. [8, 9] Numerical calculations were done to predict the thermal diffusivity of the materials, the time lag. If we represent the Onsager equation as Eq.1:

$$J = -D \times \text{grad}(X) \quad (1)$$

where J is the flux of an extensive property (eg.: atomic flux, heat flux etc.) and X is the intensive physical property (eg.: atomic concentration, temperature). By using Eq 1 we can make a connection between the first (Eq. 2) and the second law of Fick (Eq. 3) with the thermal conductivity equations (Eq 5. (Fourier) and 7.):

$$J_A = -D_A \times \text{grad}(C) \quad (2)$$

where J_A is the diffusion flux of the atoms in a given substance, D_A is the atomic diffusion coefficient and $\text{grad } C$ is the concentration gradient of the diffusing atoms. Representing the 2nd law of Fick in one dimension and assuming that there are no sources and D_A is constant, the following equation can be reached:

$$\frac{\partial C}{\partial t} = D_A \times \frac{\partial^2 C}{\partial z^2} \quad (3)$$

where t is the time and z is the direction. [9, 10]

There are some cases, when the diffusion between pure two A and B materials cannot be represented as a Gaussian function, since the concentration profiles have a complementary error function dependence on depth near the interface and can be modelled as Eq 4 [11]

$$C(z, t) = \frac{C_0}{2} \times \text{erfc}\left(\frac{z}{2 \times \sqrt{D_A t}}\right) \quad (4)$$

If one is measuring the rate of flow of a gas through a membrane in which the gas dissolves there will be an interval from the moment the gas comes into contact with the membrane until it emerges at a constant rate at the other side. In atomic diffusion is called to delay or retardation time of the diffusion barrier.

The thermal diffusion

If we represent now the main equation of the thermal conduction which is the modified Fourier's law, a form of the equation is similar to the above mentioned ones (Eq 1-2) can be found:

$$J_q = -D_T \times \text{grad}(T) \quad (5)$$

where J_q is the heat flux and $\text{grad } T$ is the temperature (T) gradient, however,

$$D_T = \frac{\lambda}{\rho \times c_p} \quad (6)$$

is the thermal diffusion coefficient, λ is the thermal conductivity, ρ is the mass density and c_p is the specific heat of the material. These predicted D_T values were taken from our previous measurements [6] If we represent the modified Fourier's law (Eq 5) in one dimension the following equation can be reached by using the following assumptions: the sample is free from heat sources and D_T is constant:

$$\frac{\partial T}{\partial t} = D_T \times \frac{\partial^2 T}{\partial z^2} \quad (7)$$

Similarly to Eq 3 and 4 the temperature profiles (Eq 7 and 8) can also be modelled as a complementary error function, too:

$$T(z,t) = \frac{T_0}{2} \times \operatorname{erfc}\left(\frac{z}{2 \times \sqrt{D_T t}}\right) \quad (8)$$

From (Eq. 8) the t (in h) as the time lag can be reached as:

$$t(h) = \frac{1}{4 \times \operatorname{Const}^2 \times D_T} : 3600 \quad (9)$$

The thermal conductivity measurements

The thermal conductivity measurements were carried out after drying the brick and aerogel samples in a VentiCell drying instrument to changeless weight. [6] With this device materials can be dried setting different air temperatures (up to 523 K). It works with hot air circulation using an inbuilt ventilator. For measuring the thermal conductivity of polystyrene samples a Lambda 2000 Heat flow meter (HFM) was applied. This equipment is designed to determine the thermal conductivity of insulation materials in accordance with standard ASTM C518 and ISO 8301 protocols. The samples with 30 cm x 30 cm area and with 1,3 (aerogel) and 10 cm (brick) height geometry were placed in the test section between two plates which are maintained at different temperatures ($T_1=285$ K and $T_2=295$ K, with $T_{\text{mean}}=290$ K) during the test. Through the measurements in the Holometrix application, if the instrument is achieving thermal equilibrium and establishing a uniform temperature gradient throughout the sample, thermal conductivity is determined. To determine the thermal conductivity of a sample, five independent measurements were carried out. The thermal conductivity of analysed material was the average of the three measured values. [6]

In table 1 the results of the thermal conductivities can be found. For the thermal conductivity of the brick 0.57 ± 0.002 , for the aerogel 0.0214 ± 0.0006 W/mK was reached. The values of the plasters were taken from the database of the manufacturer all the used input data can be found in Table 1.

Table 1. The material's constant

Material's name	Density [kg/m ³]	Specific heat capacity [J/kgK]	Thermal conductivity [W/mK]
Plaster	1650	920	0.81
Brick	1400	880	0.57
Aerogel	135	1000	0.0214
Plaster	1850	880	0.99

DETAILS AND APPLICATION OF THE METHOD

The algorithm of the calculation method

If we have a basic wall e.g.: B30 brick wall with 0.3 m thickness covered with 2*2 cm thick plaster, its time lag can be easily calculated by following the downer algorithm.

Step 1.:

Give the input data, see Table 1:

Step 2.:

By using Eq. 6, the thermal diffusivity of the wall can be reached, in this case is $D_T=4.75E-7 \text{ m}^2/\text{s}=0.00475 \text{ cm}^2/\text{s}$.

Step 3.:

If the thickness of the wall is $d=0.3 + 0.02*2 \text{ m}=34 \text{ cm}$ by the overall heat transfer coefficient (U) can be calculated as the following:

$$U = \frac{1}{\frac{1}{h_i} + \sum_j \left(\frac{d_j}{\lambda_j} \right) + \frac{1}{h_e}} = \frac{1}{R_{all}} \quad (10)$$

where, h_i, e are the internal and external heat transfer coefficients (8 and 24 W/m²K) on the surface, λ_j is the thermal conductivity and d_j is the thickness of a given layer of the wall in m, now $j=1$. For the simplification now only one layer is used. R_{all} is the overall resistance of the wall structure.

Step 4.:

If we suppose $T_i=-15 \text{ }^\circ\text{C}$ and $T_e=20 \text{ }^\circ\text{C}$ for the external and internal air temperature, the temperature at an optional surface (T_s) can be reached by using:

$$T_s = T_i - (T_i - T_e) \times \frac{R_z}{R_{all}} \quad (11)$$

By using Eq. 11 the temperatures on the sides of the wall are found to 14.07 and -13.02 °C.

Step 5.:

If we divide the temperature difference between the internal and external surface of the wall to 34 equal part:

$$\Delta T = \frac{14.07 - (-13.02)}{34} = 0.8 \tag{12}$$

we will reach a temperature profile.

This temperature profile can be seen in Figure 1.

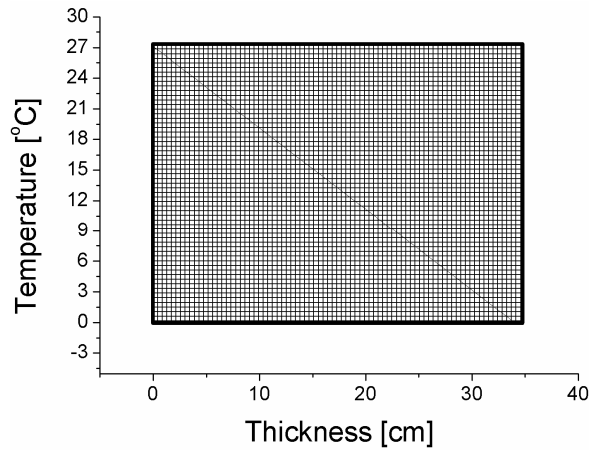


Figure 1. The temperature distribution inside the wall

Step 6.:

If we normalize the temperature values meaning we divide with the maximal temperature, in this case with 27.09 we will reach a temperature distribution. This distribution can be seen in Figure 2 with the straight line T/T_0 .

Step 7.:

Moreover, we have to generate the error function of the numbers from 0 to 34 (representing the thickness of the brick wall) with a random number error function generator and then we have to take its complementary function. Then if we plot the complementary error function numbers in function of the numbers from 0 to 34 we will reach the erfc function see Figure 2.

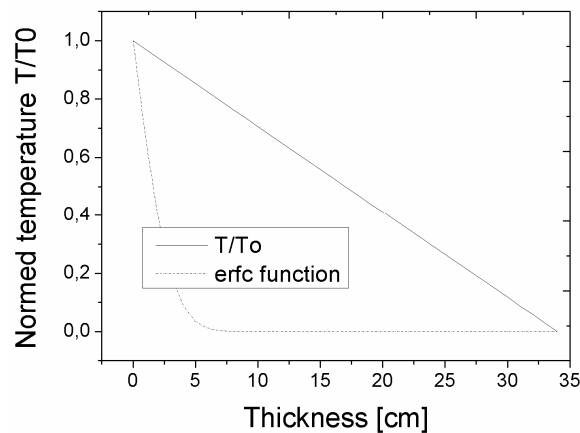


Figure 2: The complementary error function of random numbers (1 to 34) and the temperature profile of a given wall

Step 8.:

Finally, by using the thermal diffusivity $D_T=0.00475 \text{ cm}^2/\text{s}$ from Eq 6. and we change the value of the Constant in the argument iteratively, in this case $\text{Const}=0.02948$, for the time lag

$$t(h) = \frac{1}{4 \times \text{Const}^2 \times D_T} : 3600 = 16.8 \quad (13)$$

can be reached. On figure 3 we can see the overlapping of the T/T_0 and erfc functions for the Brick wall.

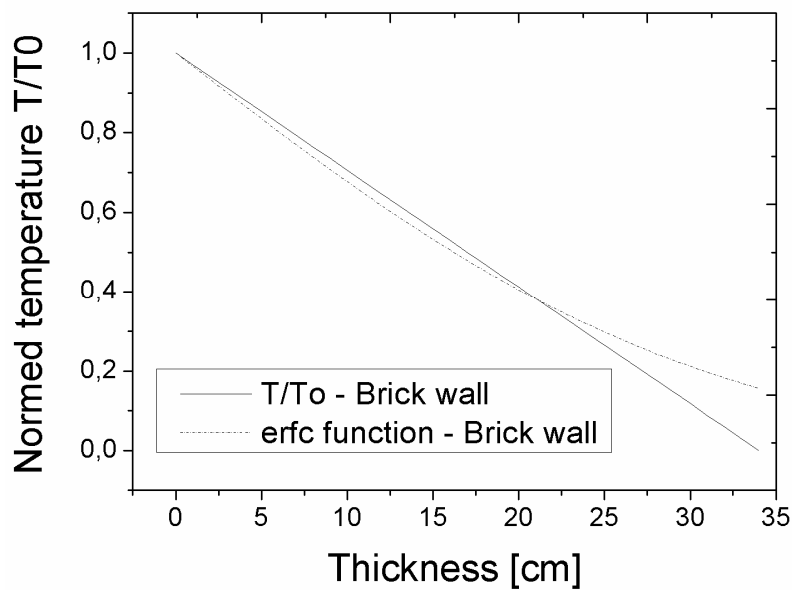


Figure 3: The complementary error function as a fit of a temperature profile of a given wall

Silica aerogel nano-insulation

The aerogel blankets were developed as an insulation material based on silica aerogels. Silica aerogel is an aggregate, nano-porous, insulation material, produced using a sol-gel process and supercritical evaporation technology. The material has reasonable insulating performance due to its nano-porous structure (several to tens of nanometers) and very small solid grain size (2–5 nm). However, the major disadvantage of using monolithic silica aerogel for thermal insulation is that it is brittle and easily broken. The heat transfer mechanisms for silica aerogel and its associated composites include solid conduction, gas conduction and thermal radiation, all of which have been extensively investigated in recent years. There are several types of aerogel samples with densities from 3 to 150 kg/m^3 , and thermal conductivities from 0.01 to 0.022 $\text{W}/\text{m}\cdot\text{K}$ at ambient temperature and atmospheric pressure. We investigated one with 135 kg/m^3 density and with 0.0214 $\text{W}/\text{m}\cdot\text{K}$. [11-13] The same algorithm was carried out on the above mentioned wall covered with 0.013 cm thick aerogel insulation with the measured thermal conductivity. The used data for the calculations can be found in Table 2. From

the calculations for the time lag of the brick wall covered with 1.3 cm aerogel insulation approximately 18 hours were reached. On figure 4 we can see the overlapping of the T/T_0 and erfc functions for the Brick wall with aerogel.

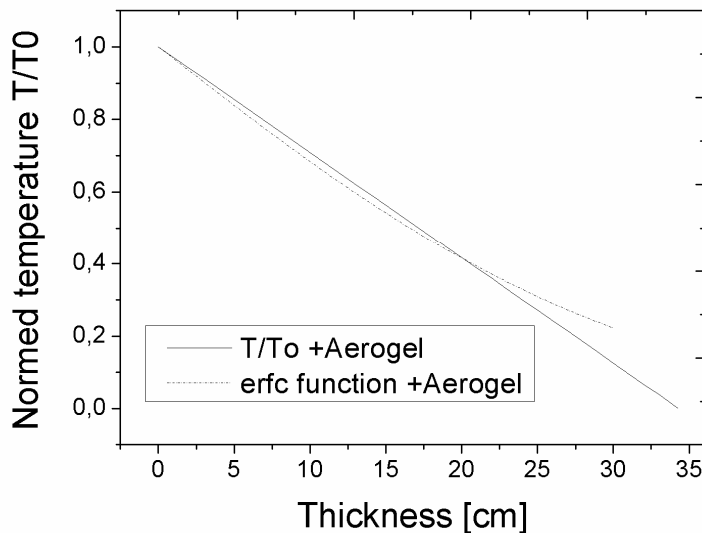


Figure 4: The complementary error function as a fit of a temperature profile of a given wall with aerogel

Table 2. The input data with the aerogel

	Dt [m^2/s]	Dt [cm^2/s]	d [m]	$U_{brick+aerogel}$ [W/m^2K]	
Aerogel	1.59E-07	0.00159	0.013	0.74	
	T_{is} [$^{\circ}C$]	T_{es} [$^{\circ}C$]	ΔT [$^{\circ}C$]	Const	t [h]
	16.733	-13.91	30.643	0.0288	18.1

CONCLUSION

Energy conscious building design consists in controlling the thermophysical characteristics of the building envelope such as, firstly, thermal transmittance (U-value). However, besides the U-value, the envelope thermal inertia should also be considered. Besides the laboratory measurements of the building and structural materials, calculations, predictions, modelling and simulations are also important when we design buildings. The effect of using heat generated during the day to warm at night in winter and vice versa in summer is known as the 'thermal flywheel' effect. The effectiveness of the flywheel depends on the time lag introduced to a building by an external wall or other boundary element. Time lag is the time delay between external maximum or minimum temperatures and internal maximum or minimum temperatures respectively. As a result, in this study a novel mathematical approach to determinate the time lag of building structures is presented. The main goal of this paper to present a clear method for the analytical solution of the time lag by using the complementary error functions as an example. The method was based on the well known equations and rules of atomic diffusion. For validating the theory measurement results were used as a base of the calculations. This method should be very useful for building scientists working in

energy conservation and savings, and for designers building nearly zero energy or passive houses as well, independently from place and residence.

ACKNOWLEDGEMENTS

This paper was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

REFERENCES

- [1] Pe´rez-Lombard, L., Ortiz, J., Pout, C. A Review on buildings energy consumption information. *Energy and Buildings* 2008;40:394–398
- [2] Directive 1 2002/91/EC of the European Parliament and of the Council of 16 December 2002 on the energy performance of buildings. *Official Journal of the European Communities*, L 1/65, 4.1. 2003
- [3] Directive 2 2010/31/EU of the European Parliament and of the Council of 19 May 2010 on the energy performance of buildings. *Official Journal of the European Union*, L 153/13, 18.6.2010.
- [4] Aste, N., Angelotti, A., Buzzetti, M. The influence of the external walls thermal inertia on the energy performance of well insulated buildings. *Energy and Buildings* 41 2009, pp 1181–1187
- [5] Asan, H. Investigation of wall’s optimum insulation position from maximum time lag and minimum decrement factor point of view *Energy and Buildings* 32, 2000, pp 197–203
- [6] Lakatos, A. Comparison of the Thermal Properties of Different Insulating Materials *Advanced Materials Research*. 899, 2014, pp 381-386.
- [7] Kalmár F: Energy analysis of building thermal insulation. In *Proceedings of the 11th conference for building physics, Dresden, Deutschland*, pp.103–112. 26–30 September (2002).
- [8] Lakatos Á – PhD disszertáció/PhD dissertation - Diffúzió és diffúzió kontrollált jelenségek vizsgálata fém/félvezető nanorétegekben SNMS technikával. PhD thesis - Investigation of diffusion and diffusion controlled processes in metal/semiconductor nanolayers by SNMS technique. 2011.
- [9] Chakraborty J, Welzel U and Mittemeijer E J. *Journal of Applied Physics* 2008, 103, pp 113-512
- [10] Hübner R, Hecker M, Mattern N, Hoffmann V, Wetzig K, Wenger Ch, Engelmann H.-J, Wenzel Ch, Zschech E. *Thin Solid Films* 458, 200, pp 237-245
- [11] Aegerter M.A., Leventis N., Koebel M.M. (Eds.), *Aerogels Handbook*, Springer 2011
- [12] Baillis D., Coquard R., Moura L.M.. Heat transfer in cellulose-based aerogels: Analytical modelling and measurements *Energy*, Volume 84, 1, 2014, pp 732-744

9. Section Green Buildings Technologies and Materials

[13] Huang Y, Niu J.L. Application of super-insulating translucent silica aerogel glazing system on commercial building envelope of humid subtropical climates – Impact on space cooling load. *Energy*, Volume 83, 1, 2015, pp 316-325