It is more probable that these faint variables escaped discovery as a consequence of an extraordinary rapid decline of the discovery chance with magnitude in a dense field. This then would seem to explain satisfactorily why only some of the rare RR Lyrae variables with absolute magnitudes as much as 1 magnitude brighter than the ordinary, were dicoverred in our survey.

Most of the work described here was carried out when the writer was still at the Radcliffe Observatory in South Africa. He should like to express his sincere thanks to Mrs. M. SHUTTLEWORTH for invaluable help during the investigation.

References:

1. A. J. WESSELINK, Applied Optics, July 1964
2. S. C. B. GASCOIGNE, IAU-URSI (1963) Symposium Nr. 20
3. M. DARTAYET and J. L. DESSY, Ap. J. **115**, 279, 1952
4. J. LANDI DESSY, PASP **71**, 435, (1959)
5. A. D. THACKERAY, The Observatory **70**, **144**, (1950)
6. W. BAADE and H. SWOPE, A. J. **66**, 300, (1961)
7. A. D. THACKERAY, MNRAS **118**, 117 (1958)
8. G. W. TIFFT, MNRAS **125**, 16 (1952)

Discussion to the paper of WESSELINK

DETRE: It seemed to me always very strange that we find RR Lyrae variables only on the continuation line of population II cepheids, but till now no RR Lyrae variables have been found on the continuation line of population I cepheids, though there are RR Lyrae stars with normal metal-content. Perhaps the bright RR Lyrae variables found by DESSY may lie in the extension of population I cepheids.

DE KORT: The determination of apparent magnitudes at faint levels in the crowded areas of the Magellanic Clouds is extraordinarily difficult. Their accuracy is of course conditional for a fruitful discussion. As Dr. DETRE suggests, these bright RR Lyrae variables may be population I objects. If one could identify galactic counterparts, the population nature could be checked on their motions and positions.

ARP: I notice that your cepheids of period greater than a day fall substantially fainter than the SMC P-L relation. Since your region is in the most sickly populated region of the SMC is it possible that this is caused by greater absorption than in the average of the rest of the cloud. If this were true it would make the „bright RR Lyrae" stars which you observe, intrinsically brighter and hence in agreement with DESSY's original results.

DE KORT: The absorption is probably less (per unit distance) in the crowded areas. DESSY's field is about as dense as mine. Further photometry will have to reduce the gap between DESSY's results and mine.

Period Changes in Variables and Evolutionary Paths in the Hertzsprung-Russell Diagram

Julia BALÁZS-DETRE and L. DETRE (Budapest)

Abstract:
The theory of random walk has been applied to the O—C diagrams of variable stars, and a method has been outlined for the determination of the standard deviation, σ, of random period-fluctuations, which is free from uncertainties in the expectation value of the period. It is shown, that σ is an important parameter of variable stars. It seems to be correlated with the rapidity of evolution: the period of a rapidly evolving star becomes strongly fluctuating.

This renders the empirical determination of evolutionary tracks from period changes in variables very difficult. But the determination of σ-values can direct attention to stages of rapid evolution along the evolutionary tracks in the HERTZSPRUNG-RUSSELL-Diagram.

σ-values for different types of periodic variables are given. The O—C diagrams of long period variables, classical Cepheids, W Virginis and dwarf Cepheids seem to be completely determined by the accumulation of random fluctuations. SZEIDL's material on the O—C diagrams of RR Lyrae variables in M 3 points at a strong dependence of σ on the period of RRab stars. A *continuous* evolution through the RR Lyrae-gap, in either direction, seems to be improbable. The structure of the O—C diagrams of RR Lyrae-stars cannot be wholly explained as the result of period-noise; some other factors, permanent period-increases and decreases may be involved. Taking all RR Lyrae variables together period-increases and decreases are of the same frequency, but on the long period branch of RRab stars evolutionary period changes may be involved.

The idea of using period changes in variable stars to infer a possible change in an individual star is very tempting. The periods can be determined with incomparably greater accuracy than any other physical data of these stars. So one may hope that changes in these periods will be the first sign of changes in other characteristic data on which the periods are dependent, provided, that the evolution does not proceed along a line of constant period.

There are already available sequences of models which have been evolved to the Cepheid region from the main sequence. We mention the fundamental work made by KIPPENHAHN, WEIGERT, HOFMEISTER and TOMAS (1964, 1965). They determined also the rate of period changes in the Cepheid-region and their results show that these period-changes are large enough, that they might be observed.

Semiempirical considerations by SANDAGE (1957) and WOOLF (1964) estimate the duration of the RR Lyrae-stage on the horizontal branch of the HR-Diagram to about some 10^7 years and if the evolution goes continuously through the RR Lyrae-gap, what we do not believe, this rate of evolution would give period-changes observable already in 10 to 20 years.

But this statement is true only if the periods are not subjected to fluctuations. If the fluctuations are large, they may obscure eventual slow period changes of evolutionary character.

Fluctuations may be of different types and EDDINGTON and PLAKIDIS (1929), who first treated this problem fourty years ago, classified them in the following way:

1) The period or phase fluctuation is independent from the preceding one. We call this type of fluctuations *period-noise.*

The individual period may be correlated to those preceding it

2) positively, so that periods larger or smaller than the average tend to occur in groups,

3) negatively if a period longer than the average is likely to be succeeded by a period shorter than the average.

PLAKIDIS (1932, 1933) investigated the period changes in Mira-variables. These stars show great variations in the period from cycle to cycle. It is easy to study the nature of the period variations for these stars, because we can determine the length of many individual cycles *). EDDINGTON's (1932) conclusion was that nearly three-quarters of the long-period variables constitute a normal class in which successive periods fluctuate accidentally, with a standard mean deviation $\sigma = 1 - 2\,\%$ of the period.

Later STERNE and CAMPELL (1937) investigated the period changes in 377 Mira-variables and concluded that in this large sample only five stars showed progressive period changes. According to an investigation by SCHNELLER (1950) such Mira variables are: RU And, R Aql, U Boo, R Hyd and RZ Sco.

Meanwhile STERNE (1934) studied in an important paper the cumulative effects of period-noise in the O—C diagrams of variable stars. He has shown that the existence of random errors of sampling, in the cycles of a variable star, prevents the inference of changes of true period from curvatures in O—C diagrams. In this way many published „changes of period" are likely to be mere statistical fluctuations. There is no shape that could not arise in the O—C diagram through accumulation of random fluctuations. SLUTZKY pointed out already

*) Naturally, this is the same in the case of ultrashortperiod variables, but they have very stable periods, even if secondary periods are involved.

in 1927, that the summation of random causes may be the source of apparently periodic processes. Therefore the physical interpretation of cycles in the O—C diagrams often lead to incorrect results, as was clearly shown for several eclipsing binaries by SCHNELLER (1964).

STERNE used the old error theory in his investigations. In the last decades the theory of MARKOV-chains has been strongly developed, and now it is possible to make a very simple treatment of the following problem: what is the structure of the O—C diagrams when they are completely determined by the period-noise.

Generally, the O—C diagrams are constructed by taking a mean value of the period (P^o) over the time interval of the observations, and then the phase-differences between the observed and predicted moments of a characteristic feature of the light curve (mostly the middle of the rising branch) are represented versus the epoch number, E. We denote the O—C value at epoch $E = n$ by $y(n)$.

We have to solve the following problem: Starting with the value $y = 0$ at epoch $E = 0$, we ask, what is the probability of reaching a value $y(n)$ at the epoch $E = n$. Let us denote the probability density function of the $y(n)$-values with $P^n(y)$, i. e., the probability that the O—C value falls within the interval $(y, y + dy)$ at the epoch n will be $P^n(y) \cdot dy$, and the time-independent probability density function of the phase-fluctuations, f, around the expectation value of the period, p, with $\psi(f)$. We suppose that f lies between the values $+a$ and $-a$, and has a mean value $\bar{f} = 0$*).

Let us suppose for a moment that $p = p_0$. Then $y(n)$ must lie between the limits $+na$ and $-na$ (s. Fig. 1). The problem is equivalent to a generalized random walk in the direction of the y-axis, the O—C diagram giving the distance from the x-axis as a function of the number of steps, i. e. epochs, with the initial condition: $y(0) = 0$, i. e. $P^p(y) = 1$ for $y = 0$, and $P^p(y) = 0$ for $y \neq 0$. $P^1(y)$ is then simply equal to $\psi(y)$.

We obtain easily a recurrence formula for $P^n(y)$ (s. Fig. 1). The value $y(n)$ can be reached from the preceding epoch starting with $y(n-1)$-values lying in the interval $[y(n+a), y(n-a)]$ with a probability density $P^{n-1}(y)$. The probability of a transition $y(n-1) \rightarrow y(n)$ is $\psi(z)$, where $z = y(n) - y(n-1)$. Hence we have

$$P^n(y) = \int_{a}^{+a} P^{n-1}(y-z)\,\psi(z)\,dz \qquad (1)$$

Starting with $P^1(y) = \psi(y)$ we successively get $P^2(y)$, $P^3(y)$, ...

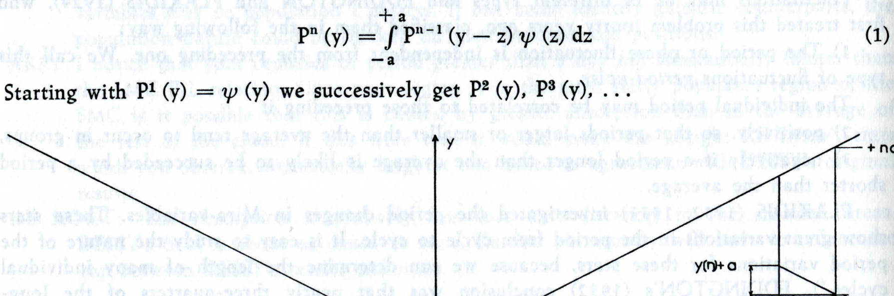

Fig. 1. To the theory of the O—C diagram.

*) This is no restriction. A mean value $\bar{f} \neq 0$ would mean, that the expectation value of the period should be not p, but $p \cdot (1 + \bar{f})$.

But we do not need this exact formula. The variable y (n) is the sum of the phase fluctuations between the epochs (0,1), (1,2), . . . (n — 1, n)

$$y(n) = f_{01} + f_{12} + \ldots + f_{n-1,\,n}.$$

As every phase-fluctuation is independent from the preceding ones, the stochastic variable y (n) is the sum of n independent stochastic variables, all having the same probability function ψ (f) with the mean value zero, and the standard deviation σ, where

$$\sigma^2 = \int_{-a}^{+a} f^2 \psi(f)\, df \qquad (2)$$

If ψ (f) is a normal distribution, it can be easily shown by (1) that the distribution of all the y (n) -s is normal. But according to the central limit theorem of the probability theory, whatever be the analytical form of this function — subject to certain very general conditions — the probability density function of their sums, i. e. P^n (y) will tend asymptotically to the normal distribution with a mean value zero, and standard deviation $\sigma \sqrt{n}$, i. e.

$$P^n(y) \rightarrow \frac{1}{\sqrt{2\pi}\,.\,\sigma\sqrt{n}}\; e^{-\frac{y^2}{2n\sigma^2}} \qquad (3)$$

The convergence to (3) is very rapid, and in practical applications to our problem we can use formula (3) already for small values of n. With this formula we have the whole probability structure of the O—C diagram in our hand, and we can calculate the probability of any track in the diagram. Generally, the diagram consists of cycles of different length and amplitude, the longer cycles having on the average greater amplitudes than the short ones.

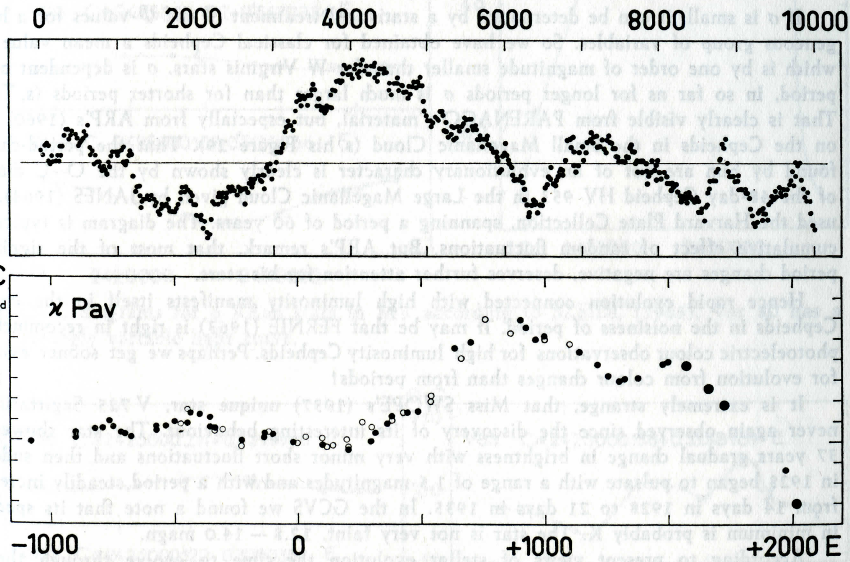

Fig. 2. Above: The record of 10,000 tosses of an ideal coin (from FELLER, 1957). The stay of the gamble is recorded after every 20th toss.
Below: O—C diagram for the disk-population Cepheid Kappa Pavonis, C = 2419619.00 + 9ᵈ09000 · *E*. PARENAGO's (1956) diagram was completed by photoelectric observations of the WALRAVEN's and TINBERGEN (1964), and by an epoch derived from radial velocities obtained by RODGERS and BELL (1963).

To illustrate that the structure of the O—C diagram of some variables is completely determined by random phase-fluctuations, let us compare the record of a typical random walk, the record of tosses of an ideal coin, with the O—C diagram of Kappa Pavonis, which is according to RODGERS' and BELL's (1963) spectroscopic work a disk population Cepheid (Fig. 2).

As the mean value of the period, p_0, calculated from observations over a certain time interval and used in the construction of the O—C diagram, is liable to errors of sampling, especially when σ is large, the difference $p - p_0$ causes a cumulative error progressing linearly with the epoch number. In this case $P^n(y)$ will have a mean value different from zero, i. e. $(p - p_0) \cdot n / p_0$. We can get rid of this difficulty considering the quantity

$$\Phi(n) = 1/2 \{ y(n) + y(-n) \}$$

instead of y (n). According to the addition theorem for normal distribution, Φ (n) will have also normal distribution, but with the mean value zero, and the standard deviation $\sigma\sqrt{\frac{n}{2}}$. Eventual evolutionary period changes, running linearly with time, are unaffected thereby, and they are superposed unchanged on the Φ-values resulting from random fluctuations.

If for a variable σ is very large, and its O—C diagram is rich in details, σ can be approximately determined for separate stars. We get in this way for the W Virginis Cepheids $\varkappa$ Pavonis, AP Herculis, RV Camelopardalis and PP Aquilae σ-values of the same order of magnitude as for long-period variables.

The O—C diagram of about 40 Cepheids was investigated by PARENAGO (1956) who, after representing the cycles in the O—C diagrams by broken lines, found several statistical relations between the length and steepness of the cycles. All these relations are understandable by the theory outlined above, and the numerical values of his relations are purely dependent on the value of σ for the variables treated by him.

If σ is small, it can be determined by a statistical treatment of the Φ-values for a homogeneous group of variables. So we have obtained for classical Cepheids a mean value of σ which is by one order of magnitude smaller than for W Virginis stars. σ is dependent on the period, in so far as for longer periods σ is much larger than for shorter periods (s. Table). That is clearly visible from PARENAGO's material, but especially from ARP's (1960) work on the Cepheids in the Small Magellanic Cloud (s. his Figure 29). That the period-changes found by him are not of an evolutionary character is clearly shown by the O—C diagram of the 48-day Cepheid HV 953 in the Large Magellanic Cloud given by JANES (1964), who used the Harvard Plate Collection, spanning a period of 60 years. The diagram is typical for cumulative effect of random fluctuations. But ARP's remark, that most of the significant period changes are negative, deserves further attention for his stars.

Hence rapid evolution connected with high luminosity manifests itself in the case of Cepheids in the noisiness of period. It may be that FERNIE (1963) is right in recommending photoelectric colour observations for high luminosity Cepheids. Perhaps we get sooner evidence for evolution from colour changes than from periods!

It is extremely strange, that Miss SWOPE's (1937) unique star, V 725 Sagittarii was never again observed since the discovery of its interesting behaviour. The star showed for 37 years gradual change in brightness with very minor short fluctuations and then suddenly in 1928 began to pulsate with a range of 1.5 magnitudes and with a period steadily increasing from 14 days in 1928 to 21 days in 1935. In the GCVS we found a note that its spectrum in minimum is probably K. The star is not very faint, 12.8 — 14.0 magn.

According to present views of stellar evolution the time to evolve through the red variable stage must be very short. Further, W Virginis stars have a low mass in comparison to their absolute magnitude, therefore, their evolution must be also rapid. We have obtained even for these two types of variables the largest σ-values. In addition σ is much larger for long-period Cepheids than for short-period Cepheids. Hence the value of σ seems to be correlated with the rapidity of evolution. Indeed, if we proceed to slower evolving stars, to RR Lyrae-variables and especially to the dwarf cepheids, we obtain smaller σ-values.

If σ is small, only the long cycles can be observed in the O—C diagrams, because the shorter ones have too small amplitudes. This is the cause of finding so many parabolas in the published O-C diagrams of RR Lyrae-stars.

Since 1948 we have obtained at the Konkoly Observatory a continuous series of observations of RR Lyrae-stars in the globular clusters M 3, M 15 and M 5, using the 24-inch

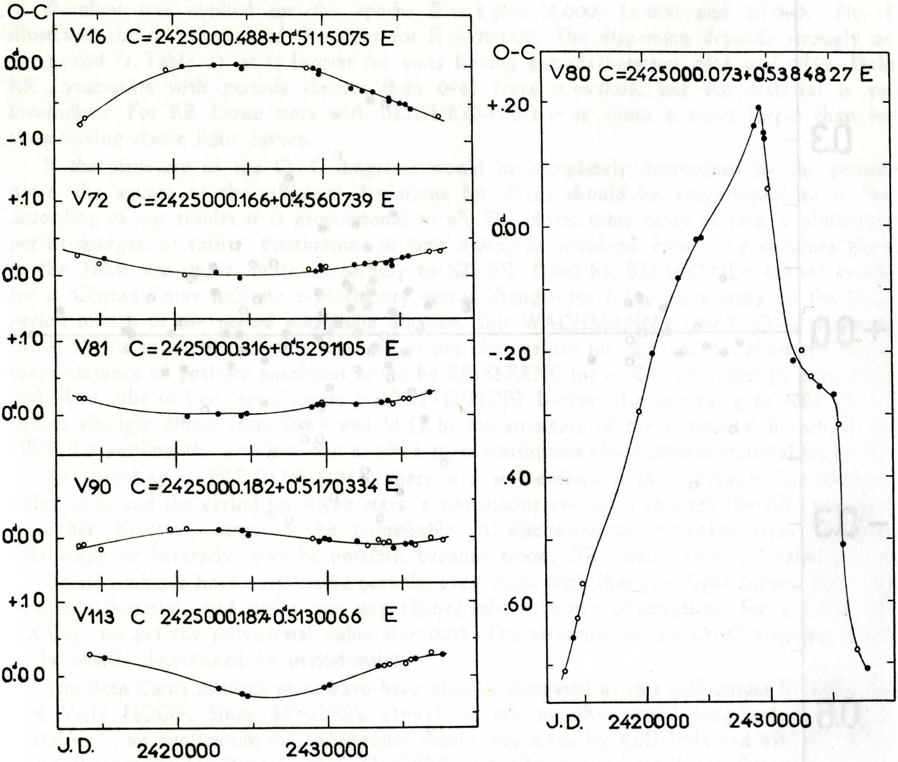

Fig. 3. O—C diagrams for 6 RRab stars in M 3 according to SZEIDL (1965). Var 80 has a strongly variable light curve.

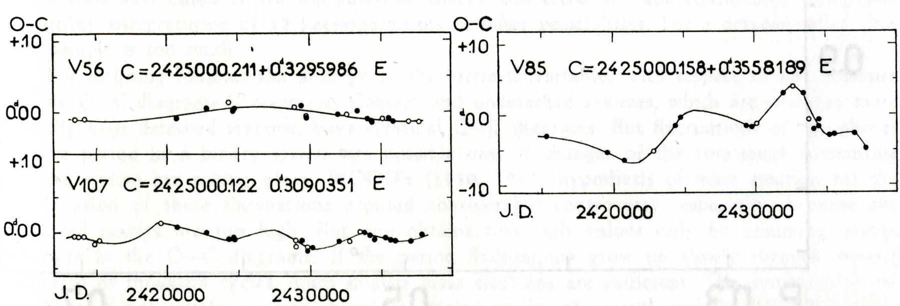

Fig. 4. O—C diagrams for 3 RRc stars in M 3 according to SZEIDL (1965). Var 85 has a variable light curve.

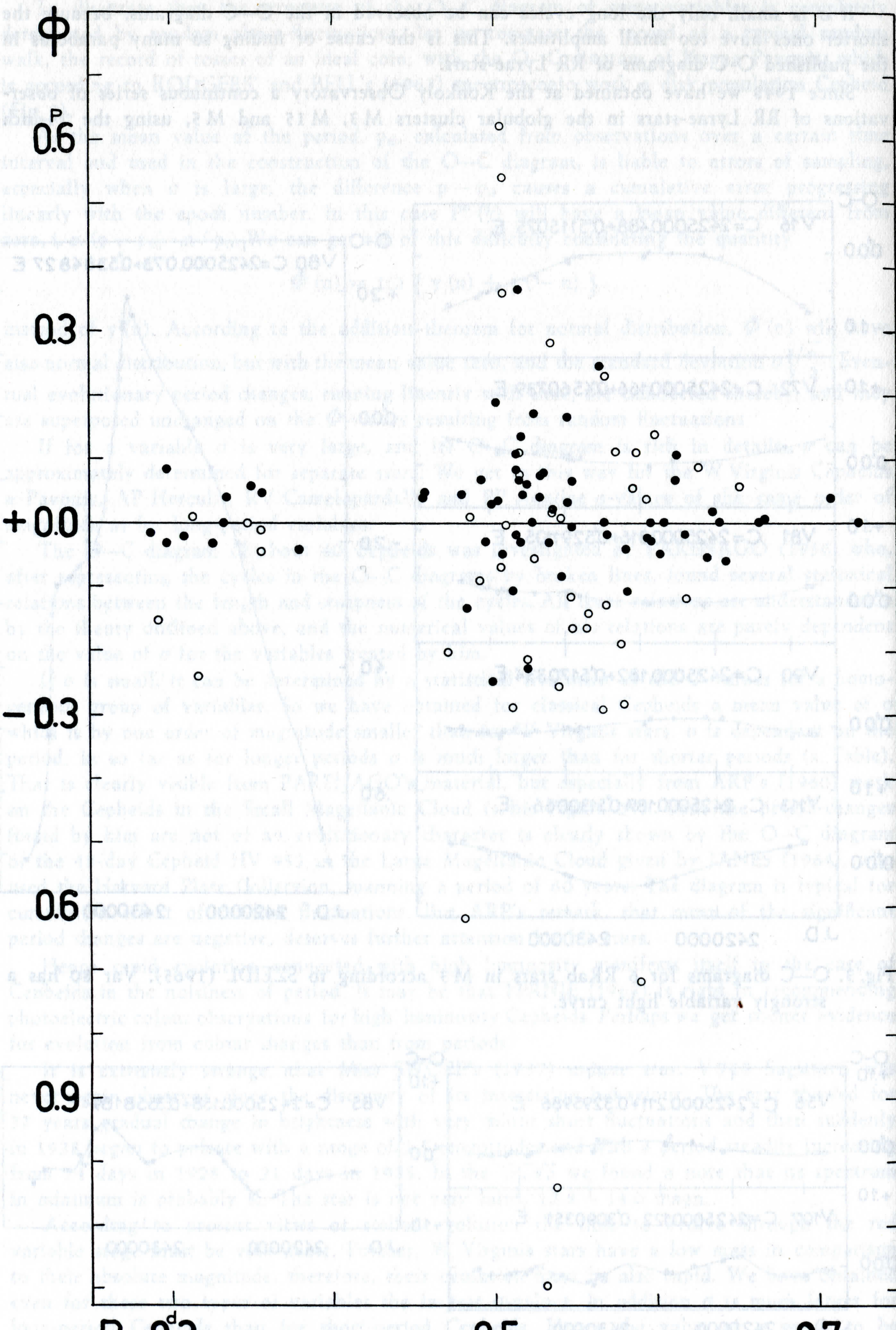

Fig. 5. The distribution of Φ-values (s. text) of 1100 RR Lyrae variables in M 3 for E = 20000. Points: variables with stable light curves; circles: variables with BLASHKO-effect.

Newtonian reflector. Recently SZEIDL (1965 a) finished the work on the variables in M 3. He was able to construct O—C diagrams for 114 RR Lyrae variables in the cluster. This is the most reliable and extensive material for the investigation of period-changes of this type of variables. In Fig. 3 and 4 several O—C diagrams of his material are presented. They contain cycles, giving evidence that period-noise has a share in their structure, especially, when the star has a variable light curve.

For the determination of σ we used SZEIDL's material and the above described method of Φ-values was applied for the epochs $E = 4\,000$, 8 000, 16 000 and 20 000. Fig. 5 illustrates the distribution of Φ-values for $E = 20\,000$. The dispersion depends strongly on the period (s. Table 1), it is largest for stars having periods between $0^d.48$ and $0^d.59$. Field RR Lyrae-stars with periods shorter than $0^d.48$ have $\sigma = 0.06$, but the material is yet insufficient. For RR Lyrae stars with BLASHKO-effect σ is about 6 times larger than for stars having stable light curves.

If the structure of the O—C diagrams would be completely determined by the period-noise, the square of the standard deviations for Φ (n) should be proportional to n. But according to our results it is proportional to n^2. Therefore, some other factors, evolutionary period-changes, or rather, fluctuations of type 2 may be involved. Hence the σ-values given in the Table are upper limits. According to SZEIDL (1965 b), BELSERENE's (1964) results for ω Centauri may indicate evolutionary period changes for RRab stars lying on the long-period branch in the period amplitude diagram. But WACHMANN's (1965) O—C diagrams for M 53 and provisional results obtained at our observatory for M 15 do not show the strong preponderance of positive parabolas found by BELSERENE for ω Centauri, though both M 15 and M 53, like ω Cen, are clusters of OOSTERHOFF II-type. But according to ARP (1965) ω Cen strongly differs from M 15 and M 53 in the structure of the horizontal branch of the HRD. For settling the problem, one needs a more continuous observational material for ω Cen.

As according to PRESTON (1959) there is a well-defined relation between his spectral-index, Δ S, and the period for RRab stars, a continuous evolution through the RR Lyrae-gap, in either direction, seems to be improbable. A discontinuous transition from RRab to RRc stage, or inversely, may be possible, because among RRc stars every Δ S-value occurs.

Dwarf cepheids have very stable periods, even those with changing light curves. From the available material, and using our unpublished photoelectric observations for CY Aqr and XX Cyg, we get the provisional value $\overline{\sigma} = 0.02$. The structure of the O—C diagrams seems to be wholly determined by period-noise.

The Beta Canis Maioris stars have been already discussed at this colloquium by ODGERS and VAN HOOF. Since STRUVE's (1955) remark on the period lengthening of these variables *), an application to evolutionary theory was made by REDDISH and SWEET (1959) using the results for Beta Cephei. ODGERS' (1965 b) new material for this star is very important, showing that the variation of gamma-velocity ist not due to duplicity but to irregular movements of the photosphere. Up to the present, evolutionary considerations for these stars were based on the old pulsation theory, but ODGERS' and KAM-CHING's (1954) beautiful interpretation of 12 Lacertae points at other possibilities. For a determination of σ the sample is too small.

For eclipsing binaries the analogy to the intrinsic variables with respect to the structure of the O—C diagrams is complete. Contact and undetached systems, which are evolving more rapidly than detached systems, have erratical O—C diagrams. But fluctuations of the photometric period in a binary system are possible only if changes of the rotational momentum of the system are taking place. WOOD's (1950, 1962) hypothesis of mass ejection for the explanation of these fluctuations aroused considerable controversy, especially, because the required masses are too high. But one obtains this high values only by assuming abrupt changes in the O—C diagrams. If the period fluctuations grow up slowly through several hundred or thousand cycles, much smaller mass ejections are sufficient. The dynamics of the problem is not simple, as was shown by recent works of several authors (HUANG 1963,

---

*) Recently MILONE (1965) found in Beta Canis Maioris a period shortening.

PIOTROWSKI 1964, KRUSZEWSKI 1964). But it is very probable that an exchange of axial and orbital angular moments in close binary systems is partly responsible for erratic period changes. According to KOCH, OLSON and YOSS (1965) a number of component stars rotate more rapidly than synchronism with orbital motion requires.

We thank Dr. B. BALÁZS and B. SZEIDL for many stimulating discussions.

Table 1. Mean σ-values for intrinsic variables expressed in percentage of the period.

Type of Variables	$\bar{\sigma}$
Long period	1.6 %
W Virginis	1.6
Classical Cepheids:	
periods between 1—18 days	0.08
„ „ 18—36 „	0.3
RR Lyrae Stars in M 3	
RRc Variables	0.04
RRab Variables:	
periods between 0.45—0.52 days	0.09
„ 0.52—0.58 „	0.11
„ 0.58—0.72 „	0.05
Dwarf Cepheids	0.02:

References:

ARP, H. C. 1960, Astron. J. **65**, 404
— 1965, private communication
BELSERENE, E. P. 1964, Astron. J. **69**, 475
EDDINGTON, A. S. and PLAKIDIS, S. 1929, Monthly Notices Roy. Astron. Soc. **90**, 65
EDDINGTON, A. S. 1932, Ib. **92**, 470
FELLER, W. 1957, An Introduction to Probability Theory and Its Applications. II. ed. p. 84
FERNIE, J. D. 1963, The Observatory **83**, 259
HUANG, S. S. 1963, Astrophys. J. **138**, 471 and 481
JANES, K. A. 1964, Astron. J. **69**, 131
KIPPENHAHN, R., HOFMEISTER, E. and WEIGERT, A. 1964, Z. Astrophys. **59**, 215 and 242; **60**, 57
KIPPENHAHN, R., THOMAS, H. C. and WEIGERT, A. 1965, Ib. **61**, 241
KOPAL, Z. 1957, I. A. U. Symp. **3**, 173
KRUSZEWSKI, A. 1963, Acta Astr. **13**, 106; **14**, 241
LEDOUX, P. and WALRAVEN, Th. 1958, Handbuch der Physik, (Springer-Verlag, Berlin), Vol. 51, p. 381—4
MARTIN, W. C. 1938, Ann. Sterrewacht Leiden **17**, Part. 2
MILONE, L. A. 1965, Z. Astrophys. **62**, 1
ODGERS, G. J. and LEUNG KAM-CHING 1965 a, Astron. J. **70**, 145
ODGERS, G. J. 1965 b, private communication
OORT, J. H. and VAN HERK, G. 1959, Bull. Astron. Inst. Neth. **14**, 299
PARENAGO, P. O. 1956, Perem. Zvezdy **11**, 236
PIOTROWSKI, S. L. 1964, Acta Astr. **14**, 251
PLAKIDIS, S. 1932, Monthly Notices Roy. Astron. Soc. **92**, 460
— 1933, Ib. **93**, 373
PRESTON, G. W. 1959, Astrophys. J. **130**, 507
REDDISH, V. C. and SWEET, P. A. 1959, Liège Inst. Astrophys. 4° No. 409
RODGERS, A. W. and BELL, R. A. 1963, Monthly Notices Roy. Astron. Soc. **125**, 487
SAHADE, J. 1960, Symp. on Stellar Evolution, La Plata. p. 185
SANDAGE, A. R. 1957, Astrophys. J. **126**, 326
SCHMALBERGER, D. C. 1960, Astrophys. J. **132**, 591
SCHNELLER, H. 1950, Astron. Nachr. **279**, 71
— 1962, Mitt. Sternw. Budapest, No. 52
— 1964, Astr. Nachr. **288**, 1
SLUTZKY, E. 1927, Problems of Economic Conditions, Moscow, **3**, No. 1
SMITH, B. 1943, Astrophys. J. **98**, 82
STERNE, Th. E. 1934, Harvard Obs. Circ. No's 386—387

STERNE, Th. E. and CAMPBELL, L. 1937, Tercentenary Papers Harvard Coll. Obs. No. 24
STIBBS, D. W. N. 1953, Monthly Notices Roy. Astron. Soc. **115**, 363
STRUVE, O. 1955, Publ. Astron. Soc. Pacific. **67**, 29
STRUVE, O., McNAMARA, D. H., KUNG, S. M. and BEYMER, C. 1953, Astrophys. J. **118**, 39
SWOPE, H. H. 1937, Tercentenary Papers Harvard Coll. Obs. No. 26
SZEIDL, B. 1965 a, Mitt. Sternw. Budapest, No. 58. (in print)
— 1965 b, private communication
VAN HOOF, A. 1964, I. A. U. Comm. 27. Inf. Bull. on Variable Stars No. 44
WACHMANN, A. A. 1965, Astr. Abh. Prof. C. Hoffmeister zum 70. Geburtstage (J. A. Barth-Verlag, Leipzig), p. 121
WALRAVEN, J. H. and TINBERGEN, Th. J. 1964, Bull. Astron. Inst. Neth. **17**, 520
WOOD, F. B. 1950, Astrophys. J. **112**, 197
— 1962, Vistas in Astr. 5, 119
— and FORMES, J. E. 1963, Astron. J. **68**, 257
WOOLF, N. J. 1964, Astrophys. J. **139**, 1081

Discussion to the paper of Mr. and Mrs. DETRE

PLAUT: V 725 Sgr has been observed recently up to 1959. The amplitude of light variation decreased to almost zero. Therefore no period could be derived from the more recent observations.

Is it possible to distinguish easily between changes in the shape of the light curve and changes in the period?

DETRE: Changes in the shape cause only erratic fluctuations in the O-C diagrams, changes in period are or may be cumulative.

PLAUT: V 725 Sgr has a cumulative change in shape, hasn't it?

DETRE: Let us say rather: a progressive change.

ODGERS: I fully agree with Dr. DETRE on the importance of FELLER's theorems on fluctuations in coin-tossing problems and its application to O-C diagrams of variable star fluctuations. Multiple periods have been asserted for some β Cephei stars such as 12 Lacertae. Do you think this can be substantiated? WEHLAU has investigated this point and concludes that these secondary periods are not real.

DETRE: The problem of secondary periods is another question. It may be that other methods of probability theory can be applied to settle this problem.

FERNIE: I should like to agree that the interpretation of period changes as an evolutionary effect is at least doubtful. At the same time, I should like to point out to photoelectric observers that both KIPPENHAHN's and DEMARQUE's theoretical models predict that in the late A supergiants the colour changes due to evolution should be directly observable. They amount to something like $0^m.02$ or more in 10 or 20 years, and the change in V is even larger. A carefully designed program of differential measurements to give accuracies of a few thousands of a magnitude should be quite capable of measuring this.

DETRE: I agree with you.

ARP: The decision whether there exists a secular period change for any group of stars is very simple. If there is an average period change in the group which is significant in terms of the scatter then there is a secular period change. The ensemble average over the group simply averages out the random variations in period and the greater the number of stars in the group, the smaller the secular evolutionary changes that can be determined. Significant evolutionary period changes have been demonstrated by Mrs. BELSERENE for the RR Lyrae stars in ω Cen, and also in my opinion for cepheids in certain period ranges in the SMC. These period changes are of the correct order of magnitude as estimated from very basic considerating of the time spent in the cepheid regions of the HR-Diagram.

DETRE: The problem is not simple. The worst type of period fluctuations is the positive correlated one. I did not say that it is impossible to find out eventual evolutionary changes, but for this we want a great number of O-C diagrams. I do not believe that BELSERENE's results are at present statistically significant, though they are very important. A large percentage of the parabolas found by her may be in reality curves of cyclic character. The cluster should be observed continuously and then we can decide the question perhaps after 10—15 years using the method of Φ-values.

KIPPENHAHN: The period of a pulsating star is mainly determined by its outer layers. The observed sporadic period changes tell us that in the outer layers of the stars some activity must take place, an activity which only by extremely sensitive means of studying period changes can be found but which may also take place in nonpulsating stars. It seems to be more effective in stars of population II. May be we vitness there an activity similar to that observed on the surface of the sun. Unfortunately the random changes obscure the evolutionary effects.

DETRE: The activity causing period fluctuations is in the case of Mira-variables just as effective for stars of population I as for those of population II. Further the activity is smaller for ordinary RR Lyrae-stars (pop. II) than for longperiod classical cepheids (pop. I). It seems to be more correlated to the position of the variables in the HR-Diagram than to population. About its nature we have some guess for eclipsing binaries, and we may assert, that for RR Lyrae variables it must be intimately connected with the BLASHKO-effect.

Three Colour Photoelectric Observations of a Number of Population II Cepheids

K. K. KWEE (Leiden)

By kindness of HILTNER of the Yerkes Observatory the author was able to make photoelectric observations of Population II Cepheids at the McDonald Observatory in Texas during the month of July 1963. The instrument used was the 36″-reflector mounted with one of Hiltner's photometers, which was equipped with a set of filters for UBV-photometry. The programme consisted of 32 galactic variables, of which 28 were classified in the General Catalogue for Variable Stars as to belong to the type "CW" or Population II Cepheids. The other four variables are so-called classical or Population I Cepheids and were included purposely in the programme for comparison. The periods of the variables as given in the General Catalogue range from 1.25 to 28 days. The selection of these variables was based solely on criteria of observability at the McDonald Observatory during July. In the accompanying table the observed variables are listed alphabetically. The four classical Cepheids, all in the Cygnus region, have been listed in the second part of the table.

Mostly 10 to 16 individual measurements in the three colours were made for each variable. Care was taken that the observations fall at different phases of the light variation. No nearby comparison stars were used, however standard stars were measured frequently during the nights. These standards also served as extinction stars. The major part of the reductions was carried out by L. BRAUN, now at the University of Toronto, during his stay at Leiden.

It soon appeared that three of the variables with periods listed in the General Catalogue of about 3 days were in fact RR Lyrae variables with periods of .426, .431 and .703 days. These three stars are listed at the end of the table.