

## PRESSURE AND TEMPERATURE DROP IN GAS TRANSPORTING PIPELINES

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**Abstract:** It is a generally accepted practice in the natural gas industry the approximation that the temperature of the flowing gas along the length of the pipeline is constant. Experimental investigations obtain the decrease of the temperature in the direction of the flow resulted by the heat transfer between the flowing gas and the soil around the pipe. The polytropic change of thermal state seems to be a more realistic assumption than the traditional isothermal approximation. The flow equations are expanded to the polytropic expansion. The pressure distribution can be determined in this way with a higher accuracy. The key problem to determine the adequate value of the polytropic power. Our paper present a method to obtain this power based on the heat transfer calculations.

**Keywords:** natural gas transporting pipeline, pressure drop, temperature drop, polytropic expansion, polytropic power

### 1. Introduction

It is a generally accepted practice in natural gas industry that the flow in gas transporting pipelines is considered to isothermal. In spite of this assumption a considerable amount of heat crosses the pipe walls of the pipe sections followed the compressor stations. The warmed up natural gas is not cooled perfectly to the temperature of the surroundings. The warm gas flowing through a pipeline buried of cold soil permanently heats the soil around itself, while the gas temperature gradually decreases. The flow cannot consider to isothermal, the change of its thermal state is polytropic. There is attempt to develop the mathematical model to determine the pressure loss of polytropic gas flows ( ZSUGA 2012).The obtained results are in rather good agreement with measured data obtained in operating pipelines. Nevertheless this description being a phenomenological treatment does not investigate the heat transfer between the flowing gas and the surroundings which is the reason of the polytropic process. The object of this paper is the detailed investigation of the heat transfer between the gas and the soil.

### 2. Formulation of the problem

Consider a one-dimensional steady flow in a pipe of circular cross section. Using log mean values which relate to the inlet and outlet cross sections, the balance of internal energy equation can be written

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$$\varepsilon_2 - \varepsilon_1 = \frac{h_{\ln} A_3 \Delta T_{\ln}}{\dot{m}} + g h_{1-2} \quad (1)$$

where  $\varepsilon_2$  and  $\varepsilon_1$  are the specific internal energy at the outlet and the inlet,  $h_{\ln}$  is the logarithmic mean heat transfer coefficient,  $A_3$  is the area of the pipe wall between the inlet and outlet,  $\Delta T_{\ln}$  is the logarithmic mean temperature difference,  $\dot{m}$  is the mass flow rate,  $g$  is the acceleration due to gravity and  $h_{1-2}$  is the head loss of the flow. Since the determination of the head loss can be done using the mechanical energy equation, the term

$$\frac{h_{\ln} A_3 \Delta T_{\ln}}{\dot{m}}$$

form the flux of the internal energy equation.

To begin our investigation of the heat transfer coefficient we examine the steady one-dimensional flow of a compressible fluid confined in a pipe which has a constant wall temperature. The bulk flow temperature  $\tilde{T}$  is greater than the wall temperature  $T_w$ . The unit normal vector  $n$  for a solid-fluid interface is taken as being directed from the fluid into the solid, thus the heat flux normal to the wall can be written as

$$q_n = h_{\text{loc}} (\tilde{T} - T_w) \quad (2)$$

where  $h_{\text{loc}}$  is the local heat transfer coefficient.

Consider now an infinitesimal length  $dl$  of the pipe. Neglecting the head loss, the internal energy balance for this control volume is obtained as

$$\rho c_v \frac{D^2 \pi}{4} c (\tilde{T} - d\tilde{T}) - \rho c_v \frac{D^2 \pi}{4} c \tilde{T} = h_{\text{loc}} D dl (\tilde{T} - T_w) \quad (3)$$

This equation can be simplified to

$$-\rho \pi c_v D c \frac{d\tilde{T}}{dl} = 4 h_{\text{loc}} (\tilde{T} - T_w) \quad (4)$$

Since  $T_w$  is constant, its derivative must be zero. Subtracting this derivative from the left-hand side of the equation we get

$$\rho c_v D c \frac{d}{dl} (\tilde{T} - T_w) = -4 h_{\text{loc}} (\tilde{T} - T_w) \quad (5)$$

This equation may be divided by  $k(\tilde{T}_1 - T_w)$ , where  $k$  is the coefficient of thermal conductivity of the fluid and  $\tilde{T}_1$  is the bulk fluid temperature at the inlet of the pipe.

Introducing the dimensionless temperature

$$\Theta = \frac{\tilde{T} - T_w}{\tilde{T}_1 - T_w} \quad (6)$$

and the dimensionless length  $l/D$ , the internal energy equation can be written as

$$\frac{\rho c_v c}{k} \frac{d\Theta}{d\left(\frac{l}{D}\right)} = -4 \frac{h_{\text{loc}} \cdot \Theta}{k} \quad (7)$$

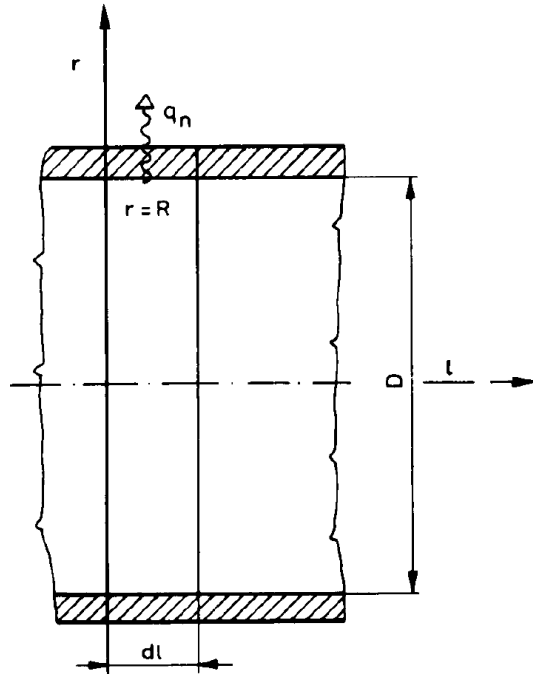


Fig. 1. Infinitesimal control volume for non-isothermal flow

Multiplying both sides of the equation by  $D$ , and the left-hand side by  $v/v$ , we get

$$\frac{\rho c_v v c D}{k} \frac{d\Theta}{v d\left(\frac{1}{D}\right)} = -4 \frac{h_{loc} D}{k} \cdot \Theta \quad (8)$$

It is clear that the equation can be expressed by three dimensionless similarity invariants: the Prandtl number, the Reynolds number and the Nusselt number. The last term, of course, only represents the local value of the latter:

$$Pr = \frac{\rho c_v v}{k}$$

$$Re = \frac{c D}{v}$$

$$Nu_{loc} = \frac{h_{loc} D}{k}$$

Thus we get a simple differential equation

$$\frac{d\Theta}{d\left(\frac{1}{D}\right)} = -4\Theta \frac{Nu_{loc}}{Pr Re} \quad (9)$$

### 3. Solution

The above differential equation can be solved easily. It is readily integrated to give

$$\ln \Theta = -4 \frac{\text{Nu}_{\text{loc}}}{\text{Re Pr D}} + \ln C$$

where C is a constant of integration. As a boundary condition to evaluate this constant we can make use of the fact that if  $l/D=0$ ,  $\Theta=1$ . Thus we get

$$\ln \Theta = -\frac{4\text{Nu}_{\text{loc}}}{\text{Re Pr D}} \quad (10)$$

From Eq. (6) the temperature distribution along the pipe axis is obtained as

$$\tilde{T} = T_w + (\tilde{T}_1 - T_w) e^{-\frac{4\text{Nu}_{\text{loc}}}{\text{Re Pr D}}} \quad (11)$$

The difference in the internal energy fluxes between the inlet and outlet can be expressed as

$$\dot{Q} = \rho c_v \frac{D^2 \pi}{4} c (\tilde{T}_2 - \tilde{T}_1) \quad (12)$$

This equation may be modified

$$\dot{Q} = \frac{\rho c_v v c D}{k} \frac{D \pi k}{v} \frac{D \pi k}{4} (\tilde{T}_2 - \tilde{T}_1) \quad (13)$$

i.e.

$$\dot{Q} = \text{Pr Re} \frac{D \pi k}{4} (\tilde{T}_2 - \tilde{T}_1)$$

At the outlet of the pipe, the temperature can be written

$$\tilde{T}_2 = T_w + (\tilde{T}_1 - T_w) e^{-\frac{4\text{Nu}_{\text{loc}} L}{\text{Re Pr D}}} \quad (14)$$

Expressing from this the product Pr Re we get

$$\text{Pr Re} = -\frac{4\text{Nu}_{\text{loc}} L}{D \ln \frac{\tilde{T}_2 - T_w}{\tilde{T}_1 - T_w}} \quad (15)$$

Substituting this into Eq. (13) we obtain

$$\dot{Q} = \frac{k L \pi \text{Nu}_{\text{loc}} (\tilde{T}_2 - \tilde{T}_1)}{\ln \frac{\tilde{T}_2 - T_w}{\tilde{T}_1 - T_w}} \quad (16)$$

After rearranging, this can be written as

$$\dot{Q} = h_{\text{loc}} D \pi L \frac{(T_w - \tilde{T}_2) - (T_w - \tilde{T}_1)}{\ln \frac{T_w - \tilde{T}_2}{T_w - \tilde{T}_1}} \quad (17)$$

Note, that this result is valid only for the case of a constant wall temperature, constant physical properties of the fluid and a constant heat transfer coefficient  $h_{loc}$ . If these conditions are satisfied, the above equation can be written in the brief form

$$\dot{Q} = h_{loc} A_3 \Delta T_{in} \quad (18)$$

The heat transfer coefficient can be determined experimentally. Measuring the flow rate  $Q$ , the bulk temperatures  $\tilde{T}_1$  and  $\tilde{T}_2$ , the wall temperature  $T_w$  in a pipe of diameter  $D$  and length  $L$ , the so-called logarithmic mean heat transfer coefficient is obtained as

$$h_{in} = \frac{Q c_v (\tilde{T}_1 - \tilde{T}_2)}{D \pi L \Delta T_{in}} \quad (19)$$

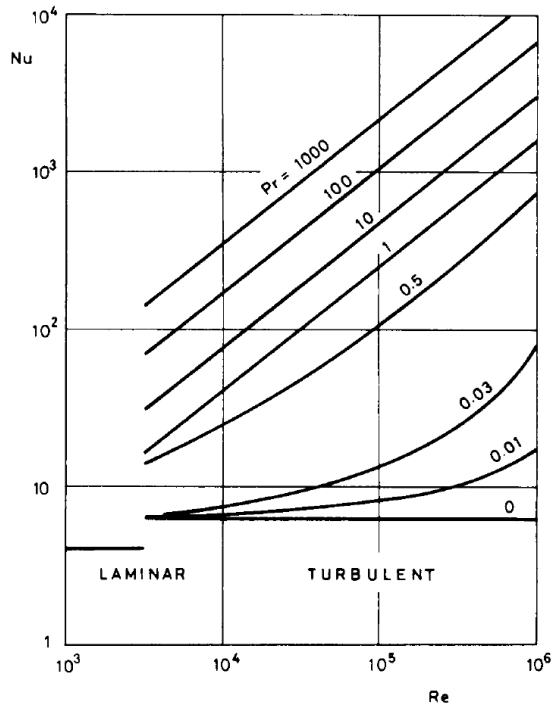


Fig. 2. Nusselt number for laminar and turbulent flow

The obtained experimental results can be interpreted in the form of a similarity invariant. As shown in Fig. 2, the laminar and turbulent regions can be distinguished easily. In the laminar region a satisfactory expression is obtained:

$$Nu_{in} = 1.86 \left( \frac{Re Pr D}{L} \right)^{\frac{1}{3}} \left( \frac{\mu}{\mu_w} \right)^{0.14} \quad (20)$$

In this equation the fluid properties ( $\rho$ ,  $c_v$ ,  $k$ ,  $\nu$ ) are evaluated at the mean bulk fluid temperature  $\frac{1}{2}(\tilde{T}_1 + \tilde{T}_2)$ . The viscosity  $\mu$  is taken at this temperature, while  $\mu_w$  is the viscosity at wall temperature.

For turbulent flow, experimental results lead to the following expression:

$$Nu_{in} = 0.015 Re^{0.83} Pr^{0.42} \left( \frac{\mu}{\mu_w} \right)^{0.14} \quad (21)$$

For case of graphical interpretation this is rearranged in the form

$$\psi = \frac{Nu_{in}}{Pr^{0.42} \left( \frac{\mu}{\mu_w} \right)^{0.14}} = 0.015 Re^{0.83} \quad (22)$$

This quantity is plotted in Fig. 3. For laminar flow the effect of  $L/D$  is considerable and the Nusselt number decreases as the dimensionless length increases. In the turbulent region all measured points fall onto a single curve. This fact shows that Eq. (22) is a reasonably satisfactory empirical formula to estimate the heat transfer coefficient for turbulent flow.

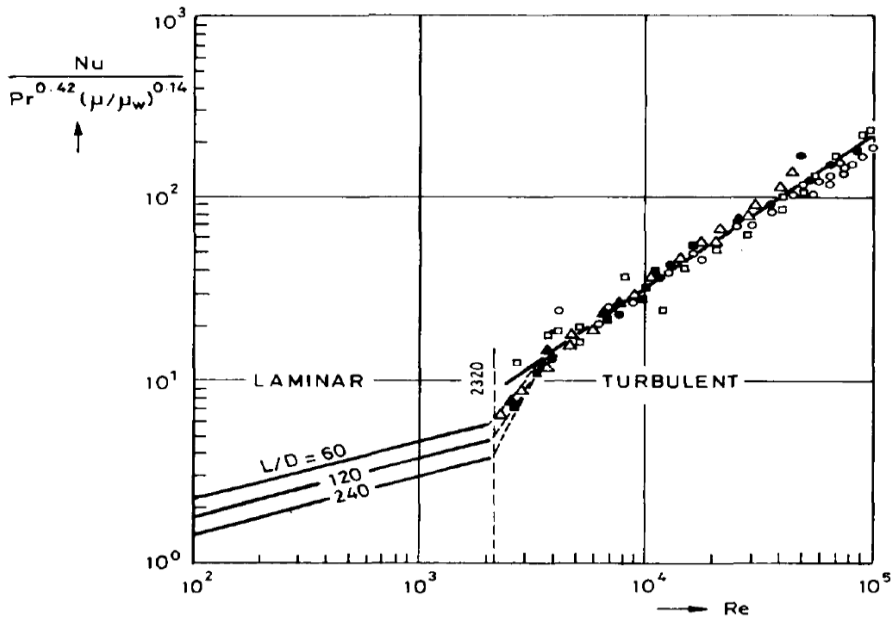


Fig. 3. Forced convection Nusselt numbers for pipe flow

The heat transfer process is investigated in the gas transporting pipeline. The temperature distribution along the length is expressed by dimensionless similarity invariant. This form is suitable for generalization particular data obtained certain pipelines. This

equation can be expressed using quantities familiar in natural gas industry. The power in Eq. (23) can be written as

$$\frac{4Nu}{Re Pr D} \frac{L}{D} = \frac{4hD}{k} \cdot \frac{v}{cD} \cdot \frac{k}{\rho c_v} \cdot \frac{L}{D} \quad (23)$$

considering that

$$\dot{m} = \rho_N Q_N = \frac{D^2 \pi}{4} \rho c \quad (24)$$

the final result is obtained as

$$T = T_w + (T_1 - T_w) \cdot e^{-\frac{D\pi L h}{\rho_N c_N Q_N}} \quad (25)$$

as

$$T = T_w + (T_1 - T_w) \cdot e^{-\frac{D\pi L h R T_N}{\rho_N c_N Q_N}} \quad (26)$$

The subscript N refers to the normal gas state, where  $p_N = 101325 \frac{N}{m^2}$  and  $T_N = 15^\circ C = 288K$ .

The temperature distribution obtained by this equation is the same as the polytropic temperature distribution along the length. Thus it is possible to determine the polytropic power for pressure calculations.

$$n = \frac{\lg \frac{p_2}{p_1}}{\lg \frac{p_2}{p_1} - \lg \frac{T_2}{T_1}} \quad (27)$$

In this formula the inlet pressure  $p_1$ , the temperature  $T_1$  and  $T_2$  are known together the distance between points 1. and 2. It seems to be an acceptable approximation to determine pressure  $p_2$  using the isothermal formula, since isothermal and polytropic pressure distributions are rather close on to another. It is convenient to choose point 2 not too far from the inlet e.g. 3-5 km.

Knowing the polytropic power the pressure, temperature, density and velocity distribution can be determined with higher accuracy.

#### 4. Summary

The pressure distribution of the flowing natural gas is determined along the length of the pipeline considering the expansion of the gas polytropic. Thus the heat transfer between the flowing gas and the soil around the pipeline is considered. The obtained equations contain the formulas referring to isothermal flow also as a particular case. It is very important the adequate evaluation of the polytropic power. This can be calculated knowing the temperature distribution along the pipe length. The temperature distribution is determined solving the differential equation referring to the heat transfer between the gas and the surrounding soil. Thus a more sophisticated equation system is obtained to describe the whole gas transporting process.

**Acknowledgements**

“The described work was carried out as part of the TÁMOP-4.2.1.B-10/2/KONV-2010-0001 project in the framework of the New Hungarian Development Plan. The realization of this project is supported by the European Union, co-financed by the European Social Fund.”

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