A Model of Corporate Liquidity*

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Abstract

We study a continuous time model of a levered firm with fixed assets generating a cash flow which fluctuates with business conditions. Since external finance is costly, the firm holds a liquid (cash) reserve to help survive periods of poor business conditions. Holding liquid assets inside the firm is costly as some of the return on such assets is dissipated due to agency problems. We solve for the firms optimal dividend, share issuance, and liquid asset holding policies.

The firm optimally targets a level of liquid assets which is a non-monotonic function of business conditions. In good times, the firm does not need a high liquidity reserve, but as conditions deteriorate, it will target higher reserve. In very poor conditions, the firm will declare bankruptcy, usually after it has depleted its liquidity reserve.

Our model can predict liquidity holdings, leverage ratios, yield spreads, expected default probabilities, expected loss given default and equity volatilities all in line with market experience.

We apply the model to examine agency conflicts associated with the liquidity reserve, and some associated debt covenants. We see that a restrictive covenant applied to the liquidity reserve will often enhance the debt value as well as the equity value.

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"Kerkorian's numbers just don't add up," said Nicholas Lobaccaro, an auto analyst with S G Warburg. "Ford says it needs double-digit billions of cash to survive the next downturn in the market. General Motors says it wants to put aside $13-15 billion. How can anyone believe Kerkorian when he says $2 billion is enough?"[for Chrysler] [1]

1 Introduction

The quotation above illustrates the range of opinions that can be found among practitioners about the levels of liquid assets that are appropriate for firms. This observation is not an isolated case - it is often remarked that many large corporations carry surprisingly large amounts of cash on their balance sheets. However, finance theory has given very little quantitative guidance as to how much cash is enough. In this paper we attempt to fill this gap by directly asking what is the optimal proportion of the firm's assets that should be held in liquid form? In attempting to answer this question, we recognize that desirable levels of cash holding may change over time according to business conditions. This leads us to couch our analysis in an explicitly dynamic setting. Furthermore, we recognize that views about what is optimal may naturally differ according to the agent's point of view–the cash holding policy which maximizes share value may not necessarily maximize the value of the firm.

The main features of our analysis and our main findings can be summarized as follows. We consider a levered firm with assets in place which generate a random revenue cash flow. Fluctuations of business conditions are captured by supposing that the rate of revenue flows is a stochastic process which reverts toward a long-term mean. Earnings after flow costs and servicing debt may be paid out as dividends or may be retained within the firm as liquid assets. The firm faces the threat of possible bankruptcy which arises if there is a spell of low cash flows that is sufficiently low or long-lasting as to exhaust all available liquid assets and to make issuance of additional securities infeasible. We assume bankruptcy takes a simple form, with creditors being awarded the value of the firm's assets minus bankruptcy costs and shareholders receiving nothing. Thus one of the objectives in holding a stock of liquid

[1]Sunday Times of London, April 23, 1995. This quote refers to the attempt by Kirk Kekorian to take over Chrysler Motors arguing that in doing so he could increase shareholder value by returning most of Chrysler's $7.5 billon cash reserve to shareholders.

assets is as a precaution against a downturn in business conditions which could threaten to wipe out all shareholder value. However, holding liquid assets inside the firm is costly as some of the return on such assets is dissipated, as there is the tendency that they will be transformed in ways which are to the advantage of managers, but detrimental to investors. The firm can also issue extra shares, but we assume that this is not perfectly efficient, due to underwriting fees and agency costs.

In this setting we solve a dynamic program, to obtain the dividend/cash retention and share issuance policy, which maximizes the value of equity. Also, we solve a similar program to obtain the corresponding debt value. We find that in the optimal policy the firm will target a level of liquid assets which varies according to the level of expected cash flows. This optimal policy is not monotonic. In business conditions when the firm is comfortably solvent, the firm does not need a high liquidity reserve, because it can expect the current favorable conditions to continue for some time into the future. However, as conditions deteriorate, the firm will target a higher level of liquid assets. As conditions deteriorate further, the firm will target a higher liquidity reserve, but the actual liquidity holding will tend to be lower, as it is being depleted by operating losses. In sufficiently poor conditions, the firm will declare bankruptcy, after its liquidity reserve has been depleted to zero. In very poor conditions the firm will declare bankrupt, even if it has a liquidity reserve, and then it will pay this reserve out as a dividend to shareholders. Whether such a discrete liquidating dividend would be legal, or whether it could be effectively prevented, are pertinent questions. We will also analyze come covenants aimed at preventing such a discrete liquidating dividend.

We find that the optimal cash policy is sensitive to the basic parameters that affect the values of debt and equity. The targeted level of cash is increasing in both the volatility of cash flow and the costs of issuing new securities. Thus shareholders optimally maintain higher levels of liquidity if they operate in less efficient capital markets; however, even reasonably low costs of security issuance are sufficient for shareholders to optimally hold substantial amounts of cash. Our model can also predict optimal leverage ratios, yield spreads, expected default probabilities and expected loss given default all in line with market experience.

We apply our model to analyzing the agency conflict associated with the liquidity reserve, and some associated debt covenants. An agency conflict arises because this reserve is determined so as to maximize the equity value, and this might be at the expense of the debt value. The "first best" policy would be to maximize the firm value. We find that under the first best policy, and for our benchmark parameter set, the firm will target a higher, not a lower, level of liquidity reserve. This makes bankruptcy less likely, and so preserves the tax shield and prevents bankruptcy costs, which fall on the debt holders. Also, the firm value is substantially higher under the first best policy. But when bankruptcy costs are very small, and there is no tax shield, then maximizing the firm value rather than the equity value leaves the liquidity reserve and the firm value much less changed. In this case the agency conflict is not much associated with a destruction of economic value, but rather with a transfer of wealth from the debt holders to the equity holders.

In light of this analysis, restrictive covenants aimed at reducing cash holding by firms may be misguided. For example, a debt covenant restricting the size of the liquidity reserve decreases the equity and firm values substantially in all cases studied. Such a covenant thus might benefit the debt holders slightly, but at a substantial economic cost. It should be emphasized that in our benchmark case we allow for a substantial conflict of interest between shareholders and managers: we assume one third of interest income on cash held inside the firm is extracted by insiders.

In contrast, we find more support for debt covenants aimed at limiting the firm's ability to pay out cash at times when the firm may be vulnerable. In particular, we find that absent any restrictions to the contrary, in the face of poor business conditions the firm may pay a large dividend, effectively plunging the firm into financial distress. When we consider curbing this behavior by imposing a covenant prohibiting the firm from paying any dividend at times when when it is not profitable, we find that firm value is increased significantly. In fact, covenants restricting dividends when accounting profits are low are common in the U.S. (See, Leuz et.al.)

We believe our analysis contributes to the literature in several important ways. First, we have simultaneously solved for the firm's optimal policies of cash holding, dividend payout, and security issuance. Furthermore, we have carried out this analysis in a dynamic model that is sufficiently rich to make realistic comparisons to a large number of quantitative benchmarks. In particular, for our reference set of parameters we show the model generates simultaneously realistic numbers for *leverage, average cash holdings, equity volatility, yield spreads, probability of default, and loss given default.* We believe that the fact that our model has been tied to realistic empirical benchmarks contributes to the credibility of our qualitative conclusions regarding optimal policies and covenant restrictions. We will now relate some of the key features of our analysis to previous literature.

Analysis of corporate holding of liquid assets goes back at least as far as Keynes (1936), who identified a "precautionary" motive and a "business" (i.e., transactions) motive for such holdings. Under the precautionary motive, the liquid assets are held to tide the firm over hard times, and under the transactions motive liquidity is held to facilitate the financing of investment opportunities which may occur in the future. The empirical study of corporate liquidity holdings by Opler, Pinkowitz, Stulz and Williamson (1999) provides some evidence in support of the precautionary motive. These authors study the determinants and uses of liquidity holdings in a panel study of listed US firms. They show that firms tend to retain a high proportion of their earnings as liquid reserves, and these reserves do not tend to be used for capital investment, but tend to be depleted by operating losses.

Opler et. al. find that corporate liquidity holdings can be a significant proportion of the firms value, averaging about 17% for their data set, which comprises all non-financial firms reported in COMPUSTAT from 1971 - 1994. This result is in accord with previous studies, for example by Kim, Mauer and Sherman (1998). Opler et.al. argue that this high liquidity, and the fact that it tends to be depleted by operating losses rather than capital acquisitions, seems incompatible with the traditional static trade-off theory of capital structure. By contrast, the "dynamic trade-off" model of the present paper is able to explain these effects. Opler et. al. also present other empirical results which agree with the solution of our model and which we will describe later. Recently, Ferreira and Vilela (2002) have extended the work of Opler et al to a comparative study of 12 European economies.

Recently, Mello and Parsons (2000), and Rochet and Villeneuve (2004) have provided theoretical treatments of precautionary corporate liquidity holdings. The models of these papers share some of the features of that of the present paper, including a firm with fixed productive asset, generating uncertain earnings, and with constraints on financing cash shortfalls, motivating the liquidity holdings. Like the present paper, these papers show that there is a liquidity target, below which the firm saves all earnings, and above which the firm pays out all excessive liquidity as a dividend. The focus of these paper is largely to study the benefits when the firm is allowed to hedge some of the uncertainty in its earnings[2]. They show that hedging can enhance the firm's value by reducing the optimal liquidity holding. We will obtain a similar result below, in our framework. Also along similar lines, Acharya, Huang, Subramaniam and Sundaram (2002) present a model of precautionary liquidity holding by a levered firm. Their main concern is to study the interaction between holding the liquidity reserve, and strategic debt service, and the credit spread.

A number of recent papers have studied the transactions motive for corporate liquidity holding. These include Almeida, Campello and Weisbach (2002), and Boyle and Guthrie (2003). These papers show that financially constrained firms tend to hold liquidity determined by the relative values of current and prospective future investment opportunities. Boyle and Guthrie show that a more constrained firm might rationally have a lower investment cash flow sensitivity, i.e. they invest a lower proportion of their earnings, than a less constrained firm.

Also related to the present paper, Myers and Rajan (1998) highlight a fundamental trade-

[2]Rochet and Villeneuve also have a separate analysis concerning the demand for insurance against the firm having to make a large payment, modelled as a Poisson jump.

off of cost and benefits of liquidity when there are frictions in capital markets. On the one hand holding liquid assets helps the firm avoid debt overhang problems or other aspects of financial distress. On the other hand, keeping liquid assets within the firm increases the chances that they will be transformed in ways that may be detrimental to investors. A similar trade-off is present in our model. First, when security issuance is costly, maintaining liquid assets is a means of reducing the chances of financial distress. Second, we model the cost of holding liquid assets by assuming liquid assets held within the firm earn less than standard money market rates. This wedge between return on inside funds and outside funds can be viewed as a reduced form representation of agency costs incurred when insiders manage to capture some of the value of financial slack. Myers and Rajan illustrate the tradeoff between the two aspects of liquidity by calculating a firm's debt capacity in a stylized 3-period setting. In contrast our paper is devoted to characterizing the optimal liquid asset holding in a steady-state continuous time model. [3]

Our modelling of the firm is related to some recent continuous time analyses of corporate debt valuation, in particular those of Leland (1994), Leland and Toft (1996), Anderson, Sundaresan and Tychon (1996), Mella-Barral and Perraudin (1996), Anderson and Sundaresan (2000) and Huang and Huang (2002). The main difference in firm's technology is that our cash flow is mean reverting to reflect business conditions, whereas in these papers it is a Geometric Brownian Motion. These papers focus on the optimal leverage, and the credit spread, and they obtain analytic solutions to the model. Two recent contributions to this field, which like the present paper use numerical solution techniques, are those of Broadie, Chernov, and Sundaresan (2004) and Titman, Tompaidis and Tsyplakov (2004). The former studies the effects of alternative bankruptcy procedures on capital structure and firm behavior. The latter includes a mean reverting aspect in modelling the business conditions, as do we also. Our analysis of the agency problems associated with the liquidity reserve is also parallel to an analysis of agency problems associated with leverage, in that paper.

The remainder of the paper is organized as follows. In Section 2 we introduce the model and the technique we use to solve it. The model does not permit an analytical solution, so in Section 2 we present the numerical solution for a benchmark case. In Section 3 we present comparative statics with respect to the principal parameters, and discuss the results obtained in the light of previous literature. In Section 4 we analyze the agency conflict associated with the liquidity reserve, and the related debt covenants, and finally, in Section 5 we summarize our results and conclusions.

[3] A more distant relation to our paper is the analysis of Holmstrom-Tirole (1998). In their model, they show that liquidity has the effect of reducing the likelihood of financial distress. However, their formal analysis and the fundamental economic questions studied are very different from our analysis.

2 The Model

2.1 Overview

Before presenting our model formally, it is useful to set out the main ideas in informal terms. We consider a firm with a fixed asset in place which has been financed by equity and long-term debt. The asset generates a random cash flow according to a stochastic process whose drift is itself random and follows a mean-reverting process. Any cash flow in excess of contractual debt service and fixed operating costs is subject to proportional corporate income tax, and the after-tax residual may either be paid out as dividends, or retained as liquid assets within the firm. Thus, the tax deductibility of interest debt service provides a rationale for debt financing. Debt is assumed to be a hard claim, and any failure to meet contractual debt service results in bankruptcy. We assume that strict priority is observed in bankruptcy, with the firm's assets in excess of bankruptcy costs being awarded to the firm's creditors. Shareholders lose all. When cash flows fall short of debt service, the firm may draw-down its liquid assets. It may also issue new equity; however, this external finance is costly so that the firm receives less than the full value of the shares it issues.

In this setting, firm faces two decisions. How much of the firm's earnings should be paid out as dividends? And how many new shares should be issued? Jointly, the two decisions will determine the firm's policy toward holding liquid assets. We assume that these decisions are under the control of shareholders who maximize the value of equity, calculated as the present discounted value of the future stream of dividends. Shareholders recognize that firm insiders will to some degree use the liquid assets of the firm for their own benefit, thus reducing the return on liquid assets. The firm's decision will depend upon two state variables - the current rate of cash flow and the current level of liquid assets. Since all the other features of the environment are constant, this is a stationary problem. The solution of the model involves solving for the optimal policy as a function of the two state variables.

The optimal dividend and share issuance strategy in this context will be of the 'bang-bang' type, under which the state space is divided into 3 regions: in the 'save' region zero dividend is paid and earnings are accumulated in the reserve of liquid assets; in the 'dividend' region, the liquid reserve is immediately paid out, until it is brought back to the 'save' region, or to abandonment or bankruptcy; and in the 'issue' region, the firm immediately issues equity until liquid reserve is brought back into the 'save' region. The solution to the problem is studied by characterizing the boundaries between these regions as free boundaries, in a dynamic program, which must be solved numerically. We now proceed to show how this can be done, and later we will show how the solution changes in relation to changes in the parameters of the model.

2.2 Model Specification

The firm has fixed assets in place, which require maintenance/operating costs at a fixed rate f, and which generate operating income at a rate dS_t which follows the Ito equation

$$dS_t = \rho_t dt + \sigma dW_t^\sigma, \tag{2.1}$$

in which the expected revenue ρ_t at time t itself obeys the Ito equation

$$d\rho_t = \mu(\rho_t)dt + \sqrt{\rho_t}\eta dW_t^\rho. \tag{2.2}$$

In these equations, dW_t^σ and dW_t^ρ are infinitesimal increments of independent, standard Brownian motions, and σ and η are constants. We model persistence in business conditions by assuming that the drift function $\mu(\rho_t)$ exhibits mean reversion. We will discuss in detail the choice of $\mu(\rho)$ when we consider the boundary conditions for the optimal policy. At this stage it suffices to say that our choice is very close to being linear, with negative slope given by $-\kappa$, so that the process ρ_t is mean reverting to $\bar{\rho}$. The factor $\sqrt{\rho_t}$ in the volatility term of ρ_t prevents ρ_t from becoming negative. Note that after deducting the operating costs the profitability of the fixed assets is given by $dS_t - f dt$.

The firm is financed by equity and debt. We assume that the firm's debt takes the form of a perpetual bond promising a continuous payment at rate q. Also, the firm cannot alter the amount of debt in issue, but it is allowed to issue more equity to cover interest payments and operating losses, if this is feasible in terms of the price at which shares can be issued. However, we assume that such equity issues will be inefficient, in that the firm will be able to sell new shares at a fraction θ of their fair value. Costly security issuance ($\theta < 1$) may be due to a variety of contracting frictions and may vary systematically with the efficiency in the capital markets where the firm operates. In a highly efficient market without frictions or agency costs, θ will be close to unity; whereas in a very underdeveloped capital market θ may be close to zero. We will see this parameter will have a large effect on the optimal liquidity policy of the firm.

In addition to its fixed asset, the firm may hold a variable amount of liquid reserves. At any time t, the value of these will be denoted by C_t. Liquid reserves held within the firm will earn an 'internal' return at rate r_{in}, which will be less than the riskless rate r earned on outside funds. This wedge between r_{in} and r reflects the moral hazard faced by the shareholders, as discussed by Myers and Rajan (1998). The shareholders recognize that the firms managers will be tempted to transform the liquid reserves for their own benefit, and this is recognized as a reduction in the rate at which the liquidity reserve is assumed to accumulate interest. The difference $r - r_{in}$ can be understood as the cost of holding liquid assets by the firm. Under these assumptions, and for the time-being ignoring the possibility of equity issues, the liquid reserve is the accumulation of total earnings net of dividends, fixed costs, and interest payments on long-term debt, and interest at r_{in}, and we can write

$$dC_t = (1-\tau)(dS_t - (f+q)dt) + r_{in}C_t dt - dD_t, \tag{2.3}$$

where τ is the rate at which the operating income is taxed, and D_t is the accumulated payment of dividends to the firm's shareholders. This equation recognizes that tax is paid on the operating income net of interest payments and fixed costs, i.e. $dS_t - (f+q)dt$, and if this negative, the equation recognizes a tax write-back on operating losses.

Finally, we assume the firm chooses the dividend policy so as to maximize equity value, which is taken to be the present value of expected dividends discounted at the risk-free rate r. The debt is also valued by discounting at the risk free rate the coupon payments until bankruptcy. This is consistent with Equations (2.1), (2.2) referring to risk the neutral probability measure. The basic risk elements in the model are represented by the processes W_t^ρ and W_t^σ in these equations. If these risks are diversifiable, then the risk neutral and the statistically realized measures will be the same. Otherwise, the risk neutral and statistically realized measures will differ by a risk premium. Referring to the risk associated with ρ, we can represent the risk premium by a parameter λ (assumed constant, for simplicity)[4], such that to obtain the statistical measure, we should replace dW_t^ρ in Equation (2.2) by $dW_t^\rho + \lambda dt$. This λ can be thought of as a Sharpe Ratio: it is the extra return required, per unit of extra exposure to the risk represented by dW_t^ρ. To see how the risk premium affects the return of the firms equity, note first that under the risk neutral measure, the expected return will be just the riskless return r. Ignoring the σ factor by assuming that $\sigma = 0$ in Equation (2.1), which we do in most of our implementations, then the equity J_t is a smooth function of ρ, and using the Ito formula, we can write $dJ_t = (drift)dt + \frac{\partial}{\partial\rho}J_t d\rho \equiv rdt + \sqrt{\rho_t}\eta\frac{\partial}{\partial\rho}J_t dW_t^\rho$. The factor $\sqrt{\rho_t}\eta\frac{\partial}{\partial\rho}J_t$ here is the equity volatility, and we will calculate this in the tables below. Substituting dW_t^ρ by $dW_t^\rho + \lambda dt$, we can see that the risk premium increases the expected return by λ times this volatility.

2.3 Solution

Under these assumptions, and ignoring for the moment the possibility of new equity issues, the value of the firm's equity is determined at any time t by the current values of ρ and C. Denoting this value by $J_t^q(\rho, C)$, then we can write the HJB equation

$$J_t^q(\rho, C) = \max_{dD_t}\left\{dD_t + e^{-rdt}E_t^{(\rho,C)}\left[J_{t+dt}^q(\rho_{t+dt}, C_{t+dt})\right]\right\}, \tag{2.4}$$

[4]Such λ has to exist, in the absence of arbitrage: see Duffie (2001).

in which dt is an infinitesimally short time step and dD_t is the optimal dividend payment over this time step, which must be non-negative. Also, $E_t^{(\rho,C)}$ means the expectation at time t, given that $(\rho_t, C_t) = (\rho, C)$. If the liquid reserve becomes low, then the firm can increase it by issuing more equity, if this is feasible in terms of the share price. However, if the liquid reserve becomes negative, then the firm is bankrupt.

Expanding $J^q_{t+dt}(\rho_{t+dt}, C_{t+dt})$ in Equation (2.4), using the Ito Formula, $E[dW_t^\sigma] = 0$, and following standard manipulations we obtain the Ito Equation

$$J_t^q(\rho, C)(1 - e^{-rdt}) = \max_{dD_t \geq 0} \left\{ dD_t + \frac{\partial}{\partial t} J_t^q + \mu(\rho) \frac{\partial}{\partial \rho} J_t^q + \frac{1}{2} \rho \eta^2 \frac{\partial^2}{\partial \rho^2} J_t^q \right.$$

$$\left. + \left[(1-\tau)(\rho - (f+q)) + r_{in} C - dD_t\right] \frac{\partial}{\partial C} J_t^q + \frac{1}{2} \sigma^2 (1-\tau)^2 \frac{\partial^2}{\partial C^2} J_t^q \right\} dt \tag{2.5}$$

We emphasize that this equation holds only for the optimal choice of dD_t, which depends on (ρ, C).

The optimal choice of dD_t here is singular: if $\frac{\partial}{\partial C} J_t^q < 1$, then then it is optimal to pay dividends as quickly as possible, reducing the cash holding until either $\frac{\partial}{\partial C} J_t^q \geq 1$, or $C = 0$, where the firm becomes bankrupt. If $\frac{\partial}{\partial C} J_t^q > 1$, then the firm will not pay dividends. If $\frac{\partial}{\partial C} J_t^q = 1$, then the firm is indifferent between paying or not paying dividends.

The optimal decision can thus be characterized in terms a "save" region $\mathcal{S}$ and a "dividend" region $\mathcal{D}$ in the state space $\{(\rho, C) : \rho > 0, C \geq 0\}$. In $\mathcal{S}$ we have $\frac{\partial}{\partial C} J_t^q > 1$, and also Equation (2.5) holds, with $dD_t/dt = 0$, i.e.

$$\frac{\partial}{\partial t} J_t^q - r J_t^q + \mu(\rho) \frac{\partial}{\partial \rho} J_t^q + \frac{1}{2} \rho \eta^2 \frac{\partial^2}{\partial \rho^2} J_t^q$$

$$+ \left[(1-\tau)(\rho - (f+q)) + r_{in} C\right] \frac{\partial}{\partial C} J_t^q + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial C^2} J_t^q = 0. \tag{2.6}$$

In $\mathcal{D}$ we have

$$\frac{\partial}{\partial C} J_t^q = 1, \tag{2.7}$$

and Equation (2.5) does not apply, since the value of an extra dollar in the liquidity reserve is just its value if immediately paid as a dividend. If the liquid reserve C_t becomes too high, so that $(\rho_t, C_t) \varepsilon \mathcal{D}$, then a dividend should immediately be paid, to take (ρ_t, C_t) back into the region S , or to $C = 0$ and bankruptcy.

If the liquid reserve becomes low, then it may be optimal for the firm to issue new equity. We have not included this possibility in the above formulation. In fact it is optimal to issue more equity if [5] $\frac{\partial}{\partial C} J_t^q > \frac{1}{\theta}$. This possibility leads to there being a third, 'issue' region, which we will denote by $\mathcal{I}$, lying below $\mathcal{S}$, and in which

$$\frac{\partial}{\partial C} J_t^q = \frac{1}{\theta}. \tag{2.8}$$

If the liquid reserve C_t becomes low, so that $(\rho_t, C_t)\varepsilon\mathcal{I}$, then new equity should immediately be issued, to take (ρ_t, C_t) back into the region S . Note that until bankruptcy occurs, the process (ρ_t, C_t) will always remain in the save region $\mathcal{S}$, since it is immediately pushed away, whenever it enters the region $\mathcal{D}$ or $\mathcal{I}$.

These regions must be chosen to maximize $J_t^q(\rho, C)$, which implies that there must be smooth pasting of the solution across the boundaries. Also, these boundaries are 'free', in that they are determined as part of the solution to Equations (2.6), (2,7), (2.8) with smooth pasting.

The regions $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{I}$ are depicted in Figure 3, which also contains other information, for a benchmark implementation, which we will describe in the following section. In this figure, the region of pluses ("+") is the dividend region $\mathcal{D}$, the region of crosses ("x") is the issue region $\mathcal{I}$, and the empty region is the save region $\mathcal{S}$. The boundary above the save region describes the target level of liquidity. Note that the region $\mathcal{I}$ is very thin, and covers the region $C = 0$ for sufficiently large ρ. Also, the region $\mathcal{S}$ extends over high values of ρ, but also becomes very thin. For such high ρ it is unnecessary to hold a liquidity reserve. It is interesting that $\mathcal{S}$ is above $\mathcal{D}$ (i.e. it contains higher C values) over some low values of ρ. If C_t declines over this region of ρ, such that the barrier between $\mathcal{S}$ and $\mathcal{D}$ is crossed, then the firm pays its remaining liquidity holding as a dividend, and is abandoned to the creditors.

In bankruptcy the creditors are awarded the firm's fixed assets net of bankruptcy costs and they then operate them optimally as unlevered shareholders. We assume bankruptcy costs are a proportion α of asset value, so that in bankruptcy, the bond holders receive[6]

[5]Proof: Suppose the current cash holding C is too low, and the firm raises δC in an equity issue, to increase the cash holding. Suppose the firm initially has N shares, and issues n more shares. Denote by s the share price after issue. Then the obtained from selling each new share is θs, and also $n = \delta C/\theta s$ and $s = J(C + \delta C)/(N + n)$. These imply that $s = [J(C + \delta C) - \delta C/\theta]/N$. Now, the firm will issue shares if it increases the share price. Before issue, the share price is $J(C)/N$, and so the firm will issue if $J(C)/N < [J(C + \delta C) - \delta C/\theta]/N$, which implies that $[J(C + \delta C) - J(C)]/\delta C > 1/\theta$. QED

[6]In our numerical implementation, we evolve the solutions J_t^q, etc, backwards from a horizon $t = T$, which is sufficiently distant that the solution has achieved a steady state, i.e. it is independent of t, for t near zero. Thus the value received on bankruptcy by the debt holders can be associated with J_t^0, for $t = 0$. In fact $\frac{\partial}{\partial t} J_t^q = 0$ in the steady state, and we could omit the t variable from our steady state equations. However, keeping the t dependence is more appropriate for our solution method, described below, and for the non-steady state calculations presented below, such as calculating the probability of bankruptcy after 20 years.

$(1-\alpha)J_0^0(\rho, 0)$, if the profitability at the time of bankruptcy is ρ. If we denote by $\mathcal{B}$ the first (random, stopping) time s beyond t for which $C_s = 0$, given that $(\rho_t, C_t) = (\rho, C)$, then the value $P_t^q(\rho, C)$ of debt can be written

$$P_t^q(\rho, C) = E_t^{(\rho,C)}\left[q\int_{s=t}^{s=\mathcal{B}} e^{-rs}ds + e^{-r(\mathcal{B}-t)}(1-\alpha)J_0^0(\rho_{\mathcal{B}}, C_{\mathcal{B}})\right]. \tag{2.9}$$

The debt value in equation (2.9) can be calculated by solving a PDE, in a similar way to the equity value $J^q(\rho, C)$ above. In fact, the calculation is simpler because the boundaries of the region in which the debt is defined, i.e. $\mathcal{S}$, have already been determined in the equity valuation. The PDE for the debt, in the region $\mathcal{S}$, is

$$q + \frac{\partial}{\partial t}P_t^q - rP_t^q + \mu(\rho)\frac{\partial}{\partial \rho}P_t^q + \frac{1}{2}\rho\eta^2\frac{\partial}{\partial \rho^2}P_t^q +$$

$$[(1-\tau)(\rho - (f+q)) + r_{in}C]\frac{\partial}{\partial C}P_t^q + \frac{1}{2}\sigma^2\frac{\partial}{\partial C^2}P_t^q = 0. \tag{2.10}$$

The boundary conditions for the debt valuation are as follows: $\frac{\partial}{\partial C}P_t^q = 0$, where $\mathcal{S}$ meets $\mathcal{I}$ or where $\mathcal{S}$ meets $\mathcal{D}$ and $\mathcal{D}$ is above $\mathcal{S}$, corresponding to the reflection of the process C_t at these boundaries; $P_t^q = (1-\alpha)J_0^0(\rho, 0)$ where $\mathcal{S}$ meets $\mathcal{D}$ and $\mathcal{D}$ is below $\mathcal{S}$, and in the region of the axis $C = 0$, corresponding to the firm becoming bankrupt.

Our strategy for valuing the equity is to solve Equation (2.6) numerically, by finite difference procedures, evolving backwards from an horizon time T, at which we assume the firm is abandoned and liquidated, so that $J_T^q(\rho, C) = C$. In particular, we take our scheme to be implicit in the ρ direction, for the sake of numerical stability, and explicit in the C direction, for ease of implementing the boundary conditions. See Ames (1992). We take T sufficiently large, that the solution has effectively reached a steady state when time has evolved back to $t = 0$[7]. To determine the regions $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{I}$, we test, at every time step and every grid point point representing (ρ, C), whether it is optimal to pay dividends, issue shares, or abandon the firm.

As well as the boundary conditions associated with the regions $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{I}$, we must also choose boundaries at high and low values of ρ and C, and we must impose corresponding boundary conditions. We take the boundaries for low ρ and low C to be at zero; denote the upper boundaries by ρ_{max} and C_{max}. At ρ_{max} we have imposed the condition $\frac{\partial}{\partial \rho}J_t^q(\rho, C) = 0$. Now, this condition is not compatible with a linear choice $\mu(\rho) = \kappa(\bar{\rho} - \rho)$ in Equation (2.2)

[7]It would be more usual to use the SOR technique to obtain our steady state solution, but our scheme is more useful for studying how quickly the steady state is achieved, and for dealing with the non-steady state problems, described below.

above, and in fact it seems unclear what boundary condition would be compatible with this choice of drift. Therefore we have taken $\mu(\rho) = -\kappa\frac{\rho_{max}-\rho_{min}}{\pi}\tan(\frac{\pi}{\rho_{max}-\rho_{min}}(\rho-\bar{\rho}))$, with ρ_{min} such that $\bar{\rho}$ is midway between ρ_{min} and ρ_{max}. With this μ, the drift is infinite at ρ_{max}, forcing our boundary condition to be respected. Also, for the parameters chosen below, this μ is very close to the linear function $\kappa(\bar{\rho}-\rho)$ for normal values of ρ. The result of our choice of μ here is that the profitability is forced very quickly away from unreasonably high values, and the boundary condition is satisfied. In our numerical implementation, we have also transformed the variable ρ to $\xi := \frac{1}{2}\rho^{\frac{1}{2}}$. This is helpful because it leads to a finer solution grid in the region for low ρ, which is the most interesting region (see the figures below). Also the boundary condition for low ρ is automatically transformed to $\frac{\partial}{\partial\xi}J_t = 0$. At $C = C_{max}$ we have imposed $\frac{\partial}{\partial C}J_t(\rho, C) = 1$, and at $C = 0$ we have imposed $\frac{\partial}{\partial C}J_t^q(\rho, C) = 1/\theta$ corresponding to "issue", or $J_t^q = 0$, if "issue" would imply $J_t^q < 0$.

Our strategy for valuing the debt is to solve Equation (2.10) by similar finite difference methods. At the horizon time T, we take the debt value to be $\min\{q/r, (1-\alpha)J_0^0(\rho, 0)\}$, reflecting the assumption that if bankruptcy has not occurred by time T, the productive asset is sold, incurring the bankruptcy costs, and the proceeds are used to pay pay of the value of the debt valued as a risk free flow at rate q, if the proceeds are sufficient to do this[8].

2.4 Implementation for a benchmark case

As discussed in the Introduction, one of our objectives was to develop a model which can deliver quantitative predictions about liquid asset holding in a reasonably realistic setting. While our model is simple enough to permit solution, we will show in this section that it can be calibrated to match a number of empirical benchmarks. One of the advantages of working with a structural contingent claims model of the firm is that we are able to look at liquid asset holding in a model that also has implications for leverage, equity volatility and quantitative measures typically used in credit market analysis, namely, credit spreads, probability of default and loss-given-default (or equivalently, recovery rates).

We take our benchmark parameter set to be $r_{in} = 4\%, r = 6\%, \bar{\rho} = 0.15, \eta = 0.09, \kappa = 0.9, \sigma = 0.0, \tau = 30\%, \theta = 0.8, q = 0.0045, f = 0.14$ and $\alpha = 0.3$ (all taken on an annual basis). Also, we take $\lambda = 0.3$. Concerning the mathematical parameters, we represent $\xi \equiv \frac{1}{2}\rho^{\frac{1}{2}}$ by a grid with 201 points, ranging from 0 to $\rho_{max} = 5$, and we represent C by a grid with 201 points ranging from 0 to $C_{max} = 0.5$. Also, we take $T = 50$ years. Our solutions are insensitive to first order variations of these mathematical parameters.

[8] Theoretically, the choice of terminal time T condition does not matter, if T is sufficiently far away. This choice prevents arbitrage and ensures continuity at time T.

Some of our parameters have a direct economic interpretation. Notice that by setting $r_{in} = 0.04$ we are assuming that one third of the market return on cash is dissipated by keeping the cash inside the firm and under the control of management. We view this as a reasonably severe problem of managerial moral hazard and a rather strong disincentive to holding cash. In this sense, the levels of cash holding our model predicts might be viewed as conservative. By setting $\theta = 0.8$ we assume 20% of the market value of newly issued equity is lost through transactions costs of one form or another. Given the direct costs plus underpricing of equity issues, we view these costs as substantial but not unreasonable in many settings. Similarly, our assumption of bankruptcy costs of 30% is consistent with empirical evidence and the assumptions of other researchers. The sensitivity of our results to these parameter choices is examined in the next section devoted to comparative statics.

A reasonable value for the risk premium (Sharpe Ratio) of the market itself in $\lambda = 0.5$, corresponding to a market excess return of 8%, and market volatility to be 16%. On the other hand a completely diversifiable risk would imply $\lambda = 0$. Thus, our choice $\lambda = 0.3$ is reasonable, if we assume that the risk of the firm has a systematic component, i.e. it is somewhat correlated with the market.

The values of the technological parameters $\bar{\rho}, \eta, \kappa$ enable the process ρ_t to give a realistic representation of a business cycle. This is evidenced by the simulation presented in Figure 2A. These values also give realistic equity volatility, leverage, credit spreads and default probabilities. These implications of the model are derived according to the following method.

For each set of parameters we solve the model for the optimal regions $, \mathcal{D}, \mathcal{I}$ and $\mathcal{S}$. Then for each solution we perform 300 simulations as in Figure 2, each one starting from $\rho = 0.2, C = 0.0$ and running to the firm's bankruptcy. We then calculate the overall average liquidity until bankruptcy. Based on the realizations of the simulations we also calculate non-parametrically the average liquidity as a function of expected revenue flow, denoted $\overline{C}(\rho)$. Using this average liquidity function, we study the model conditional on 4 levels of profitability - $\rho = 0.10$ ('low'), $\rho = 0.15$ ('normal'), $\rho = 0.20$ ('high'), and $\rho = 0.25$ ('very high'). At each level of ρ and average liquidity $\overline{C}(\rho)$, we present the net equity[9] value $J^q(\rho, \overline{C}(\rho)) - \overline{C}(\rho)$ (i.e. equity, net of the liquid reserve), debt value $D^q(\rho, \overline{C}(\rho))$, net firm value $J^q(\rho, \overline{C}(\rho)) + D^q(\rho, \overline{C}(\rho)) - \overline{C}(\rho)$, leverage (debt value divided by the value of the firm), and the yield spread (yield on debt less r). Finally, we give the equity volatility, calculated as $\sqrt{\left(\sqrt{\rho}\eta\frac{\partial}{\partial\rho}J\right)^2 + \left(\sigma\frac{\partial}{\partial C}J\right)^2}/J$.

In addition we calculate the yield spread on zero-coupon bonds of 5 and 20 years until

[9]Taking net equity and firm values is appropriate when we compare valuations across different scenarios. To change between scenarios with different liquidity reserves, one would have to make up the difference with cash.

maturity. For this calculation, and following Duffie and Lando (2001), we assume that the perpetual debt is made up of a continuum of zero coupon bonds, and if the firm defaults, the these bonds are paid off in proportion to their value weight in the total debt. This calculation is done by adapting the perpetual debt valuation to accommodate this default rule, a payment of one dollar if there is not default before maturity, and the coupon being zero. We also calculate the probability of of bankruptcy at 1, 5 and 20 year horizons. This calculation is again done by adapting the perpetual bond valuation, and we include the risk premium λ, since this probability is not risk neutral, but objectively realized.

By this methodology we have calculated the values of liquidity, debt value, equity, and leverage as in Table 1a and the credit relevant statistics as in Table 1b. We take as the main reference for our calibration the case $q = 0.0045$ and $\rho = 0.2$ which corresponds to quite good business conditions. From Table 1a, we see that with the other benchmark parameters as given, and with $\rho = 0.2$, this value of q maximizes the value of the firm. Also from Table 1a, we see that with these parameters, our model generates a leverage of 40% and an equity volatility of 30%. For this firm, the average liquid asset holding is something over 7% of total asset value. From Table 1b, we see that this firm has a credit spread of 70 basis points over the risk free rate. Under our assumption about the risk premium, the probability of default at the five-year horizon is 1.7%. This is corresponds to Standard and Poor's historical experience for a bond rated BBB or perhaps a bit below (see, de Servigny and Renault). Our results on leverage and equity volatility are realistic for a firm in that rating class. Our results for credit spreads may seem a bit low for this rating class, but we need to recognize that our model does not include any allowance for a liquidity component in yield spreads. A number of analysts suggest that liquidity may account for a large fraction of observed yield spreads and can easily amount to 100 basis points for many instruments[10]. Accepting this, we arrive at a total yield spread of 170 basis points which is very plausible for bonds rated at the bottom of the investment grade. When we discuss Figure 4, we will see in addition that the model generates very plausible figures for loss-given-default as well.

Figure 1, Panels A and B, graph the equity and debt values as functions of ρ and C, over the region $\mathcal{S}$. As we have explained, the process (ρ_t, C_t) will never stray from this region. In the Figure the ρ axis ranges from 0.0 to 0.5, and C varies from 0.00 to 0.07. As expected, the equity value increases with ρ and C, as also does the debt value, but less steeply, and with less upside for high ρ and C.

[10] In their calibration of several contingent claims models Huang and Huang report an average yield spread of 194 basis points for 10-year bonds rated BBB. In their benchmark calibration they find the credit component accounts for 56 basis points leaving 138 basis points accounted for by liquidity and possibly other factors.

Figure 2, Panel A graphs a time series simulation of the profitability ρ_t, and Panel B graphs the corresponding optimal cash holding C_t, the equity value J_t and the total firm value $J_t + D_t$. This simulation starts from $\rho_0 = 0.2$ at $t = 0$, and continues until the firm becomes bankrupt, and we have started the figure at $t = 110$ years.

This figure nicely illustrates some of the properties of the optimal policy for the firm. Starting at about period 118 and lasting until period 125 the firm suffers a sharp drop in profitability. This results in a reduction of cash reserves so that by period 121 the firm has zero cash. Between period 121 and 124 the firm issues equity to survive. It does so because by then profitability had recovered somewhat. After period 124 profitability has improved sufficiently for the firm to stop issuing equity and to start to build up its cash reserve. Later, from period 129, the firm once again suffers a drop in profitability. This time the business conditions are so severe that when the firm exhausts its cash, no equity is issued and the firm declares bankruptcy.

Figure 3 gives the regions ,$\mathcal{D}, \mathcal{I}$ and $\mathcal{S}$ ('dividend', 'issue', and 'save'), indicated respectively by pluses ("+"), crosses ("x"), and empty space. It also gives as a line of stars, the average realized liquidity holdings as a function of ρ, resulting from 300 simulations of the benchmark case. For low values of ρ, the average cash holding is less than the target represented by the upper boundary of $\mathcal{S}$, though it is still a substantial fraction of the firm value, at reasonable levels of ρ.

We also see in Figure 3, that the region $\mathcal{S}$ bulges to the left, above the value ρ around 0.08. At such values of ρ, and for C around 0.05, then $(\rho, C)\epsilon\mathcal{S}$, and the firm will use the liquidity represented by C to pay operating losses, in the hope of surviving until business conditions improve. But at such ρ and for C around say 0.01, then in our model, this cash will be paid out, and the firm will be abandoned. As we have mentioned above, whether the shareholders are allowed to do this, or whether they could be effectively prevented, is a pertinent question.

Figure 4 gives the equity valuations in terms of ρ, at $C = 0$ for our benchmark case (pluses), and for the corresponding unlevered case (crosses). It also gives the debt value in terms of ρ, and for all values of C (stars). The lowest of these stars gives the debt values at $C = 0$. We see from Figure 3 that the benchmark firm will go bankrupt for ρ in the region from 0.07 to 0.09, depending on the trajectory of the liquidity reserve. For higher ρ the crosses indicate that the firm will issue equity if $C = 0$, and the region $\mathcal{S}$ does not extend to lower ρ. We see from Figure 4, that the debt value is quite insensitive to the profitability ρ, when ρ is relatively high, and has value about 0.070, but when the firm goes bankrupt, the debt value is about 0.035. Assuming that the firm was initiated when the profitability was high, we thus see that the debts recovery rate on bankruptcy is about $\frac{0.035}{0.070} \approx 50\%$. This number is consistent with empirical studies of BBB rated firms. Over the period 1988-2002 Standard and Poors found that the recovery rates on defaulted bonds were 30 per cent for Senior Subordinated Notes, 38 per cent for Senior Unsecured Notes, and 50 per cent for Senior Secured Notes. (See de Servigny and Renault (2004) Chapter 4.)

3 Some Comparative Statics

In this section we explore the sensitivity of the model to the most significant economic parameters.

3.1 Leverage, q:

Table 1 summarizes our results for the parameter values of the benchmark case above, except for q, which ranges over the values 0.0000, 0.0005, ..., 0.0060. Our benchmark parameter set includes $q = 0.0045$, and we see from Panel A, that for $\rho = 0.2$ ('high'), the firm value is at the maximum at this value of q. The firm chooses its leverage when it is initiated, and it should rationally choose q to maximize its value. Assuming that the firm is initiated in a favorable business climate (high ρ), then it should choose this value of q, and this is why we have included this in our benchmark parameter set. As already discussed this value of q also gives realistic credit spreads and default probabilities for a BBB rated firm.

The average values of the liquidity reserve are non-monotonic in q, and amounts to about 15% of the firm value, for normal ρ. This figure is in line with the empirical evidence presented by Opler et al (1999).

3.2 Volatilities η and σ:

The parameters η and σ represent different sources of volatility in our model: η refers to the dynamic of the profitability ρ. Given the mean reverting nature of the profitability relation, η shocks are *persistent*. In contrast, σ refers to a white noise type of volatility which represents a *non-persistent* shock to profitability.

In Table 2, Columns 2 and 3, we examine the effect of varying the level of the persistent volatility while the other parameters are at their benchmark levels. These columns should be compared with each other, and with Column 1, which repeats the results for the benchmark case. We see that liquid cash holdings conditional on this level of profitability is strongly increasing in η. Also, as η increases, the probability of bankruptcy increases, and the net firm value decreases, as expected, since bankruptcy is costly. However, most of this reduction in net firm value comes at the expense of debt, which decreases much more than the equity value. This is consistent with the asset substitution effect: higher volatility means a higher upside potential, which mostly accrues to equity, and higher downside risk, which most accrues to the debt. This effect is superposed on the effect of the increased probability of bankruptcy, which reduces the value of debt and equity, and the cost of the higher liquidity reserve, which reduces the value of equity[11].

The effects of changes in *nonpersistent* volatility, σ, are seen in columns 4 and 5 of Table 2. We see that increasing σ again causes the firm to hold a higher liquid reserve. This is true even for $\rho = 0.25$ ('very high'), and in fact for $\sigma > 0$ there is no upper limit of ρ above which no liquid reserve is held[12]. We also see that with $\sigma > 0$, the probability of bankruptcy is higher, and the firm value is lower. However, in contrast to increasing persistent volatility η, the loss in firm value is distributed more evenly between debt and equity. Since σ has no dynamic implication on the profitability ρ of the firm, there is no asset substitution effect associated with this parameter.

The results for varying σ are consistent with those of Mello and Parsons (2000), and Rochet and Villeneuve (2004), if we assume that the shock σdW_t^σ to the earnings cash flow can be hedged. This may be the case for a firm heavily dependent upon inputs of commodities traded in futures markets which typically are liquid only for relatively short time horizons. If so, then hedging corresponds to setting $\sigma = 0$. Note that σdW_t^σ is a martingale difference, corresponding to the assumption that there is no risk premium associated with this risk. The modelling set up is different in these papers, but the results are consistent with ours, in finding that hedging decreases the optimal liquidity reserve, and increases the firm value[13].

These results for η and σ are also consistent with the empirical findings of Opler et al, who document that higher volatility is associated with a higher liquidity reserve.

3.3 Speed of mean reversion κ:

The effects of changes in the mean parameter κ are given in Columns 6 and 7 of Table 2. As this parameter is higher, the debt value, the credit spreads and the probability of bankruptcy

[11]If the bankruptcy cost is is taken to be lower, at 5%, and the tax rate is set at 0, to obviate tax shield effects, then higher volatility η increases the equity value, and decreases the debt value, consistent with the usual asset substitution effect. To conserve space, this is not shown.

[12]This is not shown. This can presumably be explained in terms of the infinite variational nature of the earnings cash flow for $\sigma > 0$, under which the realized earning cash flow can be negative over a very short time, even if ρ is very high.

[13]One might also be able to hedge the risk associated with ρ, but this would not be modelled by simply taking $\eta = 0$, but rather by subtracting the martingale component of the process ρ.

are all lower. Also, the firm value is higher, but this benefit falls to the debt, and not the equity, which is slightly lower. In fact higher κ has much the same effect as lower η. This is intuitively reasonable, since increasing the mean reversion and decreasing the volatility in equation (2.2) both make the process ρ_t less likely to stray far from the mean.

3.4 Bankruptcy costs α:

In Columns 8 and 9 of Table 2, we study the effect of changing bankruptcy costs in our model. The question of the level of bankruptcy costs has been much discussed in the literature. Early estimates based on US railroad bankruptcies suggests direct bankruptcy costs for firms with predominantly fixed assets may be relatively low (see Warner (1977)). For example as a proportion of the firm value, 5 per cent might be reasonable. More recent research based on formal reorganizations (Chapter 11) as well as liquidations (Chapter 7) and working with a wider range of industries (including those with substantial intangible assets) suggests that total bankruptcy costs (including both direct and indirect costs) may be very substantial. (See Franks and Torous (1989) and Weiss (1990)). This has led some debt analysts to assume bankruptcy costs as high as 50 per cent of the firm's assets (e.g., Leland (1994)). In light of this we have taken $\alpha = 0.3$ as our benchmark, and in Table 2 we have evaluated our model for values $\alpha = 0.05$ and $\alpha = 0.50$.

The first thing to note from Columns 8 and 9 of Table 2, is that changes in bankruptcy costs have no affect whatsoever on the equity value, and the holding of liquid assets. This is a direct reflection of the fact that bankruptcy costs accrue completely to debt holders, because in our model we assume bankruptcy respects absolute priority of financial claims. As a result bankruptcy costs do not affect the shareholders' choice of liquid asset holding, once the choice of capital structure, represented by the payment rate q to debt, has been fixed. This in turn implies that the value of equity is unaffected by changes in bankruptcy costs.

Changes in bankruptcy costs also have no effect on the probability of bankruptcy, in Columns 8 and 9 of Table 2, panel B. Consequently, the effect of higher bankruptcy costs is simply to reduce the value of creditor's collateral, and thus the debt value and firm value[14].

Our assumption that in bankruptcy absolute priority of claims is maintained contrasts with the literature on strategic debt service (e.g., Anderson and Sundaresan (1996), Mella-

[14] However, higher bankruptcy costs will reduce the value of the debt, and will lead the firm to choose a lower q to maximize the firm value, and a higher liquidity reserve. (To conserve space, this is not shown.) This is consistent with the empirical finding of Kim et al (1998) who find weak evidence of a positive association between bankruptcy costs and liquidity holdings.

Barral and Perraudin (1997) and Acharya et.al. (2002)). From this literature, it seems likely that allowing for deviations from absolute priority in our setting would make equity value sensitive to bankruptcy costs. Not only would this directly affect the values received by shareholders in the event of bankruptcy, but it would also likely affect the cash management policy which indirectly would affect the probability of bankruptcy.

3.5 Efficiency of external finance θ:

In our model the efficiency of external capital markets is captured by the parameter θ which is the fraction of the value of securities issued by the firm which accrues to the firm. The fraction $1 - \theta$ is lost through the underwriting process as commissions, fees or excessive dilution.

In Column 10 of Table 2, we consider the extreme case of $\theta = 0.01$, which may be taken as the case when capital markets are so inefficient as to be virtually useless. In Column 11 we take the opposite extreme of $\theta = 0.95$ which would seem to correspond to a very efficient capital market and small agency costs. While θ cannot be directly interpreted as the degree of underpricing, the estimates of the underpricing of IPO's, range from 5 per cent to often greater than 20 per cent. In the U.S. seasoned issues typically involve somewhat lower costs. From the estimates of Lee et.al. (1996) using the U.S. data, reasonable estimates might be 11% for IPO's and 7% for seasoned issues. This suggests that $\theta = 0.95$ would be appropriate only for very highly developed capital markets, and where there is no information asymmetry. Our benchmark value is $\theta = 0.8$.

From Columns 10 and 11 of Table 2 we see that when capital market efficiency increases, the average level of liquid asset holding decreases. Focusing on the average profit case $\rho = 0.15$, liquid assets average about 50 per cent of the net firm value in the very underdeveloped capital market context. In the 95 per cent efficient capital market, optimal liquid assets correspond to only about 3 per cent of firm value. This result is consistent with that of Opler et al (1999), who document empirically that firms with greater access to capital markets carry smaller amounts of liquidity reserve. This result is also consistent with the findings of Ditmar et.al. who study an international comparison of large firms and find a negative association between cash holdings and an index of investor protections. A similar result is obtained in study of listed firms in twelve continental European countries by Ferreira and Vilela (2002) who find a negative association between liquidity and capital market development, proxied by the ratio of the country's free float (i.e., total value of stocks held by minority shareholders) to GDP.

Continuing the analysis of the normal profit case we see that the value of the firm is increasing in θ which is as we would expect. Interestingly, most of the benefits of this increased capital market efficiency accrues to debt holders. That is, when equity issuance is more efficient, shareholders are slower to abandon the firm, so that creditors stand a better chance of being paid. This is reflected in the credit spreads and bankruptcy probabilities being much lower in the efficient capital market case.

Finally, it is interesting that a more efficient capital market is associated with higher levels of equity volatility. An intuitive explanation for this is that the higher liquidity associated with market inefficiency serves to dampen the equity volatility. This intuition will be supported in Subsection 4.2 below, where we will see that the equity volatility is much higher, when the firm is not allowed to hold a liquidity reserve.

3.6 Internal cost of financial slack $r - r_{in}$:

In our framework the cost of holding liquidity inside the firm is represented by the $r - r_{in}$. This is a proxy for the amount of rent extraction that is obtained through various forms of managerial moral hazard. In Columns 12 and 13 of Table 2 we take $r_{in} = 3\%$ and 5% while we maintain our other benchmark parameters including $r = 6\%$. In our view $r_{in} = 3\%$ would indicate severe agency problems since in this case half the income flow from liquid assets would be diverted or wasted by managers.

First we note that the average level of liquid asset holding is strongly increasing in r_{in}. This is reasonable, since a reduction in the costs of holding liquid assets should lead shareholders to hold greater amounts of such assets. Focusing on the normal profit case ($\rho = 0.15$) we see that the value of the firm is strongly increasing in r_{in} as we would expect. At $r_{in} = 0.05$ when insiders extract only 1/6 of the return on liquid assets, the optimal level of liquid asset holding is about 25 per cent of net firm value.

The values of both equity and debt are increasing in r_{in}. The benefit to shareholders is direct since as residual claims on the firm's cash flows any increase in the return on a given level of liquid securities accrues to them. The benefit to creditors is indirect through the fact that the higher level of cash holding by the firm reduces the chances of bankruptcy. The sharing of the benefits of higher r_{in} is fairly evenly spread across debt and equity. As with varying θ, the higher level of liquid asset holding here is associated with a lower level of equity volatility.

4 Agency Conflicts, Debt Covenants and the Liquidity Reserve

In our model, the liquidity reserve is chosen by the shareholders to maximize the share price. This choice is made simultaneously with the default strategy, and the debt holders are assumed to have no influence in these choices. It is thus possible that the liquidity reserve strategy of the shareholders is detrimental to the debt holders, or to the economic efficiency of the firm.

In this section we will address a number of issues arising from this debt-equity conflict. First we ask what the consequences would be for the debt, equity and firm values, if the managers were maximizing the firm value, rather than the equity value. The answer to this tells us the severity of the conflict in terms of how much the equity holders are gaining from the bond holders via their liquidity strategy. Also, since the debt holders can anticipate this gain when they originally buy the bonds, the consequences for the firm value are the appropriate measure of the economic cost of the liquidity strategy. Our answer will be that, at our benchmark parameters, the managers would maintain a higher, not a lower liquidity reserve, if they were maximizing the firm value. This can be explained by noting that the liquidity reserve is protecting against bankruptcy, as well as protecting the productive asset, and bankruptcy costs fall on the bond holders. Also in our model, the tax shield is lost on bankruptcy. We also see that this higher liquidity greatly benefits the debt value and the firm value, with relatively little erosion of the equity value. To gain more insight, we also investigate the case when the bankruptcy cost α is lower, at 5%, and $\tau = 0$, so that there is no tax shield to protect. In this last case, maximizing the firm value, rather than the equity value has a relatively small effect the firm value, and mostly has the effect of transferring value from the equity holder to the debt holders.

Second, we ask what would be the consequences if firm were prohibited from holding a liquidity reserve. Under such a prohibition, the equity holders must lose, since they are forced to declare bankruptcy suboptimally. But it is unclear whether the debt holders will lose or gain: they will tend to gain from the suboptimal bankruptcy, but they will tend to lose from the more imminent bankruptcy costs. At our benchmark parameters, the debt holders lose slightly under this prohibition, but with $\tau = 0, \alpha = 5\%$, they gain slightly, though greatly at the expense of the equity and the firm value.

Third, we analyze two debt covenants, and discuss these in relation to the possibility that the firm will make a discrete liquidating dividend at bankruptcy. The first covenant requires the firm to keep a minimum liquidity reserve, which the debt holders can claim on bankruptcy. The second covenant prohibits the firm from paying a dividend when it is not profitable. Both of these enhance the debt value at the expense of the equity value. But the first usually erodes the firm value, and so it is of no economic value; and it does not obviate the possibility of the discrete liquidating dividend. By contrast, the second enhances the firm value, and does obviate the possibility of the discrete liquidating dividend, and so it is to be preferred.

Finally, we make some more remarks on the discrete liquidating dividend, and show that it is not the cash flow associated with this dividend which is significantly detrimental to the bond holders, but the fact that in making it, the shareholders trigger bankruptcy prematurely.

4.1 Taking the liquidity Reserve to Maximize the Firm value:

The levered firm value is the discounted expectation of dividends and payments to debt, up to bankruptcy, at which time its value is $1-\alpha$ times its unlevered value. Following our discussion in Section 2.3, if the firm value is being maximized, then its value $F_t^q(\rho, C)$ will satisfy

$$q + \frac{\partial}{\partial t}F_t^q - rF_t^q + \mu(\rho)\frac{\partial}{\partial \rho}F_t^q + \frac{1}{2}\rho\eta^2\frac{\partial^2}{\partial \rho^2}F_t^q$$
$$+ \left[(1-\tau)(\rho-(f+q)) + r_{in}C\right]\frac{\partial}{\partial C}F_t^q + \frac{1}{2}\sigma^2\frac{\partial^2}{\partial C^2}F_t^q = 0 \tag{4.1}$$

in the 'save' region $\mathcal{S}^F$;

$$\frac{\partial}{\partial C}F_t^q = 1, \tag{4.2}$$

in the 'dividend' region $\mathcal{D}^F$; and

$$\frac{\partial}{\partial C}F_t^q = \frac{1}{\theta}. \tag{4.3}$$

in the 'issue' region $\mathcal{I}^F$. Consistent with the boundary conditions in Section 2.3, if the firm become bankrupt, then it has value $(1-\alpha)J_0^0(\rho, 0)$, and at the horizon time T, we take the value to be $C + \min\{q/r, (1-\alpha)J_0^0(\rho, 0)\}$.

The numerical procedure described in Subsection 2.3 is easily adapted to this case, and the result is summarized in Table 3. In this table, the first two columns correspond to our benchmark case; the first column repeats the relevant numbers from Table 1, except that we also present results for $\rho = 0.05$('very low'). The second column corresponds to maximizing the firm value. In this column, we calculate the firm value as just described, and then the debt value as in Section 2.3, based on the regions $\mathcal{S}^F, \mathcal{D}^F$ and $\mathcal{I}^F$, and the share value is then the firm value minus the debt value. We see, as expected, that the net firm value (i.e. net of the value of the liquidity reserve) is higher, the debt value his higher, and the net equity value is lower in the second column, than in the first. In fact the gain to debt is much higher than the loss to equity. Also, the liquidity level is higher. In fact when maximizing the firm value, and for very low ρ, the equity value is negative, but the firm still operates, whereas the equity maximizing firm has already gone bankrupt.

The results corresponding to Table 1 Panel B, i.e. the credit spreads and bankruptcy probabilities, and all zero for maximizing the firm value, in the 4 cases discussed in this subsection, and so these are not displayed. These indicate that the firm never goes bankrupt when the firm value is being maximized.

Columns 3 and 4 repeat the case of columns 1 and 2, but with bankruptcy cost at 5%, rather than 30%. Column 3 here is a repeat of Column 8 of Table 2, and comparing Columns 1 and 3, the equity value and liquidity holdings are the same, but the debt value is higher, as discussed in Subsection 3.4. Comparing Columns 3 and 4, the result is much the same as the comparison between Columns 1 and 2, except that the gain to debt in Column 4 is not so much, since the avoidance of the bankruptcy cost is not so crucial.

Columns 5 and 6 again repeat the case of columns 1 and 2, but with $\tau = 0$, so that there is no tax shield. Comparing these again gives much the same result as comparing columns 1 and 2.

Columns 7 and 8 present the case $\alpha = 0.05$ and $\tau = 0$. Comparing Column 7 with Column 1, the equity and debt values are both higher, as expected. Also, the liquidity reserve is higher. Comparing Columns 7 with 8, the firm value and liquidity reserve are very little different, and we see that the differences in debt and equity values are substantially more than the change in the net firm value.

To summarize this subsection: Maximizing the firm value, rather than the equity value, is particularly valuable to debt holders, and to the firm value itself, when bankruptcy costs and the tax shield are significant. In this case, the agency conflict results in a significant erosion of economic value. But when bankruptcy costs and the tax shield are not high, the agency conflict is associated with a transfer of value between the equity and debt holders, more than a destruction of economic value[15].

[15] This result is in tune with the findings of Titman, Tompaidis and Tayplakov (2004), who study the debt and equity values of a levered firm, and compare these values under an equity-optimizing investment strategy, with the values if the investment strategy were constrained to be as if the firm were unlevered. They show that the equity-optimal strategy transfers value from debt to equity, but does not destroy much value. These authors have in mind real estate project financing, and 'investment' in their context corresponds to maintenance of the real estate. They take the bankruptcy cost to be zero in their benchmark case, and they do not have a tax shield effect in their model, corresponding to our condition $\tau = 0$.

4.2 What if the Liquidity Reserve is Prohibited?:

In this case, the equation for the equity value reduces to

$$\frac{\partial}{\partial t}J_t^q - rJ_t^q + \frac{dD_t}{dt} + \mu(\rho)\frac{\partial}{\partial \rho}J_t^q + \frac{1}{2}\rho\eta^2\frac{\partial^2}{\partial \rho^2}J_t^q + \Theta(\rho - (f+q)) = 0, \tag{4.4}$$

in the region $\rho > B_t$ in which $J_t^q(\rho) > 0$, and $J_t^q(\rho) = 0$ in the region $\rho \leq B_t$, where B_t is the profitability level below which the firm declares bankrupt. Also, we take $\Theta(d) = (1-\tau)d$ for $d > 0$ corresponding to paying dividends, and $\Theta(d) = ((1-\tau)/\theta)d$ for $d < 0$, corresponding to issuing equity to cover cash shortfalls. The factor $(1-\tau)$ here reflects that tax is paid on operating income net of interest payments and fixed costs and tax loss write-backs are allowed. The solution is developed backwards in time t from $J_t^q(\rho) = 0$ at a horizon time $t = T$, and the barrier B_t is chosen to maximize the value of $J_t^q(\rho)$ at each t and ρ.

The equation for the debt reduces to

$$q + \frac{\partial}{\partial t}P_t^q - rP_t^q + \mu(\rho)\frac{\partial}{\partial \rho}P_t^q + \frac{1}{2}\rho\eta^2\frac{\partial}{\partial \rho^2}P_t^q = 0, \tag{4.5}$$

in the region $\rho_t > B_t$ and $P_t^q(\rho) = (1-\tau)J^0(\rho)$ in the region $\rho_t < B_t$. These reduced equations are been solved in a similar way to the full equations.

In comparing the case when liquidity holdings are allowed, and when they are prohibited, we calculate the bankrupt value in both cases assuming that the debt holders can efficiently cover any cash shortfalls, if they take over the firm, i.e. that they have 'deep pockets'. With deep pockets, the bond holders do not need a liquidity reserve, and the bankrupt valuation is is achieved by solving Equation (4.4), with $\theta = 1$. Our purpose in doing this is to ensure that the bankrupt firm value is the same, whether or not the share holders are allowed to hold liquidity.

The results are given in Table 4. The first column repeats the benchmark case of Table 1, Column 6, or Table 2, Column 1, except that the debt is more valuable, since the debt holders having 'deep pockets' leads to the firm being more valuable in bankruptcy. The second column uses the same parameters, but with liquidity reserve prohibited. We see that the prohibition substantially reduces the values of both equity and debt. In Columns 3 and 4 the bankruptcy cost is reduced to 5%, and as expected, this change leaves the equity value is unchanged, but the debt value is increased. The prohibition still reduces the equity value substantially, but reduces the debt value much less. Columns 5 and 6 correspond to $\tau = 0, \alpha = 30\%$. Decreasing τ has made the equity more valuable, but the effect of the prohibition is again to reduce the equity value substantially, and to leave the debt value virtually unchanged. Columns 7 and 8 correspond to $\tau = 0, \alpha = 5\%$, and now we see that the liquidity prohibition reduces the equity value but slightly increases the debt value.

To summarize: Equity holders will always lose from a debt covenant prohibiting a liquidity holding, since they will be forced to declare bankrupt suboptimally, losing the productive asset and the tax shield, if $\tau > 0$. The bondholders tend to gain from the suboptimal bankruptcy, but lose from the more imminent bankruptcy costs, under such a covenant, and if these costs are low, the net effect might be a slight gain. However, the firm value is always lower under such a prohibition[16].

The debt values in Table 4 are surprisingly high, and the credit spreads correspondingly low, and in fact negative in Columns 7 and 8. This is because the firm is more valuable to the debt holders than to the equity holders, even if the equity holders are unlevered, since the debt holders have 'deep pockets' ($\theta = 1$), and can finance any cash shortfalls efficiently. It seems unlikely that the debt would actually carry a negative credit spread, and more likely that there would be a departure from absolute default priority rules, in this case. However, these calculations serve to demonstrate that the debt can sometimes gain from a liquidity prohibition.

4.3 Two Debt Covenants:

To address agency conflicts analyzed above, it is natural for the debt holders to impose covenants on the liquidity that the firm can hold, or relating to the possible discrete liquidating dividend. We have seen in Sections 4.1 and 4.2 that allowing the firm to hold a liquidity reserve usually benefits debt holders as well as equity holders. Therefore a covenant prohibiting the liquidity reserve would usually be of no value. A more promising covenant might therefore be to require that the liquidity reserve be maintained at not less than a minimum level say $\underline{C}$, which the debt holders can claim in bankruptcy. This covenant can easily be incorporated into our dynamic programme by altering the boundary conditions, so that the firm becomes bankrupt at $C = \underline{C}$, and the debt value on bankruptcy is $(1-\alpha)J_0^0(\rho, 0)+\underline{C}$, in which ρ is the profitability at bankruptcy.

The results are given in Table 5. In this table, Column 1 repeats the benchmark case of Section 2.4, and Columns 2 and 3 implement the covenant, respectively with $\underline{C} = 0.005$ and $\underline{C} = 0.010$. We see that the effect of the covenant is that the firm holds extra liquidity, only slightly less than the amount reserved for the bond holders, and the net equity value (i.e. net of all liquidity) is eroded. This erosion can be explained since this extra liquidity is imputed to earn interest at the inefficient rate r_{in}. In fact, apart form being an extra burden,

[16] This result is again in tune with the analysis of Titman, Tompaidis and Tayplakov (2004), concerning the effects of debt covenants that prohibit the injection of extra cash unless the project value is less than the debt value. These cause the equity value to be lower, but the debt value can be higher.

this bond holder reserved liquidity does not have a significant dynamic effect on the liquidity reserve, and in particular, it does not postpone bankruptcy. Also, the prospect of the discrete liquidating dividend is largely unaltered. The debt value is higher, since the debt holders can expect a higher payoff in bankruptcy, but the firm value is usually eroded, reflecting simply that the firm is carrying an extra liquidity amount $\underline{C}$, which earns an inefficient return.

We next analyze a covenant which prohibits the firm from paying a dividend when it is not currently sufficiently profitable. Leuz, Deller and Stubenrath (1998) note that debt covenants restricting dividend payouts when the firm is in distress are common in the US, but not in the UK. They argue that such covenants are considered unnecessary in the UK because company law already restricts the conditions under which firms can pay dividends, largely based on recent earnings.

In detail, we consider a covenant which prohibits dividend payments when when $\rho_t < f+q-L$, where L represents a measure of lee-way allowed under the covenant. This covenant will automatically obviate the possibility of the discrete liquidating dividend, at least if L is not chosen to be so big that the firm is allowed to pay dividends when it is not worthwhile to continue operating. This covenant can be incorporated into our dynamic programme, by imposing Equation (2.6) above for all (ρ, C) such that $\rho < f+q-L$, irrespective of whether $\frac{\partial}{\partial C} J_t^q > 1$.

The results are given in Table 5, Columns 4, 5, 6 corresponding respectively to $L = 0.00, 0.02, 0.04$. Comparing these columns with Column 1, which corresponds to there being no covenant, we see that the covenant enhances the debt and firm values significantly, and much more than it erodes the equity value.

The enhancement of the debt at the expense of the equity might be attributed at least partially to the direct value of the prospective liquidation dividend, which the covenant has saved for the debt holders at the expense of the equity holders. However, the value of the liquidating dividend is very small, when the firm is not currently in distress. This contingent cash flow can be valued by simulating its discounted risk neutral expectation, and if currently $\rho = 0.2$, then its value is about 0.00008[17]. This is less than the wealth transfer between debt and equity, associated with the covenant.

Preventing the transfer associated with the liquidating dividend cannot in any case account for the enhancement of the firm value, and this must be attributed to the fact that the covenant makes bankruptcy less imminent, since the firm will continue operating at any ρ value, if there is some liquidity reserve. In fact we see from the table that the firm might

[17]Simulation also shows that the a liquidating dividend will be paid in about one quarter of bankruptcies, and if it is paid, then its average value will be about 0.01. The value at $\rho = 0.2$ is so low because at such ρ bankruptcy is not expected for decades.

continue operating at $\rho = 0.05$, when the net equity value is negative.

It is interesting that the average amount of the liquidity reserve is not much affected by this covenant, and the lee-way value L does not have much effect on the valuations. Overall, our results are consistent with the view that covenants imposing a profitability condition for dividends are value increasing.

5 Summary and Conclusions

We have developed a structural dynamic model of a company's optimal holding of liquid reserve, together with its optimal debt and equity issuance and dividend policy. Our model allows for a stochastic mean reverting earnings rate, and we it solve numerically, as a non-linear PDE with free boundaries, with the expected earnings and cash levels as the state variables.

We have shown with this model that it is in shareholders' interests to hold relatively large amounts of cash inside the firm the even if they have access to relatively efficient capital markets and even if some of the return cash is dissipated by insiders. The reason that forcing the firm to pay out "idle" cash in relatively good times may be a bad idea, is that it is myopic– it does not properly weight the future dilution costs to shareholders of raising funds, if the firm later were to approach financial distress.

While the interests of shareholders and creditors regarding cash holding policy are not perfectly aligned, they are not totally divergent either. Our comparison of a share-value-maximizing policy with a firm-value-maximizing policy reveals that the targeted levels of cash are often quite similar under both policies. The one area where their interests clearly diverge is when the firm's earnings are persistently low. In such circumstances, it is optimal for shareholders to pay out available cash in the form of special dividend, thus exposing the firm to financial distress. In this regard we have shown that it may enhance firm value to prevent the firm from paying dividends when earnings are low, as is often done through bond covenants or as seen in some systems of corporate law.

In order to assure the relevance of our solutions we have been careful in calibrating our model. For plausible parameter values, our model is able to simultaneously hit many empirical benchmarks including: firm leverage, liquidity holdings, credit spreads, default probabilities at various time horizons, debt recovery rates given default, and equity volatility. Our model also exhibits realistic comparative statics behavior. For example, it has been documented that liquidity holdings increases when external finance becomes more costly, and our model reflects this.

Our model also gives some new insights: First, we corroborate the recent conclusion of

Mello and Parsons (2000) and Rochet and Villeneuve (2004), that hedging can enhance firm value, by decreasing the optimal liquidity reserve. Regarding the asset substitution effect, whereby increasing earnings volatility tends to benefit equity at the expense of debt, we see that this effect only occurs if the extra volatility affects the earnings state variable in a dynamic way, and will not occur if the earnings cash flow is merely subjected to a random perturbation. Finally, the liquidity reserve has the effect of dampening the equity price volatility. Thus, if the market is more efficient in terms of external finance being less costly, then the equity volatility will be higher, since the liquidity holding will be lower.

TABLE 1

Panel A - Values of Equity, Debt and Liquidity

Payment to debt, q	0.0000	0.0005	0.0010	0.0015	0.0020	0.0025	0.0030	0.0035	0.0040	0.0045	0.0050	0.0055	0.0060
Measures at $\rho = 0.10$													
Liquidity	0.0071	0.0072	0.0074	0.0074	0.0076	0.0073	0.0084	0.0079	0.0078	0.0077	0.0071	0.0072	0.0103
Equity val	0.0598	0.0539	0.0479	0.0420	0.0362	0.0303	0.0249	0.0195	0.0146	0.0104	0.0068	0.0042	0.0027
Debt val	0.0000	0.0083	0.0167	0.0250	0.0332	0.0411	0.0481	0.0536	0.0564	0.0585	0.0588	0.0546	0.0516
Net firm val	0.0598	0.0622	0.0647	0.0670	0.0694	0.0714	0.0731	0.0731	0.0711	0.0689	0.0657	0.0588	0.0543
Leverage	0.0000	0.1202	0.2317	0.3353	0.4310	0.5218	0.5906	0.6615	0.7142	0.7625	0.8072	0.8266	0.7983
Equity volatility	0.3616	0.3961	0.4377	0.4899	0.5538	0.6427	0.7171	0.8545	0.9947	1.1412	1.3049	1.3514	0.9485
Measures at $\rho = 0.15$													
Liquidity	0.0185	0.0190	0.0194	0.0201	0.0204	0.0202	0.0212	0.0207	0.0203	0.0201	0.0196	0.0194	0.0191
Equity val	0.1009	0.0951	0.0892	0.0835	0.0777	0.0720	0.0664	0.0609	0.0557	0.0509	0.0463	0.0422	0.0385
Debt val	0.0000	0.0083	0.0167	0.0250	0.0333	0.0414	0.0490	0.0558	0.0611	0.0659	0.0696	0.0709	0.0719
Net firm val	0.1009	0.1034	0.1060	0.1085	0.1110	0.1134	0.1155	0.1168	0.1169	0.1168	0.1160	0.1132	0.1104
Leverage	0.0000	0.0682	0.1332	0.1946	0.2534	0.3100	0.3588	0.4060	0.4455	0.4813	0.5134	0.5348	0.5548
Equity volatility	0.2186	0.2291	0.2407	0.2527	0.2670	0.2847	0.2993	0.3217	0.3454	0.3695	0.3962	0.4210	0.4469
Measures at $\rho = 0.20$													
Liquidity	0.0059	0.0060	0.0072	0.0082	0.0083	0.0091	0.0103	0.0103	0.0116	0.0125	0.0125	0.0124	0.0111
Equity val	0.1381	0.1323	0.1265	0.1207	0.1150	0.1093	0.1037	0.0983	0.0931	0.0881	0.0835	0.0793	0.0754
Debt val	0.0000	0.0083	0.0167	0.0250	0.0333	0.0415	0.0492	0.0561	0.0616	0.0667	0.0708	0.0727	0.0743
Net firm val	0.1381	0.1407	0.1433	0.1458	0.1483	0.1508	0.1529	0.1544	0.1547	0.1549	0.1544	0.1521	0.1497
Leverage	0.0000	0.0570	0.1111	0.1626	0.2128	0.2594	0.3014	0.3406	0.3705	0.3985	0.4243	0.4421	0.4616
Equity volatility	0.2087	0.2173	0.2247	0.2331	0.2440	0.2540	0.2638	0.2770	0.2871	0.2983	0.3122	0.3262	0.3449
Measures at $\rho = 0.25$													
Liquidity	0.0024	0.0024	0.0024	0.0024	0.0024	0.0024	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025
Equity val	0.1753	0.1695	0.1638	0.1580	0.1522	0.1466	0.1410	0.1355	0.1303	0.1254	0.1207	0.1165	0.1125
Debt val	0.0000	0.0083	0.0167	0.0250	0.0333	0.0415	0.0492	0.0562	0.0618	0.0670	0.0713	0.0734	0.0753
Net firm val	0.1753	0.1779	0.1805	0.1831	0.1856	0.1881	0.1902	0.1918	0.1922	0.1925	0.1921	0.1900	0.1878
Leverage	0.0000	0.0464	0.0914	0.1351	0.1773	0.2179	0.2556	0.2894	0.3178	0.3440	0.3666	0.3817	0.3955
Equity volatility	0.1876	0.1940	0.2008	0.2081	0.2158	0.2241	0.2329	0.2421	0.2515	0.2612	0.2708	0.2802	0.2894

TABLE 1

Panel B - Credit Spreads and Bankruptcy Probabilities

Payment to debt, q	0.0000	0.0005	0.0010	0.0015	0.0020	0.0025	0.0030	0.0035	0.0040	0.0045	0.0050	0.0055	0.0060
Measures at $\rho = 0.10$													
Credit spreads:													
Perptual bond	N/A	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.010	0.016	0.024	0.040	0.056
5 year PDB	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.019	0.040	0.061	0.088	0.155	0.221
20 year PDB	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.018	0.027	0.039	0.066	0.094
Prob. bankrupt after:													
1 year	0.000	0.000	0.000	0.000	0.002	0.006	0.016	0.042	0.096	0.151	0.229	0.386	0.518
5 years	0.000	0.000	0.000	0.000	0.003	0.009	0.025	0.060	0.128	0.194	0.280	0.451	0.585
20 years	0.000	0.000	0.000	0.001	0.005	0.014	0.037	0.087	0.180	0.270	0.376	0.565	0.706
Measures at $\rho = 0.15$													
Credit spreads:													
Perptual bond	N/A	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.008	0.011	0.017	0.023
5 year PDB	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.008	0.013	0.019	0.031	0.046
20 year PDB	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.010	0.016	0.022	0.035	0.050
Prob. bankrupt after:													
1 year	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.004	0.007	0.014
5 years	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.018	0.029	0.044	0.071	0.106
20 years	0.000	0.000	0.000	0.000	0.002	0.006	0.015	0.037	0.076	0.119	0.171	0.264	0.366
Measures at $\rho = 0.20$													
Credit spreads:													
Perptual bond	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.007	0.010	0.015	0.020
5 year PDB	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.009	0.013	0.021	0.032
20 year PDB	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.009	0.014	0.020	0.033	0.047
Prob. bankrupt after:													
1 year	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5 years	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.005	0.010	0.017	0.027	0.044	0.067
20 years	0.000	0.000	0.000	0.000	0.002	0.005	0.014	0.033	0.069	0.108	0.156	0.242	0.338
Measures at $\rho = 0.25$													
Credit spreads:													
Perptual bond	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.007	0.010	0.014	0.019
5 year PDB	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.005	0.008	0.011	0.019	0.028
20 year PDB	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.004	0.009	0.014	0.020	0.032	0.046
Prob. bankrupt after:													
1 year	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5 years	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.004	0.009	0.014	0.022	0.037	0.057
20 years	0.000	0.000	0.000	0.000	0.001	0.005	0.013	0.032	0.067	0.105	0.152	0.236	0.330

TABLE 2 - Comparative Statics

Panel A; Values of Equity, Debt and Liquidity

Parameter being varied	None	η		σ		κ		α		θ		r_{in}	
Parameter value	(Repeat	0.08	0.10	0.01	0.02	0.8	1.0	0.05	0.50	0.01	0.95	0.03	0.05
Benchmark value	Benchmark)	0.09	0.09	0.00	0.00	0.9	0.9	0.30	0.30	0.80	0.80	0.04	0.04
TS average liquidity	0.016	0.015	0.015	0.026	0.023	0.015	0.015	0.016	0.016	0.041	0.004	0.015	0.031
Measures at $\rho = 0.10$													
Liquidity	0.0077	0.0068	0.0093	0.0104	0.0178	0.00816	0.0083	0.0077	0.0077	0.0347	0.0000	0.0027	0.0250
Net equity val	0.0104	0.0105	0.0107	0.0093	0.0078	0.00812	0.0132	0.0104	0.0104	0.0064	0.0209	0.0082	0.0153
Debt val	0.0585	0.0635	0.0534	0.0560	0.0510	0.05065	0.0653	0.0624	0.0551	0.0210	0.0721	0.0541	0.0640
Net firm val	0.0689	0.0740	0.0642	0.0654	0.0588	0.05877	0.0786	0.0729	0.0655	0.0274	0.0931	0.0624	0.0794
Leverage	0.7625	0.7853	0.7258	0.7390	0.6657	0.75660	0.7508	0.7741	0.7514	0.3381	0.7739	0.8316	0.6132
Equity volatility	1.1412	1.1049	1.1121	1.1791	1.1539	1.25228	0.9446	1.1412	1.1412	0.4323	0.9944	1.8182	0.5338
Measures at $\rho = 0.15$													
Liquidity	0.0201	0.0186	0.0218	0.0238	0.0308	0.02130	0.0205	0.0201	0.0201	0.0452	0.0044	0.0130	0.0371
Net equity val	0.0509	0.0515	0.0505	0.0494	0.0466	0.05181	0.0506	0.0509	0.0509	0.0452	0.0583	0.0490	0.0542
Debt val	0.0659	0.0696	0.0614	0.0646	0.0620	0.06146	0.0699	0.0681	0.0638	0.0428	0.0734	0.0635	0.0689
Net firm val	0.1168	0.1211	0.1120	0.1141	0.1087	0.11327	0.1206	0.1190	0.1147	0.0880	0.1317	0.1125	0.1231
Leverage	0.4813	0.4981	0.4591	0.4685	0.4444	0.45670	0.4957	0.4897	0.4732	0.3212	0.5388	0.5058	0.4301
Equity volatility	0.3695	0.3327	0.4012	0.3841	0.4294	0.39853	0.3342	0.3695	0.3695	0.2875	0.4101	0.4269	0.2829
Measures at $\rho = 0.20$													
Liquidity	0.0125	0.0083	0.0145	0.0172	0.0247	0.01257	0.0126	0.0125	0.0125	0.0347	0.0026	0.0046	0.0288
Net equity val	0.0881	0.0888	0.0877	0.0867	0.0839	0.09333	0.0844	0.0881	0.0881	0.0823	0.0952	0.0864	0.0911
Debt val	0.0667	0.0701	0.0626	0.0655	0.0632	0.06273	0.0704	0.0687	0.0647	0.0452	0.0736	0.0645	0.0695
Net firm val	0.1549	0.1590	0.1503	0.1523	0.1471	0.15606	0.1548	0.1569	0.1529	0.1276	0.1688	0.1510	0.1607
Leverage	0.3985	0.4191	0.3796	0.3867	0.3677	0.37202	0.4203	0.4058	0.3913	0.2789	0.4292	0.4146	0.3668
Equity volatility	0.2983	0.2747	0.3255	0.3043	0.3322	0.31645	0.2802	0.2983	0.2983	0.2564	0.3044	0.3311	0.2489
Measures at $\rho = 0.25$													
Liquidity	0.0025	0.0025	0.0025	0.0079	0.0138	0.00250	0.0025	0.0025	0.0025	0.0154	0.0025	0.0025	0.0067
Net equity val	0.1254	0.1260	0.1249	0.1239	0.1211	0.13486	0.1181	0.1254	0.1254	0.1195	0.1322	0.1238	0.1282
Debt val	0.0670	0.0704	0.0630	0.0659	0.0636	0.06329	0.0706	0.0690	0.0651	0.0459	0.0737	0.0650	0.0697
Net firm val	0.1925	0.1965	0.1880	0.1899	0.1848	0.19815	0.1888	0.1945	0.1905	0.1655	0.2059	0.1888	0.1979
Leverage	0.3440	0.3538	0.3308	0.3334	0.3205	0.31542	0.3691	0.3506	0.3375	0.2537	0.3537	0.3398	0.3406
Equity volatility	0.2612	0.2307	0.2911	0.2643	0.2885	0.27137	0.2507	0.2612	0.2612	0.2481	0.2464	0.2648	0.2469

TABLE 2 - Comparative Statics

Panel B; Credit Spreads and Bankruptcy Probabilities

Parameter being varied	None	η		σ		κ		α		θ		r_{in}	
Parameter value	(Repeat	0.08	0.10	0.01	0.02	0.8	1.0	0.05	0.50	0.01	0.95	0.03	0.05
Benchmark value	Benchmark)	0.09	0.09	0.00	0.00	0.9	0.9	0.30	0.30	0.80	0.80	0.04	0.04
Measures at $\rho = 0.10$													
Credit spreads:													
Perptual bond	0.0168	0.0108	0.0242	0.0202	0.0280	0.02884	0.0088	0.0120	0.0216	0.1537	0.0023	0.0230	0.0102
5 year PDB	0.0612	0.0433	0.0803	0.0728	0.0994	0.09686	0.0337	0.0612	0.0613	0.1983	0.0083	0.0875	0.0343
20 year PDB	0.0279	0.0178	0.0400	0.0327	0.0433	0.04317	0.0150	0.0279	0.0279	0.0907	0.0040	0.0388	0.0163
Prob. bankrupt after:													
1 year	0.1517	0.1167	0.1836	0.1804	0.2495	0.24764	0.0872	0.1517	0.1517	0.3259	0.0169	0.2289	0.0726
5 years	0.1949	0.1450	0.2434	0.2292	0.3067	0.30249	0.1119	0.1949	0.1949	0.4786	0.0253	0.2785	0.1054
20 years	0.2701	0.1887	0.3528	0.3114	0.4019	0.39635	0.1555	0.2701	0.2701	0.6105	0.0386	0.3719	0.1519
Measures at $\rho = 0.15$													
Credit spreads:													
Perptual bond	0.0082	0.0046	0.0131	0.0096	0.0125	0.01322	0.0042	0.0060	0.0105	0.0450	0.0012	0.0108	0.0052
5 year PDB	0.0137	0.0072	0.0223	0.0157	0.0204	0.02113	0.0070	0.0137	0.0137	0.0374	0.0020	0.0191	0.0074
20 year PDB	0.0160	0.0087	0.0254	0.0183	0.0235	0.02418	0.0083	0.0160	0.0160	0.0492	0.0024	0.0217	0.0093
Prob. bankrupt after:													
1 year	0.0024	0.0010	0.0046	0.0027	0.0042	0.00418	0.0010	0.0024	0.0024	0.0057	0.0002	0.0041	0.0008
5 years	0.0294	0.0153	0.0479	0.0337	0.0450	0.04496	0.0147	0.0294	0.0294	0.0752	0.0039	0.0426	0.0148
20 years	0.1196	0.0654	0.1849	0.1361	0.1754	0.17269	0.0627	0.1196	0.1196	0.3027	0.0175	0.1662	0.0646
Measures at $\rho = 0.20$													
Credit spreads:													
Perptual bond	0.0074	0.0041	0.0118	0.0086	0.0111	0.01173	0.0039	0.0054	0.0094	0.0393	0.0011	0.0097	0.0047
5 year PDB	0.0093	0.0049	0.0154	0.0106	0.0137	0.01404	0.0050	0.0093	0.0093	0.0252	0.0013	0.0132	0.0049
20 year PDB	0.0148	0.0081	0.0237	0.0170	0.0218	0.02235	0.0078	0.0148	0.0149	0.0459	0.0023	0.0202	0.0087
Prob. bankrupt after:													
1 year	0.0000	0.0000	0.0002	0.0001	0.0001	0.00015	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000
5 years	0.0178	0.0091	0.0298	0.0203	0.0270	0.02576	0.0092	0.0178	0.0178	0.0461	0.0023	0.0263	0.0087
20 years	0.1087	0.0593	0.1690	0.1237	0.1594	0.15548	0.0574	0.1087	0.1087	0.2793	0.0158	0.1518	0.0585
Measures at $\rho = 0.25$													
Credit spreads:													
Perptual bond	0.0070	0.0039	0.0113	0.0082	0.0106	0.01110	0.0037	0.0051	0.0090	0.0379	0.0010	0.0092	0.0045
5 year PDB	0.0080	0.0041	0.0134	0.0090	0.0118	0.01182	0.0044	0.0080	0.0080	0.0239	0.0011	0.0113	0.0044
20 year PDB	0.0145	0.0079	0.0231	0.0166	0.0213	0.02177	0.0076	0.0145	0.0145	0.0455	0.0022	0.0197	0.0085
Prob. bankrupt after:													
1 year	0.0000	0.0000	0.0000	0.0000	0.0000	0.00001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5 years	0.0148	0.0074	0.0245	0.0168	0.0225	0.02054	0.0078	0.0148	0.0148	0.0417	0.0019	0.0218	0.0074
20 years	0.1059	0.0576	0.1642	0.1204	0.1554	0.15061	0.0560	0.1059	0.1059	0.2757	0.0154	0.1477	0.0572

TABLE 3

Maximizing Equity versus Maximizing the Firm Value

Tax rate τ	30%		30%		0%		0%	
Bankruptcy cost α	30%		5%		30%		5%	
Value being maximized	Equity	Firm	Equity	Firm	Equity	Firm	Equity	Firm
Measures at $\rho = 0.05$								
Liquidity	N/A	0.0000	N/A	0.0000	N/A	0.0000	N/A	0.0000
Net equity val	0.0000	-0.0403	0.0000	-0.0397	0.0000	-0.0555	0.0000	-0.0542
Debt val	0.0104	0.0747	0.0134	0.0750	0.0154	0.0750	0.0198	0.0750
Net firm val	0.0104	0.0343	0.0134	0.0353	0.0154	0.0195	0.0198	0.0207
Leverage	0.4047	2.1728	0.4663	2.1260	0.4343	3.8418	0.4966	3.6089
Perp. cr. spread	0.369	0.000	0.274	0.000	0.230	0.000	0.166	0.000
Measures at $\rho = 0.10$								
Liquidity	0.0077	0.0086	0.0077	0.0090	0.0103	0.0114	0.0103	0.0114
Net equity val	0.0104	0.0052	0.0104	0.0059	0.0153	0.0092	0.0153	0.0105
Debt val	0.0585	0.0747	0.0617	0.0750	0.0632	0.0751	0.0679	0.0751
Net firm val	0.0689	0.0799	0.0721	0.0810	0.0785	0.0844	0.0832	0.0857
Leverage	0.7625	0.8431	0.7719	0.8335	0.7112	0.7841	0.7256	0.7740
Perp. cr. spread	0.016	0.000	0.012	0.000	0.011	0.000	0.006	0.000
Measures at $\rho = 0.15$								
Liquidity	0.0201	0.0225	0.0201	0.0228	0.0279	0.0286	0.0279	0.0286
Net equity val	0.0509	0.0472	0.0509	0.0478	0.0731	0.0692	0.0731	0.0704
Debt val	0.0659	0.0747	0.0677	0.0751	0.0686	0.0752	0.0711	0.0752
Net firm val	0.1168	0.1220	0.1186	0.1229	0.1417	0.1444	0.1443	0.1457
Leverage	0.4813	0.5171	0.4881	0.5152	0.4044	0.4346	0.4132	0.4315
Perp. cr. spread	0.008	0.000	0.006	0.000	0.005	0.000	0.003	0.000
Measures at $\rho = 0.20$								
Liquidity	0.0125	0.0127	0.0125	0.0127	0.0153	0.0165	0.0153	0.0165
Net equity val	0.0881	0.0846	0.0881	0.0852	0.1263	0.1226	0.1263	0.1238
Debt val	0.0667	0.0748	0.0683	0.0751	0.0692	0.0753	0.0715	0.0753
Net firm val	0.1549	0.1594	0.1565	0.1603	0.1956	0.1979	0.1979	0.1991
Perp. cr. spread	0.3985	0.4345	0.4044	0.4341	0.3282	0.3511	0.3356	0.3491
Leverage	0.007	0.000	0.005	0.000	0.004	0.000	0.002	0.000
Measures at $\rho = 0.25$								
Liquidity	0.0025	0.0024	0.0025	0.0024	0.0025	0.0024	0.0025	0.0024
Net equity val	0.1254	0.1218	0.1254	0.1224	0.1795	0.1758	0.1795	0.1770
Debt val	0.0670	0.0748	0.0686	0.0751	0.0695	0.0753	0.0718	0.0753
Net firm val	0.1925	0.1967	0.1941	0.1976	0.2491	0.2512	0.2513	0.2524
Leverage	0.3440	0.3757	0.3493	0.3757	0.2763	0.2970	0.2828	0.2956
Perp. cr. spread	0.007	0.000	0.005	0.000	0.004	0.000	0.002	0.000

TABLE 4

Liquidity holding versus no liquidity holding

Tax rate τ	30%		30%		0%		0%	
Bankruptcy cost α	30%		5%		30%		5%	
Firm holds liquidity reserve	Yes	No	Yes	No	Yes	No	Yes	No
Measures at $\rho = 0.10$								
Liquidity	0.0077	N/A	0.0077	N/A	0.0103	N/A	0.0103	N/A
Net equity val	0.0104	0.0069	0.0104	0.0069	0.0153	0.0099	0.0153	0.0099
Debt val	0.0638	0.0610	0.0696	0.0690	0.0707	0.0706	0.0792	0.0821
Net firm val	0.0743	0.0680	0.0800	0.0760	0.0860	0.0805	0.0946	0.0920
Leverage	0.7779	0.8981	0.7925	0.9088	0.7338	0.8770	0.7554	0.8923
Perp. cr. spread	0.010	0.013	0.004	0.005	0.005	0.003	-0.003	-0.005
Equit volatility	1.1412	2.7108	1.1412	2.7108	1.1655	2.7096	1.1655	2.7096
Measures at $\rho = 0.15$								
Liquidity	0.0201	N/A	0.0201	N/A	0.0279	N/A	0.0279	N/A
Net equity val	0.0509	0.0462	0.0509	0.0462	0.0731	0.0661	0.0731	0.0661
Debt val	0.0689	0.0671	0.0721	0.0719	0.0728	0.0728	0.0774	0.0797
Net firm val	0.1198	0.1133	0.1230	0.1182	0.1459	0.1390	0.1506	0.1458
Leverage	0.4924	0.5919	0.5038	0.6085	0.4187	0.5244	0.4338	0.5468
Perp. cr. spread	0.005	0.007	0.002	0.002	0.001	0.001	-0.001	-0.003
Equity volatility	0.3695	0.5898	0.3695	0.5898	0.3708	0.5897	0.3708	0.5897
Measures at $\rho = 0.20$								
Liquidity	0.0125	N/A	0.0125	N/A	0.0153	N/A	0.0153	N/A
Net equity val	0.0881	0.0843	0.0881	0.0843	0.1263	0.1205	0.1263	0.1205
Debt val	0.0694	0.0677	0.0724	0.0722	0.0730	0.0731	0.0773	0.0795
Net firm val	0.1576	0.1521	0.1606	0.1566	0.1994	0.1936	0.2037	0.2000
Leverage	0.4081	0.4455	0.4182	0.4613	0.3402	0.3776	0.3530	0.3975
Perp. cr. spread	0.004	0.006	0.002	0.002	0.001	0.001	-0.001	-0.003
Equity volatility	0.2983	0.3595	0.2983	0.3595	0.3029	0.3595	0.3029	0.3595
Measures at $\rho = 0.25$								
Liquidity	0.0025	N/A	0.0025	N/A	0.0025	N/A	0.0025	N/A
Net equity val	0.1254	0.1218	0.1254	0.1218	0.1795	0.1740	0.1795	0.1740
Debt val	0.0697	0.0680	0.0725	0.0723	0.0732	0.0732	0.0773	0.0794
Net firm val	0.1951	0.1898	0.1980	0.1941	0.2528	0.2472	0.2569	0.2534
Leverage	0.3527	0.3584	0.3620	0.3727	0.2868	0.2962	0.2981	0.3134
Perp. cr. spread	0.004	0.006	0.002	0.002	0.001	0.001	-0.001	-0.003
Equity volatility	0.2612	0.2753	0.2612	0.2753	0.2621	0.2753	0.2621	0.2753

TABLE 5

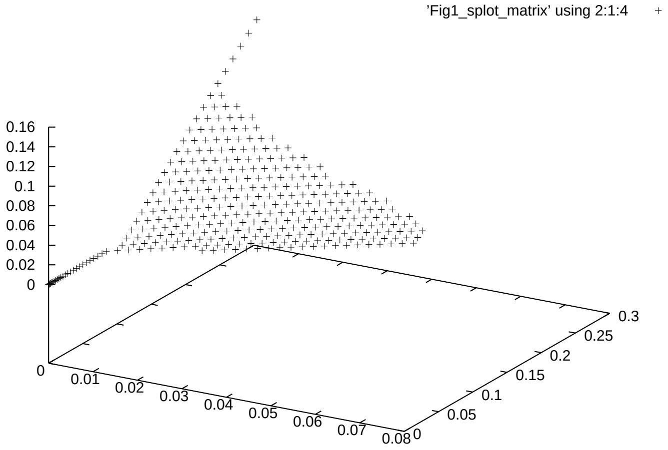
Analyzing Two Debt Covenants

	Benchmark (No covenant)	Debt holders claim liquidity $\underline{C}$: $\underline{C} = 0.005$	$\underline{C} = 0.010$	No dividend if $\rho_t < f + q - L$: $L = 0.00$	$L = 0.02$	$L = 0.04$
Measures at $\rho = 0.05$						
Liquidity	N/A	N/A	N/A	0.0002	0.0002	0.0002
Net equity val	0.0000	0.0000	0.0000	-0.0002	-0.0002	-0.0002
Debt val	0.0104	0.0050	0.0204	0.0107	0.0107	0.0107
Net firm val	0.0104	0.0050	0.0204	0.0104	0.0104	0.0104
Leverage	1.0000	1.0000	1.0000	0.9998	0.9998	0.9998
Perp. cr. spread	0.369	0.840	0.159	0.359	0.359	0.359
Equit volatility	N/A	N/A	N/A	3.4991	3.4991	3.4990
Measures at $\rho = 0.10$						
Liquidity	0.0077	0.0116	0.0165	0.0073	0.0074	0.0077
Net equity val	0.0104	0.0094	0.0086	0.0100	0.0100	0.0101
Debt val	0.0585	0.0603	0.0600	0.0597	0.0597	0.0597
Net firm val	0.0690	0.0697	0.0687	0.0697	0.0697	0.0698
Leverage	0.7624	0.7407	0.7041	0.7748	0.7731	0.7703
Perp. cr. spread	0.016	0.014	0.014	0.015	0.015	0.015
Equit volatility	1.1407	0.9634	0.7861	1.2195	1.2080	1.1900
Measures at $\rho = 0.15$						
Liquidity	0.0201	0.0243	0.0293	0.0196	0.0197	0.0200
Net equity val	0.0509	0.0498	0.0487	0.0505	0.0505	0.0506
Debt val	0.0659	0.0669	0.0670	0.0669	0.0669	0.0669
Net firm val	0.1168	0.1167	0.1157	0.1175	0.1175	0.1175
Leverage	0.4814	0.4744	0.4620	0.4879	0.4875	0.4864
Perp. cr. spread	0.008	0.007	0.007	0.007	0.007	0.007
Equity volatility	0.3695	0.3535	0.3351	0.3738	0.3732	0.3713
Measures at $\rho = 0.20$						
Liquidity	0.0125	0.0156	0.0206	0.0115	0.0116	0.0126
Net equity val	0.0881	0.0870	0.0859	0.0878	0.0878	0.0879
Debt val	0.0667	0.0676	0.0677	0.0676	0.0676	0.0676
Net firm val	0.1549	0.1547	0.1537	0.1555	0.1555	0.1556
Leverage	0.3984	0.3972	0.3886	0.4049	0.4047	0.4024
Perp. cr. spread	0.007	0.006	0.006	0.006	0.006	0.006
Equity volatility	0.2981	0.2924	0.2817	0.3023	0.3020	0.2990
Measures at $\rho = 0.25$						
Liquidity	0.0025	0.0074	0.0124	0.0025	0.0025	0.0025
Net equity val	0.1254	0.1242	0.1231	0.1251	0.1251	0.1251
Debt val	0.0670	0.0680	0.0680	0.0679	0.0679	0.0679
Net firm val	0.1925	0.1922	0.1912	0.1931	0.1931	0.1931
Leverage	0.3440	0.3404	0.3341	0.3475	0.3475	0.3475
Perp. cr. spread	0.007	0.006	0.006	0.006	0.006	0.006
Equity volatility	0.2612	0.2534	0.2461	0.2618	0.2618	0.2617

Figure 1:

Equity and debt values, in terms of ρ and C, over the region $\mathcal{S}$, for a benchmark example:

Panel A - Equity value, J:



Panel B - Debt value, P:

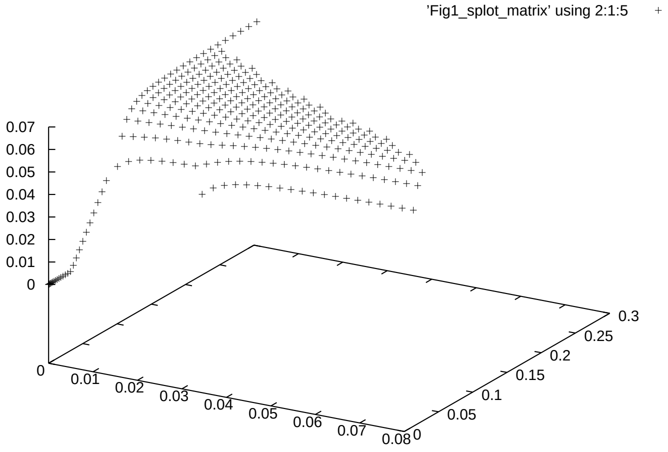
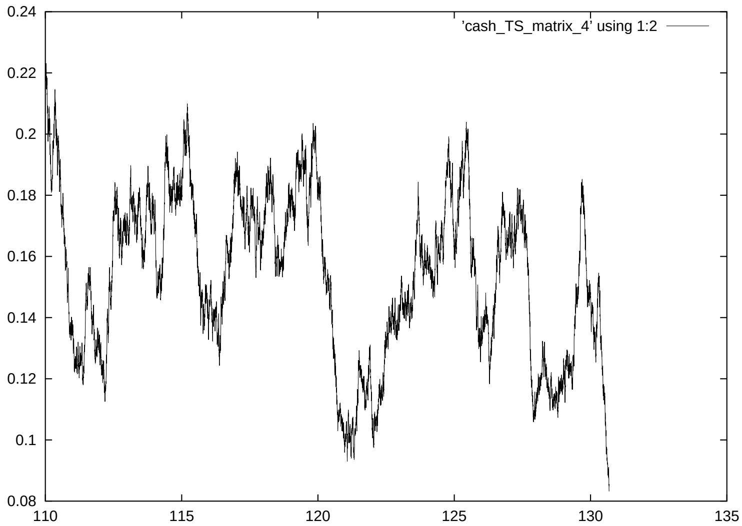


Figure 2:

Simulated time series of the profitability ρ_t, (top graph), and the liquidity reserve C_t, debt valueP_t, and total firm value $J_t + D_t$ (all on the bottom graph, in increasing order). Note that J_t represents a claim on the productive asset, and the liquidity reserve.

Panel A



Panel B

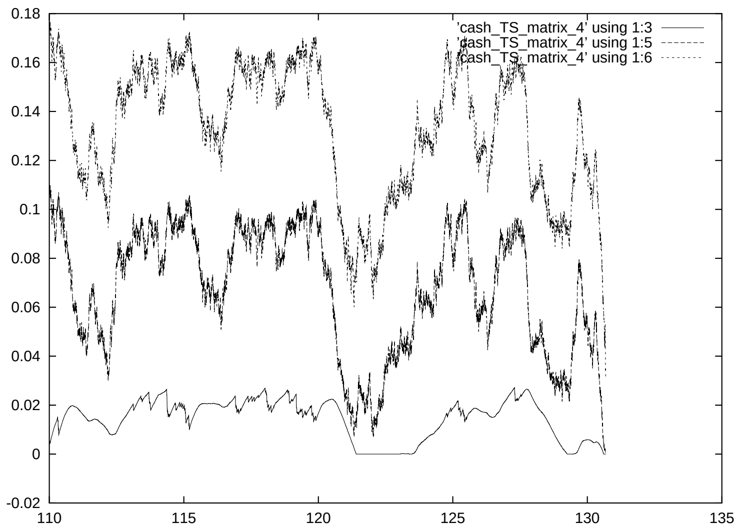


Figure 3:

The regions $\mathcal{D}, \mathcal{I}$ and $\mathcal{S}$ are depicted by dots, crosses ('x'), and empty space, respectively. The target liquid asset holding is the upper boundary of the $\mathcal{S}$ region. The realized liquid asset holding, averaged over 300 simulations of the firm history, as a function of ρ, is depicted by crosses ('+').

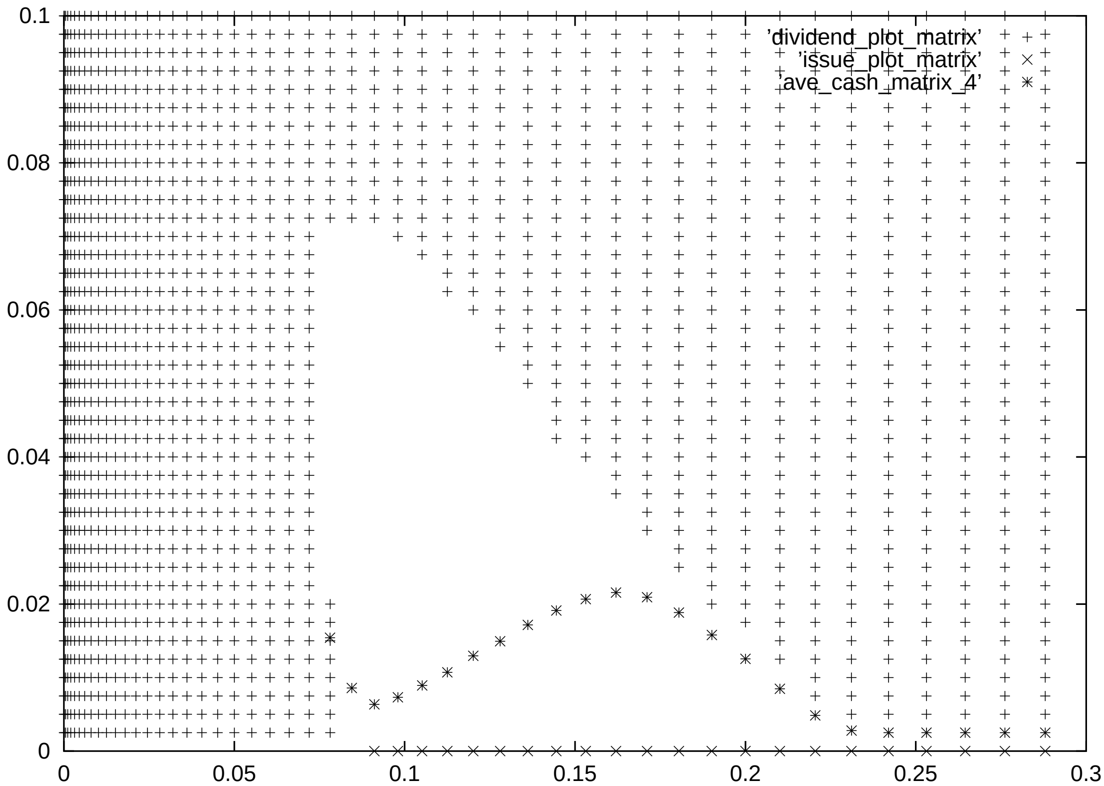
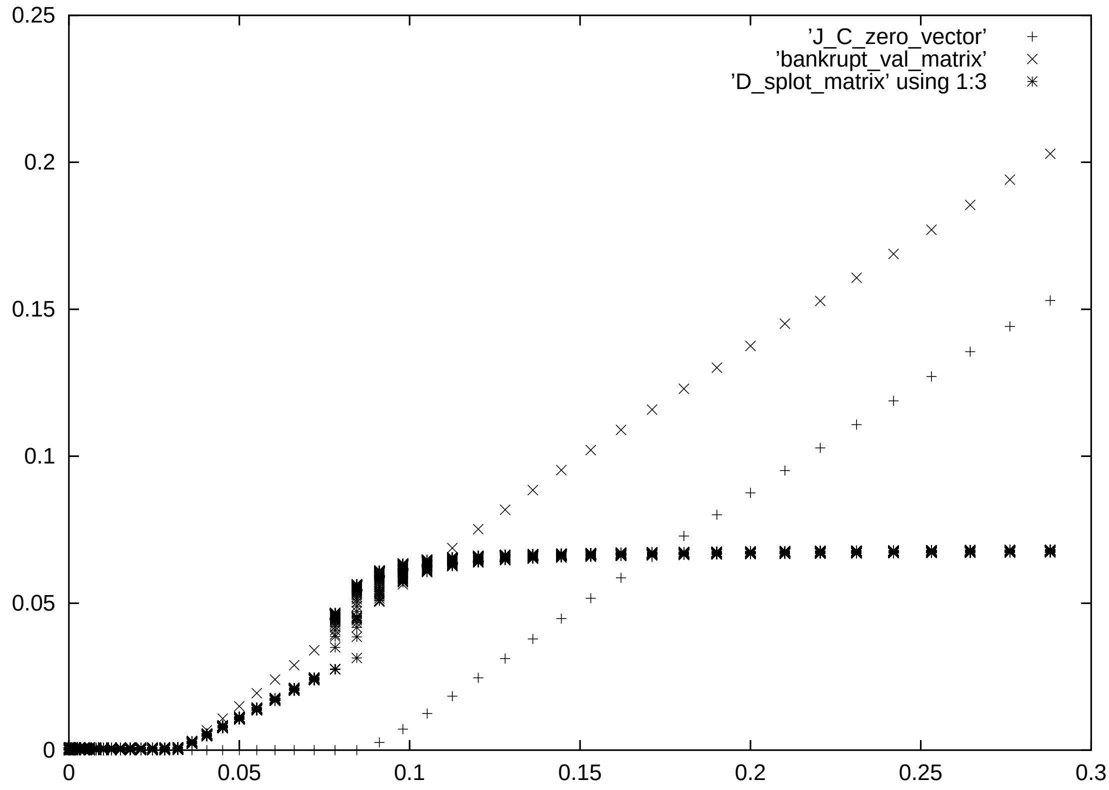


Figure 4:

Here the crosses ("+") represent the value of the equity in the benchmark example, when the liquidity reserve is zero. Over the region where this is zero, the firm is bankrupt, and the debt holders get $1 - \alpha$ time the corresponding unlevered equity value, which is represented by crosses ("x"). The stars represent the debt value in the benchmark case, for all levels of liquidity.



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