# 3-5 MAN MUTUAL ZUGZWANGS IN CHESS 

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#### Abstract

This note reports the work of Wirth and Karrer in twin-sourcing all mutual zugzwang positions, mzugs, in 2-5-man endgames. This paper tabulates the mzug statistical data, gives examples of maximal mzugs and refers to a chess endgame website where further data is to be found.


## 1. INTRODUCTION

Wirth and Nalimov have generated all 2-to-5-man endgame tables (EGTs) to the Depth-to-Conversion (DTC) and Depth-to-Mate (DTM) metrics respectively (Wirth and Nievergelt, 1999; Nalimov, Haworth and Heinz, 2000). Wirth and Karrer scanned the DTC and DTM EGTs respectively, second-sourcing all the mzugs.

A reciprocal or mutual zugzwang, mzug, in chess is a position where, ironically, each side could get a better result in theory if it were the other side's turn to move. There are three types of mzug:

$$
\begin{array}{lll}
w w & \text { ' }=/ 1-0 \text { ', } & \text { a 'White win' mzug ... the position is a wtm draw and a btm win for White } \\
b w & ' 0-1 /=, & \text { a 'Black win' mzug ... the position is a wtm win for Black and a btm draw } \\
f p & ‘ 0-1 / 1-0 & \text { a 'full point' mzug ... the side that has to move loses. }
\end{array}
$$

They are relatively rare and a running theme in Nunn's endgame trilogy (1992, 1994, 1995). Many endgame studies exploit the mzug theme at some point (Roycroft, 1972; Beasley, 1996; Elkies, 1998). Many counts of mzugs, principally by Rasmussen (1991-2000), have already been published: they confirm and are confirmed by the data here. This article completes the coverage of the 2-to-5-man domain, providing DTC and DTM data and examples of maximal mzugs in these two metrics.

## 2. THE RESULTS

Karrer (2000) scanned Nalimov’s 3-to-5-man EGTs (Hyatt, 2001; Tamplin, 2001b) for mzug positions, giving:

- a list of mzugs together with statistics about counts and DTM-maximal depths
n.b. for a full-point mzug, the depth is taken to be the sum of the wtm and btm depths
- a list of the DTM-maximal mzugs

The lists were then passed to workers in this field including Elkies, Rasmussen, Roycroft, Tamplin and Wirth. Wirth then computed the 4-1 EGTs to complete his suite of 2-to-5-man EGTs and executed the analogous mzug scan on them. Haworth collated the resulting statistics, confirmed full agreement between the results of Karrer and Wirth, and with the DTM data for the EGTs which Rasmussen (2000) had computed. He also identified mzugs which were maximal in terns of both the DTC and DTM metrics.

Tamplin (2001a) confirmed that the scans of DTC and DTM tables had yielded exactly the same sets of mzugs of each type, valuable confirmation in itself of the integrity of the EGTs. There are $21,677 w w, 3,395 \mathrm{bw}$ and 33 fp 2 -to- 5 -man-mzugs. These occur in 59 of the 146 endgames. The full point mzugs occur in just six endgames, namely KBPKP, KNPKP, KPKP, KPPKP, KPPKR and KRPKP: each features at least two Pawns. The complete data, statistics and various sets of positions, are available (Tamplin, 2001b). Here, the statistics are in Table 1 and the examples of maximal mzugs are in Table 2. Some explanatory notes are appropriate:

- White has at least as many men as Black: the men are listed in the standard K-Q-R-B-N-P order and endgames are listed in alphabetical order. The GBR code qrcn.wb indicates an endgame with $w$ white and $b$ black Pawns, $q$ mod 3 white and $\lfloor q / 3\rfloor$ black Queens. $r, c$ and $n$ similarly define the Rs, Bs and Ns.

[^0]- with one exception, all positions are essentially unique, i.e. they cannot be transformed into another listed position by board transformation or by switching colours. The exception is that in symmetric endings, only KPKP here, the set of $b w$ mzugs is acknowledged even though it is transformed by colour-reversal into the set of $w w$ mzugs. The brackets in Table 1 highlight this equivalence.
- depths are in side-to-move moves. In Table 2, $c, m$ and $c m$ denote a DTC-, DTM- and DTC-_\&_DTMmaximal position respectively. $w w, b w$ and $f p$ denote the three types of mzug as above.
- the positions are in canonical form in the sense that:
the wK is confined to a-d for endgames with Pawns, and to a1-d1-d4 for endgames without Pawns if there are no Pawns and the wK is on a1-d4, the bK is confined to a1-h1-h8 if both Kings are on a1-h8, only one of probably two equivalent positions is included in the count.
- a/the DTC-maximal mzug with maximal DTM depth has been cited. A DTM-maximal mzug with maximal DTC depth has also been cited if different, as it is in 16 endgames.
- the following 55 2-to-4-man and 3-2 endgames have no mzugs:

KBBK, KBBKB, KBK, KBKB/N, KBNK, KK, KNK, KNKN, KNNK, KNNKB, KQBK, KQBKB/N/P/R, KQK, KQKB/N/P/Q/R, KQNK, KQNKB/N/P/R, KQPK, KQPB/N/P, KQQK, KQQKB/N/P/Q/R, KQRK, KQRKB/N/P/R, KRBK, KRBKB/N, KRK, KRKR, KRNK, KRNKB/P, KRPK, KRRK, KRRKN/P/R.

- the only 4-1 endgames with mzugs are KBPPK, KNPPK and KPPPK, echoing the fact that the only 3-1 endgames with mzugs are KBPK, KNPK and KPPK.


MZ1. The deepest mzug


MZ5. $\mathrm{wtm}, d c=63$


MZ2. The deepest $f p$ mzug


MZ6. $\mathrm{btm}, d c=58$


MZ3. $b$ tm, $d c=91$


MZ7. $\mathbf{w t m}, d c=53$


MZ4. $\mathrm{btm}, d c=67$


MZ8. Le Trébuchet

## 3. HIGHLIGHTS

KNNKP MZ1 max 3-to-5-man mzug: $d c=d m=105$ moves.
KNPKP MZ2 max $f p$ mzug: $(\mathrm{wtm}) d c=1$ and $d m=17$, $(\mathrm{btm}) d c=13$ and $d m=28$.
KQPKQ MZ3 max KQPKQ mzug: $d c=91$ and $d m=102$.
KBNKN MZ4 max KBNKN mzug: $d c=67$ and $d m=97$.
KBNKQ 8/8/q7/8/3K4/2N5/8/k1B5: only maximal P-less mzug with both Kings on a1-h8.
KPPKR only fp mzugs not having Pawns on both sides: one is maxDTC, the other maxDTM.
KPKP MZ8 the max fp mzug: like most KPKP positions, it has a twin with colours exchanged.
KPKP provides 15 of the 33 full-point mzugs, all of the well known Le Trébuchet type.
KRKN $\quad 8 / 8 / 8 / 4 \mathrm{k} 3 / 3 \mathrm{R} 4 / 2 \mathrm{~K} 5 / 1 \mathrm{n} 6 / 8$ : diagonally symmetric mzug.

All full-point mzugs feature at least two pawns: a conjecture is that there is a 6-man fpz with only one pawn. The sets of maxDTC mzugs and maxDTM mzugs are either disjoint, identical or one is contained within the other: further details are given below.

|  |  |  |  |  | _- DTC-maximal mzugs -_ |  |  |  |  |  |  |  |  | -_ DTM-maximal mzugs -_ |  |  |  |  |  |  |  |  | $\mathbf{Z C} \wedge \mathbf{Z M}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Endgame |  | mzug count |  |  | counts |  |  | max DTC |  |  | $\begin{gathered} \text { max-DTC } \\ \text { mzugs' } \\ \text { max DTM } \end{gathered}$ |  |  | counts |  |  | max DTM |  |  | $\begin{aligned} & \text { max-DTM } \\ & \text { mzugs' } \\ & \text { max-DTC } \end{aligned}$ |  |  | counts |  |  |
| Title | w-b | $\xi$ | 3 | - | $3$ | $3$ | $\approx$ | $\text { \} }$ | $3$ | 안 | $3$ | $3$ | - | $\text { \} }$ | $3$ | $\approx$ | $3$ | $3$ | 는 | $3$ | $3$ | - | $\text { \} }$ | 3 | - |
| KBBKN | 3-2 | 1 | 0 | 0 | 1 | 0 | 0 | 2 | --- | --- | 2 | --- | --- | 1 | 0 | 0 | 2 | --- | --- | 2 | --- | --- | 1 | 0 | 0 |
| KBBKP | 3-2 | 1 | 0 | 0 | 1 | 0 | 0 | 2 | --- | --- | 2 | --- | --- | 1 | 0 | 0 | 2 | --- | --- | 2 | --- | --- | 1 | 0 | 0 |
| KBBKQ | 3-2 | 0 | 1 | 0 | 0 | 1 | 0 | --- | 5 | --- | --- | 17 | --- | 0 | 1 | 0 | --- | 17 | --- | --- | 5 | --- | 0 | 1 | 0 |
| KBBKR | 3-2 | 3 | 0 | 0 | 3 | 0 | 0 | 1 | --- | --- | 19 | --- | --- | 1 | 0 | 0 | 19 | --- | --- | 1 | --- | --- | 1 | 0 | 0 |
| KBKP | 2-2 | 0 | 1 | 0 | 0 | 1 | 0 | --- | 1 | --- | --- | 12 | --- | 0 | 1 | 0 | --- | 12 | --- | --- | 1 | --- | 0 | 1 | 0 |
| KBNKB | 3-2 | 45 | 0 | 0 | 1 | 0 | 0 | 10 | --- | --- | 36 | --- | --- | 1 | 0 | 0 | 36 | --- | --- | 10 | --- | --- | 1 | 0 | 0 |
| KBNKN | 3-2 | 922 | 0 | 0 | 1 | 0 | 0 | 67 | --- | --- | 97 | --- | --- | 1 | 0 | 0 | 97 | --- | --- | 67 | --- | --- | 1 | 0 | 0 |
| KBNKP | 3-2 | 61 | 1 | 0 | 1 | 1 | 0 | 13 | 1 | --- | 36 | 45 | --- | 4 | 1 | 0 | 37 | 45 | --- | 8 | 1 | --- | 0 | 1 | 0 |
| KBNKQ | 3-2 | 0 | 1 | 0 | 0 | 1 | 0 | --- | 33 | --- | --- | 44 | --- | 0 | 1 | 0 | --- | 44 | --- | --- | 33 | --- | 0 | 1 | 0 |
| KBNKR | 3-2 | 6 | 2 | 0 | 1 | 1 | 0 | 5 | 8 | --- | 26 | 28 | --- | 1 | 1 | 0 | 32 | 28 | --- | 1 | 8 | --- | 0 | 1 | 0 |
| KBP K | 3-1 | 6 | 0 | 0 | 1 | 0 | 0 | 14 | --- | --- | 20 | --- | --- | 1 | 0 | 0 | 20 | --- | --- | 14 | --- | --- | 1 | 0 | 0 |
| KBP KB | 3-2 | 160 | 0 | 0 | 2 | 0 | 0 | 33 | --- | --- | 44 | --- | --- | 2 | 0 | 0 | 44 | --- | --- | 33 | --- | --- | 2 | 0 | 0 |
| KBP KN | 3-2 | 2112 | 13 | 0 | 2 | 12 | 0 | 37 | 2 | --- | 47 | 2 | --- | 2 | 12 | 0 | 50 | 2 | --- | 19 | 2 | --- | 0 | 12 | 0 |
| KBP KP | 3-2 | 403 | 2 | 1 | 1 | 1 | 1 | 20 | 2 | 2+8 | 31 | 11 | 16+22 | 1 | 1 | 1 | 31 | 14 | 16+22 | 20 | 1 | 2+8 | 1 | 0 | 1 |
| KBP KQ | 3-2 | 0 | 16 | 0 | 0 | 2 | 0 | --- | 20 | --- | --- | 28 | --- | 0 | 1 | 0 | --- | 28 | --- | --- | 20 | --- | 0 | 1 | 0 |
| KBP KR | 3-2 | 4 | 302 | 0 | 1 | 1 | 0 | 3 | 15 | --- | 32 | 29 | --- | 1 | 1 | 0 | 32 | 33 | --- | 3 | 14 | --- | 1 | 0 | 0 |
| KBPPK | 4-1 | 6 | 0 | 0 | 1 | 0 | 0 | 6 | --- | --- | 12 | --- | --- | 1 | 0 | 0 | 16 | --- | --- | 1 | --- | --- | 0 | 0 | 0 |
| KNKP | 2-2 | 22 | 7 | 0 | 6 | 1 | 0 | 6 | 9 | --- | 6 | 24 | --- | 6 | 1 | 0 | 6 | 24 | --- | 6 | 9 | --- | 6 | 1 | 0 |
| KNNKN | 3-2 | 362 | 0 | 0 | 85 | 0 | 0 | 4 | --- | --- | 4 | --- | --- | 85 | 0 | 0 | 4 | --- | --- | 4 | --- | --- | 85 | 0 | 0 |
| KNNKP | 3-2 | 3124 | 19 | 0 | 1 | 2 | 0 | 105 | 3 | --- | 105 | 39 | --- | 1 | 1 | 0 | 105 | 53 | --- | 105 | 1 | --- | 1 | 0 | 0 |
| KNNKQ | 3-2 | 0 | 229 | 0 | 0 | 1 | 0 | --- | 53 | --- | --- | 62 | --- | 0 | 1 | 0 | --- | 62 | --- | --- | 53 | --- | 0 | 1 | 0 |
| KNNKR | 3-2 | 0 | 25 | 0 | 0 | 1 | 0 | --- | 5 | --- | --- | 28 | --- | 0 | 1 | 0 | --- | 35 | --- | --- | 2 | --- | 0 | 0 | 0 |
| KNP K | 3-1 | 75 | 0 | 0 | 1 | 0 | 0 | 14 | --- | --- | 19 | --- | --- | 2 | 0 | 0 | 19 | --- | --- | 14 | --- | --- | 1 | 0 | 0 |
| KNP KB | 3-2 | 640 | 2 | 0 | 1 | 1 | 0 | 28 | 2 | --- | 40 | 2 | --- | 1 | 1 | 0 | 40 | 2 | --- | 28 | 2 | --- | 1 | 1 | 0 |
| KNP KN | 3-2 | 4128 | 63 | 0 | 3 | 42 | 0 | 40 | 2 | --- | 56 | 2 | --- | 1 | 42 | 0 | 56 | 2 | --- | 40 | 2 | --- | 1 | 42 | 0 |
| KNP KP | 3-2 | 2281 | 14 | 8 | 1 | 6 | 1 | 21 | 5 | 1+13 | 29 | 24 | 17+28 | 1 | 1 | 1 | 33 | 24 | 17+28 | 19 | 5 | 1+13 | 0 | 1 | 1 |
| KNP KQ | 3-2 | 0 | 52 | 0 | 0 | 1 | 0 | --- | 29 | --- | --- | 45 | --- | 0 | 1 | 0 | --- | 45 | --- | --- | 29 | --- | 0 | 1 | 0 |
| KNP KR | 3-2 | 23 | 1158 | 0 | 2 | 1 | 0 | 4 | 34 | --- | 19 | 58 | --- | 1 | 1 | 0 | 28 | 58 | --- | 3 | 34 | --- | 0 | 1 | 0 |
| KNP P K | 4-1 | 93 | 0 | 0 | 6 | 0 | 0 | 5 | --- | --- | 11 | --- | --- | 1 | 0 | 0 | 16 | --- | --- | 1 | --- | --- | 0 | 0 | 0 |
| KP K | 2-1 | 80 | 0 | 0 | 1 | 0 | 0 | 18 | --- | --- | 27 | --- | --- | 1 | 0 | 0 | 27 | --- | --- | 18 | --- | --- | 1 | 0 | 0 |
| KP KP | 2-2 | 106 | 106 | 15 | 1 | 1 | 15 | 10 | 10 | 1+1 | 24 | 24 | 19+12 | 2 | 2 | 1 | 27 | 27 | 19+12 | 6 | 6 | 1+1 | 0 | 0 | 1 |
| KP P K | 3-1 | 43 | 0 | 0 | 6 | 0 | 0 | 11 | --- | --- | 26 | --- | --- | 1 | 0 | 0 | 26 | --- | --- | 11 | --- | --- | 1 | 0 | 0 |
| KP P KB | 3-2 | 211 | 1 | 0 | 2 | 1 | 0 | 14 | 2 | --- | 25 | 2 | --- | 1 | 1 | 0 | 28 | 2 | --- | 12 | 2 | --- | 0 | 1 | 0 |
| KP P KN | 3-2 | 920 | 157 | 0 | 1 | 11 | 0 | 22 | 12 | --- | 36 | 12 | --- | 2 | 15 | 0 | 36 | 12 | --- | 22 | 12 | --- | 1 | 11 | 0 |
| KP P KP | 3-2 | 4179 | 52 | 6 | 3 | 2 | 1 | 24 | 7 | 6+3 | 38 | 18 | 15+19 | 1 | , | 1 | 86 | 23 | 16+20 | 10 | 1 | 4+3 | 0 | 0 | 0 |
| KP P KQ | 3-2 | 0 | 2 | 0 | 0 | 1 | 0 | --- | 8 | --- | --- | 13 | --- | 0 | 1 | 0 | --- | 13 | --- | --- | 8 | --- | 0 | 1 | 0 |
| KP P KR | 3-2 | 18 | 99 | 2 | 1 | 1 | 1 | 11 | 21 | 1+4 | 39 | 36 | 18+17 | 1 | 1 | 1 | 39 | 36 | 15+21 | 11 | 21 | 3+1 | 1 | 1 | 0 |
| KP PPK | 4-1 | 11 | 0 | 0 | 2 | 0 | 0 | 7 | --- | --- | 17 | --- | --- | 1 | 0 | 0 | 17 | --- | --- | 7 | --- | --- | 1 | 0 | 0 |
| KQBKQ | 3-2 | 25 | 0 | 0 | 1 | 0 | 0 | 9 | --- | --- | 11 | --- | --- | 1 | 0 | 0 | 14 | --- | --- | 6 | --- | --- | 0 | 0 | 0 |
| KQNKQ | 3-2 | 38 | 0 | 0 | 1 | 0 | 0 | 24 | --- | --- | 30 | --- | --- | 1 | 0 | 0 | 30 | --- | --- | 24 | --- | --- | 1 | 0 | 0 |
| KQP KQ | 3-2 | 640 | 0 | 0 | 1 | 0 | 0 | 91 | --- | --- | 102 | --- | --- | 1 | 0 | 0 | 102 | --- | --- | 91 | --- | --- | 1 | 0 | 0 |
| KQP KR | 3-2 | 1 | 0 | 0 | 1 | 0 | 0 | 2 | --- | --- | 11 | --- | --- | 1 | 0 | 0 | 11 | --- | --- | 2 | --- | --- | 1 | 0 | 0 |
| KQRKQ | 3-2 | 1 | 0 | 0 | 1 | 0 | 0 | 11 | --- | --- | 18 | --- | --- | 1 | 0 | 0 | 18 | --- | --- | 11 | --- | --- | 1 | 0 | 0 |
| KRBKP | 3-2 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | --- | --- | 8 | --- | --- | 1 | 0 | 0 | 8 | --- | --- | 1 | --- | --- | 1 | 0 | 0 |
| KRBKQ | 3-2 | 0 | 372 | 0 | 0 | 1 | 0 | --- | 38 | --- | --- | 67 | --- | 0 | 1 | 0 | --- | 67 | --- | --- | 38 | --- | 0 | 1 | 0 |
| KRBKR | 3-2 | 17 | 0 | 0 | 1 | 0 | 0 | 49 | --- | --- | 55 | --- | --- | 1 | 0 | 0 | 55 | --- | --- | 49 | --- | --- | 1 | 0 | 0 |
| KRKB | 2-2 | 5 | 0 | 0 | 1 | 0 | 0 | 14 | --- | --- | 25 | --- | --- | 1 | 0 | 0 | 25 | --- | --- | 14 | --- | --- | 1 | 0 | 0 |
| KRKN | 2-2 | 18 | 0 | 0 | 2 | 0 | 0 | 10 | --- | --- | 24 | --- | --- | 2 | 0 | 0 | 24 | --- | --- | 10 | --- | --- | 2 | 0 | 0 |
| KRKP | 2-2 | 12 | 0 | 0 | 6 | 0 | 0 | 6 | --- | --- | 20 | --- | --- | 4 | 0 | 0 | 20 | --- | --- | 6 | --- | --- | 4 | 0 | 0 |
| KRNKN | 3-2 | 3 | 0 | 0 | 1 | 0 | 0 | 19 | --- | --- | 24 | --- | --- | 2 | 0 | 0 | 24 | --- | --- | 19 | --- | --- | 1 | 0 | 0 |
| KRNKQ | 3-2 | 0 | 455 | 0 | 0 | 1 | 0 | --- | 42 | --- | --- | 63 | --- | 0 | 1 | 0 | --- | 63 | --- | --- | 42 | --- | 0 | 1 | 0 |
| KRNKR | 3-2 | 10 | 0 | 0 | 2 | 0 | 0 | 22 | --- | --- | 23 | --- | --- | 2 | 0 | 0 | 23 | --- | --- | 22 | --- | --- | 2 | 0 | 0 |
| KRP KB | 3-2 | 225 | 0 | 0 | 1 | 0 | 0 | 58 | --- | --- | 67 | --- | --- | 1 | 0 | 0 | 67 | --- | --- | 58 | --- | --- | 1 | 0 | 0 |
| KRP KN | 3-2 | 413 | 0 | 0 | 4 | 0 | 0 | 37 | --- | --- | 47 | --- | --- | 1 | 0 | 0 | 47 | -- | --- | 37 | --- | -- | 1 | 0 | 0 |
| KRP KP | 3-2 | 0 | 2 | 1 | 0 | 2 | 1 | --- | 1 | 1+4 | --- | 15 | 15+15 | 0 | 1 | 1 | --- | 15 | 15+15 | --- | 1 | 1+4 | 0 | 1 | 1 |
| KRP KQ | 3-2 | 2 | 241 | 0 | 2 | 1 | 0 | 2 | 63 | --- | 12 | 91 | --- | 1 | 1 | 0 | 12 | 91 | --- | 2 | 63 | --- | 1 | 1 | 0 |
| KRP KR | 3-2 | 209 | 0 | 0 | 1 | 0 | 0 | 43 | --- | --- | 54 | --- | --- | 2 | 0 | 0 | 54 | -- | --- | 43 | --- | --- | 1 | 0 | 0 |
| KRRKB | 3-2 | 1 | 0 | 0 | 1 | 0 | 0 | 8 | --- | --- | 16 | --- | --- | 1 | 0 | 0 | 16 | --- | --- | 8 | --- | --- | 1 | 0 | 0 |
| KRRKQ | 3-2 | 10 | 0 | 0 | 1 | 0 | 0 | 7 | --- | --- | 20 | --- | --- | 1 | 0 | 0 | 20 | --- | --- | 7 | --- | --- | 1 | 0 | 0 |

Table 1. Mutual zugzwangs: the statistics.

As is usual, the statistics of Table 1 may be affected by the presence of unreachable positions in which the side-not-to-move is not in check. Elkies (2000) and van der Heijden (2000) have both pointed out such positions:

KBPKP: 8/8/8/8/1p6/1k6/1P6/BK6
KBPPK: kB6/8/1PK5/1P6/8/8/8/8, kB6/8/KP6/1P6/8/8/8/8 ... leaving 4 of the 6
KNPPK: kN6/8/KP6/1P6/8/8/8/8, kN6/8/1PK5/1P6/8/8/8/8, K1k5/P1PN4/8/8/8/8/8/8 ... leaving 90 of 93.

| Endgame |  | type | dc | dm | a maximal mzug | Endgame |  | type | dc | dm | a maximal mzug |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KBBKN | 0023 | cm ww | 2 | 2 | 8/8/8/8/8/6n12K4B/kB6 | KNPPK 0 | 000120 | ww | 5 | 11 | K7/P 1k5/P 7/8/8/8/8/N7 |
| KBBKP | 0020.01 | cm ww | 2 | 2 | B1k5/ıpB5/3K4/8/8/8/8/8 |  |  | m ww | 1 | 16 | kN6/8/1P K5/ $\mathrm{P}^{\text {6/8/8/8/8 }}$ |
| KBBKQ | 3020 | cm bw | 5 | 17 | 8/8/8/8/q7/2BB4/1K6/3k4 | KPK 0 | 0000.10 | cm ww | 18 | 27 | 8/8/8/1k6/8/1K6/6P 18 |
| KBBKR | 0320 | cm ww | 1 | 19 | 8/8/8/B7/8/3k4/2r5/KB6 | KP KP 0 | 0000.11 | ww | 10 | 24 | 8/k7/6p1/K5P 18/8/8/8 |
| KBKP | 0010.01 | cm bw | 1 | 12 | 8/8/8/8/8/8/1pK5/kB6 |  |  | m ww | 6 | 27 | 8/8/k7/8/K5p18/5P $2 / 8$ |
| KBNKB | 0041 | cm ww | 10 | 36 | 8/8/8/8/1b6/8/2K5/k1BN4 |  |  | bw | 10 | 24 | 8/8/8/8/k5p16P 1K7/8 |
| KBNKN | 0014 | cm ww | 67 | 97 | 8/8/2B2n2/8/N7/8/3K4/kk |  |  | m bw | 6 | 27 | 8/5p2/8/k5P 1/8/K7/8/8 |
| KBNKP | 001101 | c ww | 13 | 36 | 8/8/2K5/8/2k5/5p2/8/2N1B3 |  |  | cm fp | 1+1 | 19+12 | 8/1pK5/kP 6/8/8/8/8/8 |
|  |  | m | 8 | 37 | 8/8/8/8/8/B7/p3N3/k2K4 | KPPK 0000 | 0000.20 | cm ww | 11 | 26 | 8/8/8/8/1k6/1P 6/P1P K5/8 |
|  |  | cm bw | 1 | 45 | 8/8/8/1N6/3K4/B7/5p2/k7 | КРРКВ 003 | 0030.20 | ww | 14 | 25 | 1b6/8/3P 4/4P 3/8/8/8/3K1k2 |
| KBNKQ | 3011 | cm bw | 33 | 44 | 8/8/q7/8/3K4/2N5/8/k1B5 |  |  | $m$ ww | 12 | 28 | 8/8/5b1P /8/1K6/1P 6/1k6/8 |
| KBNKR | 0311 | c ww | 5 | 26 | 3r4/8/2B5/8/1N6/8/8/k1K5 |  |  | cm bw | 2 | 2 | 8/8/8/8/8/b2k4/P 2P 4/1K6 |
|  |  | m | 1 | 32 | 8/8/8/8/B7/1r6/N1k5/K7 | KPPKN 00 | 0003.20 | cm ww | 22 | 36 | 8/5n2/8/K6P /3P 4/k7/8/8 |
|  |  | cm | 8 | 28 | 8/r7/8/B7/8/8/N1k5/K7 |  |  | cm bw | 12 | 12 | K7/2k5/8/P 7/P 7/5n2/8/8 |
| KBP K | 0010.10 | cm w | 14 | 20 | 1B1K4/8/8/k7/8/P 7/8/8 | KPPKP 0000 | 0000.21 | ww | 24 | 38 | 1K3k2/8/7p/8/8/7P /6P 18 |
| КВР КВ | 0040.10 | cm | 33 | 44 | 8/8/8/2P 5/4b3/1B6/8/k1K5 |  |  | $m$ | 10 | 86 | 8/2p5/8/8/8/2k1P 3/P 7/3K4 |
| KBP KN | 0013.10 | c w | 37 | 47 | 8/8/8/8/8/K5n1/B5P 1/k7 |  |  | bw | 7 | 18 | K1k5/8/7p/P 7/2P 5/8/8/8 |
|  |  | m | 19 | 50 | k7/8/Kn1BP 3/8/8/8/8/8 |  |  | m bw | 1 | 23 | 8/2p5/8/8/1P $\mathbb{P} 4 / 2 \mathrm{k} 5 / 8 / 2 \mathrm{~K} 5$ |
|  |  | cm b | 2 | 2 | K7/P 1k5/8/8/8/1B6/ln6/8 |  |  | c fp | 6+3 | 15+19 | 8/8/8/2k5/K1p5/P 3P 3/8/8 |
| KBP KP | 0010.11 | cm w | 20 | 31 | 2K5/8/3k4/8/7p/5P 2/8/7B |  |  | m fp | 4+3 | 16+20 | 8/8/8/8/5k2/3K1p2/3P 3P/8 |
|  |  | c | 2 | 11 | 8/8/8/8/8/1k6/1P 1p4/KB6 | KP P KQ 300 | 3000.20 | cm bw | 8 | 13 | 8/2KP 3q/8/2P 3k1/8/8/8/8 |
|  |  | m | 1 | 14 | 8/8/8/8/8/2p5/2P 5/kBK5 | KP PKR 0 | 0300.20 | cm ww | 11 | 39 | 2k5/K6P /6P r/8/8/8/8/8 |
|  |  | cm fp | 2+8 | 16+22 | 8/8/8/8/8/k1p5/2P 5/1BK5 |  |  | cm bw | 21 | 36 | 8/8/8/8/2P 5/2K1P 3/4r3/2k5 |
| KBP KQ | 3010.10 | cm bw | 20 | 28 | 5k2/PP1K4/1qB5/8/8/8/8/8 |  |  | c fp | 1+4 | 18+17 | 1rk $4 / \mathbb{P}$ 6/P1P K5/8/8/8/8/8 |
| KBP KR | 0310.10 | cm ww | 3 | 32 | K7/rBPP 3/k7/8/8/8/8/8 |  |  | $m \quad \mathrm{fp}$ | 3+1 | 15+21 | 8/8/8/8/k7/r1P 5/1KP 5/8 |
|  |  | c | 15 | 29 | 8/8/4r3/8/2k5/K6P/5B2/8 | KPPPK 00 | 0000.30 | cm ww | 7 | 17 | 8/8/8/1k1P 4/8/P K6/P 7/8 |
|  |  | m bw | 14 | 33 | 8/8/8/8/k7/rPP 5/1K6/B7 | KQBKQ | 4010 | ww | 9 | 11 | 8/3K4/3B4/8/k7/3Q4/8/2q5 |
| KBP P K | 0010.20 | c | 6 | 12 | 8/B1k5/K7/P 7/P 7/8/8/8 |  |  | $m$ ww | 6 | 14 | 1q6/8/2Q5/B7/8/1k6/8/1K6 |
|  |  | m ww | 1 | 16 | kB6/8/PP K5/ $\mathbb{P}$ 6/8/8/8/8 | KQNKQ | 4001 | cm ww | 24 | 30 | 8/3q4/1Q1N4/8/k7/8/3K4/8 |
| KNKP | 000101 | cm w | 6 | 6 | 8/8/8/8/8/p7/k2N4/2K5 | KQP KQ 4 | 4000.10 | cm | 91 | 102 | 8/8/8/8/2K3Q12P 1q3/8/4k3 |
|  |  | cm | 9 | 24 | 8/K7/N1k5/p7/8/8/8/8 | KQP KR | 1300.10 | cm | 2 | 11 | k7/8/KQ1r4/P 7/8/8/8/8 |
| KNNKN | 0005 | cm | 4 | 4 | 8/8/8/8/n7/8/2KN4/kN6 | KQRKQ | 4100 | cm | 11 | 18 | 8/8/8/8/1R6/k4q2/8/1K2Q3 |
| KNNKP | 0002.01 | cm w | 105 | 105 | 8/8/8/p7/K7/4k3/8/6NN | KRBKP | 0110.01 | cm | 1 | 8 | 1k1K4/7R/8/8/8/8/6p17B |
|  |  |  | 3 | 39 | 7N/8/K1N5/8/8/1pk5/8/8 | KRBKQ | 3110 | bw | 38 | 67 | 1q6/8/1B3R2/8/k7/8/8/1K6 |
|  |  | m b | 1 | 53 | 1N6/8/8/N7/K7/8/kp6/8 | KRBKR | 0410 | cm ww | 49 | 55 | 5R2/8/8/8/8/3K4/5Br1/2k5 |
| KNNKQ | 3002 | cm bw | 53 | 62 | 8/8/1q6/8/4N3/3K2N18/4k3 | KRKB | 0130 | cm ww | 14 | 25 | 8/8/1b6/5R2/8/3K4/8/2k5 |
| KNNKR | 0302 | c bw | 5 | 28 | 6rN/5N2/8/8/8/2k5/8/1K6 | KRKN | 0103 | cm | 10 | 24 | 8/8/8/6n13K4/4R3/3k4/8 |
|  |  | m bw | 2 | 35 | 5N2/1N6/8/3r4/8/2k5/8/2K5 | KRKP 0 | 0100.01 | cm ww | 6 | 20 | 8/K7/8/k7/lp6/8/8/1R6 |
| KNP K | 0001.10 | cm ww | 14 | 19 | 8/8/8/8/8/1kP1P 4/8/KN6 | KRNKN | 0104 | cm ww | 19 | 24 | 8/8/8/8/8/3n4/N2k4/RK6 |
| KNP KB | 0031.10 | cm ww | 28 | 40 | 8/6b18/8/1N5P /8/2K5/k7 | KRNKQ | 3101 | cm bw | 42 | 63 | 1Nk5/8/8/8/8/ $\mathbb{R} 6 / 6 \mathrm{q} 12 \mathrm{~K} 5$ |
|  |  | cm bw | 2 | 2 | K5b1P 1k2N2/8/8/8/8/8/8 | KRNKR | 0401 | cm ww | 22 | 23 | 8/8/8/8/8/2KRN3/8/2klr3 |
| KNP KN | 0004.10 | cm w | 40 | 56 | 2n5/8/8/1N6/8/6P 18/k1K5 | KRP KB 0 | 0130.10 | cm ww | 58 | 67 | 8/8/8/8/7R/1k2P 3/2b5/K7 |
|  |  | cm bw | 2 | 2 | K7/P 2n4/1k6/N7/8/8/8/8 | KRP KN 0 | 0103.10 | cm ww | 37 | 47 | K7/1R6/2n5/4k3/8/4P 3/8/8 |
| KNP KP | 000111 | c | 21 | 29 | 8/8/8/8/8/2k2p2/P 7/1KN5 | KRP KP | 0100.11 | cm bw | 1 | 15 | 8/8/8/8/8/1p6/kP 6/RRK5 |
|  |  |  | 19 | 33 | K7/N6p/k7/8/8/8/7P/8 |  |  | cm fp | 1+4 | 15+15 | 8/8/8/8/8/2p5/1kP 5/2RK4 |
|  |  | cm bw | 5 | 24 | 8/8/8/8/6P 13k2p18/2KN4 | KRP KQ 3 | 3100.10 | cm ww |  | 12 | 7k/2K2P 1q/8/8/8/8/5R2/8 |
|  |  | cm fp | 1+13 | 17+28 | 8/8/8/8/8/1k2p3/4P 3/KN6 |  |  | bw | 63 | 91 | 8/8/q1k5/8/R1K 1 4/8/PP $6 / 8$ |
| KNP KQ | 3001.10 | cm bw | 29 | 45 | 4N3/3P 4/2K5/q7/1k6/8/8/8 | KRP KR 0 | 0400.10 | cm ww | 43 | 54 | 8/8/8/8/8/2RPr3/8/2K1k3 |
| KNP KR | 0301.10 | ww | 4 | 19 | k6r/5P 2/K7/3N4/8/8/8/8 | KRRKB | 0230 | cm ww | 8 | 16 | 8/8/8/8/8/b1k5/ 1 R6/RK5 |
|  |  | $\begin{array}{cc} \mathrm{m} & \mathrm{ww} \\ \mathrm{~cm} & \mathrm{bw} \end{array}$ | $\begin{gathered} 3 \\ 34 \end{gathered}$ | $28$ | 3k4/r1N5/2KP 4/8/8/8/8/8 8/8/8/5N2/K7/2k5/P2r4/8 | KRRKQ | 3200 | cm ww | 7 | 20 | 6R1/8/8/8/6R1/7q/1K5k/8 |

Table 2. Maximal mutual zugs: examples.

For a given endgame and type of mzug, the sets $Z C$ and $Z M$ are the sets of DTC-maximal and DTM-maximal mzugs respectively. ZC \& ZM are either disjoint, identical or one is a subset of the other. There is no endgame for which $\mathrm{ZC}-\mathrm{ZM} \neq \varnothing$ and $\mathrm{ZM}-\mathrm{ZC} \neq \varnothing$.

```
ZC \cap ZM = \varnothing KBNKP ww, KBNKR ww, KBPKN ww, KBPKP bw, KBPKR bw, KNNKP bw,
    KNNKR bw, KNPKP ww, KNPKR ww, KPKP ww (and therefore bw), KPPKB ww,
    KPPKP ww, bw and fp, KPPKR fp and KQBKQ ww.
ZC}\subset\textrm{ZM}\quad\mathrm{ KNPK ww (1-2), KPPKN ww (1-2) and bw (11-15), KRNKN ww (1-2) and
    KRPKR ww (1-2)
ZC }\supset\textrm{ZM}\quad\mathrm{ KBBKR ww (3-1), KBPKQ bw (2-1), KNPKN ww (3-1), KNPKP bw (6-1), KPKP fp (15-1),
    KPPK ww (6-1), KRKP ww (6-4), KRPKN ww (4-1), KRPKP bw (2-1), KRPKQ ww (2-1)
```


## 4 SUMMARY

A complete set of data about 3-to-5-man mzugs in chess has been compiled, second-sourced, summarised here and made available via the web. This includes maximal mzug and depth statistics to both the DTC and DTM metrics.

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