3-5 Man Mutual Zugzwangs in Chess

3-5 MAN MUTUAL ZUGZWANGS IN CHESS

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ABSTRACT

This note reports the work of Wirth and Karrer in twin-sourcing all mutual zugzwang positions, *mzugs*, in 2-5-man endgames. This paper tabulates the mzug statistical data, gives examples of maximal mzugs and refers to a chess endgame website where further data is to be found.

1. INTRODUCTION

Wirth and Nalimov have generated all 2-to-5-man endgame tables (EGTs) to the Depth-to-Conversion (DTC) and Depth-to-Mate (DTM) metrics respectively (Wirth and Nievergelt, 1999; Nalimov, Haworth and Heinz, 2000). Wirth and Karrer scanned the DTC and DTM EGTs respectively, second-sourcing all the mzugs.

A reciprocal or mutual zugzwang, *mzug*, in chess is a position where, ironically, each side could get a better result in theory if it were the other side's turn to move. There are three types of mzug:

ww	'= /1-0'	a 'White win' mzug the position is a wtm draw and a btm win for White
bw	' 0-1/= '	a 'Black win' mzug the position is a wtm win for Black and a btm draw
fp	'0-1/1-0'	a 'full point' mzug the side that has to move loses.

They are relatively rare and a running theme in Nunn's endgame trilogy (1992, 1994, 1995). Many endgame studies exploit the mzug theme at some point (Roycroft, 1972; Beasley, 1996; Elkies, 1998). Many counts of mzugs, principally by Rasmussen (1991-2000), have already been published: they confirm and are confirmed by the data here. This article completes the coverage of the 2-to-5-man domain, providing DTC and DTM data and examples of maximal mzugs in these two metrics.

2. THE RESULTS

Karrer (2000) scanned Nalimov's 3-to-5-man EGTs (Hyatt, 2001; Tamplin, 2001b) for mzug positions, giving:

- a list of mzugs together with statistics about counts and DTM-maximal depths
- n.b. for a full-point mzug, the *depth* is taken to be the sum of the wtm and btm depths
- a list of the DTM-maximal mzugs

The lists were then passed to workers in this field including Elkies, Rasmussen, Roycroft, Tamplin and Wirth. Wirth then computed the 4-1 EGTs to complete his suite of 2-to-5-man EGTs and executed the analogous mzug scan on them. Haworth collated the resulting statistics, confirmed full agreement between the results of Karrer and Wirth, and with the DTM data for the EGTs which Rasmussen (2000) had computed. He also identified mzugs which were maximal in terns of both the DTC and DTM metrics.

Tamplin (2001a) confirmed that the scans of DTC and DTM tables had yielded exactly the same sets of mzugs of each type, valuable confirmation in itself of the integrity of the EGTs. There are 21,677 *ww*, 3,395 *bw* and 33 *fp* 2-to-5-man-mzugs. These occur in 59 of the 146 endgames. The full point mzugs occur in just six end-games, namely KBPKP, KNPKP, KPPKP, KPPKP, KPPKR and KRPKP: each features at least two Pawns. The complete data, statistics and various sets of positions, are available (Tamplin, 2001b). Here, the statistics are in Table 1 and the examples of maximal mzugs are in Table 2. Some explanatory notes are appropriate:

• White has at least as many men as Black: the men are listed in the standard K-Q-R-B-N-P order and endgames are listed in alphabetical order. The GBR code *qrcn.wb* indicates an endgame with *w* white and *b* black Pawns, $q \mod 3$ white and $\lfloor q/3 \rfloor$ black Queens. *r*, *c* and *n* similarly define the Rs, Bs and Ns.

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- with one exception, all positions are essentially unique, i.e. they cannot be transformed into another listed position by board transformation or by switching colours. The exception is that in symmetric endings, only KPKP here, the set of bw mzugs is acknowledged even though it is transformed by colour-reversal into the set of ww mzugs. The brackets in Table 1 highlight this equivalence.
- depths are in side-to-move moves. In Table 2, c, m and cm denote a DTC-, DTM- and DTC-_&_DTMmaximal position respectively. ww, bw and fp denote the three types of mzug as above.
- the positions are in *canonical form* in the sense that:

the wK is confined to a-d for endgames with Pawns, and to a1-d1-d4 for endgames without Pawns if there are no Pawns and the wK is on a1-d4, the bK is confined to a1-h1-h8 if both Kings are on a1-h8, only one of probably two equivalent positions is included in the count.

- a/the DTC-maximal mzug with maximal DTM depth has been cited. A DTM-maximal mzug with maximal DTC depth has also been cited if different, as it is in 16 endgames.
- the following 55 2-to-4-man and 3-2 endgames have no mzugs:

KBBK, KBBKB, KBK, KBKB/N, KBNK, KK, KNK, KNKN, KNNK, KNNKB, KQBK, KQBKB/N/P/R, KQK, KQKB/N/P/Q/R, KQNK, KQNKB/N/P/R, KQPK, KQPB/N/P, KQQK, KQQKB/N/P/Q/R, KQRK, KQRKB/N/P/R, KRBK, KRBKB/N, KRK, KRKR, KRNK, KRNKB/P, KRPK, KRRK, KRRKN/P/R.

the only 4-1 endgames with mzugs are KBPPK, KNPPK and KPPPK, echoing the fact that the only 3-1 endgames with mzugs are KBPK, KNPK and KPPK.









MZ1. The deepest mzug MZ2. The deepest *fp* mzug



MZ4. btm, dc = 67

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MZ5. wtm, dc = 63

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MZ6. btm, *dc* = 58



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MZ8. Le Trébuchet

HIGHLIGHTS 3.

KNNKP MZ1 max 3-to-5-man mzug: dc = dm = 105 moves. KNPKP MZ2 max fp mzug: (wtm) dc = 1 and dm = 17, (btm) dc = 13 and dm = 28. KQPKQ MZ3 max KQPKQ mzug: dc = 91 and dm = 102. KBNKN MZ4 max KBNKN mzug: dc = 67 and dm = 97. 8/8/q7/8/3K4/2N5/8/k1B5: only maximal P-less mzug with both Kings on a1-h8. **KBNKQ** KPPKR only fp mzugs not having Pawns on both sides: one is maxDTC, the other maxDTM. MZ8 the max fp mzug: like most KPKP positions, it has a twin with colours exchanged. KPKP KPKP provides 15 of the 33 full-point mzugs, all of the well known Le Trébuchet type. 8/8/8/4k3/3R4/2K5/1n6/8: diagonally symmetric mzug. KRKN

All full-point mzugs feature at least two pawns: a conjecture is that there is a 6-man fpz with only one pawn. The sets of maxDTC mzugs and maxDTM mzugs are either disjoint, identical or one is contained within the other: further details are given below.

					_		DT	'C-n	naxi	mal r	nzu	gs –		_		DT	M-r	nax	imal r	nzu	gs -		ZC	^	ZM
Endgame		mzug count			counts			max DTC		max-DTC mzugs' max DTM		counts		max DTM			max-DTM mzugs' max-DTC			counts					
Title	w-b	мм	bw	fp	WW	bw	fp	мм	bw	fp	мм	bw	fp	мм	bw	fp	ММ	bw	fp	мм	bw	fp	мм	bw	fp
KBBKN	3-2	1	0	0	1	0	0	2			2			1	0	0	2			2			1	0	0
KBBKP	3-2	1	0	0	1	0	0	2			2			1	0	0	2			2			1	0	0
KBBKQ	3-2	0	1					1	5		10	1/					10	1/		1	5				
KBKP	2-2	0	1			1			1			12			1	0		12			1		0	1	0
KBNKB	3-2	45	0	0	1	0	0	10			36			1	0	0	36			10			1	0	0
KBNKN	3-2	922	0	0	1	0	0	67			97			1	0	0	97			67			1	0	0
KBNKP	3-2	61	1	0	1	1	0	13	1		36	45		4	1	0	37	45		8	1		0	1	0
KBNKQ	3-2	0	1			1			33		26	44					20	44		1	33				0
KBNKK	3-2	6	2	0				12	8		20	28					20	28		14	8				
KBPKB	3-2	160	0	0	$\begin{vmatrix} 1\\2 \end{vmatrix}$	0	0	33			44			$\begin{vmatrix} 1\\2 \end{vmatrix}$	0	0	44			33			2		0
KB P KN	3-2	2112	13	0	2	12	0	37	2		47	2		2	12	0	50	2		19	2		0	12	0
KB P KP	3-2	403	2	1	1	1	1	20	2	2+8	31	11	16+22	1	1	1	31	14	16+22	20	1	2+8	1	0	1
KB P KQ	3-2	0	16	0	0	2	0		20			28		0	1	0		28			20		0	1	0
KBPKR	3-2	4	302		1	1	0	3	15		32	29			1	0	32	33		3	14		1		0
KBPPK	4-1	6 22						6	0		6	24					16	24		1	0		0		
KNNKN	3-2	362	0	0	85	0		4			4	24		85	0	0	4			4			85	0	0
KNNKP	3-2	3124	19	0	1	2	0	105	3		105	39		1	1	0	105	53		105	1		1	0	0
KNNKQ	3-2	0	229	0	0	1	0		53			62		0	1	0		62			53		0	1	0
KNNKR	3-2	0	25	0	0	1	0		5			28		0	1	0		35			2		0	0	0
KNP K	3-1	75	0	0	1	0	0	14			19			2	0	0	19			14			1	0	0
KNP KB	3-2	640	2			1	0	28	$\begin{vmatrix} 2 \\ 2 \end{vmatrix}$		40	2			1	0	40	2		28	2			1	0
KNP KN VND VD	3-2	4128	14			42		40	2	1,12	20	$\frac{2}{24}$	17, 28		42	1	30	$\frac{2}{24}$	17,08	40	2	1.12		42	
KNP KO	3-2	0	52	0		1	0		29			45			1	0		45			29			1	0
KNP KR	3-2	23	1158	0	2	1	0	4	34		19	58		1	1	0	28	58		3	34		0	1	0
KNP P K	4-1	93	0	0	6	0	0	5			11			1	0	0	16			1			0	0	0
KP K	2-1	80	0	0	1	0	0	18			27			1	0	0	27			18			1	0	0
KP KP	2-2	106	106	15	1	1	15	10	10	1+1	24	24	19+12	2	2	1	27	27	19+12	6	6	1+1	0		1
KPPK	3-1	43	0		6				2		26	2					26	2		n II	2				0
KP P KN	3-2	920	157	0		11		22	$\frac{2}{12}$		36	$\frac{2}{12}$		$\begin{vmatrix} 1 \\ 2 \end{vmatrix}$	15		36	$\frac{2}{12}$		22	$\frac{2}{12}$		1	11	0
KPPKP	3-2	4179	52	6	3	2	1	24	7	6+3	38	18	15+19	1	2	1	86	23	16+20	10	1	4+3	0	0	0
KP P KQ	3-2	0	2	0	0	1	0		8			13		0	1	0		13			8		0	1	0
KP P KR	3-2	18	99	2	1	1	1	11	21	1+4	39	36	18+17	1	1	1	39	36	15+21	11	21	3+1	1	1	0
KPPPK	4-1	11	0	0	2	0	0	7			17			1	0	0	17			7			1	0	0
KQBKQ	3-2	25	0					9									4			$\begin{vmatrix} 6 \\ 2^{4} \end{vmatrix}$					
KOPKO	3-2	640	0	0				91			102				0		102			91					
KQP KR	3-2	1	0	0		0	0	2			11				0	0	11			2			1	0	0
KQRKQ	3-2	1	0	0	1	0	0	11			18			1	0	0	18			11			1	0	0
KRBKP	3-2	1	0	0	1	0	0	1			8			1	0	0	8			1			1	0	0
KRBKQ	3-2	0	372	0	0				38		=	67		0		0		67			38		0		0
KRBKR	3-2	17	0			$\begin{vmatrix} 0 \\ 0 \end{vmatrix}$		49							$\begin{vmatrix} 0 \\ c \end{vmatrix}$		55			49					
KKKB	2-2	5 19	0					14	<u></u>		25						25			14					
KRKP	2-2	12	0	0	$\begin{vmatrix} 2\\6 \end{vmatrix}$	0	0	6			$\begin{vmatrix} 24 \\ 20 \end{vmatrix}$			$\begin{vmatrix} 2\\4 \end{vmatrix}$	0	0	$\frac{24}{20}$			6			$\begin{vmatrix} 2\\4 \end{vmatrix}$	0	0
KRNKN	3-2	3	õ	0	1	0	0	19			24				0	0	24			19				0	ŏ
KRNKQ	3-2	0	455	0	0	1	0		42			63		0	1	0		63			42		0	1	0
KRNKR	3-2	10	0	0	2	0	0	22			23			2	0	0	23			22			2	0	0
KR P KB	3-2	225	0	0	1	0	0	58			67			1	0	0	67			58			1	0	0
KRPKN	3-2	413	0		4	$\begin{vmatrix} 0\\ 2 \end{vmatrix}$		37	 1	 1 · 4	47	15	15 . 15				47	15	15 . 15	37		 1 · 4			
KRPKP	3-2		$\frac{2}{241}$			2		2	62	1+4	17	01 01	D+D					01 01	D+D	2	62	H4			
KRPKR	3-2	209^{2}	0	0	$\begin{vmatrix} 2\\1 \end{vmatrix}$			$\begin{vmatrix} 2 \\ 43 \end{vmatrix}$	0.5		54	91		$\begin{vmatrix} 1\\2 \end{vmatrix}$			54	91		$\begin{vmatrix} 2 \\ 43 \end{vmatrix}$	0.5				
KRRKB	3-2	1	0	0		0	0	8			16				0	0	16			8				0	0
KRRKO	3_2	10	ő	ŏ		lŏ	ŏ	7			20				ŏ	ŏ	20			7				ŏ	ŏ

Table 1. Mutual zugzwangs: the statistics.

As is usual, the statistics of Table 1 may be affected by the presence of unreachable positions in which the side-not-to-move is not in check. Elkies (2000) and van der Heijden (2000) have both pointed out such positions:

KBPKP: 8/8/8/1p6/1k6/1P6/BK6 KBPPK: kB6/8/1PK5/1P6/8/8/8, kB6/8/KP6/1P6/8/8/8/8 ... leaving 4 of the 6 KNPPK: kN6/8/KP6/1P6/8/8/8, kN6/8/1PK5/1P6/8/8/8/8, K1k5/P1PN4/8/8/8/8/8 ... leaving 90 of 93.

Endş	Endgame		pe	e dc dm		a maximal mzug	Endg	type		dc	dm	a maximal mzug	
KBBKN	0023	cm	ww	2	2	8/8/8/8/8/6n1/2K4B/kB6	KNP P K	000120	с	ww	5	11	K7/P 1k5/P7/8/8/8/8/N7
KBBKP	0020.01	cm	ww	2	2	B lk 5/ lnB 5/3K4/8/8/8/8/8		000120	m	ww	1	16	kN6/8/1P K5/1P 6/8/8/8/8
кввко	3020	cm	bw	5	17	8/8/8/8/a7/2BB4/1K6/3k4	KP K	0000.10	cm	ww	18	27	8/8/8/1k6/8/1K6/6P 1/8
KBBKR	0320	cm	ww	1	19	8/8/8/B7/8/3k4/2r5/KB6	KP KP	0000.11	с	ww	10	24	8/k7/6p1/K5P1/8/8/8/8
KBKP	0010.01	cm	bw	1	12	8/8/8/8/8/8/bK5/kB6			m	ww	6	27	8/8/k7/8/K5p1/8/5P2/8
KBNKB	0041	cm	ww	10	36	8/8/8/8/106/8/2K5/k1BN4			с	bw	10	24	8/8/8/8/k5p1/6P1/K7/8
KBNKN	0014	cm	ww	67	97	8/8/2B2n2/8/N7/8/3K4/1k6			m	bw	6	27	8/5p2/8/k5P1/8/K7/8/8
KBNKP	0011.01	с	ww	13	36	8/8/2K5/8/2k5/5p2/8/2NIB3			cm	fp	1+1	19+12	8/lpK5/kP 6/8/8/8/8/8
		m	ww	8	37	8/8/8/8/8/B7/p3N3/k2K4	KPPK	0000.20	cm	ww	11	26	8/8/8/8/1k6/1P6/1PK5/8
		cm	bw	1	45	8/8/8/1N6/3K4/B7/5p2/k7	KP P KB	0030.20	с	ww	14	25	1b6/8/3P4/4P3/8/8/8/3K1k2
KBNKO	3011	cm	bw	33	44	8/8/g7/8/3K4/2N5/8/k1B5			m	ww	12	28	8/8/5b1P/8/1K6/1P6/1k6/8
KBNKR	0311	c	ww	5	26	3r4/8/2B5/8/1N6/8/8/k1K5			cm	hw	2	2	8/8/8/8/8/b2k4/P2P4/1K6
	0011	m	ww	1	32	8/8/8/8/B7/h6/N1k5/K7	KP P KN	0003 20	cm	ww	22	36	8/5n2/8/K6P/3P4/k7/8/8
		cm	bw	8	28	8/r7/8/B7/8/8/N1k5/K7		0000.20	cm	hw	12	12	K7/2k5/8/P7/P7/5n2/8/8
KBPK	001010	cm	ww	14	20	1B 1K4/8/8/k7/8/P7/8/8	KP P KP	0000.21	c	ww	24	38	1K3k2/8/7p/8/8/7P/6P1/8
KBPKB	0040 10	cm	ww	33	44	8/8/8/2P 5/4b3/1B6/8/k1K5		0000.21	m	ww	10	86	8/2p5/8/8/8/2k1P3/P7/3K4
KBPKN	0013 10	c	ww	37	47	8/8/8/8/8/K5n 1/B5P 1/k7			c	hw	7	18	K1k5/8/7p/P7/2P5/8/8/8
	0010110	m	ww	19	50	k7/8/Kn 1BP 3/8/8/8/8/8			m	bw	1	23	8/2p5/8/8/1P 1P 4/2k5/8/2K5
		cm	bw	2	2	K7/P lk5/8/8/8/1B6/1n6/8			c	fn	6+3	15+19	8/8/8/2k5/Kln5/P3P3/8/8
KBP KP	001011	cm	ww	20	31	2K5/8/3k4/8/7p/5P2/8/7B			m	fn	4+3	16+20	8/8/8/8/5k2/3Klp2/3P3P/8
	0010111	c	bw	2	11	8/8/8/8/8/1k6/1P 1n4/KB6	KPP KO	3000.20	cm	hw	8	13	8/2KP 3g/8/2P 3k 1/8/8/8/8
		m	bw	1	14	8/8/8/8/8/2p5/2P5/kBK5	KPPKR	0300.20	cm	ww	11	39	2k5/K6P/6Pr/8/8/8/8/8
		cm	fn	2+8	16+22	8/8/8/8/8/k/n5/2P5/1BK5		0000.20	cm	hw	21	36	8/8/8/8/2P 5/2KIP 3/4r3/2k5
KBP KO	301010	cm	hw	20	28	5k2/1P 1K4/1aB5/8/8/8/8/8			c	fn	1+4	18+17	1r1k4/1P6/1PK5/8/8/8/8/8
KBPKR	0310.10	cm	ww	3	32	K7/hB 1P 3/k7/8/8/8/8/8			m	fn	3+1	15+21	8/8/8/8/k7/r1P5/1KP5/8
	001010	c	bw	15	29	8/8/4r3/8/2k5/K6P/5B2/8	KPPPK	0000 30	cm	ww	7	17	8/8/8/1k 1P 4/8/P K6/P 7/8
		m	hw	14	33	8/8/8/8/k7/r1P 5/1K6/B7	KOBKO	4010	c	ww	9	11	8/3K4/3B4/8/k7/3O4/8/2g5
KBPPK	0010.20	c	ww	6	12	8/B 1k5/K7/P 7/P 7/8/8/8	1.25.12	1010	m	ww	6	14	la6/8/205/B7/8/1k6/8/1K6
	0010120	m	ww	1	16	kB6/8/1PK5/1P6/8/8/8/8	KONKO	4001	cm	ww	24	30	8/3a4/101N4/8/k7/8/3K4/8
KNKP	000101	cm	ww	6	6	8/8/8/8/8/p7/k2N4/2K5	KOP KO	4000 10	cm	ww	91	102	8/8/8/8/2K3O1/2P la3/8/4k3
12,12	0001.01	cm	hw	9	24	8/K7/N1k5/n7/8/8/8/8	KOPKR	1300.10	cm	ww	2	102	k7/8/K01r4/P7/8/8/8/8
KNNKN	0005	cm	ww	4	4	8/8/8/8/n7/8/2KN4/kN6	KORKO	4100	cm	ww	11	18	8/8/8/8/1R6/k4a2/8/1K2O3
KNNKP	0002.01	cm	ww	105	105	8/8/8/p7/K7/4k3/8/6NN	KRBKP	0110.01	cm	ww	1	8	1k 1K4/7R/8/8/8/8/6n1/7B
	0002101	c	bw	3	39	7N/8/K1N5/8/8/1nk5/8/8	KRBKO	3110	cm	hw	38	67	$1_{16}/8/1B 3R 2/8/k7/8/8/1K6$
		m	bw	1	53	1N6/8/8/N7/K7/8/kp6/8	KRBKR	0410	cm	ww	49	55	5R2/8/8/8/8/3K4/5Br1/2k5
KNNKO	3002	cm	bw	53	62	8/8/h6/8/4N3/3K2N1/8/4k3	KRKB	0130	cm	ww	14	25	8/8/1b6/5R2/8/3K4/8/2k5
KNNKR	0302	c	bw	5	28	6rN/5N2/8/8/8/2k5/8/1K6	KRKN	0103	cm	ww	10	24	8/8/8/6n1/3K4/4R3/3k4/8
	0002	m	bw	2	35	5N2/1N6/8/3r4/8/2k5/8/2K5	KRKP	0100.01	cm	ww	6	20	8/K7/8/k7/1n6/8/8/1R6
KNP K	0001.10	cm	ww	14	19	8/8/8/8/8/1k 1P 4/8/KN6	KRNKN	0104	cm	ww	19	24	8/8/8/8/8/3n4/N2k4/RK6
KNP KB	003110	cm	ww	28	40	8/6b1/8/8/1N5P/8/2K5/k7	KRNKO	3101	cm	hw	42	63	1Nk5/8/8/8/8/1R6/6a1/2K5
	000 110	cm	bw	2	2	K5h1/P lk2N2/8/8/8/8/8/8/8	KRNKR	0401	cm	ww	22	23	8/8/8/8/8/2KRN3/8/2k Ir3
KNP KN	0004 10	cm	ww	40	56	2n5/8/8/1N6/8/6P 1/8/k1K5	KRPKB	013010	cm	ww	58	67	8/8/8/8/7R/1k2P3/2b5/K7
	0001.10	cm	hw	2	2	K7/P2n4/1k6/N7/8/8/8/8	KRPKN	0103.10	cm	ww	37	47	K7/1R6/2n5/4k3/8/4P3/8/8
KNP KP	000111	c	ww	21	29	8/8/8/8/8/2k2p2/P7/1KN5	KRPKP	0100.11	cm	hw	1	15	8/8/8/8/8/106/kP 6/1R K5
	0001.11	m	ww	19	33	K7/N6n/k7/8/8/8/7P/8		0100.11	cm	fn	1+4	15+15	8/8/8/8/8/2n5/1kP5/2RK4
		cm	hw	5	24	8/8/8/8/6P 1/3k2n1/8/2KN4	KRP KO	3 100 10	cm	ww	2	12	$7k/2K2P \ln/8/8/8/8/8/5R2/8$
		cm	fn	1+13	17+28	8/8/8/8/8/8/1k2p3/4P3/KN6		2.20.10	cm	hw	63	91	8/8/a lk 5/8/ lR 1K4/8/ lP 6/8
KNP KO	300110	cm	hw	29	45	$4N_3/3P_4/2K_5/a^7/k_6/8/8/8$	KRPKR	0400 10	cm	ww	43	54	8/8/8/8/8/2RPr3/8/2K1k3
KNP KR	030110	c	ww	4	19	k6r/5P 2/K7/3N4/8/8/8/8	KRRKR	0230	cm	ww	8	16	8/8/8/8/8/blk5/1R6/1RK5
	5551.10	m	ww	3	28	3k4/r1N5/2KP4/8/8/8/8/8	KRRKO	3200	cm	ww	7	20	6R 1/8/8/8/6R 1/7g/1K5k/8
		cm	bw	34	58	8/8/8/5N2/K7/2k5/P2r4/8		0200	•				

Table 2. Maximal mutual zugs: examples.

For a given endgame and type of mzug, the sets *ZC* and *ZM* are the sets of DTC-maximal and DTM-maximal mzugs respectively. ZC & ZM are either disjoint, identical or one is a subset of the other. There is no endgame for which ZC - ZM $\neq \emptyset$ and ZM - ZC $\neq \emptyset$.

$ZC \cap ZM = \emptyset$	KBNKP ww, KBNKR ww, KBPKN ww, KBPKP bw, KBPKR bw, KNNKP bw,
	KNNKR bw, KNPKP ww, KNPKR ww, KPKP ww (and therefore bw), KPPKB ww,
	KPPKP ww, bw and fp, KPPKR fp and KQBKQ ww.
$ZC \subset ZM$	KNPK ww (1-2), KPPKN ww (1-2) and bw (11-15), KRNKN ww (1-2) and
	KRPKR ww (1-2)
$ZC \supset ZM$	KBBKR ww (3-1), KBPKQ bw (2-1), KNPKN ww (3-1), KNPKP bw (6-1), KPKP fp (15-1),
	KPPK ww (6-1), KRKP ww (6-4), KRPKN ww (4-1), KRPKP bw (2-1), KRPKQ ww (2-1)

4 SUMMARY

A complete set of data about 3-to-5-man mzugs in chess has been compiled, second-sourced, summarised here and made available via the web. This includes maximal mzug and depth statistics to both the DTC and DTM metrics.

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