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COEFFICIENTS OF REFLECTION AND TRANSMISSION OF TRANSVERSE AND LONGITUDINAL ACOUSTIC WAVE IN THE BLATZ-KO MATERIAL

Abstract

The purpose of this paper is to analyze the propagation of transverse and longitudinal acoustic wave in a composite made of hyperelastic Blatz-Ko material. Composite consists of a homogeneous layer of predetermined thickness d separating two infinite homogeneous material areas. In the paper it is assumed that the middle layer is filled with a homogeneous rubber (f=1), whereas the external areas with foam rubber (f=0). The final effect of paper are graphs of coefficients reflection of transverse and longitudinal acoustic wave, propagating in this composite.

Keywords

Acoustic wave, the layer composite, Blatz-Ko material, hyperelastic material.

1 INTRODUCTION

In the paper is considered longitudinal and lateral acoustic wave propagated in the layer composite. The composite is made from the transition layer of a thickness d filled by a homogeneous rubber (f=1) and external homogeneous material areas *0* and *2*, filled by foam rubber (f=0). In the end of the paper are graphs of coefficients of transmission and reflection of transverse and longitudinal acoustic wave dependent on the parameter of initial deformation λ for the selected frequency ω. The analysis of discussed harmonic wave are based on the work [1], assuming the maximal value in the range of Poisson's ratio according to work [2] ν=0.493. Constant value of Poisson's ratio for infinitesimal deformation of foam rubber was assumed as ν=0.25.

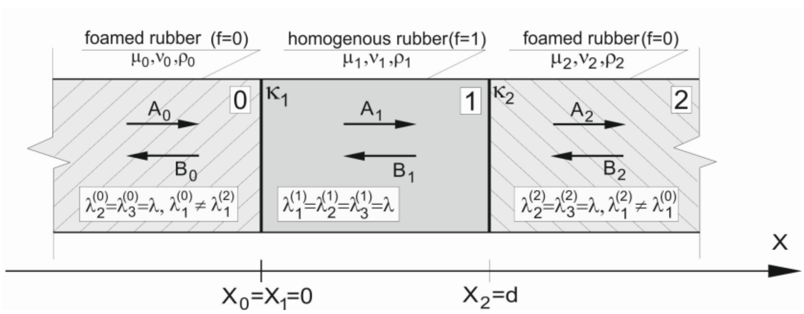

Fig. 1: Rubber composite consisting of the transition layer (homogeneous rubber) separating the two infinite material areas (foam rubber)

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2 BLATZ–KO MATERIAL

The Blatz-Ko models for rubber have been extensively used to describe the behaviour of compressible isotropic hyperelastic materials undergoing finite deformations (see [3,4,5]).

Composite considered in the paper was composed of hyperelastic material with Blatz-Ko potential [6,7]:

$$W(I_1, I_2, I_3) = \frac{\mu f}{2}\left\{I_1 - 3 + \frac{1-2\nu}{\nu}\left[I_3^{\frac{-\nu}{1-2\nu}} - 1\right]\right\} +$$
$$\frac{\mu(1-f)}{2}\left\{\frac{I_2}{I_3} - 3 + \frac{1-2\nu}{\nu}\left[I_3^{\frac{\nu}{1-2\nu}} - 1\right]\right\} \quad (1)$$

where: I_1, I_2, I_3 - invariants of the left Cauchy Green deformation tensor [8], μ- shear modulus [MPa], ν- Poisson ratio (relating to infinitesimal deformation). The value of the parameter f describing the share of pores in material is in the range $0 \leq f \leq 1$. Special attention was devoted to two models of the material, when f=0 (foam rubber) and f=1 (homogeneous rubber) in the literature for which the equation (1) reduces to the following form [2]:

when f=0 (foamed rubber):

$$W(I_2, I_3) = \frac{\mu}{2}\left\{\frac{I_2}{I_3} - 3 + \frac{1-2\nu}{\nu}\left[I_3^{\frac{\nu}{1-2\nu}} - 1\right]\right\} \quad (2)$$

when f=1 (homogenous rubber):

$$W(I_2, I_3) = \frac{\mu}{2}\left\{\frac{I_2}{I_3} - 3 + \frac{1-2\nu}{\nu}\left[I_3^{\frac{\nu}{1-2\nu}} - 1\right]\right\} \quad (3)$$

3 THE BASIC EQUATIONS DESCRIBING THE PROPAGATION OF LONGITUDINAL AND TRANSVERSE ACOUSTIC WAVES IN A LAYERED COMPOSITE

It was assumed that the motion associated with the propagation of a plane wave accept the form [1]:

$$x_1 = \lambda_1^{(k)} X'_1 + u_1^{(k)}(X, t), x_2 = \lambda_2^{(k)} X'_2, x_3 = \lambda_3^{(k)} X'_3 + u_3^{(k)}(X, t) \quad (4)$$

where: X'_1, X'_2, X'_3 - the coordinates of the material, upper index (k) is a variable defined in the layer k, whereas $\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}$- the main elongations of the static homogeneous initial deformation in the area k. Layout of equations of motion is reduced to two non-conjugated wave equations for the foamed and homogenous rubber [1]:

$$u_{1,11}^{(k)} = \frac{1}{c_k^2}\ddot{u}_1^{(k)}, \; u_{3,11}^{(k)} = \frac{1}{c'^2_k}\ddot{u}_3^{(k)} \quad (5)$$

Where velocity of propagation of longitudinal waves for foamed rubber (f=0) and homogenous rubber (f=1) are [1]:

$$c_k^2 = \frac{\mu}{\rho_R^{(k)}}\left[3\lambda_1^{-4} + \frac{4\nu-1}{1-2\nu}(\lambda_2, \lambda_3)^{\frac{2\nu}{1-2\nu}}\lambda_1^{-\frac{6\nu-2}{1-2\nu}}\right] \quad (6)$$

$$c_k^2 = \frac{\mu}{\rho_R^{(k)}}\left[1 + \frac{1}{1-2\nu}(\lambda_2, \lambda_3)^{-\frac{2\nu}{1-2\nu}}\lambda_1^{-\frac{2-2\nu}{1-2\nu}}\right] \quad (7)$$

For the values of Poisson's ratio ν=0.439 for the area 1 and ν=0.25 for the area 0 and 2 we get:

$$c_0^2 = \frac{3\mu_0}{\rho_R^{(0)}}\left(\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{4}{3}}, c'^2_0 = \frac{\mu_0}{\rho_R^{(0)}}\lambda^{-2}\left(\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{2}{3}} \quad (8)$$

$$c_1^2 = \frac{\mu_1}{\rho_R^{(1)}}\left[1 + 71.429\lambda^{-\frac{1493}{7}}\right], c'^2_1 = \frac{\mu_1}{\rho_R^{(1)}} \quad (9)$$

$$c_2^2 = \frac{3\mu_2}{\rho_R^{(2)}}\left(\lambda^2 + \frac{\mu_1}{\mu_2}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{4}{3}}, c'^2_2 = \frac{\mu_2}{\rho_R^{(2)}}\lambda^{-2}\left(\lambda^2 + \frac{\mu_1}{\mu_2}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{2}{3}} \tag{10}$$

The density of the foamed rubber is less than the density of homogeneous rubber and may change. Analysis was based on two values of density for the foam rubber $\rho_R^{(0)} = 0.9\rho_R^{(1)}$ and $\rho_R^{(0)} = 0.3\rho_R^{(1)}$, wherein the density of the homogeneous rubber: $\rho_R^{(1)} = 911\text{kg/m}^3$.

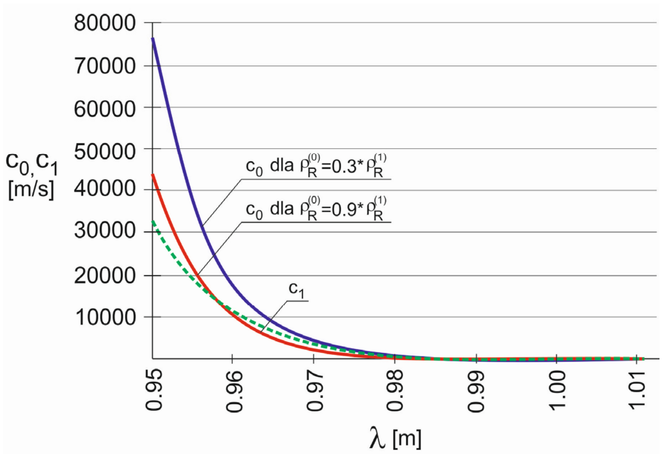

Fig. 2: Graphs of longitudinal propagation velocity of acoustic waves in foam rubber (c_0) and homogeneous rubber (c_1)

Velocity of propagation of transverse waves for foamed rubber (f=0) and homogenous rubber (f=1) is described by following equation [1]:

$$c'^2_k = \frac{\mu}{\rho_R^{(k)}}\frac{1}{\lambda_1^2\lambda_3^2} \tag{11}$$

$$c'^2_k = \frac{\mu}{\rho_R^{(k)}} \tag{12}$$

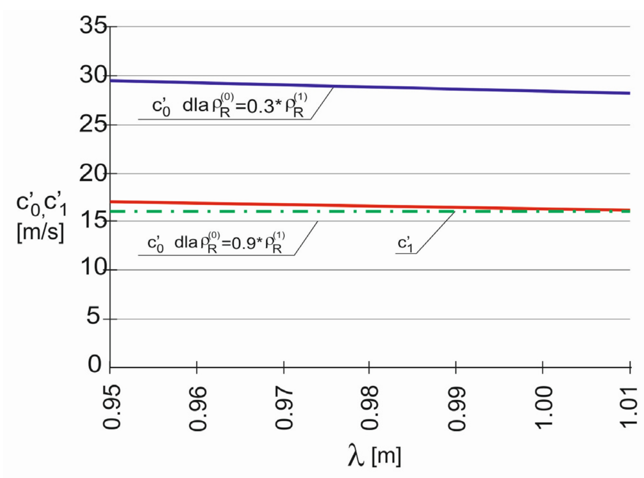

Fig. 3: Graphs of transverse propagation velocity of acoustic waves in foam rubber (c'_0) and homogeneous rubber (c'_1)

For the values of Poisson's ratio v=0.439 for the area 1 and v=0.25 for the area 0 and 2 we get:

$$c'^2_0 = \frac{\mu_0}{\rho_R^{(0)}}\lambda^{-2}\left(\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{2}{3}} \tag{13}$$

$$c'^2_1 = \frac{\mu_1}{\rho_R^{(1)}} \tag{14}$$

$$c'^2_2 = \frac{\mu_2}{\rho_R^{(2)}}\lambda^{-2}\left(\lambda^2 + \frac{\mu_1}{\mu_2}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{2}{3}} \tag{15}$$

It is assumed that the material k-1 in the left side of the plane $X_0 = X_1$ is foamed rubber (f=0), and k material lying in the right side is homogenous rubber (f=1). For two adjacent layers should be considered dependence of extensions of the main deformation $\lambda_1^{(k-1)}$, $\lambda_1^{(k)}$ in the form [1]:

$$\frac{\mu_k}{\mu_{k-1}} = \frac{\left(\lambda_1^{(k-1)}\right)^{\frac{4\nu_{k-1}-1}{1-2\nu_{k-1}}}\left(\lambda_2^{(k-1)}\lambda_3^{(k-1)}\right)^{\frac{\nu_{k-1}}{1-\nu_{k-1}}} - \left(\lambda_1^{(k-1)}\right)^{-3}}{\lambda_1^{(k)} - \left(\lambda_1^{(k)}\right)^{\frac{-1}{1-2\nu_k}}\left(\lambda_2^{(k)}\lambda_3^{(k)}\right)^{\frac{-2\nu_k}{1-2\nu_k}}} \tag{16}$$

At admission the Poisson's ratio for foam rubber $\nu = 0.25$ defined for infinitesimal deformation [2], and assuming that the layer on the right side of the plane (homogeneous rubber) is subjected to uniform dilatation, where $\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)} = \lambda$ equation (16) takes the form [2]:

$$\lambda_1^{(k-1)} = \left\{\lambda^2 + \frac{\mu_k}{\mu_{k-1}}\left[\lambda^{-\frac{4\nu_k+1}{1-2\nu_k}} - \lambda\right]\right\}^{-\frac{1}{3}} \tag{17}$$

After inserting the Poisson's ratio for homogeneous rubber v_1=0.493 into equation (17) we obtain component of the gradient of static deformation for extrenal areas equal to:

$$\lambda_1^{(0)} = \left\{\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right\}^{-\frac{1}{3}} \tag{18}$$

$$\lambda_1^{(2)} = \left\{\lambda^2 + \frac{\mu_1}{\mu_2}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right\}^{-\frac{1}{3}} \tag{19}$$

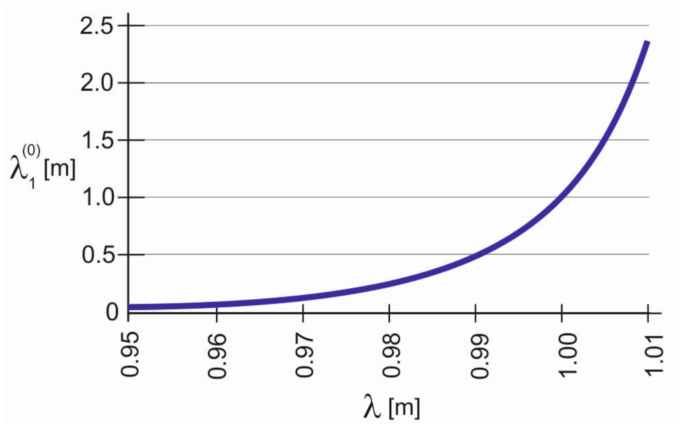

Fig. 4: Graphs the relationship between the components of the deformation gradient in homogeneous and foamed rubber.

It is assumed, that the harmonic wave motion propagating in the analyzed composite in the direction perpendicular to the layers, has the form [1]:

$$u_1^{(k)}(X,t) = A_k expi\omega\left(t - \frac{X-X_k}{c_k}\right) + B_k expi\omega\left(t + \frac{X-X_k}{c_k}\right) \tag{20}$$

$$u_3^{(k)}(X,t) = A'_k expi\omega'\left(t - \frac{X-X_k}{c'_k}\right) + B'_k expi\omega\left(t + \frac{X-X_k}{c'_k}\right) \tag{21}$$

where: ω, ω'- the frequency of longitudinal and transverse waves; c_k, c'_k – velocity of propagation of longitudinal and transverse waves, A_k, B_k, A'_k, B'_k – reciprocally incorporated amplitude of the longitudinal and transverse waves in the layer k. The relationship between the complex amplitudes of the sinusoidal waves of the longitudinal and transversal in layer *k -1* and *k* is [1]:

$$\begin{bmatrix} A_k \\ B_K \end{bmatrix} = M_k \begin{bmatrix} A_{k-1} \\ B_{k-1} \end{bmatrix}, \begin{bmatrix} A'_k \\ B'_K \end{bmatrix} = M'_k \begin{bmatrix} A'_{k-1} \\ B'_{k-1} \end{bmatrix} \tag{22}$$

where:

$$M_k = \frac{1}{2}\begin{bmatrix} (1+\kappa_2)\exp(-i\alpha_2) & (1-\kappa_2)\exp(-i\alpha_2) \\ (1-\kappa_2)\exp(-i\alpha_2) & (1+\kappa_2)\exp(-i\alpha_2) \end{bmatrix} \tag{23}$$

$$M'_k = \frac{1}{2}\begin{bmatrix} (1+\kappa'_2)\exp(-i\alpha'_2) & (1-\kappa'_2)\exp(-i\alpha'_2) \\ (1-\kappa'_2)\exp(-i\alpha'_2) & (1+\kappa'_2)\exp(-i\alpha'_2) \end{bmatrix} \tag{24}$$

and:

$$\kappa_k = \frac{\rho_R^{(k-1)} c_{k-1}}{\rho_R^{(k)} c_k}, \alpha_k = \omega \frac{X_k - X_{k-1}}{c_{k-1}} = \frac{\omega d_{k-1}}{c_{k-1}} \tag{25}$$

$$\kappa'_k = \frac{\rho_R^{(k-1)} c'_{k-1}}{\rho_R^{(k)} c'_k}, \alpha'_k = \omega' \frac{X_k - X_{k-1}}{c'_{k-1}} = \frac{\omega' d_{k-1}}{c'_{k-1}} \tag{26}$$

Parameter of transition matrix of longitudinal wave κ_k, that describes the jump surface of discontinuity in the layers of the composite for *X=0* and for $X_2=d$ after substitution of velocity propagation is:

$$\kappa_1(\lambda) = \frac{\rho_R^{(0)} c_0(\lambda)}{\rho_R^{(1)} c_1(\lambda)} = \left\{ 3\frac{\rho_R^{(0)} \mu_0}{\rho_R^{(1)} \mu_1} \frac{\left(\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{4}{3}},}{1 + 71.429\lambda^{-\frac{1493}{7}}} \right\}^{\frac{1}{2}} \tag{27}$$

$$\kappa_2(\lambda) = \frac{\rho_R^{(1)} c_1(\lambda)}{\rho_R^{(2)} c_2(\lambda)} = \left\{ \frac{1}{3}\frac{\rho_R^{(1)} \mu_1}{\rho_R^{(2)} \mu_2} \frac{1 + 71429\lambda^{-\frac{1493}{7}}}{\left(\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{4}{3}}} \right\}^{\frac{1}{2}} \tag{28}$$

Parameter of transition matrix of transverse wave κ'_k, that describes the jump surface of discontinuity in the layers of the composite for *X*=0 and for $X_2=d$ after substitution of velocity propagation is:

$$\kappa'_1(\lambda) = \frac{\rho_R^{(0)} c'_0}{\rho_R^{(1)} c'_1} = \left\{ \frac{\rho_R^{(0)} \mu_0}{\rho_R^{(1)} \mu_1} \lambda^{-2} \left(\lambda^2 + \frac{\mu_1}{\mu_0}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{\frac{2}{3}} \right\}^{\frac{1}{2}} \tag{29}$$

$$\kappa'_2(\lambda) = \frac{\rho_R^{(1)} c'_0}{\rho_R^{(2)} c'_1} = \left\{ \frac{\rho_R^{(1)} \mu_1}{\rho_R^{(2)} \mu_2} \lambda^{2} \left(\lambda^2 + \frac{\mu_1}{\mu_2}\left[\lambda^{-\frac{1486}{7}} - \lambda\right]\right)^{-\frac{2}{3}} \right\}^{\frac{1}{2}} \tag{30}$$

Assuming the same external infinite material (0 and 2), in the present case the following identity holds: $\kappa_2 = \kappa_1^{-1}$. According to the paper [9] in addition to the symmetry of the reflection coefficients $r^{(0)} = r^{(2)}$ is introduced the symmetry of the transmission coefficients $t^{(0)} = t^{(2)}$ (This is due to symmetry of the arrangement of materials in the compositions). Coefficients of reflection $r^{(0)}$ and transmission $t^{(0)}$ for the transverse wave takes the form [1]:

$$r'^{(0)} = \sqrt{\frac{\left(\kappa'_1-\kappa'^{-1}_1\right)^2(1-cos2\alpha'_2)}{8+\left(\kappa'_1-\kappa'^{-1}_1\right)^2(1-cos2\alpha'_2)}} \tag{31}$$

$$t'^{(0)} = \frac{2\sqrt{2}}{\sqrt{8+\left(\kappa'_1-\kappa'^{-1}_1\right)^2(1-cos2\alpha'_2)}} \tag{32}$$

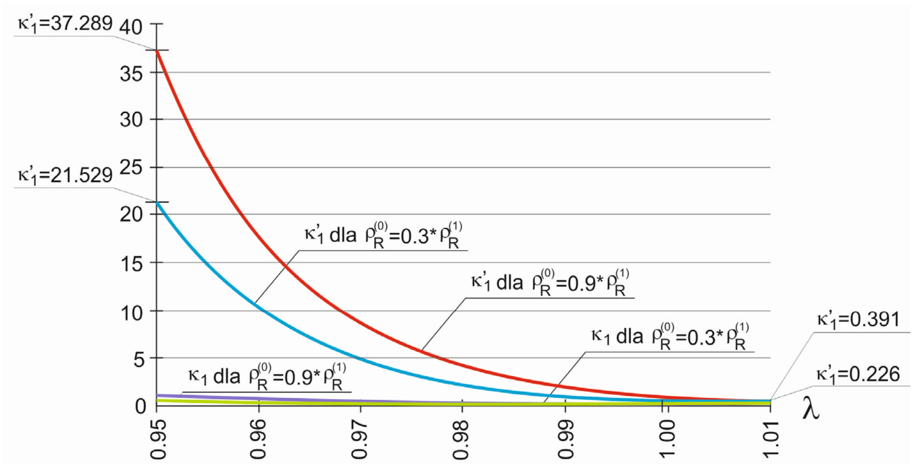

Fig. 5: Graph of the relationship of quotient impedance of adjacent layers from initial deformation of λ

After inserting the two extreme values of quotient of impedance maximum $\kappa'_k = 21.529$ and minimum $\kappa'_k = 0.226$ (Fig. 5) (designated for proportion of density $\frac{\rho_R^{(0)}}{\rho_R^{(1)}}$ =0.3 to formula (31) and (32) obtained the following graph:

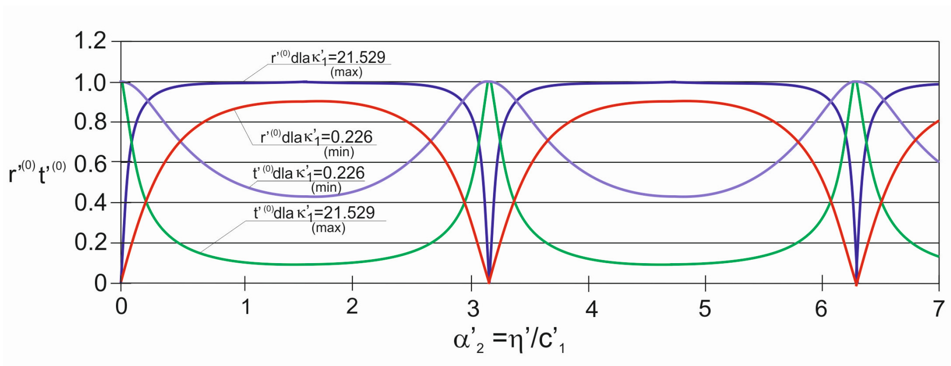

Fig. 6: Graph of coefficients of reflection and transmission for the proportion of the density of $\frac{\rho_R^{(0)}}{\rho_R^{(1)}}$ =0.3

The graph shows coefficients of the reflection and transmission of transverse wave as a function dependent from variable parameter $\alpha'_2 = \frac{\eta'}{c'_1}$ (η'=ω'd). Similarly, after inserting the two extreme values of quotient of impedance ratio maximum $\kappa'_k = 37.289$ and minimum $\kappa'_k = 0.391$ (Fig. 5.), designated for proportion of density $\frac{\rho_R^{(0)}}{\rho_R^{(1)}}$ =0.9 to formula (30) and (31) we obtained the following graph:

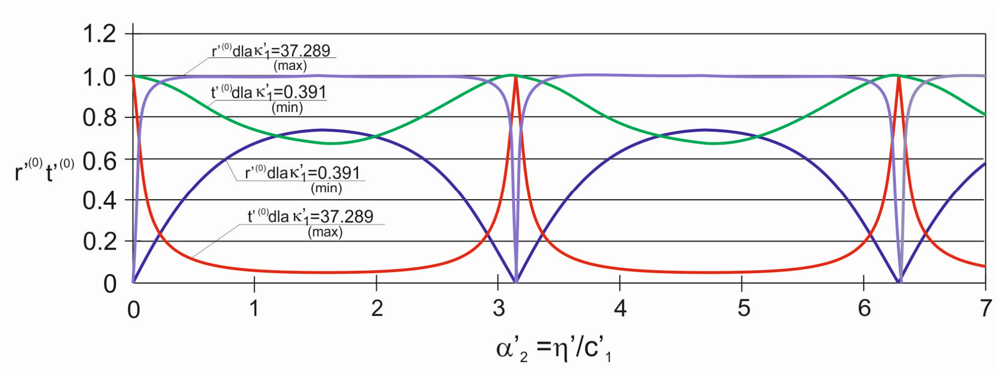

Fig. 7: Graph of coefficients of reflection and transmission

for the proportion of the density of $\frac{\rho_R^{(0)}}{\rho_R^{(1)}}$ =0.9

4 CONCLUSION

Analysis of the graphs dependence between components of deformation gradient in rubber $\lambda_1^{(0)} = \lambda_1^{(2)}$ on the parameter λ (Fig. 4) shows, that small changes of value $\lambda = (0.95; 1.02)$ accompanied large fluctuations of value of the component $\lambda_1^{(0)} = \lambda_1^{(2)}$.

Figure 5 representing the relationship the impedance of adjacent layers shows that in the range of examined variation of the parameter λ , initial deformation affects the quotient of impedance of the transverse waves more than the longitudinal waves. Graphs (Fig. 6) and (Fig.7), showing coefficients of reflection and transmission of the transverse wave as a function dependent from variable parameter $\alpha'_2 = \frac{\eta'}{c'_1}$ *(η'=ω'd)*. In both cases for the proportion of density 0.3 or 0.9 for $\alpha'_2 = \pi$ or $\alpha'_2 = 2\pi$, we have $r'^{(0)} = 0$ and $t'^{(0)} = 0$ according to the formula (31) and (32). In the above formulas show that in the general case where acoustic transverse wave (from any physically acceptable frequency) is transmitted in shown composite - coefficients of reflection and transmission are periodic functions of the frequency of the incident wave. They depend also on the initial deformation. As shown in the graph (Fig. 6) and (Fig. 7) the impact of the initial deformation on the values of the coefficients of reflection and transmission increases with decreasing density of areas filled by foamed rubber while keeping constant values of shear modulus and Poisson's ratio. Calculation of parameters serves broader researches and observing behavior of wave propagation in a layered elastic medium made of Blatz-Ko materials.

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