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Modelování volatility devizových kurzů

Modelling Exchange Rate Volatility

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Description:

1. Introduction
2. Characteristics of the Foreign Exchange Market and its Volatility
3. Basic Approaches of Modeling and Prediction of Volatility
4. Estimation of Volatility Models and their Predictions
5. Summary of the Results
6. Conclusion
Bibliography
List of Abbreviations
Declaration of Utilization of Results from the Diploma Thesis
List of Annexes
Annexes

References:

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Extent and terms of a thesis are specified in directions for its elaboration that are opened to the public on the web sites of the faculty.
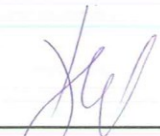
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"Herewith I declare that I elaborated the entire thesis including annexes, independently."

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CONTENT

1 INTRODUCTION

Modelling the financial time series is nowadays widely used not only in finances itself but also in insurance industry, investment decision making process or generally economy forecasting. Interest in this area is gradually increasing because its possibilities are large and this field is developing very fast. Modelling the volatility plays an important part because it reflects the risk of investments to assets, commodities or, as analyzed in this thesis, exchange rates.

Through the time and growth in international trade the foreign exchange market became the largest market according the volume of trades and its importance in nowadays globalized world is unquestionable. The trades go on twenty-four hours a day and the amount of data and changes is very high, so the financial time series which are established are called high-frequency data. Autoregressive conditional heteroskedasticity model established in 1982 by Robert Engle presented a great tool to model, estimate and forecast volatility. Also other models are mentioned and used in this thesis, for instance general autoregressive conditional heteroskedasticity model and other nonlinear models derived.

The aim of this Diploma thesis is modelling and in-sample forecasting the volatility of selected exchange rates using linear and nonlinear conditional heteroskedasticity models. The exchange rates used for the purpose of this thesis represent countries from different European regions and their domestic currency is always rated against Euro to see how volatile are their exchange rates in different periods with connection to European Union, Slovenian tolar to Euro for Slovenia (SIT/EUR), Cyprus pound to Euro for Cyprus (CYP/EUR), Slovakian koruna to Euro for Slovakia (SKK/EUR) and Latvian Lat to Euro for Latvia (LVL/EUR). Time period which is used for modelling the volatility is from 1/01/1999 as the year when Euro was established as a non-physical currency to the year in which each of the observed countries joined European monetary union, Slovenia 1/01/2007, Cyprus 1/01/2008, Slovakia 1/01/2009 and Latvia 1/01/2014.

The main aim of the thesis is supported by two partial aims. The first partial aim is to compare whether linear or nonlinear models are more efficient for modelling conditional heteroskedasticity for exchange rates. The second partial aim is to assess the suitability of estimated models to predict volatility.

Thesis is divided into six parts including Introduction and Conclusion. Second and third chapter is theoretical and methodological while the fourth and fifth chapter is practical and empirical.

Second part gives us information about the foreign exchange market and trading of the currencies. This chapter includes the description of various exchange rate systems. The last part of the second chapter provides us introduction and elementary information about volatility.

We can find the basic approaches of modelling and prediction of volatility in the third part. Firstly the assumptions and features of the financial time series are mentioned and basic univariate linear models are described. The chapter continues with explaining the topic of linear conditional heteroskedasticity models from ARCH, GARCH to their modifications leading to nonlinear models such as EGARCH or GJR-GARCH. Information about forecast construction and generally about the volatility models construction follows. The last parts of this chapter are about diagnostic tests and criteria for model selection.

Fourth part uses the theoretical background of the previous chapters and firstly analyzes and verifies the data used in this thesis. The data are represented by time series of daily returns of observed exchange rates. These time series are adjusted into time series of daily logarithmic returns and undergo the tests of normality, stationarity and heteroskedasticity. Then they are used to estimate the best possible models of conditional heteroskedasticity. Estimated models are diagnosed whether the standardized residuals are autocorrelated, or shows normality or homoskedasticity. Volatility of the estimated model is expressed by GARCH graphs of conditional variance. The last part tests the in-sample forecasting ability of estimated models.

Fifth chapter contains a commented summary of the results given by the previous chapter with synoptic Tables with a summary of the diagnostic tests of standardized residuals.

Conclusion is dedicated to summarize the whole Diploma thesis, evaluate whether the focus of the thesis was fulfilled and comment on the possible improvements.

For the calculations and outputs like Charts, Figures, Tables etc. in this Diploma thesis will be used the statistical software Eviews7 and Microsoft Excel 2010.

2 CHARACTERISTICS OF THE FOREIGN EXCHANGE MARKET AND ITS VOLATILITY

This chapter devotes itself to the characteristics of the foreign exchange market. Firstly describing what the foreign exchange market is and how is it organized, then continue with information about the participants of the foreign exchange market, explaining who and how are the currencies traded. All the different markets and their characteristics are mentioned to understand the background of currency trading. We can find information about various exchange rate systems in the following part and the last part of this chapter describes basic information about volatility.

2.1 The Foreign Exchange Market

A market generally represents an actual or nominal place where the forces of demand and supply meet, where buyers and sellers interact and by trading goods, services, contracts or instruments, they are determining the price of the traded items. The foreign exchange market (forex, FX, or currency market) is a kind of global decentralized market where especially the currencies are traded and the conversion rates are determined. Levinson (2005) says: „The value of the currency itself, however, can be judged only against an external reference. This reference, the exchange rate, thus becomes the fundamental price in any economy. Most often, the references against which a currency's value is measured are other currencies. Determining the relative values of different currencies is the role of the foreign-exchange markets. "As seen on the Figure 2.1., to importers and exporters the foreign exchange market basically fulfills two elementary functions. The first one is the transfer of purchase power into the foreign currency, which is because importers need foreign currency to buy goods, services or other instruments from abroad while exporters are selling foreign currency to pay for their liabilities in domestic currency. The second function is to hedge against the exchange rate risk.

The foreign exchange market is the largest market in the world by the volume with average daily turnover around 4 trillion USD. Because of this high volume, the foreign exchange market is also the most liquid market in the world. A unique feature of the foreign exchange market is that the trading goes on twenty-four hours a day, excluding weekends, ongoing on

trading sessions in each local market all around the world with the trading centers in London, New York, Tokyo and Hong Kong. This makes it possible for all participants of the market to immediately respond to the actual situation.

Figure 2.1 Organization of the foreign exchange market
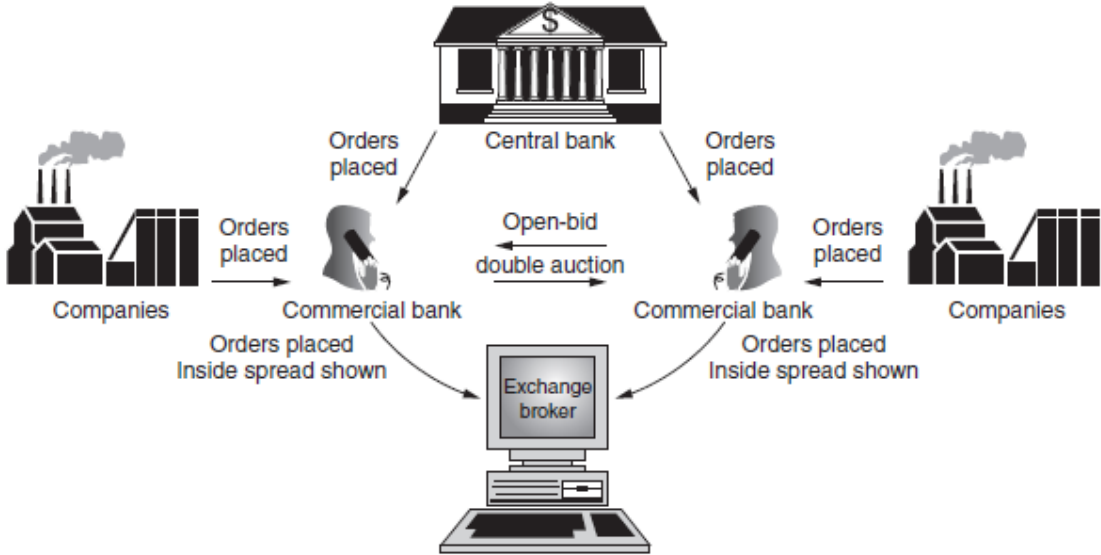
Source: Levi, 2005, p. 35

2.2 The Foreign Exchange Market Spot, Futures and Swaps

According to the technique of conducted trades on the foreign exchange market, we distinguish the three different kinds of markets, which works separately but in the same time they are strongly linked together. The foreign exchange markets are spot, futures and swap. Nowadays the biggest traded volume is in the swap market with more than a half of all foreign exchange market trades. This thesis is especially focused on the spot market transactions. The assorted ways of how the exchange rates can be quoted will be explained together with some of the terms used in association with the foreign exchange market.

2.2.1 Spot Market

The trading operations on the spot market are settled usually in two trading days after the trade is agreed. The difference between the days of agreement and the day when the contract is closed serves for the transfer from the account of the seller the buyer. Spot

operations represents around 60% of volume of all foreign exchange trades in 80s, but currently the forward and swap operations are more important even in the Czech foreign exchange market. Purchasing or selling of the foreign currency is settled with the spot rate which is usually quoted directly.

Quotation basically means the determination of the exchange rate. An exchange rate is the amount of the currency that one needs in order to buy one unit of another currency, or it is the amount of a currency that one receives when selling one unit of another currency. We distinguish two kinds of quotations: direct and indirect quotation.

Direct quotation represents a record which determinates an amount of domestic currency per unit of foreign currency. For example a quote like USD/EUR 1,263 is natural for American citizens and express, that for 1 Euro you have to pay 1,263 United state dollars.

Indirect quotation represents a record which determinates an amount of foreign currency needed to buy or sell one unit of domestic currency. It is also known as a "quantity quotation". Generally there are three groups of people using this kind of quotation, mostly for their own purpose or from historical reasons. The first group are professional traders in the United states of America. The second group is represented by the Commonwealth countries such as Australia, New Zealand or Ireland before accepting Euro as a domestic currency. The third group is Eurolanders who are always quoting USD/EUR or JPY/EUR because for some reason when Euro was established it was foreign to all existing currencies. (Levi, 2005)

Generally we can distinguish two different prices while quoting currencies. It is a "bid price" and an "ask price". The bid price represents the highest price that a buyer accepts to pay for a currency. It is the price in which a buyer purchases the foreign currency from a bank or in the foreign exchange market. The ask price represents the price which a seller is willing to accept for the currency. It is the price in which a seller sells the foreign currency to the bank or in the foreign exchange market.

The difference between bid and ask price is called a "spread". The value of the spread depends greatly on the liquidity of the asset. In the foreign exchange markets, it is the currencies which are traded so the spread is very low compared for example to exotic commodities. The more traded is the currency and the bigger is the traded volume the lower spread the transaction has and vice versa. In the spot interbank market the spread can reach values around 0, 1% and on the client market around 1%. (Sercu, 2008)

2.2.2   Futures Market

In the futures market the trades are agreed in the present time and settled to the specific date in the future with the pre-set futures exchange rate which was agreed by both parties. Because the trades on the futures market have different characteristics, we distinguish three types of trades: forward, futures and swap transactions. (Johnson, 1999)

Forward

Forward contracts are being traded in the OTC markets and they are not standardized at an amount or at a time of settlement. The most frequent forward transactions are settled in one year or less. The second frequent forward transactions are settled between one and three years and the longest and least used are up to five years. The further is the date of settlement of the forward contract the higher is the risk because it is getting more difficult to predict or track a spot rate from which the forward rate is derived. The forward transaction is realized with the forward rate, which depends on the current level of supply and demand. Forward rate under direct quotation should be equal to the product of spot rate and the ratio of interest rates for domestic and exchange currency. The following equations (2.1) and (2.2) are showing a relation between the forward rate and the spot rate:

$$FR_{BID} = SR_{BID} * \frac{\left(1 + IR_{DOM\,D} * \frac{t}{360}\right)}{\left(1 + IR_{EXC\,L} * \frac{t}{360}\right)}, \tag{2.1}$$

$$FR_{ASK} = SR_{ASK} * \frac{\left(1 + IR_{DOM\,L} * \frac{t}{360}\right)}{\left(1 + IR_{EXC\,D} * \frac{t}{360}\right)}, \tag{2.2}$$

where *F* represents a forward rate, *SR* the spot rate, $IR_{x\,D}$, $IR_{x\,L}$ are interests rate lending and borrowing and *t* is the time to repay the forward contract.

Futures

Futures contracts share a lot of same features with Forward contracts. The specific contract or in other words the agreement about the price and amount is opened in present and settled in the specific date at the future. The difference is that Futures contracts are traded in the organized market, in the stock exchange. Thus Futures contracts are standardized. Minimum tradable amount of currency futures is called "lot", while only integer multiplies of lots are allowed to be traded. In some stock exchanges also the delivery time of contracts are

standardized. The purpose of standardization is to create larger concentration of supply and demand and thus better liquidity.

Another difference is that the subjects who conclude a contract have no direct legal relationship to each other, but to the Clearing control. The trading subjects have to deposit a "margin" to the Clearing control. It brings an advantage, because unlike Forward contracts it is not necessary to wait with the settlement till the maturity of the contract. It is possible to close the contract and immediately use the profit for next trading. Another advantage of Futures contracts is lower transaction costs because brokerage fees are smaller than spread in forward market.

Options

Currency options have also basic features of future transactions. Unlike previously mentioned transactions, options represent an agreement between the seller/writer of an option, who cannot revoke his offer for the set time, and holder of an option, who has the right to withdraw from the contract. The seller can write a call option or a put option. Option price is basically the price which the holder pays to the seller for the right to not fulfill the contract. Fulfilling the contract means to deliver or to take an amount of foreign exchange currency or commodity etc. Option premium is then set according to the unit of underlying currency. The price for standardized currency option is the product of option premium and the size of on lot of underlying currency. Option contracts can be settled via the stock exchange and also on the OTC market. Also standardized times for realization of the contracts are more frequent. Another difference for option market is that the margin is paid only by one side of the contract, the seller of an option, because only he has the obligation to fulfill the contract and is opened to unlimited loss.

Generally we distinguish two elementary types of option contracts which differ in the terms of settling the contract. It is American and European option. The holder of an American option has the right to require the fulfillment of the contract anytime between opening the contract and the maturity of the contract. The settlement of European option is always at the time of the maturity of the contract. The seller will require higher option premium for the American option, because the time of the settlement is not set.

2.2.3   **Swap**

Foreign exchange swap is created by two not-separable operations in the same time with the same business partner and at least one of the operations is forward transaction. So there are two elementary types of transactions: spot – forward, forward – forward. In the first case, the dealer is buying the foreign currency in spot market together with selling it on the Futures market, or vice versa. In the second case the dealer is buying a shorter forward contract (with maturity yield e.g. one month) together with selling a longer forward contract (with maturity yield e.g. three months), or vice versa.

We know also other forms of swaps – currency swaps and cross currency interest swaps. These transactions enable to hedge currency or interest rate risk in case of medium-term and long-term credits. E.g. currency swaps enable a company with receiving dollar credit to exchange all the future dollar payments into Czech crowns. The cross currency interest swap enables to change also the interest rate (e.g. from fix to floating or vice versa). This market is organized by the swap houses, which are usually subsidiaries of large banks. (Levinson, 2005)

2.3   Exchange Rate Systems

The term exchange rate system is closely connected to the way of how the monetary policy in each state is made. Generally it represents how domestic currency is related to other currencies and foreign exchange market. Arranging exchange relations within the system of exchange rates are mainly associated with the different role of central banks and governments in the determination of market rate and varying levels of elasticity and exchange rate stability. Different properties of each of the foreign exchange rate systems have different influence on adaptability of domestic and foreign prices, balance of payments and foreign exchange reserves. They also tend to create different assumptions for monetary and fiscal policy and handling the exchange rate risk. Each of the exchange rate systems has its advantages and disadvantages. The summary of the different exchange rate systems is captured in Figure 2.2 and further description of each of the exchange rate systems follows. (Durčáková, Mandel, 2010)

Figure 2.2 Exchange rate systems
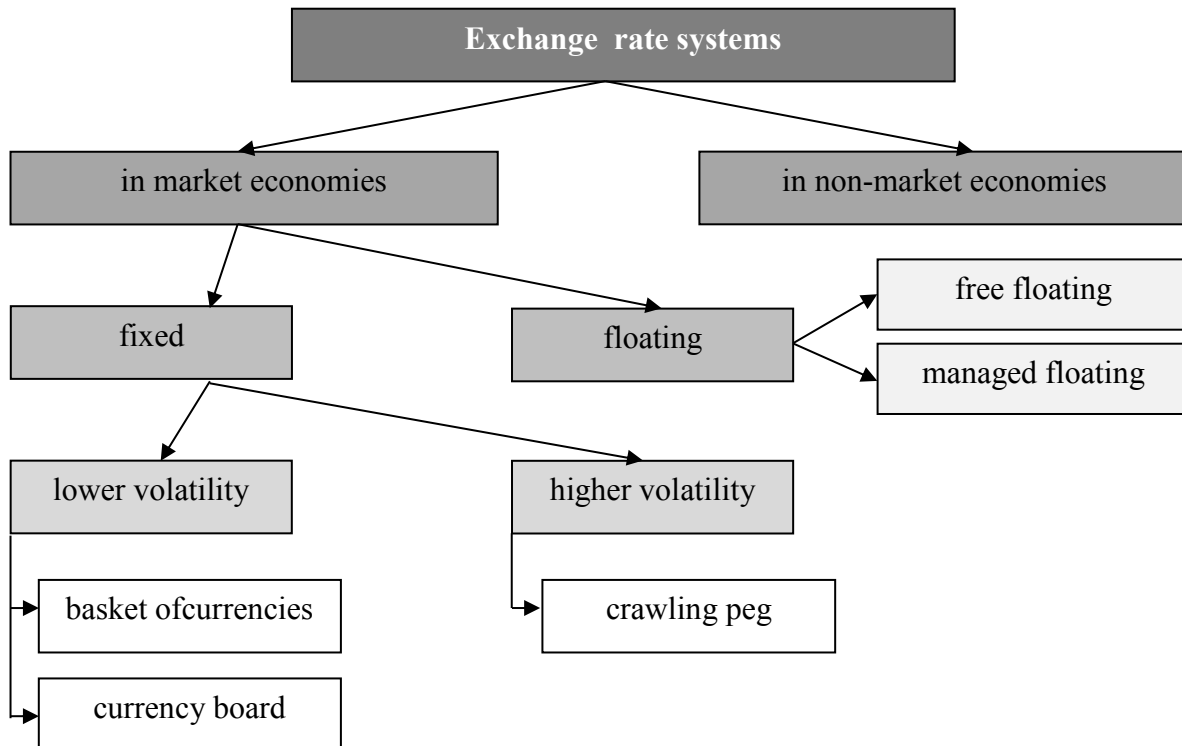
Source: Durčáková, Mandel, 2010, p. 368, modified

2.3.1 Fixed Exchange Rate Systems

Fixed exchange rate systems are characteristic by the fact, that the central bank sets a fixed value of the nominal exchange rate usually also with fixed limits of oscillation. This fixed value of the nominal exchange rate is then being kept by official interventions of the central bank on the foreign exchange market using devaluation or revaluation of domestic currency. Single fixed exchange rate systems differ according to the set limits of oscillation and regularity or revocability of fixed central exchange rate. The first fixed exchange rate systems were gold standard and its various forms and Bretton Woods system. Nowadays we can distinguish several types of fixed exchange rate systems.

One of the advantages of the fixed exchange rate systems is that it creates a stable environment for international investments. A fixed exchange rate lowers volatility and fluctuations in relative prices. It may also help to decrease transaction costs and to keep the inflation on a lower level. On the other hand disadvantages include arguments that the fixed exchange rate leads to the loss of autonomic monetary policy and constant need to maintain

the exchange rate of the domestic currency on the officially set value. Also having the fixed exchange rate system may lead to the worse adaptability to ongoing economic processes, the difference between nominal and real value of the currency and for smaller countries a risk of speculative attack on the currency. (Levinson, 2005)

A special case of fixing a currency to Euro is European exchange rate mechanism II (ERM II). Countries involved in the ERM II must maintain their exchange rates in the fluctuation limits ± 15% from the central parity against Euro. At the same time there must be no devaluation of the central parity. Revaluation of the central parity is allowed. Staying in the ERM II at least for two years while fulfilling the convergence criteria mentioned above is a necessary condition for a country to accept Euro as a domestic currency. (Faure, 2013)

Basket of currencies

Domestic currency is tied to one or more currencies in the basket of currencies in the given exchange rate. The flexibility of these systems is restrained by the fixed limits of oscillation. The central bank intervenes in the foreign exchange market in case these limits are exceeded and moves the exchange rate of the domestic currency back inside the limits of oscillation. These limits can change in a time and good-timing for spreading or moving the limits of oscillation is crucial for the success of this type of fixed exchange rate.

Crawling pegs

The system of fixed exchange rate with periodical changes of the set fixed exchange rate. The fixed exchange rate has no limits of oscillation; it just changes in the pre-set periodical intervals, which are usually announced in advance. For the successful function of this fixed exchange rate system is crucial to optimally set the interval for the change of the fixed exchange rate. This system is suitable for countries with high level of inflation.

Currency boards

A currency board represents a fixed exchange rate system in which the exchange rate is absolutely fixed to other currency without any limits of oscillation. The central bank is giving up all its monetary tools to influence the domestic currency. To make this system stabilized, the central bank has to use unsterilized foreign exchange interventions in order to keep the balance of the monetary supply and demand. Extreme version of currency board is Dollarization/Euroization. The best example is the Eurozone, where 18 countries adopted euro as a common currency instead of their own domestic currencies. Their exchange rates are

effectively fixed to each other. The situation is similar with the countries which adopted the U.S. dollar as their domestic currency, such as Caribbean Netherlands, East Timor, Marshall Islands and others

2.3.2   Floating Exchange Rate Systems

Floating exchange rate systems overspread in the time, when the gold standard or Bretton Woods system ended and only paper money not covered by any precious metal became the most important part of the state currency. These systems are also called self-regulating because the value of the exchange rate changes according to the currency supply and demand on the foreign exchange market. Floating exchange rate systems are more resistant against major economic changes in other countries.

The main advantage of the floating exchange rate systems is that they allow the level of domestic prices and costs flexibly and effectively adapt to the current economic situation. In the long term it helps to create a balance between supply and demand of the currency and according to some economists it helps to stabilize the economic growth by using the market forces. Also the central banks don't have to hold as high foreign exchange reserves as in the fixed rate systems. Conversely the disadvantages connected to the floating exchange rate systems are higher inflation because of the influence of appreciation/depreciation on the domestic price level. Generally the disadvantages are also higher volatility and for the smaller countries the risk of speculations on the development of the currency.

Free floating

The main idea behind free floating exchange rate system is that the exchange rate is given only and solely by the currency supply and demand on the foreign exchange markets without any interventions of the central bank. This exchange rate system is not very common in nowadays world because of its high risk and the possibility of high volatility of the currency because of speculations.

Managed floating

This floating exchange rate system represents a compromise between the free floating and fixed exchange rate systems. In the managed floating exchange rate system is the rate given by the currency supply and demand on the foreign exchange market, but in case of high volatility, the central bank issues certain interventions in order to bring back the stability.

These interventions shouldn't be regular or frequent. The managed floating lowers the risk and insecurity in foreign exchange market about the currency by the guarantee of the central bank to act if the situation or the exchange rate of the domestic currency is concerned.

2.4 Volatility and its Characteristics

One of the most important factors in trading on the foreign exchange markets is a risk. A risk is an unobservable quantity, which leads to many problems with its measuring, modelling and predicting. The investors tends to focus on obtaining a high returns but they should also ask how high risk they can expect in exchange for these returns. Even though only risk in a general sense is taken into account, there are also formal expressions of the risk-reward relationship.

Volatility is generally used to measure risk. Volatility determines the rate of variability of observed variables, i.e. the amplitude and speed of changes. Technically volatility is annualized standard deviation of returns. The main difference between risk and volatility is that a risk is connected only with unfavorable events, while volatility gives the rate of variability in negative and also positive sense. The higher is the volatility the higher is uncertainty about the future progress of the underlying asset. We can distinguish two different variations of volatility, historical and implied. (Tsay, 2005)

Volatility as described above refers to actual current volatility. Typical measure of this volatility is standard deviation of the asset's price during given time period. Unfortunately in order to be an accurate risk measure, it has to fulfill certain assumptions such as normal distribution of measured data. Because this assumption is not always fulfilled, we can measure the volatility or basically risk simply by generating a histogram of returns. Following equation shows how to calculate standard deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2}, \qquad (2.3)$$

where σ represents a standard deviation and μ an arithmetic mean.

2.4.1 Historical and Implied Volatility

The first variation of volatility is **historical volatility**. It is calculated from the historical prices of given asset, mostly as a standard deviation of logarithmic daily returns, usually based on daily closing prices. Mathematically the historical volatility represents annualized standard deviation of returns. The main advantage of the historical volatility is the simplicity of calculation and clarity of data used for the calculation. Disadvantage is that the real future volatility can be different than the historical one calculated as following

$$\sigma_h = \sigma\sqrt{T}, \tag{2.4}$$

where σ_h represents a historical volatility and T the number of trading days in a measured period of time.

The second variation of volatility is **implied volatility**. The main difference between implied and historical volatility lies in the calculation. While historical volatility is using historical data, implied volatility is calculated more sophisticatedly by using evaluation models based on the current prices linked to the given asset, mostly options or similar derivatives. The other difference is the interpretation and a period of time in which the volatility is being calculated. Implied volatility is the volatility expected by the market in a given future period of time. This period of time is usually from present to the day of maturity of the derivative.

2.4.2 Volatility Properties

Volatility shows some specific features, which every volatility model should inherit and reflect. This topic is covered in the article by Engle and Patton, 2001. According to the article, volatility exhibits following properties.

1. Persistence – the changes in the prices tend to cluster together, which means that large volatility connected with large price changes often comes after other large changes and vice versa. The assumption of volatility clustering is that nowadays volatility influences the expectation of volatility in the future. We can see the clustering of volatility in the Figure 2.3 showing the time series of daily logarithmic returns time series of Cyprus pound to Euro.

2. Mean reverting – volatility changes through a time, so after the period of low volatility eventually comes a period of normal volatility and similarly after the period of high volatility will come a drop. This volatility feature basically means that volatility is trending.

Asymmetric impact of innovations – also known as the leverage effect. Positive and negative shocks don't have the same influence on volatility. When the prices are increasing, they tend to be less volatile in many models and in the most empirical estimates. Negative equity returns were found to be related to volatility, while this doesn't apply for exchange rates.

3. Influencing exogenous variables –the prices are developing under the influence of the market around them and also its volatility can be altered by certain deterministic events such as company reports, macroeconomic announcements or even time-of-day effects.

Figure 2.3 Clustering of volatility in the time series of daily logarithmic returns CYP/EUR
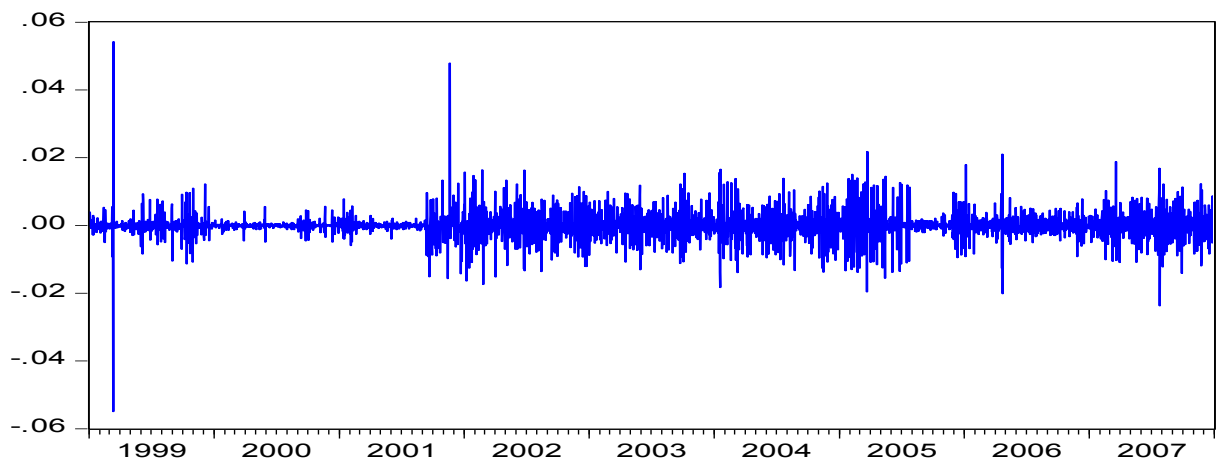
Source: Own calculations in EViews 7

3 BASIC APPROACHES OF MODELLING AND PREDICTION OF VOLATILITY

This thesis investigates the volatility of exchange rates of chosen currencies, so firstly this chapter will be dedicated to the basic characteristics of financial time series. Since the assumption of stationarity is necessary for financial time series analysis the most used Dickey-Fuller test is described. The second part of this chapter describes the models which purpose is to analyze, model and estimate the volatility by using historical samples of data and ways how to estimate, verify and predict these models.

3.1 Characteristics of Financial Time Series

Financial time series can be generally described as empirical observations in financial markets, which are structured in time from past to presence. We can distinguish the time series into two categories: long term time series which observed values are in yearly or longer intervals and short term time series which observed values are shorter than one year. A shape of the financial time series is connected with the length of the interval; the longer is the interval, the smoother is the time series. A time series is a special type of stochastic process. Basic attributes of the financial time series are trend, seasonality, nonlinearity, conditional heteroskedasticity and properties common for more time series like a common trend etc. These attributes usually don't approach in the time series jointly. The presence of any of these attributes depends on the type of the time series.

Financial time series are high-frequency economical time series, which belongs into short term time series, which frequency of observing is shorter than a year. Usually the short term time series data are yearly, quarterly or monthly, but especially the time series based on the exchange rates for foreign exchange trading are daily or even shorter such as 4-hourly or hourly. Observed feature of financial time series is the key information of financial markets, which is the price. Financial time series are based on the price or they describe its development. The amount of data allows us to use nonlinear models. Also these time series have higher and changing variability. The next features are long trend and cyclical component, while seasonal component is barely observable. (Wooldridge, 2009)

The Assumptions and Features of Financial Time Series

In this chapter are covered elementary assumptions and features required for creation of volatility and financial time series models. The assumptions are normality, linearity, stationarity and leptokurtic distribution. (Cipra, 2008)

The assumption of normality – the elementary assumption used in working with financial time series, which is saying that the logarithms of returns have normal distribution with constant mean value μ and constant variance σ_r^2, i.e. the assumption $r_t \sim N$ (μ, σ_r^2). This distribution is symmetrical and its skewness is equal to zero and its kurtosis is equal to number 3.

Skewness is calculated by following formulas

$$SE_r = E\left[\frac{(r_t - \mu)^3}{\sigma_r^3}\right],$$

(3.1)

while kurtosis is calculated by following formula

$$KU_r = E\left[\frac{(r_t - \mu)^4}{\sigma_r^4}\right].$$

(3.2)

In most of the cases the estimation of kurtosis is higher than 3, which means, that the real distribution of logarithms of returns is more pointed the normal distribution. It can be explained that lower positive and negative returns occur more often than the normal distribution assumed. Also financial time series tend to have higher incidence of extreme values.

The assumption of linearity – another important assumption for financial time series analysis is that the logarithms of returns are uncorrelated equally distributed random variables with zero mean value and constant variance, also called the white noise process, or independent equally distributed random variables with zero mean value and constant variance, also called the strict white noise process. Linear models can describe only one type of dependency, which is why we use also nonlinear models to analyze financial time series volatility.

The assumption of stationarity – any time series is stationary if its probability distribution is constant in time, which means, that the changes in the time series are not constant in time. One cannot differentiate one part of stationary time series from another according to the mean

value, variance or other statistical parameters. It is necessary for some kinds of analysis to have a stationary time series. Non-stationary time series have different statistical parameters in a various parts.

Leptokurtic distribution – this type of distribution is more pointed around the center and thicker on ends than the normal distribution. This is connected with higher probability of extreme changes. Compared to the normal distribution it brings higher risk of potential profits or losses for investors. In the Figure 3.1 we can see that leptokurtic distribution is focused more around the middle than the normal distribution.

Figure 3.1 Leptokurtic distribution
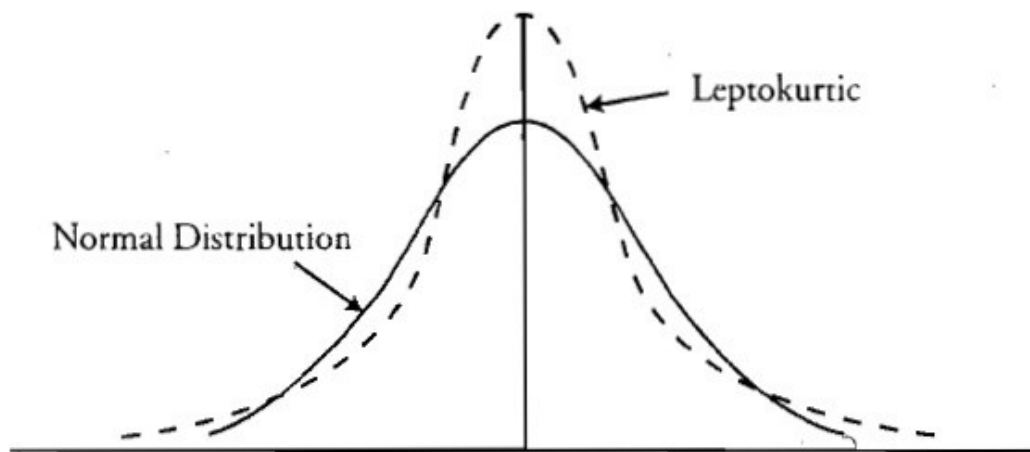
Source: Arlt and Arltová, 2003

3.2 Testing the Stationarity

The assumption of dynamic economic time series models is that they are constructed from observations of economic values that are stationary. In case that this ability is not fulfilled, the non-stationary time series of original observations are transformed by the first or higher differentiations and then called homogenous or integrated time series of first, second or higher order. The stationarity of time series is important for any econometric operations and its existence is required when building dynamic economic time series models. The tests used to analyze whether the used data are stationary are the unit root test, Dickey-Fuller (DF) test or adjusted Dickey-Fuller (ADF) test. (Brooks, 2008)

Most of the economic time series, mostly value indicators such as GDP, consumption, wages, investments, export, import etc. expressed in nominal prices is non-stationary. It is because these macro-data shows clear trend. In econometric analysis the trend is usually eliminated by one of the two following ways:

1. By including a variable time to the regression model as one of explanatory variables.
2. By replacing original data by its first or higher differentiations.

The model is trend stationary (TS) when the variable Y_t is stationary around the trend. It is indicated by residuals e_t being stationary or in other words showing no trend.

$$Y_t = \delta_1 + \delta_2 t + u_t, \qquad u_t = \alpha u_{t-1} + \varepsilon_t. \qquad (3.3)$$

Unlike in the TS model the random parts u_t in the model which is differential stationary (DS) are not stationary, because their variance σ^2 is increasing in time, so the mean value is not changing, but the variance shows trend.

$$Y_t = \delta + Y_{t-1} + u_t, \qquad resp. \ Y_t = \delta + u_t. \qquad (3.4)$$

To verify whether is the time series TS or DS we can use the following equation:

$$\Delta Y_t = \gamma + \delta t + (\alpha - 1)Y_{t-1} + u_t. \qquad (3.5)$$

If $\alpha < 1$ then the equation is equal to TS model. If $\alpha = 1$ then the equation is equal to DS model.

Dickey-Fuller test

Testing statistics for the DF test were derived according to the regression relationship which includes constant and trend.

$$\Delta Y_t = (\alpha - 1)Y_{t-1} + u_t, \qquad (3.6)$$

$$\Delta Y_t = \gamma + (\alpha - 1)Y_{t-1} + u_t. \qquad (3.7)$$

The equation (3.7) without the level constant is the least complicated, but it is not applied to economic time series very often. The regression is important especially when the variable Y_t has no trend. Dickey and Fuller derived three test statistics for each kind of regressions. Considering the assumptions that the characteristics of random parts u_t are white noise, they confirm the validity of the hypothesis $\alpha - 1 = 0$ or $\alpha = 1$. The first of them is analogy of standard t statistics: (Gujarati, 2003)

$$\tau = \frac{a-1}{s_a},\tag{3.8}$$

where a is the estimation of parameter α by ordinary least square method (OLS), s_a is estimated standard error of estimation a.

Test statistics based on regressions (3.6, 3.7, 3.8) are usually signed as τ_{cl}, τ_{nc}, τ_c. Their distribution even with large samples is different from t. This mean that they are not suitable for testing the significance of the point estimates OLS.

Augmented Dickey-Fuller test

It is impossible to use DF test in case that the dependent variable ΔY_t contains autocorrelation, because of the mistake of the first order. Thus was the previous model adjusted and extended to augmented DF test which can be expressed as

$$\Delta Y_t = (\alpha - 1)Y_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta Y_{t-1} + u_t\tag{3.9}$$

where $(\alpha - 1) = 0$ and the test statistic and critical values for each other way of calculation remains the same as before extension. (Wooldridge, 2009)

3.3 Univariate Linear Models

This chapter will explain a few concepts generally important for the creation of financial time series models before the description of the volatility models. Basically the autocorrelation function and partial autocorrelation are the most important for giving us the information about the character of stochastic process. For example the simple autoregressive model (AR), moving average (MA), which combined are extended into autoregressive moving average (ARMA). Technically, there are two sets of elementary constraints connected with time series for viability of inference and forecasting. To fulfill the first one, the time invariance of the first two moments of the marginal distribution is needed. The second one anticipates the same kind of temporal dependence across the sample and is focused on the dynamics. All the time series, that meets with both criteria mentioned above are called second-order stationary or simply stationary. (Gourieroux and Jasiak 2011)

Definition and dynamic properties of the AR process and its MA representation

The series $(y_t, t \in Z)$ follows an autoregressive process of order 1, denoted AR(1), if and only if it can be written as

$$Y_t = \rho Y_{t-1} + \varepsilon_t \tag{3.10}$$

where $(\varepsilon_t, t \in Z)$ is a weak white noise with variance $\varepsilon_t = \sigma^2$, and ρ is a real number of absolute value strictly less than 1. The coefficient ρ is called the autoregressive coefficient.

The dynamics of AR model are quite clear. The actual value of the series (y_t) composes of two elements. The first element expresses the effect of past events and is given by the history of the process. The history which we take into account is limited only to the last event y_{t-1} and the influence of this variable is affected by the autoregressive coefficient $|\rho| > 1$. The second element represents a random shock which is taking place at time t, called the white noise. It is called the innovation and is not observable. Considering the dynamics vice versa, the current value y_t is affected by the current and lagged shocks.

The autoregressive process of order 1 can be written as

$$Y_t = \varepsilon_t + \rho \varepsilon_{t-1} + \rho^2 \varepsilon_{t-2} + \cdots$$

$$= \sum_{h=0}^{\infty} \rho^h \varepsilon_{t-h}. \tag{3.11}$$

The formula used above represents the (infinite) moving average (MA(∞)) of the AR(1) process and ρ^h is the moving average coefficient of order h.

The formula of basic autoregressive model of order p is defined as

$$Y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t, \tag{3.12}$$

where $\phi_1, \dots, \phi_i$ are parameters, c represents a constant variable and ε_t is white noise. The values of the parameters have to fulfill certain conditions for the mode to be stationary.

One way to calculate a simple moving average is

$$SMA = \frac{Y_t + Y_{t-1} + Y_{t-2} + \dots + Y_{t-(k-1)}}{n}. \tag{3.13}$$

The term moving average is explained that every time the average is actualized by deleting one observation at the beginning and adding one in the end of the selected period it is updated. The MA method can be considered relevant only in case of sizeable randomness in the data series. Moving average model of the order q can be explained as

$$Y_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-1},$$
(3.14)

where $\theta_1, \dots, \theta_q$ are the parameters of the model, μ is the expected value of y_t and ε_t is the white noise.

The model which combines the condition of stationarity from the autoregressive part of the process and invertability from the moving average part of the process is called autoregressive moving average model (ARMA). Firstly described by P. Whittle and later improved by G. Box and G. Jenkins. (Hušek 2007) The formula of ARMA(p,q) model is

$$Y_t = c + \varepsilon_t + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-1}.$$
(3.15)

3.4 Linear Conditional Volatility Models

The analysis of volatility was firstly described by Robert F. Engle (1982). The model which he used to analyze the inflation of Great Britain was autoregressive conditional heteroskedasticity model (ARCH). This model and models derived are called linear, because conditional variance is a linear function of values $\varepsilon_{t-1}^2, \varepsilon_{t-2}^2, \dots, \varepsilon_{t-m}^2$.

Conditional heteroskedasticity is the attribute, which can be described as the series of logarithmic returns with normal distribution and variance, which changes in time. Unconditional distribution of logarithmic returns is a combination of normal distributions, where the ones with small conditional variance concentrate returns close to the mean value and vice versa the ones with large conditional variance move returns to the sides of the distribution. The result is unconditional pointy distribution with wide sides or in other words leptokurtic distribution described in chapter 3.1.1.

The nature of the alternative heteroskedasticity scheme which does not require any previous knowledge about the specific dependency if variance on other variable or variables is

to express the conditional variance of random variable u_t or residues e_t of the regression model. The difference between conditional and unconditional variance of the random variable is the same as between conditional and unconditional mean value. We can express the conditional variance u_t, marked as σ_t^2, as follows

$$
\begin{aligned}
\sigma_t^2 &= var(u_t|\,u_{t-1}, u_{t-2}, \ldots) \\
&= E[(u_t - E(u_t))^2|\,u_{t-1}, u_{t-2}, \ldots],
\end{aligned}
\tag{3.16}
$$

while we assume that $E(u_t) = 0$, which leads to

$$
\begin{aligned}
\sigma_t^2 &= var(u_t|\,u_{t-1}, u_{t-2}, \ldots) \\
&= E[{u_t}^2|\,u_{t-1}, u_{t-2}, \ldots].
\end{aligned}
\tag{3.17}
$$

The formula (3.17) basically says that the conditional variance of normally distributed random variable u_t is equal to conditional expected value ${u_t}^2$. (Brooks, 2008)

3.4.1 Autoregressive Conditional Heteroskedasticity Models (ARCH)

The elementary model of autoregressive conditional heteroskedasticity is used to model auto-correlated volatility in its simplest form is

$$
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2.
\tag{3.18}
$$

It is then easy to prove that conditional variance of the ARCH(1) process random variable is

$$
var(u_t|\,u_{t-1}) = E({u_t}^2|\,u_{t-1}) = \alpha_0 + \alpha_1 u_{t-1}^2 = \sigma_t^2.
\tag{3.19}
$$

The formula (3.18) is called ARCH(1) model because conditional variance of the random variable depends only on one lagged value. It is a partial model, which does not testify about the change of dependent variable Y_t in the linear regression model. A complete model can be expressed for instance as:

$$
Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t, \qquad u_t \sim NI(0,\ \sigma_t^2),
\tag{3.20}
$$

$$
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2.
\tag{3.21}
$$

Because $\sigma_t^2 \geq 0$ we have to conclude that $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$. Theoretically, following conclusions may occur:

1. $\alpha_0 \geq 0$: if $\alpha_1 = 0$, then conditional variance is equal to α_0, so the parameter α_0 has to be nonnegative, usually positive.

2. $\alpha_1 \geq 0$: if $\alpha_0 = 0$, then conditional variance grows with growing u_{t-1}^2. For $0 \leq \alpha_1 < 1$ the ARCH(1) process generating u_t is covariance stationary, while for $\alpha_1 = 0$ is σ_t^2 constant.

The model ARCH(1) implies that there is a high probability that also u_t has a high absolute value if during the period $t - 1$ any great shock occurs. Which means that for a high u_{t-1}^2 the conditional variance of following innovation u_t is also high.

The formula (3.18) does not imply that ARCH(1) process is non-stationary. It is just showing, that $u_t{}^2$ and u_{t-1}^2 are correlated. If we express the unconditional variance u_t and marked it as σ^2, then

$$\sigma^2 = E(u_t{}^2) = \alpha_0 + \alpha_1 E(u_{t-1}^2), \qquad (3.22)$$

while for $0 \leq \alpha_1 < 1$ the formula above has stationary solution, so the unconditional variance looks as

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1}, \qquad (3.23)$$

in other expression, σ^2 does not depend on time t (covariance stationarity). The process ARCH(1) is therefore homoscedastic. (Hušek, 2007)

A simple ARCH(1) model can be expressed also in an alternative form which is suitable for application of general autoregressive conditional heteroskedasticity (GARCH) process to simulation.

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t, \qquad (3.24)$$

$$u_t = v_t \sigma_t, \qquad v_t \sim NI(0, 1) \qquad (3.25)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2. \qquad (3.26)$$

A simple extension of ARCH(1) process is the is the ARCH process of q order. The model ARCH(q) can be expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2, \qquad (3.27)$$

so the conditional variance u_t depends on the q lagged values. The condition of non-negativity of conditional variance validates of relations $\alpha_0 \geq 0$ and $\alpha_1 \geq 0$ ($i = 1, 2, \ldots q$). The

effect of a shock created before j periods on a volatility in the current period is expressed by the coefficient α_j. Shocks older than q periods do not affect volatility in a current period.

For the conditional variance after the transition to expectations we get

$$E(\sigma_t^2) = E\big(\alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2\big),$$
$$= \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \cdots + \alpha_q \sigma_{t-q}^2, \tag{3.28}$$

where $E(u_{t-i}^2) = \sigma_{t-i}^2$. If we fulfill the conditions of covariance stationarity of ARCH(q) process, the long term conditional variances σ_{t-i}^2 are consistent and equal to unconditional variance σ^2. Because of this fact, we can claim

$$\sigma^2 = \alpha_0 + \alpha_1 \sigma^2 + \cdots + \alpha_q \sigma^2, \tag{3.29}$$

or constant and finite unconditional variance

$$\sigma^2 = \frac{\alpha_0}{1 - \sum_{i=1}^q \alpha_i}, \tag{3.30}$$

so also the process of ARCH(q) is homoscedastic.

The application of ARCH models in volatility analysis brings certain problems, from which we can mention:

1. The first problem is to determine the length of delay of squared residuals q. The length can be determined in many ways, for instance by the credibility test, but it does not necessarily has to be the best procedure.
2. The length of delay q can be large, which leads to a considerable amount of parameters in the final model (over-parametrization). Engle (1982, in Hušek 2007) proposed to solve this problem prior limitation of parameters by specifying the linearly decreasing coefficient α_i.
3. Breaching the conditions of non-negativity of all coefficients in the conditional variance equation. One or more coefficients can be estimated as negative.

The fact that regressive or autoregressive model contain random variables generated by ARCH process does not mean that it is impossible to estimate its parameters by ordinary least squares method (OLS). Unlike the common forms of heteroskedasticity, the application of OLS on the model which does not contain lagged variables of dependent variable in the set

of regressors gives the estimations with optimal abilities even for smaller samples. There are also other estimation procedures which are more efficient than OLS. (Tsay, 2005)

3.4.2 Generalized Autoregressive Conditional Heteroskedasticity Models (GARCH)

Autoregressive conditional heteroskedasticity models were generalized in many ways. One of the most used generalized ARCH models is generalized autoregressive conditional heteroskedasticity (GARCH), which were invented interdependently in the year 1986 by Bollerslev and Taylor. Unlike the ARCH model, the model GARCH extended to lagged values of conditional variance, so the simplest equation of GARCH model can be expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{3.31}$$

The formula (3.31) is model GARCH(1,1) which is widely used because it appropriately describes volatility clusters in data. Non-negativity conditional variance σ_t^2 requires non-negativity of all three parameters, specifically $\alpha_0 \geq 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$. The main advantage of GARCH model is that we can replace the model ARCH with infinite length of delay q and coefficients decreasing by geometric progression just with three parameters in equation of conditional variance. This fact is a great advantage especially in case of small samples.

If we define the process of white noise $\varepsilon_t = u_t^2 - \sigma_t^2$ or $\sigma_t^2 = u_t^2 - \varepsilon_t$, then by substituting this expression to the equation of conditional variance and with simple alternation we get

$$u_t^2 = \alpha_0 + (\alpha_1 + \beta_1)u_{t-1}^2 - \beta_1\varepsilon_{t-1} + \varepsilon_t, \tag{3.32}$$

which is ARMA(1,1) process for squared random variables of estimated model. AR is represented by lagged value u_t^2 and parts of the moving average are $u_t^2 - \sigma_t^2$ and its lagged value. The expression $u_t^2 - \sigma_t^2$ has zero average, unlikely u_{t-1}^2 in equation for GARCH(1,1) model which does not have the zero average. Random variable ε_t is uncorrelated in time, however it shows heteroskedasticity. The root of autoregressive part is $\alpha_1 + \beta_1$, so the condition of stationarity for unconditional variance u_t is validity of relation $\alpha_1 + \beta_1 > 1$. The values of the sum $\alpha_1 + \beta_1$ close to one means that the persistence of volatility is significant.

In case of unit root $\alpha_1 + \beta_1 = 1$, the volatility shocks have permanent effect and the process is called integrated GARCH(1,1) or IGARCH(1,1)

$$u_t^2 = \alpha_0 + u_{t-1}^2 - \beta_1 \varepsilon_{t-1} + \varepsilon_t. \tag{3.33}$$

It is the process integrated in variance, ARIMA(0,1,1). Non-stationary model GARCH in case of $\alpha_1 + \beta_1 > 1$ is not applicable, because for example conditional variance with increasing length of estimation does not converge but grows without limitation. (Gourieroux and Jasiak 2011)

In case of stationarity $E(u_{t-1}^2) = E(\sigma_t^2) = \sigma^2$ the unconditional variance u_t can be expressed as

$$\sigma^2 = \alpha_0 + \alpha_0 \sigma^2 + \beta_1 \sigma^2, \tag{3.34}$$

or

$$\sigma^2 = \frac{\alpha_0}{1 - (\alpha_1 + \beta_1)}. \tag{3.35}$$

Which proves that also in GARCH(1,1) process the unconditional variance is homoskedastic. Similarly to ARCH models, it is possible to avoid a considerable length of delay u_t^2 by including lagged values σ_t^2, because for example σ_{t-1}^2 is implicitly infinite delay of u_t^2.

Conditional variance for GARCH(p,q) model can be specified as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{3.36}$$

where q is the length of delay u_t^2 and p represents maximal length of delay σ_t^2. Non-negativity of conditional variance requires to fulfill the conditions $\alpha_0 \geq 0$, $\alpha_1 \geq 0$ for $i = 1, 2, ...,q$ and $\beta_j \geq 0$ for j = 1, 2, ..., p.

GARCH(p,q) model can be applied in praxis only for relatively low values of delay p and q. Similarly to GARCH(1,1) process also GARCH(p,q) process can be interpreted as ARMA process

$$u_t^2 = \alpha_0 + \sum_{i=1}^{m} (\alpha_i + \beta_i) u_{t-i}^2 - \sum_{j=1}^{p} \beta_j \varepsilon_{t-j} + \varepsilon_t, \tag{3.37}$$

where $\varepsilon_t = u_t^2 - \sigma_t^2$ and $m = \max(p,q)$, while $\alpha_i = 0$ for $i > q$ and $\beta_j = 0$ for $j > p$.

The expression (3.37) is ARMA(m,p) model for u_t^2, in which every i^{th} autoregressive coefficient is a sum of $\alpha_i + \beta_i$ and every j^{th} coefficient as a part of moving average is equal to β_j with negative mark.

Because of the condition of non-negativity the u_t^2 in GARCH(p,q) model is covariance stationary if the following equation is valid:

$$(\alpha_1 + \beta_2) + (\alpha_2 + \beta_2) + \cdots + (\alpha_m + \beta_m) < 1. \tag{3.38}$$

If the condition (3.38) is fulfilled, then conditional variance u_t is

$$\sigma^2 = \frac{\alpha_0}{1 - [(\alpha_1 + \beta_2) - (\alpha_2 + \beta_2) - \cdots - (\alpha_m + \beta_m)]}. \tag{3.39}$$

3.4.3 Modifications of Symmetric ARCH and GARCH Models

An important feature restricting to fully specify previously mentioned ARCH and GARCH models is their symmetry. The symmetry causes the volatility to react only to absolute size of shocks, not to their signs. This means that a negative shock has the same effect on future volatility as a positive shock. It is caused by the fact, that conditional variance in equation (3.36) is a function of the squared lagged values of residuals, so their signs do not make any difference. (Gourieroux and Jasiak 2011)

As said in subchapter 2.6.1., it is typical for the assets' returns that because of the leverage effects the negative shocks cause usually higher volatility growth than positive shocks of the same size. This asymmetric impact of innovations leads to invention of derived asymmetric models of conditional heteroskedasticity, in which the favorable and unfavorable events have different effect on the future development of volatility. This distinction between positive and negative effect is used especially for the stock market, less often for exchange rates. In the foreign exchange market the information favorable for one subject can be unfavorable for the second subject. An asymmetric model is used also in case, that unexpected drop in prices (unfavorable event) cause a larger effect in the development of volatility than unexpected increase in prices (favorable event) of the same size.

Nelson (1991, in Gaynor and Kickpatrick, 1994) invented one of the first models describing asymmetric effect of shocks on the development of volatility. The model is

exponential GARCH model, marked as EGARCH. This model can show different effect of negative and positive shocks, so it respects the asymmetry of the financial markets. It means, that it distinguishes the effect of unfavorable events (negative shocks) and favorable events (positive shocks) even when their size in absolute value is identical.

Natural logarithm of conditional variance EGARCH(1,1) model

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{u_{t-1}}{\sigma_{t-1}} + \gamma_1 \left(\left| \frac{u_{t-1}}{\sigma_{t-1}} \right| - \mu \right) + \beta_1 \ln(\sigma_{t-1}^2), \tag{3.40}$$

where $\alpha_1, \beta_1, \gamma_1$ are parameters and $\mu = E\left(\left| \frac{u_t}{\sigma_t} \right| \right) = \sqrt{\frac{2}{\pi}}$ for $u_t \sim NI(0,1)$.

The equation of conditional variance with logarithm ensures non-negativity of σ_t^2 also in case that some of the parameters are negative, so it is not needed to limit their values. We get the standardized random part (shock) by dividing random parts u_{t-1} by conditional standard error σ_{t-1}. The second part with the absolute value of standardized shocks is decreased by the mean value μ.

The general EGARCH(p,q) model in analogous form

$$\ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \frac{u_{t-1}}{\sigma_{t-1}} + \sum_{i=1}^{q} \gamma_i \left(\left| \frac{u_{t-1}}{\sigma_{t-1}} \right| - \mu \right) + \sum_{j=1}^{p} \beta_j \ln(\sigma_{t-j}^2). \tag{3.41}$$

Simple extension of GARCH model with one more part which allows us to take a possible asymmetry into account is the specification of conditional variance of random variable, for instance in GARCH(1,1) model

$$\sigma_t^2 = \alpha_0 + \alpha_i u_{t-i}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 I_{t-1} u_{t-1}^2, \tag{3.42}$$

where $I_{t-1} = 1$ for $u_{t-1} < 0$ (negative shock) and $I_{t-1} = 0$ in all other cases.

If the leverage effect exists, then $\gamma_1 > 0$. The conditions of non-negativity are secured by the values $\alpha_0 \geq 0, \alpha_1 \geq 0, \beta_1 \geq 0$ and $\alpha_1 + \gamma_1 \geq 0$. The variable I_{t-1} is included for negative shocks because they have higher effect on volatility than positive shocks.

Following model is called after its inventors Glosten, Jagannathan and Runkle (1993) as GJR-GARCH model.

General GJR-GARCH(p,q) model can be expressed as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, u_{t-i}^2 + \sum_{j=1}^{p} \beta_j\, \sigma_{t-j}^2 + \sum_{i=1}^{q} \gamma_i\, I_{t-1} u_{t-1}^2. \tag{3.43}$$

The model which is created by modifying GJR-GARCH model which is based on conditional standard error instead of conditional variance is TGARCH, invented by Zakoian (1994). Some of the less known asymmetric models are IEGARCH, FIEGARCH, STGARCH or LSTGARCH etc., described by Arlt and Arltová (2007), Tsay (2005).

3.5 The Forecast Construction Based on Volatility Models

Estimating the volatility model is important for their use in forecasting the volatility and it is one of the main goals of volatility model construction. These forecasts are used in many various financial operations, for example option evaluation, researching the relationship between the volatility of assets market and the business cycle etc. They are used also in construction of interval forecasts based on linear and nonlinear models.

Consider GARCH(p,q) model in a form

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2 + \beta_1\, \sigma_{t-1}^2 + \cdots + \beta_j\, \sigma_{t-j}^2, \tag{3.44}$$

for $t = T + \sigma^2$ we get

$$\sigma_{T+\sigma^2}^2 = \alpha_0 + \alpha_1 u_{T+\sigma^2-1}^2 + \cdots + \alpha_q u_{T+\sigma^2-q}^2 + \beta_1\, \sigma_{T+\sigma^2-1}^2 + \cdots$$
$$+ \beta_j\, \sigma_{T+\sigma^2-j}^2. \tag{3.45}$$

The forecast with minimal mean squared error (MSE) of the value $\sigma_{T+\sigma^2}^2$ can be expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_T^2(\sigma^2 - 1) + \cdots + \alpha_q u_T^2(\sigma^2 - q)$$
$$+ \beta_1\, \sigma_T^2(\sigma^2 - 1) + \cdots + \beta_j\, \sigma_T^2(\sigma^2 - j). \tag{3.46}$$

Assume the forecast construction based on GARCH(1,1) model, where the conditional variance is expressed as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1\, \sigma_{t-1}^2. \tag{3.47}$$

For $t = T + \sigma^2$ we can rewrite the equation as

$$\sigma_{T+\sigma^2}^2 = \alpha_0 + \alpha_1 u_{T+\sigma^2-1}^2 + \beta_1\, \sigma_{T+\sigma^2-1}^2. \tag{3.48}$$

Than the forecast of future conditional variance can be expressed as

$$\sigma_T^2(\sigma^2) = \alpha_0 \sum_{j=0}^{\sigma^2-1} (\alpha_1 + \beta_1)^j + (\alpha_1 + \beta_1)^{\sigma^2-1} \alpha_1 u_T^2 + (\alpha_1$$

$$+ \beta_1)^{\sigma^2-1} \beta_1 \sigma_T^2 \qquad (3.49)$$

$$= \alpha_0 \sum_{j=0}^{\sigma^2-2} (\alpha_1 + \beta_1)^j + (\alpha_1 + \beta_1)^{\sigma^2-1} \sigma_{T+1}^2.$$

In many practical applications in financial economy, the goal of analysis and calculations is the forecast of conditional variance itself instead of forecasting the future level of time series with focus on the MSE. It is important to determine the accuracy of such a forecast in these situations.

Assume the forecast of conditional variance based on the GARCH(1,1) model in the form (3.47). The forecast error can be expressed as

$$l_T(\sigma^2) = \alpha_1 \sum_{j=1}^{\sigma^2-1} (\alpha_1 + \beta_1)^{j-1} v_{T+h-j}. \qquad (3.50)$$

The conditional MSE corresponding to previous equation is:

$$MSE[\sigma_T^2(\sigma^2)|\Omega_T] = (K_e - 1)\alpha_1^2 \sum_{j=1}^{\sigma^2-1} (\alpha_1 + \beta_1)^{2(j-1)} E(\sigma_{T+h-j}^2|\Omega_T). \qquad (3.51)$$

To calculate the MSE one needs to know the conditional mean value of the future conditional fourth moment $E(\sigma_{T+h-j}^2|\Omega_T)$. The conditional MSE (3.51) can be applied also to the calculation of interval forecasts of conditional variance. However conditional stochastic distribution of forecast errors $l_T(s)$ is not normal, so the construction is problematic.

A certain tools are used to evaluate which model is the most optimal for forecasting. They are called loss functions. They have to be calculated individually for each model. The lower is the value of loss function the better is the model. We can distinguish three different functions for the purpose of this work. It is the root mean square error (RMSE), mean absolute error (MAE) and Theil inequality coefficient (Theil).

RMSE can be calculated by using following equation

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\varepsilon_t^2} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(\hat{\sigma}_t - \sigma_t)^2}. \qquad (3.52)$$

MAE can be calculated by using following equation

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\varepsilon_t| = \frac{1}{T}\sum_{t=1}^{T}|\hat{\sigma}_t - \sigma_t|. \qquad (3.53)$$

And Theil inequality coefficient can be calculated by using following equation

$$Theil = \frac{\sqrt{\sum_{t=T+1}^{T}(\hat{\sigma}_t - \sigma_t)^2}}{\sqrt{\sum_{t=T+1}^{T}\hat{\sigma}_t} + \sqrt{\sum_{t=T+1}^{T}\sigma_t^2}}. \qquad (3.54)$$

3.6 The Construction of Volatility Models

The process of linear and nonlinear volatility models construction can be summarized into following points:

1) Determine an appropriate linear or nonlinear model for the specific time series.
2) Test the hypothesis of conditional homoskedasticity against the alternative hypothesis of conditional heteroskedasticity of linear or nonlinear type.
3) Estimation of parameters of chosen linear or nonlinear model of conditional heteroskedasticity.
4) Verify the suitability of given model by diagnostic tests.
5) Modify the model, if necessary.
6) Use the model for descriptive or predictive purposes.

3.6.1   Testing the Conditional Heteroskedasticity in the Time Series

The test of conditional heteroskedasticity is based on the ARCH model formulation and follows the principles of Lagrange's multiplications (LM). Conditional variance σ_t^2 in the model ARCH (3.27) is constant, if the parameters comply the values $u_{t-1}^2, \ldots, u_{t-q}^2$ are equal to zero. The null hypothesis, i.e. the hypothesis of conditional homoskedasticity is $H_0 = \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$. The alternative hypothesis, i.e. the hypothesis of conditional

heteroskedasticityH_1 is that at least one parameter is different to zero. The test includes following steps:

1) Estimate the parameters of the linear or nonlinear model and obtain error values $\hat{u}_t$ and the residual sum of squares, also known as sum of squared errors of prediction SSE_0.

2) Construct a regression model

$$\hat{u}_t^2 = \alpha_0 + \alpha_1 \hat{u}_{t-1}^2 + \alpha_2 \hat{u}_{t-2}^2 + \ldots + \alpha_q \hat{u}_{t-q}^2 + \varepsilon_t, \tag{3.55}$$

and obtain the residual sum of squares SSE_1 and coefficient of determination R^2.

3) The test criteria LM in the form $T * R^2$ have asymptotically the distribution $\chi^2(q)$ if the null hypothesis is applied.

4) The F-version of the test criteria for small samples can be written as

$$F_{LM} = \frac{(SSE_0 - SSE_1)/q}{SSE_1/(T - q - 1)}, \tag{3.56}$$

and the distribution can be approximated to $F(q, T - q - 1)$ if the null hypothesis is applied.

This test is often called the ARCH test. The other interpretation speaks about this test as autocorrelation of unsystematic component. It was proved that it is identical with the test based on the formulation of GARCH(p,q) model.

3.6.2 Estimation of Parameters

A typical model of financial time series returns consists of two parts: linear or nonlinear model of the level of the time series and linear or nonlinear model of time series volatility. This model can be generally expressed as

$$X_t = G(X_t, \eta) + u_t, \tag{3.57}$$

where$X_t = (1, X_{t-1}, \ldots, X_{t-p})'$ and $G(X_t, \eta)$ is the core of the linear or nonlinear autoregressive model with parameters η and u_t is the process with zero conditional mean value and conditional variance σ_t^2 of GARCH model with parameters φ. The vector of parameters of the complete model (3.57) is $\theta = (\eta', \varphi')'$.

These parameters can be estimated by the maximum likelihood estimation (MLE). If e_t has standardized normal distribution, the logarithm of MLE function for the time series with T observations can be expressed as

$$L(\theta) = \sum_{t=1}^{T} l_t(\theta), \qquad (3.58)$$

where

$$l_t(\theta) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln \sigma_t^2 - \frac{u_t^2}{2\sigma_t^2}. \qquad (3.59)$$

The MLE $\hat{\theta}$ is obtained by maximizing the logarithm (3.59). This estimation is solved by the equation

$$\sum_{t=1}^{T} \frac{\vartheta l_t(\theta)}{\vartheta \theta} = 0, \qquad (3.60)$$

the relation $\frac{\vartheta l_t(\theta)}{\vartheta \theta}$ is often called the score function $s_t(\theta)$.

If the linear regression model in the role of regressors contains any lagged values of the variable Y, the OLS method keeps the properties of consistency. On the other hand if the random parts of linear regression model are generated by the ARCH process, then estimated standard errors are not consistent and it is unable to use them. It is caused by the fact that the ARCH process contains squares of random variables which are functions of lagged variables Y and thus the squares of random variables are correlated with the squares of lagged values of dependent variable. We can use the White standard error to achieve consistent estimation of covariance matrix, because they are robust against the heteroskedasticity of ARCH or GARCH type. We can get asymptotically more efficient estimations using the MLE instead of using the OLS estimation for estimation of regression model with ARCH or GARCH structured random parts.

The estimated parameters guarantee the most credible data generated while using the MLE to estimate the linear regression model. Considering the assumption of normality of conditional distribution $(Y_t|X_t = x_t)$ for the simple paired linear regression model with constant conditional variance we specify the natural logarithm of likelihood function L as

$$\ln L\left(\beta_1, \beta_2, \sigma_{Y|x}^2; Y_t | x_t\right) = \sum_{t=1}^{T} f\left(Y_t | x_t, \beta_1, \beta_2, \sigma_{Y|x}^2\right)$$

(3.61)

$$= -\frac{T}{2}(\ln 2\pi + \ln \sigma^2) - \frac{1}{2\sigma^2}\sum_{t=1}^{T}(Y_t - \beta_1 - \beta_2 x_t)^2.$$

Parameter $\sigma_{Y|x}^2$ was substituted by σ^2 for simplification. Since in ARCH model σ^2 is not constant, we have to use σ_t^2. The logarithm of likelihood function for GARCH model is

$$\ln L(ARCH) = -\frac{T}{2}(\ln 2\pi + \ln \sigma_t^2) - \frac{1}{2\sigma_t^2}\sum_{t=1}^{T}(Y_t - \beta_1 - \beta_2 x_t)^2$$

(3.62)

$$= -\frac{T}{2}(\ln 2\pi + \ln \sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{u_t^2}{\sigma_t^2},$$

where $\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2$,

$u_t = Y_t - \beta_1 - \beta_2 x_t$.

Logarithm of likelihood function L transfers its multiplicative form into additive function of observed data for a sample of T observations which are easier to estimate in order to maximize it. Estimation functions MLE are derived by differentiating ln L according to unknown parameters of linear regression model. In this case according to β_1, β_2 and ARCH(q) parameters $\alpha_0, \alpha_1, \dots, \alpha_q$.

Derivation of estimation function MLE for ARCH and GARCH models of higher orders is made by using matrix algebra (Bauwens, Hafner and Laurent 2012). The assessment of partial derivations of logarithmic likelihood function analytically to specify the GARCH type is available only for the simplest GARCH models, because resulting formulas are overcomplicated. Thus we use numerical procedures to maximize the function. Sophisticated software products use different iterative techniques. The process of estimating ARCH or GARCH model according to MLE usually contains of following steps:

1. Estimate the linear regression model according to OLS and use estimated parameters as initial values for MLE.
2. Sum up the residuals of OLS, choose initial values of parameters of conditional variance σ_t^2 and specify logarithmic likelihood function ln L, which maximum we want to find considering the assumption of normal distribution of random parts.

3. Using available software we maximize the ln L function. In other words by iterative techniques we generate the values of parameters for which ln L function is maximized and calculate their standard errors.

Iterative techniques of numerical optimization are based on initial values of all estimated parameters and they improve these values for every iteration until they reach the maximum, in this case it is the maximum of natural logarithm of L function. If the estimated function includes only one maximum, we can find it after more or less steps depending on used method and convergence criteria. The likelihood function when estimating GARCH model can include more local maximums, so different algorithms of numerical optimization can find different local maximums of ln L function.

The most used iterative optimization methods are Berndt-Hall-Hall-Hausman (BHHH) algorithm which is based only on numerically calculated first derivations of ln L function according to the values of parameters in every iteration and approximated values of second partial derivations. Let $\hat{\theta}^{(i)}$ be the estimation of parameter obtained in i iteration, then $\hat{\theta}^{(i+1)}$ is obtained as

$$\hat{\theta}^{(i+1)} = \hat{\theta}^{(i)} - \lambda \left(\sum_{t=1}^{T} \frac{\vartheta l_t(\hat{\theta}^{(i)})}{\vartheta \theta} \frac{\vartheta l_t(\hat{\theta}^{(i)})}{\vartheta \theta'} \right)^{-1} \sum_{t=1}^{T} s_t(\hat{\theta}^{(i)}), \qquad (3.63)$$

3.7 Diagnostic Tests

The estimations of parameters for linear or nonlinear volatility models have to fulfill certain conditions. Diagnostic tests are mostly focused on the unsystematic component and checks whether the conditions are applied. Volatility models work with an assumption that residuals are independent random variables with zero mean value and unit variance and in some models normed normal distribution. Usually the purpose of diagnostic tests is to test the unsystematic component. If the volatility model is estimated right then the standardized residuals should indicate following ability

$$e_t = u_t \sigma_t^{-1}. \qquad (3.64)$$

3.7.1 Variance of the Unsystematic Component

The simplest way to analyze whether the unsystematic component has a constant variance is the chart of residuals. However in praxis we use often the test to analyze whether the unsystematic component exhibits so called ARCH(q) effect. This test is based on the creation of artificial regression with added constant. The model can be expressed as

$$\hat{u}_t^2 = \alpha_0 + \alpha_1 \hat{u}_{t-1}^2 + \varepsilon_t. \tag{3.65}$$

The parameters are estimated by using OLS method. We will create a model based on the artificial regression

$$\hat{u}_t^2 = \alpha_0 + \alpha_1 \hat{u}_{t-1}^2 + \alpha_2 \hat{u}_{t-2}^2 + \cdots + \alpha_q \hat{u}_{t-q}^2 + \varepsilon_t, \tag{3.66}$$

which can be used to test whether the unsystematic component exhibits ARCH(q)effect. Under the assumption of zero hypotheses for conditional homoskedasticity$H_0 = \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$ of unsystematic component the statistic $T * R^2$ has distribution $\chi^2(q)$.

3.7.2 Autocorrelation of the Unsystematic Component

Autocorrelation of the unsystematic component can be analyzed by the selective autocorrelation function

$$\hat{\rho}_k = \frac{\sum_t \hat{u}_t \hat{u}_{t-k}}{\sum_t \hat{u}_t^2}. \tag{3.67}$$

In case the unsystematic component is not autocorrelated, the values of the function should lie within the interval $\pm 2\sqrt{T}$(95% confidence interval).

Another option to analyze whether the unsystematic component is not autocorrelated is to use portmanteau test. The null hypothesis $H_0: \rho_1 = \rho_2 = \cdots = \rho_K = 0$ is tested against the hypothesis H_1non H_0, where $\rho_k, k = 1, \dots, K$ are autocorrelated unsystematic components of the model for the lag k. The statistic of the well-constructed model is

$$Q = T \sum_{k=1}^{K} \hat{\rho}_k^2 \tag{3.68}$$

for large T and K with distribution approximately χ^2 with $(K - p - q)$ degrees of freedom. We can test the autocorrelation of unsystematic component, if we compare the values of test criteria (3.73)to the quantiles of distributionχ^2 $(K - p - q)$.

However it was proved that for the small samples the statistic (3.73) is not effective. Ljung and Box invented the statistic

$$Q' = T(T + 2) \sum_{k=1}^{K} (T - k)^{-1} \hat{\rho}_k^2, \tag{3.69}$$

which is called modified portmanteau statistic. Its values are also compared with the quantiles of distribution χ^2 $(K - p - q)$, when testing the autocorrelation. (Arlt, Arltová, 2003)

3.7.3   Normality of the Unsystematic Component

Normality of the unsystematic component is important assumption for the autocorrelation test, construction of point forecasts and for interpretation of the estimated parameters. Jarque-Bera test is often used for the test of normality. It is based on the idea of testing skewness and kurtosis simultaneously. The assumption is that the third moment of skewness in normal distribution is zero and fourth moment of kurtosis in normal distribution is three. The test criteria can be expressed as

$$JB = (SK^2 + KU^2), \tag{3.70}$$

where *SK* is the test criteria for skewness of the distribution

$$SK = (\frac{T}{6})^{\frac{1}{2}} (\frac{\hat{m}_3^2}{\hat{m}_2^3})^{\frac{1}{2}} \tag{3.71}$$

and *KU* is the test criteria for kurtosis of the distribution

$$KU = (\frac{T}{24})^{\frac{1}{2}} (\frac{\hat{m}_4}{\hat{m}_2^2} - 3) \tag{3.72}$$

while

$$\hat{m}_j = \frac{1}{T} \hat{u}_t^j, \qquad j = 2,3,4 . \tag{3.73}$$

Statistics *SK* and *KU* have asymptotically normed normal distribution N (0,1) under the assumption of null hypothesis, which is the normality of unsystematic component. *JB* statistic

has distribution χ^2 (2). Not only non-normality of unsystematic component but also the fact that the unsystematic component is heteroskedastic can lead to refusal of the null hypothesis.

3.8 Criteria for the Model Selection

There can be more than one acceptable estimated model while analyzing the time series. Following paragraph is dedicated to the ways of choosing the most optimal one. Few criteria were invented to solve this problem. These criteria compare the residuals of each estimated model with the summary statistics. They assume that the order of differentiation was chosen well. The criteria are Akaike AIC and BIC and Schwartz-bayes SBC. (Arlt, Arltová, 2003)

Akaike information criteria can be expressed as

$$AIC(M) = T \ln \hat{\sigma}_u^2 + 2M, \tag{3.74}$$

where $M = (p + q)$ is the number of parameters in ARMA(p,q) model and $\hat{\sigma}_u^2$ is the residual variance of this model. The model which leads to minimum value of this criterion is chosen. This criterion was extended because AIC can lead to overestimation of autoregression order. The Schwartz-bayes criteria can be expressed as

$$SBC(M) = T \ln \hat{\sigma}_u^2 + M \ln T, \tag{3.75}$$

where $M = (p + q)$ is the number of parameters, $\hat{\sigma}_u^2$ is the residual variance of the model and T is the number of observations. T is equal to the number of residuals obtained from the model. Again the model which leads to minimum value of this criterion is chosen.

4 ESTIMATION OF VOLATILITY MODELS AND THEIR PREDICTION

The previous chapters of the thesis were focused on the theoretical information. Firstly about the foreign exchange market and the system of how the trading with the currencies works and what is the volatility. The next chapter gave us closer information about volatility assumptions and properties. The chapter also set up a theoretical background of volatility modelling and estimation which will be applied on the real financial time series in this chapter.

4.1 Data Samples Characteristics

Data samples used for the purpose of this thesis are financial time series of exchange rates SIT/EUR Slovenian tolar to Euro for Slovenia, CYP/EUR Cyprus pound to Euro for Cyprus, SKK/EUR Slovakian koruna to Euro for Slovakia and LVL/EUR Latvian Lat to Euro for Latvia. The purpose of the thesis is to estimate the conditional heteroskedasticity models and predict the volatility of the observed exchange rates. Data samples mentioned above are the data source for this thesis. All data used for calculations in this chapter were downloaded from the financial webpage focused on electronic foreign exchange trading Oanda.com (http://www.oanda.com/currency/historical-rates/). Observed interval is different for each exchange rate and the observed period is always from 1/01/1999 as a year when Euro began to exist in the electronic form till the year when the observed country accepted Euro as their domestic currency or in other words till the country became part of the European monetary union (EMU). Specifically Slovenia became part of EMU 1/01/2007, Cyprus 1/01/2008, Slovakia 1/01/2009 and Latvia with a few years distance 1/01/2014.

To make the analysis of the data more extensive and the prediction more precise the observed data will be divided into three different time periods. The first period (Period 1) begins 1/01/1999 and finishes before the date when all of the observed countries entered the European Union (EU) which was 1/05/2004 and because the observed exchange rates are from the countries representing central, south, east/central and Baltic states the expected volatility may vary. The second period (Period 2) begins 1/05/2004 and lasts differently for every country. The end of the second period is exactly two years before each country entered

EMU. For Slovenia it is just the rest of the year 2004, for Cyprus it is period from 1/05/2004 to 31/12/2005, for Slovakia from 1/05/2004 to 31/12/2006 and for Latvia from 1/05/2004 to 31/12/2011. The third period (Period 3) represents the two years before each country joined EMU, because one of the conditions to join EMU is to participate at least two years in ERM II in which the candidate currencies demonstrate economic convergence by maintaining limited deviation from their target rate against Euro. For Slovenia it is the years 2005 and 2006, for Cyprus 2006 and 2007, for Slovakia 2007 and 2008 and for Latvia 2012 and 2013. The third period is expected to be less volatile than the previous observed periods.

Statistical software EViews is used to edit the large amount of data, show following Figures or other visual output and for most of the calculations and tests in the following chapters. The charts 4.1, 4.2, 4.3 and 4.4 are showing the development of observed exchange rates. From the Figure 4.1 we can see that SIT was steadily decreasing its value against EUR practically during the whole observed period. In the period of last two years we can see almost flat development SIT joined the ERM II described closer in subchapter 2.3.1. Unlike SIT/EUR, SKK/EUR in the Figure 4.3 was strengthening its value from almost 43 SKK/EUR to 30 SKK/EUR and participation in ERM II is visible only from the second half of the year 2008. The Figure 4.2 shows the development of CYP/EUR which exchange rate did not show any strong trend, just a relatively large short swing in the end of 2001. The Figure 4.4 LVL/EUR shows that Latvia held the exchange rate close to 0.7LAT/EUR shortly after becoming a member of EU in 1/05/2004. All following Figures and Tables are own calculations and output via EViews 7 and Microsoft Excel.

Figure 4.1 SIT/EUR exchange rate development

42
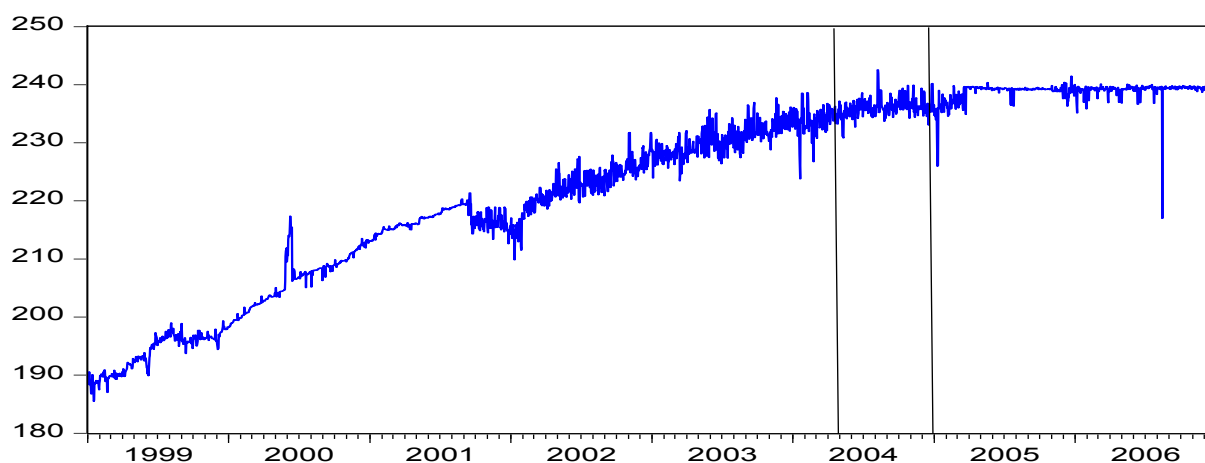
Figure 4.2 CYP/EUR exchange rate development
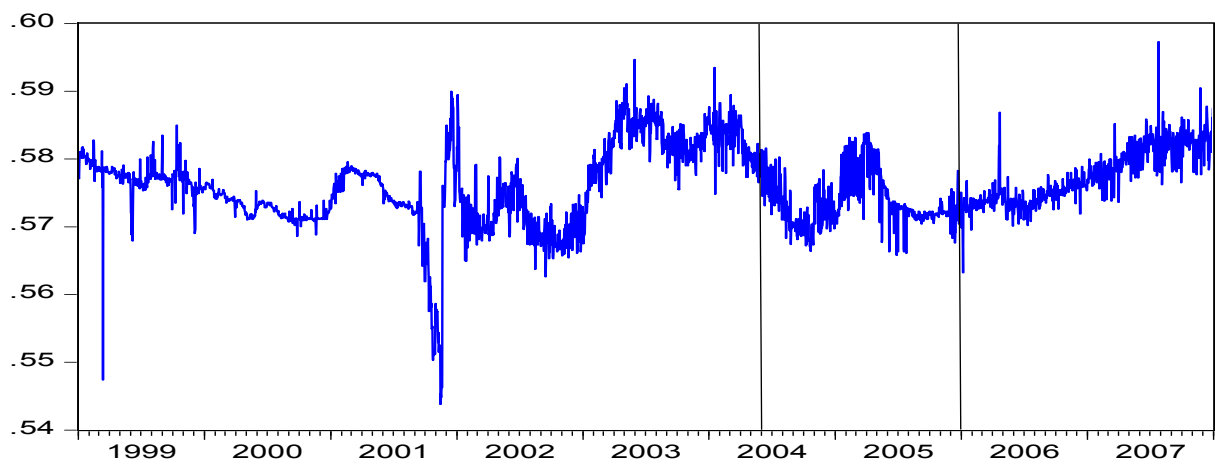
Figure 4.3 SKK/EUR exchange rate development
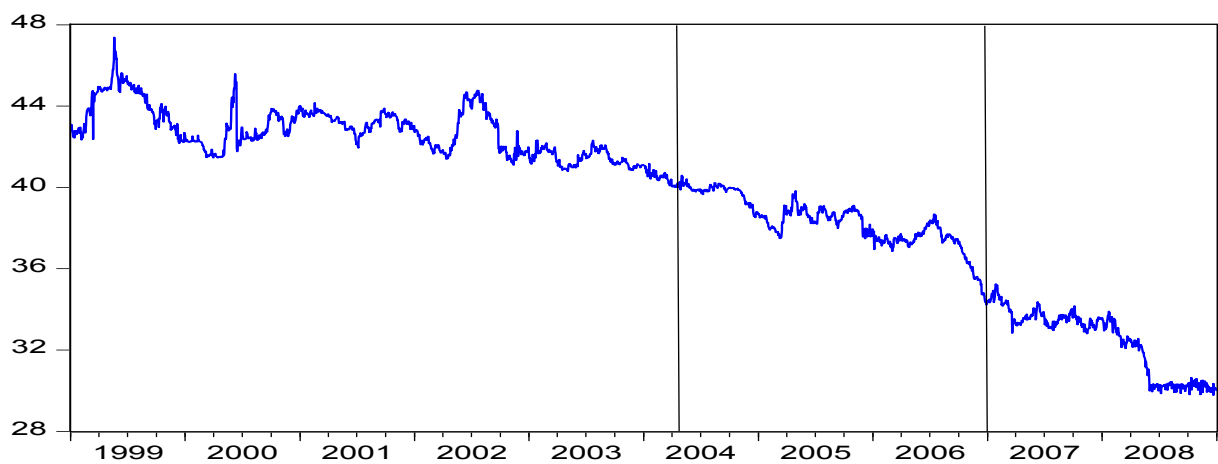
Figure 4.4 LAT/EUR exchange rate development
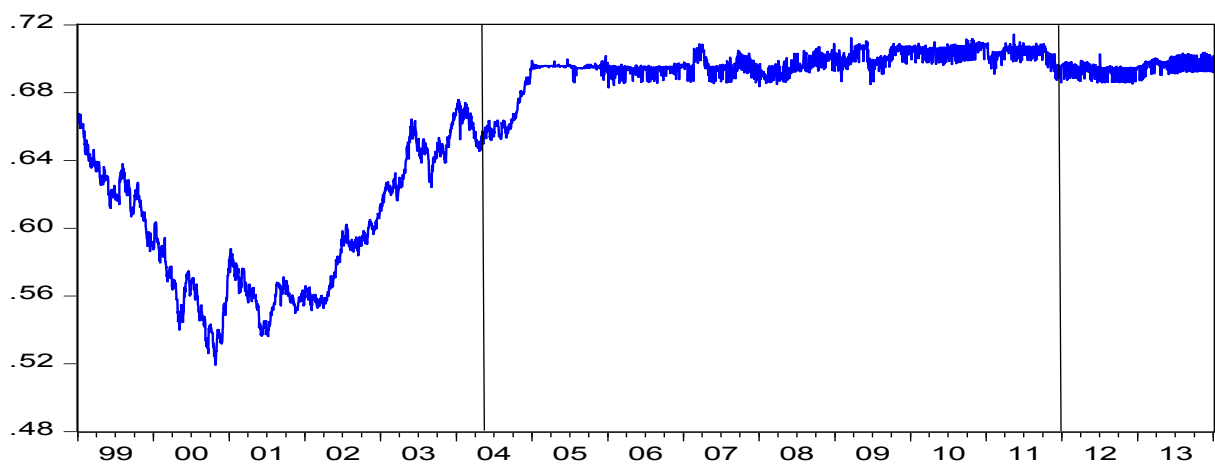
4.2 Logarithmic Returns

Figures 4.1, 4.2, 4.3 and 4.4 showing the development of the observed exchange rates are non-stationary. We need to work with stationary data to be able to meet with the focus of this thesis. Elementary way of changing the data from non-stationary financial time series to stationary data is to create a time series of daily returns and continue using the time series of daily returns in all following calculations.

Generally P_t represents the price of an assetat the time t, in our case the exchange rate, or in the other words the price we need to pay in domestic currency to receive one unit of foreign currency. Then holding the asset from the time $t - 1$brings the investor a brutto return defined by the relation

$$(1 + R_t) = \frac{P_t}{P_{t-1}}. \tag{4.1}$$

Netto return can be derived as

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}. \tag{4.2}$$

This time series of daily returns does not confirm the presence of normal distribution. The following inference into logarithmic returns confirms stationarity of time series of daily returns

$$r_t = \ln(1 + R_t) = \ln\frac{P_t}{P_{t-1}} = \ln(P_t) - \ln(P_{t-1}), \tag{4.3}$$

wherer_t represents the absolute increment of logarithmic price and is called a logarithmic return.

If we apply the equation (4.3) on the observed exchange rates we will receive the time series of daily logarithmic returns. The Figure 4.5 shows the time series of daily logarithmic returns SIT/EUR. Only SIT/EUR graph is shown because of the similarity of each currency's time series of daily logarithmic returns.

Figure 4.5 Time series of daily logarithmic returns SIT/EUR
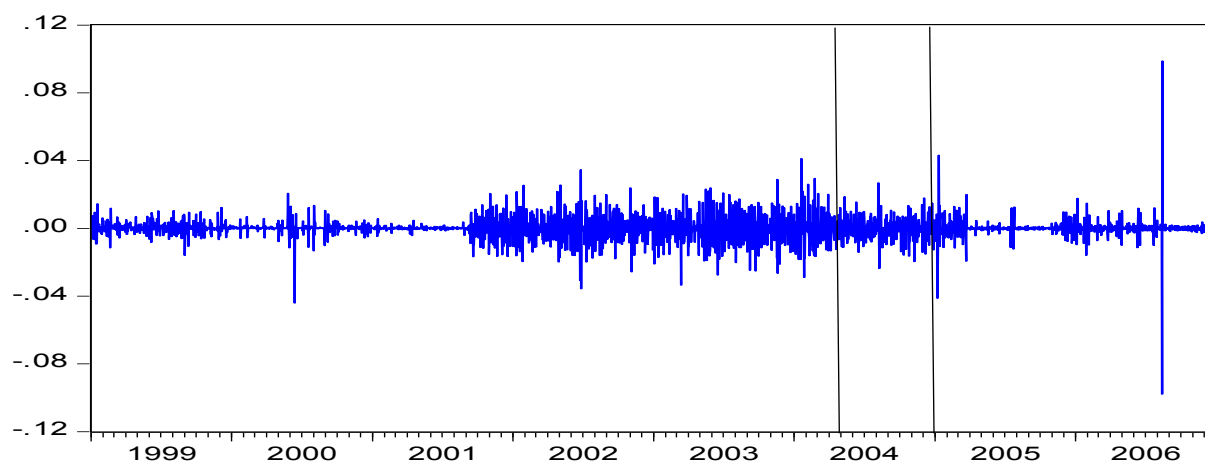
4.3 Normality, Stationarity and Heteroskedasticity Tests

The normality of the time series of daily logarithmic returns of each exchange rate is tested by Jarque-Bera test described in the paragraph 3.7.3. The Table 4.1 is showing us the results. We are testing the null hypothesis which is the normality of unsystematic component and we can conclude that the null hypothesis is refused in all observations.

Table 4.1 Jarque-Bera normality test

	JB statistic	probability
SIT/EUR1	3100.372	0.000000
SIT/EUR2	39.61206	0.000000
SIT/EUR3	657982.5	0.000000
CYP/EUR1	159497.6	0.000000
CYP/EUR2	244.8873	0.000000
CYP/EUR3	1060.049	0.000000
SKK/EUR1	839506.9	0.000000
SKK/EUR2	1242.212	0.000000
SKK/EUR3	739.218	0.000000
LVL/EUR1	307.4136	0.000000
LVL/EUR2	1069.394	0.000000
LVL/EUR3	60.14207	0.000000

Following Tables 4.2, 4.3, 4.4 and 4.5 show the descriptive statistics of each exchange rate calculated for each observed period individually. Descriptive statistics include mean value, median, maximum and minimum value of the tested sample, standard deviation, skewness, kurtosis and number of observations in each period.

Table 4.2 Descriptive statistics for daily logarithmic returns SIT/EUR

	Period 1	Period 2	Period 3
Mean	0.000118362	0.000009107	0.000028470
Median	0.000064426	0.000283684	-0.000033421
Maximum	0.041141532	0.026851233	0.098720858
Minimum	-0.044083280	-0.023653806	-0.097797715
Std. Dev.	0.006415638	0.006483344	0.006489816
Skewness	-0.114711930	0.020372279	0.232285112
Kurtosis	9.179338191	4.973476172	150.078500500
Observations	1946	244	730

Table 4.3 Descriptive statistics for daily logarithmic returns CYP/EUR

	Period 1	Period 2	Period 3
Mean	0.000003283	-0.000016801	0.000036766
Median	0	0	0
Maximum	0.054221047	0.021822827	0.021006282
Minimum	-0.054912969	-0.019593171	-0.023717866
Std. Dev.	0.004076493	0.00467569	0.003911119
Skewness	0.795548478	0.068247651	-0.024613827
Kurtosis	47.32324742	6.103562782	8.903261357
Observations	1946	609	730

Table 4.4 Descriptive statistics for daily logarithmic returns SKK/EUR

	Period 1	Period 2	Period 3
Mean	-0.000029675	-0.000170006	-0.000182295
Median	-0.000020152	-0.000145215	0.000051359
Maximum	0.053324708	0.016853623	0.020009606
Minimum	-0.07433135	-0.018119947	-0.03174946
Std. Dev.	0.003974201	0.002782073	0.004654213
Skewness	-3.178665481	-0.114042808	-0.676193738
Kurtosis	104.5540022	8.527829034	7.737178296
Observations	1946	974	730

Table 4.5 Descriptive statistics for daily logarithmic returns LVL/EUR

	Period 1	Period 2	Period 3
Mean	-0.000006489	0.000022505	0.000011366
Median	0	0	0
Maximum	0.02238599	0.019429183	0.015076745
Minimum	-0.018229276	-0.021850209	-0.01589722
Std. Dev.	0.004283147	0.004724599	0.005412784
Skewness	0.073633325	-0.077754379	0.370917582
Kurtosis	4.941556636	6.023040655	4.194551896
Observations	1946	2801	730

The observed data and shown descriptive statistics are not showing any specific common features. This was supposed to happen due to different position, economic performance and a year of joining EMU. We can conclude that the monetary measures before joining EU in 2004 had to be similar in all observed countries or not affecting the exchange rate of domestic currency versus Euro differently for each country. Interesting fact is that the standard deviation was expected to be the lowest in the Period 3 and showed up to be very similar or in cases of SKK/EUR and LVL/EUR even higher than in previous periods. The skewness for normal distribution is equal to zero and kurtosis to three. These assumptions are far from being fulfilled. Kurtosis is always higher than three which is typical for financial time series with leptokurtic instead of normal distribution. In few cases even higher than one hundred which is extremely high and means that almost all the observed data are very close to the middle.

For further use of regression analysis is required that the data are stationary. We will use ADF test described in the chapter 3.2 to check whether the data are stationary. The test is performed for each of the observed exchange rates and periods separately.

As we can see in the Figure 4.7, all ADF test statistics are statistically significant for all observed exchange rates in all observed periods. Critical values on all significance levels are higher than the ADF test statistics. We can refuse the null hypothesis about non-stationarity of the data and conclude that all observed time series of daily logarithmic returns are stationary.

Table 4.6 Augmented Dickey-Fuller test

	ADF test statistic	Probability	Critical values on certain significance level		
			1%	5%	10%
SIT/EUR1	-12.23588	0.000000	-3,43624	-2,86403	-2,56814
SIT/EUR2	-6.121721	0.000000			
SIT/EUR3	-10.04892	0.000000			
CYP/EUR1	-8.984962	0.000000			
CYP/EUR2	-7.744504	0.000000			
CYP/EUR3	-10.98672	0.000000			
SKK/EUR1	-12.41758	0.000000			
SKK/EUR2	-7.999103	0.000000			
SKK/EUR3	-6.010794	0.000000			
LVL/EUR1	-28.20318	0.000000			
LVL/EUR2	-11.8904	0.000000			
LVL/EUR3	-9.868332	0.000000			

Another of the conditions for using the conditional heteroskedasticity models is that the time series should not have constant variance or in other words do not be homoskedastic. The test used to verify the presence of heteroskedasticity in the residuals is ARCH test described in the subchapter 3.6.1. The dependent variable is represented by the squared residuals and independent variable is a constant and lagged squared residuals. In the Table 4.7 we can see, that the residuals are tested up to three day lag.

Bold marked numbers lead to refusal of the null hypothesis on the 5% significance level and thus refusal of homoskedasticity. The Table 4.7 shows that in every observed time series of daily logarithmic returns is at least one lag which leads to confirmation of heteroskedasticity. That means that the chosen time series are appropriate for modelling the conditional variance.

Table 4.7 ARCH test of conditional heteroskedasticity

	RESID^2(-1)	Prob.	RESID^2(-2)	Prob.	RESID^2(-3)	Prob.
SIT/EUR1	**8.0532**	**0.0000**	**1.9937**	**0.0463**	**3.0819**	**0.0021**
SIT/EUR2	-1.4331	0.1532	-1.3108	0.1912	**3.9600**	**0.0001**
SIT/EUR3	**20.0603**	**0.0000**	-12.0062	**0.0000**	**8.1019**	**0.0000**
CYP/EUR1	**19.5111**	**0.0000**	-7.5342	**0.0000**	**3.9059**	**0.0001**
CYP/EUR2	3.1416	0.0018	3.1006	**0.0020**	-0.8305	0.4066
CYP/EUR3	**10.1630**	**0.0000**	1.3597	0.1744	-1.6215	0.1054
SKK/EUR1	**7.9565**	**0.0000**	-1.3875	0.1655	1.1943	0.2325
SKK/EUR2	**6.0358**	**0.0000**	**5.2543**	**0.0000**	-1.7201	0.0857
SKK/EUR3	**6.6756**	**0.0000**	-1.1336	0.2573	-0.7571	0.4493
LVL/EUR1	**6.1403**	**0.0000**	-0.3496	0.7267	0.1817	0.8558
LVL/EUR2	**6.8518**	**0.0000**	**3.7528**	**0.0002**	**-5.5092**	**0.0000**
LVL/EUR3	**4.3848**	**0.0000**	**-2.4572**	**0.0142**	**-5.1990**	**0.0000**

4.4 Estimation of Volatility Models

This chapter is dedicated to estimation of volatility models with the best possible features. Both linear and nonlinear models are tested. The most optimal model is chosen individually for each of the observed exchange rates and for each of the observed periods. To optimize the model estimations we use the BHHH algorithm described in the subchapter 3.6.2. The best model is always determined by the statistical significance of AIC and SBC criteria described in the chapter 3.8. Every model is estimated using the generalized error distribution (GED)

which showed the best results for most of the models. Other possible distributions are for instance normal (Gaussian) distribution or student's distribution.

Estimation of volatility models of SIT/EUR

This subchapter is showing the results of the best possible model estimations of linear or nonlinear volatility models for all three observed periods of SIT/EUR.

Period 1

The best estimated model for the Period 1 of the SIT/EUR exchange rates GARCH(4,1) model with added independent variables of a constant value and one day lagged time series of daily logarithmic returns of Period 1. The result of estimation is shown in the Table 4.8.

Table 4.8 The best estimated model for SIT/EUR in Period 1

Variable	Coefficient	Std. Error	z-statistics	Prob.
RESID(-1)^2	0.5766	0.1374	4.1970	0,0000
RESID(-2)^2	-0.5017	0.1314	-3.8176	0,0001
RESID(-3)^2	0.2798	0.0878	3.1870	0,0014
RESID(-4)^2	-0.2481	0.0800	-3.1005	0,0019
GARCH(-1)	0.9321	0.0123	75.8255	0,0000
Log likelihood		8250.3210		
Durbin-Watson stat.		2.4391		
AIC		-8.4744		
SBC		-8.4486		

Period 2

The best estimated model for the Period 2 of the SIT/EUR exchange rate is nonlinear model EGARCH(1,3). The result of estimation is shown in Table 4.9. The constant value C(1) is statistically insignificant and thus can be left out of the model. Negative value of the variable C(2) indicates that this model has leverage effect and so reflect positive and negative changes differently.

Table 4.9 The best estimated model for SIT/EUR in Period 2

Variable	Coefficient	Std. Error	z-statistics	Prob.
C(1)	-2.9525	1.6216	-1.8207	0,0686
C(2)	-0.5263	0.1783	-2.9525	0,0032
C(3)	0.5212	0.1016	5.1320	0,0000
C(4)	-0.5999	0.0853	-7.0345	0,0000
C(5)	0.7523	0.1195	6.2966	0,0000
Log likelihood	916.8891			
Durbin-Watson stat.	2.3232			
AIC	-7.4663			
SBC	-7.3803			

Period 3

The best estimated model for the Period 3 of the SIT/EUR exchange rate is nonlinear model GJR-GARCH(1,1)with one threshold order and with added independent variable of one day lagged time series of daily logarithmic returns from the Period 3. The result of estimation is shown in the Table 4.10. The constant value is statistically significant.

Table 4.10 The best estimated model for SIT/EUR in Period 3

Variable	Coefficient	Std. Error	z-statistics	Prob.
C	6.51E-07	1.64E-07	3.9605	0,0001
RESID(-1)^2	0.6712	0.2810	2.3886	0,0169
RESID(-1)^2*I_{t-1}	0.9588	0.4457	2.1512	0,0315
GARCH(-1)	0.4172	0.0448	9.3092	0,0000
Log likelihood	3510.9210			
Durbin-Watson stat.	2.3445			
AIC	-9.6157			
SBC	-9.5779			

Estimation of volatility models of CYP/EUR

This subchapter is showing the results of the best possible model estimations of linear or nonlinear volatility models for all three observed periods of CYP/EUR.

Period 1

The best estimated model for the Period 1 of the CYP/EUR exchange rate is linear model GARCH(2,2) with added independent variable of one day lagged time series of daily

logarithmic returns from the Period 1. The constant value is statistically significant. The result of estimation is shown in the Table 4.11.

Table 4.11 The best estimated model for CYP/EUR in Period 1

Variable	Coefficient	Std. Error	z-statistics	Prob.
C	1.67E-07	4.48E-08	3.7359	0,0002
RESID(-1)^2	0.3251	0.0418	7.7735	0,0000
RESID(-2)^2	0.3187	0.0427	7.4547	0,0000
GARCH(-1)	-0.2094	0.0490	-4.2736	0,0000
GARCH(-2)	0.7431	0.0387	19.1976	0,0000
Log likelihood		9188.3130		
Durbin-Watson stat.		2.3171		
AIC		-9.4409		
SBC		-9.4209		

Period 2

The best estimated model for the Period 2 of the CYP/EUR exchange rate is nonlinear model GARCH(1,1) with added independent variables of time series of daily logarithmic returns of Period 2 lagged up to three days. The result of estimation is shown in the Table 4.12. The constant value is statistically insignificant.

Table 4.12 The best estimated model for CYP/EUR in Period 2

Variable	Coefficient	Std. Error	z-statistics	Prob.
RESID(-1)^2	0.1809	0.0539	3.3558	0.0008
GARCH(-1)	0.8512	0.0275	31.0020	0.0000
Log likelihood		2595.5540		
Durbin-Watson stat		2.4084		
AIC		-8.5431		
SBC		-8.4922		

Period 3

The best estimated model for the Period 3 of the CYP/EUR exchange rate is linear model GARCH(1,1) with added independent variables of time series of daily logarithmic returns of Period 3 lagged up to three days similarly to previous period. The result of estimation is shown in the Table 4.13.The constant value is statistically significant.

Table 4.13 The best estimated model for CYP/EUR in Period 3

Variable	Coefficient	Std. Error	z-statistics	Prob.
C	8.46E-07	2.60E-07	3.2542	0,0011
RESID(-1)^2	0.2073	0.0454	4.5636	0,0000
GARCH(-1)	0.7276	0.0479	15.1920	0,0000
Log likelihood		3199.7620		
Durbin-Watson stat.		2.1267		
AIC		-8.7834		
SBC		-8.7392		

Estimation of volatility models of SKK/EUR

This subchapter is showing the results of the best possible model estimations of linear or nonlinear volatility models for all three observed periods of SKK/EUR.

Period 1

The best estimated model for the Period 1 of the SKK/EUR exchange rate is nonlinear model GJR-EGARCH(1,2).The result of estimation is shown in the Table 4.14. Positive value of the variable C(2) indicates that this model does not have a leverage effect and so do not reflect positive and negative changes differently. All the estimated parameters are statistically significant.

Table 4.14 The best estimated model for SKK/EUR in Period 1

Variable	Coefficient	Std. Error	z-statistics	Prob.
C(1)	-1.8376	0.3040	-6.0449	0,0000
C(2)	0.4615	0.0526	8.7666	0,0000
C(3)	0.1156	0.0389	2.9689	0,0030
C(4)	0.3706	0.0987	3.7555	0,0002
C(5)	0.4971	0.0942	5.2793	0,0000
Log likelihood		8914.6450		
Durbin-Watson stat.		2.2557		
AIC		-9.1559		
SBC		-9.1387		

Period 2

The best estimated model for the Period 2 of the SKK/EUR exchange rate is linear model GARCH(2,1) with added independent variables of a constant value and one day lagged

time series of daily logarithmic returns of Period 2. The result of estimation is shown in the Table 4.15.

Table 4.15 The best estimated model for SKK/EUR in Period 2

Variable	Coefficient	Std. Error	z-statistics	Prob.
RESID(-1)^2	0.3644	0.0808	4.5125	0,0000
RESID(-2)^2	-0.3015	0.0773	-3.8985	0,0001
GARCH(-1)	0.9364	0.0178	52.5941	0,0000
Log likelihood		4500.2440		
Durbin-Watson stat.		1.9298		
AIC		-9.2359		
SBC		-9.2007		

Period 3

The best estimated model for the Period 3 of the SKK/EUR exchange rate is linear model GARCH(1,0) or in other words ARCH(1) model with added independent variables of time series of daily logarithmic returns of Period 3 lagged up to three days. The result of estimation is shown in the Table 4.16. The constant value is statistically significant.

Table 4.16 The best estimated model for SKK/EUR in Period 3

Variable	Coefficient	Std. Error	z-statistics	Prob.
C	1.20E-05	1.14E-06	10.5609	0,0000
RESID(-1)^2	0.4736	0.1266	3.7425	0,0002
Log likelihood		2985.8610		
Durbin-Watson stat.		2.2189		
AIC		-8.1864		
SBC		-8.1486		

Estimation of volatility models of LVL/EUR

This subchapter is showing the results of the best possible model estimations of linear or nonlinear volatility models for all three observed periods of LVL/EUR.

Period 1

The best estimated model for the Period 1 of the LVL/EUR exchange rate is linear model GARCH(2,1) with added independent variable of time series of daily logarithmic returns of Period 1 lagged up to one days. The result of estimation is shown in the Table 4.17. The constant value is statistically insignificant and thus can be left out of the model.

Table 4.17 The best estimated model for LVL/EUR in Period 1

Variable	Coefficient	Std. Error	z-statistics	Prob.
RESID(-1)^2	0.1126	0.0274	4.1085	0,0000
RESID(-2)^2	-0.1091	0.0273	-3.9963	0,0001
GARCH(-1)	0.9945	0.0027	365.0334	0,0000
Log likelihood		7981.5120		
Durbin-Watson stat.		1.9982		
AIC		-8.2010		
SBC		-8.1838		

Period 2

The best estimated model for the Period 2 of the LVL/EUR exchange rate is linear model GARCH(1,2) with added independent variables of time series of daily logarithmic returns of Period 3 lagged up to four days. The result of estimation is shown in the Table 4.18. Nonlinear model EGARCH(1,1) showed slightly better results in estimation but contained one or more than one insignificant variables and thus the linear model GARCH(1,2) was prefered.

Table 4.18 The best estimated model for LVL/EUR in Period 2

Variable	Coefficient	Std. Error	z-statistics	Prob.
C	5.04E-08	7.05E-09	7.1483	0,0000
RESID(-1)^2	0.1266	0.0087	14.4802	0,0000
GARCH(-1)	0.1429	0.0416	3.4321	0,0006
GARCH(-2)	0.7315	0.0400	18.2676	0,0000
Log likelihood		11698.9700		
Durbin-Watson stat.		2.0258		
AIC		-8.3589		
SBC		-8.3398		

Period 3

The best estimated model for the Period 3 of the LVL/EUR exchange rate is linear model GARCH(3,1) just like in the Period 1.The result of estimation is shown in the Table 4.19. The constant value is statistically significant.

Table 4.19 The best estimated model for LVL/EUR in Period 3

Variable	Coefficient	Std. Error	z-statistics	Prob.
C	4.54E-06	1.15E-06	3.9306	0,0001
RESID(-1)^2	0.4694	0.1522	3.0837	0,0020
RESID(-2)^2	-0.3059	0.1126	-2.7161	0,0066
RESID(-3)^2	-0.1293	0.0549	-2.3560	0,0185
GARCH(-1)	0.8402	0.0458	18.3556	0,0000
Log likelihood		3029.9010		
Durbin-Watson stat.		2.5707		
AIC		-8.2733		
SBC		-8.2356		

4.5 Diagnostic Tests of Estimated Models

The standardized residuals of the estimated models need to fulfill certain conditions. Appropriate standardized residuals have a constant variance or in other words are homoskedastic, are not autocorrelated and do not have normal distribution. Homoskedasticity, autocorrelation and normality tests are described in the chapter 3.7.

4.5.1 Normality Test and Descriptive Statistics

Normality test is based on Jarque-Bera test described in the chapter 3.7.3. Descriptive statistics such as mean value, median, maximum, minimum, standard deviation, skewness and kurtosis are shown together with a histogram of standardized residuals for each observed exchange rate and all observed periods.

SIT/EUR

Following Figures 4.6, 4.7 and 4.8 are showing a Jarque-Bera normality test, descriptive statistics and a histogram of standardized residuals of SIT/EUR exchange rate for all three observed periods.

The Figures 4.6, 4.7 and 4.8 are pointed more around the center with high values of kurtosis which leads to refusal of null hypothesis confirming that estimated models do not have normal distribution in any of the observed periods. The Figures 4.6 and 4.7 standardized residuals are distributed around the middle with skewness almost equal to zero while the

Figure 4.8 shows high negative coefficient of skewness and the standardized residuals are clustered in the right side.
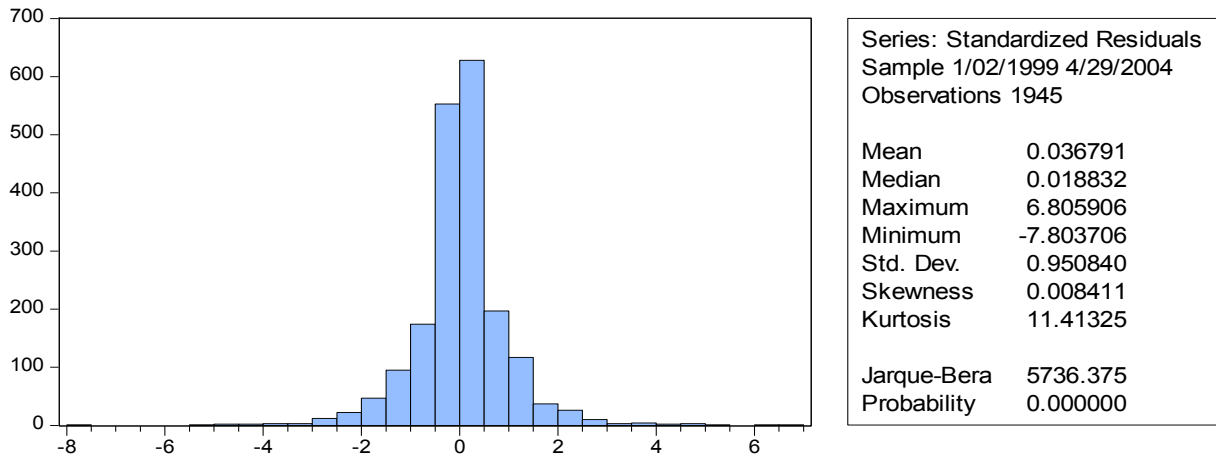
Figure 4.6 SIT/EUR Period 1 GARCH(4,1) model

Series: Standardized Residuals	
Sample 1/02/1999 4/29/2004	
Observations 1945	
Mean	0.036791
Median	0.018832
Maximum	6.805906
Minimum	-7.803706
Std. Dev.	0.950840
Skewness	0.008411
Kurtosis	11.41325
Jarque-Bera	5736.375
Probability	0.000000

Figure 4.7 SIT/EUR Period 2 EGARCH(1,3) model

Series: Standardized Residuals	
Sample 5/01/2004 12/30/2004	
Observations 244	
Mean	0.030805
Median	0.050282
Maximum	3.109870
Minimum	-3.230929
Std. Dev.	0.962773
Skewness	-0.084622
Kurtosis	4.221583
Jarque-Bera	15.46257
Probability	0.000439
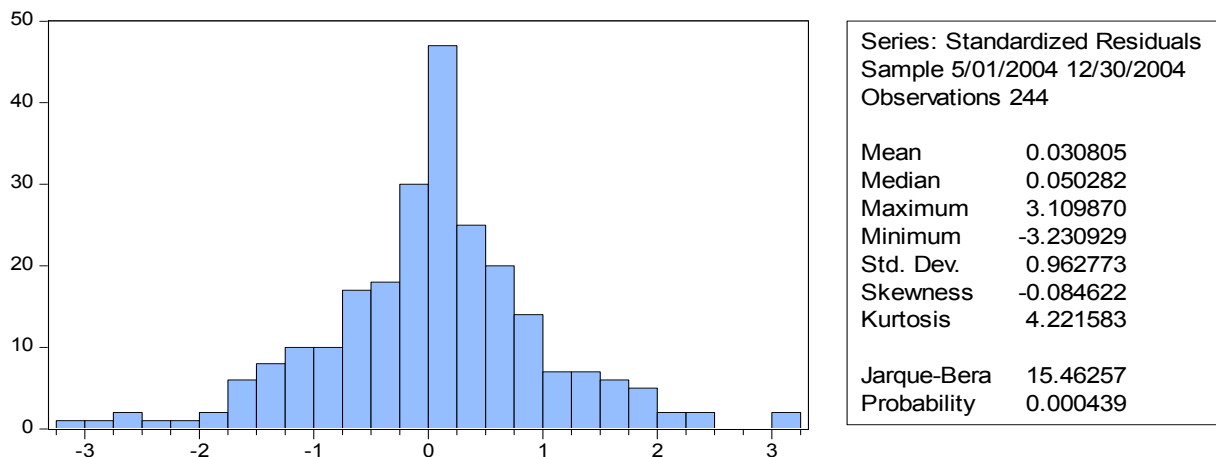
Figure 4.8 SIT/EUR Period 3 GJR-GARCH(1,1) model

Series: Standardized Residuals	
Sample 1/02/2005 12/31/2006	
Observations 729	
Mean	-0.148650
Median	-0.000464
Maximum	6.139769
Minimum	-36.90964
Std. Dev.	1.752117
Skewness	-13.42167
Kurtosis	270.3253
Jarque-Bera	2192570.
Probability	0.000000
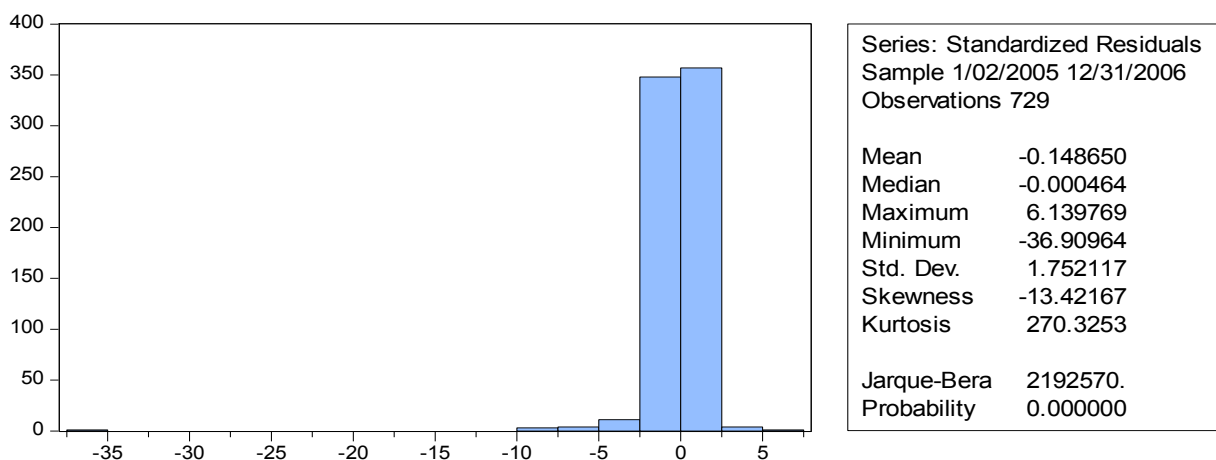
CYP/EUR

Following Figures 4.9, 4.10 and 4.11 are showing a Jarque-Bera normality test, descriptive statistics and a histogram of standardized residuals of CYP/EUR exchange rate for all three observed periods.

The Figures 4.9, 4.10 and 4.11 are pointed more around the center with high values of kurtosis which leads to refusal of null hypothesis confirming that estimated models do not have normal distribution in any of the observed periods. Standardized residuals are distributed around the middle in all of the observed periods.
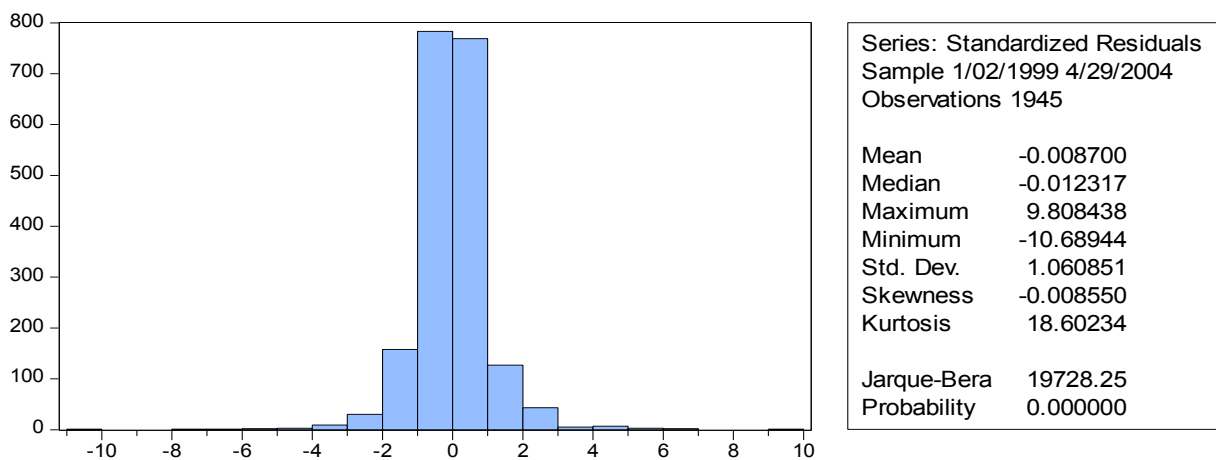
Figure 4.9CYP/EUR Period 1 GARCH(2,2) model
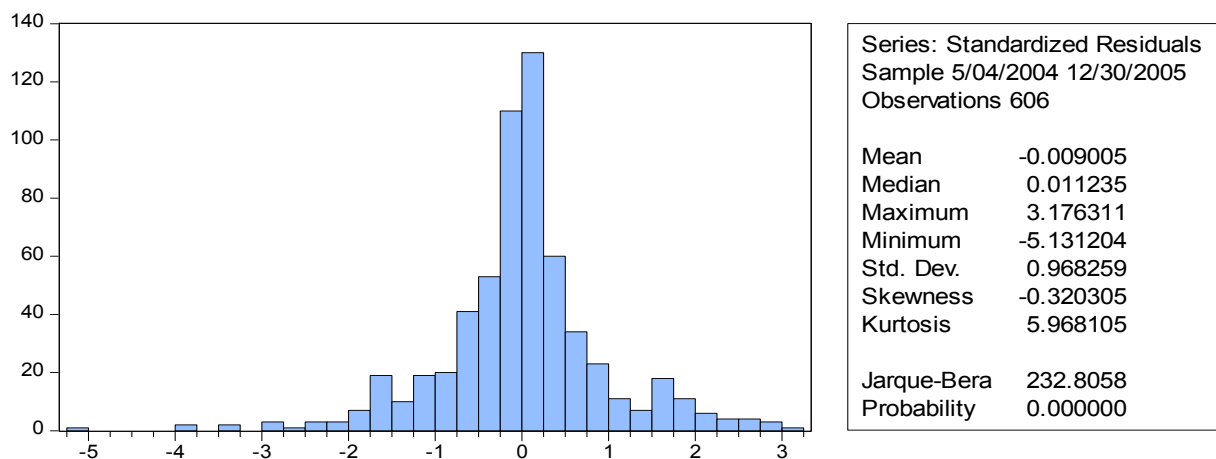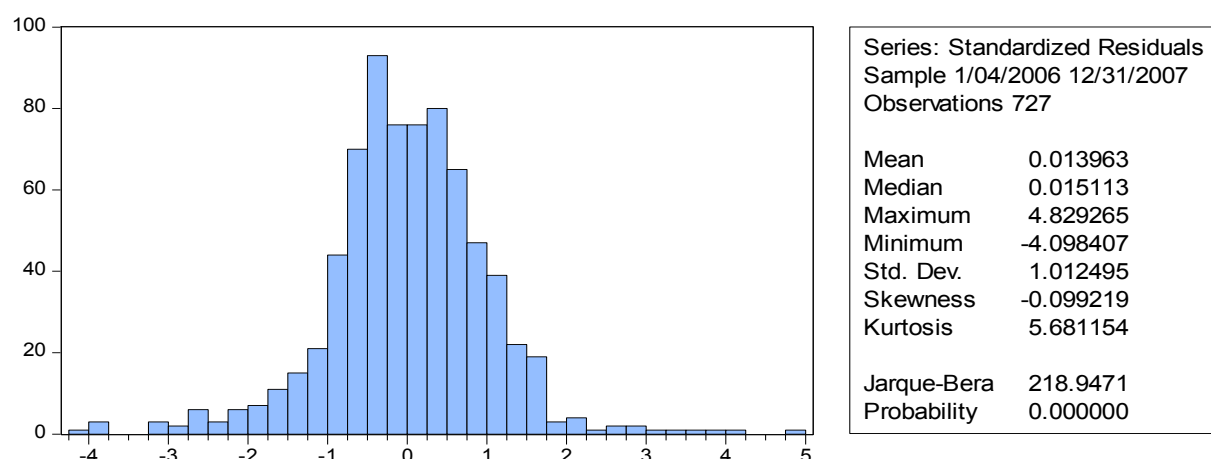

Figure 4.10 CYP/EUR Period 2 GARCH(1,1) model

Figure 4.11 CYP/EUR Period 3 GARCH(1,1) model

Series: Standardized Residuals		
Sample 1/04/2006 12/31/2007		
Observations 727		
Mean		0.013963
Median		0.015113
Maximum		4.829265
Minimum		-4.098407
Std. Dev.		1.012495
Skewness		-0.099219
Kurtosis		5.681154
Jarque-Bera		218.9471
Probability		0.000000

SKK/EUR

Following Figures 4.12, 4.13 and 4.14 are showing a Jarque-Bera normality test, descriptive statistics and a histogram of standardized residuals of SKK/EUR exchange rate for all three observed periods.

The Figures 4.12, 4.13 and 4.14 are pointed more around the center with high values of kurtosis which leads to refusal of null hypothesis confirming that estimated models do not have normal distribution in any of the observed periods. Standardized residuals are skewed to the right with a negative value of skewness in all of the observed periods.
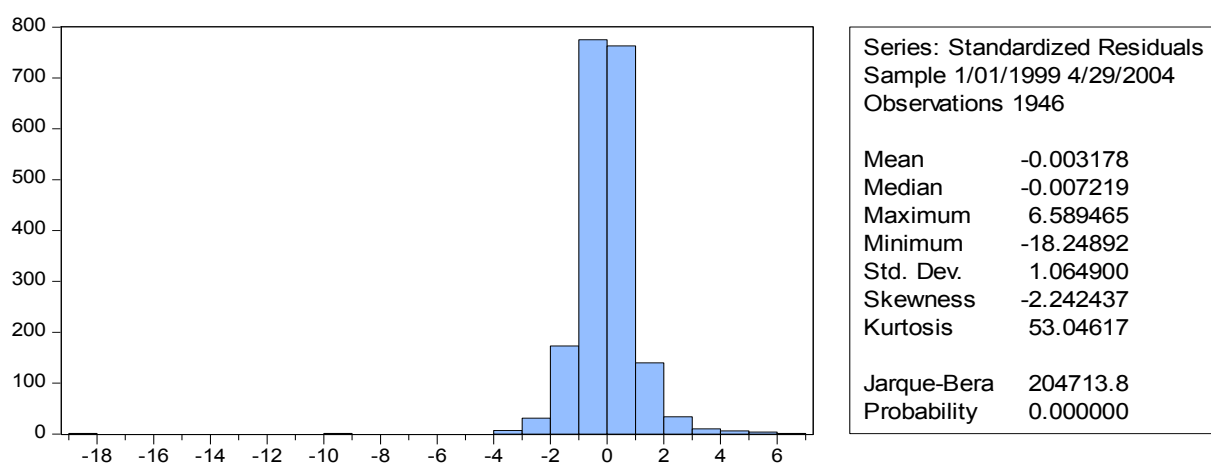
Figure 4.12 SKK/EUR Period 1 GJR-EGARCH(1,2) model

Series: Standardized Residuals		
Sample 1/01/1999 4/29/2004		
Observations 1946		
Mean		-0.003178
Median		-0.007219
Maximum		6.589465
Minimum		-18.24892
Std. Dev.		1.064900
Skewness		-2.242437
Kurtosis		53.04617
Jarque-Bera		204713.8
Probability		0.000000

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Figure 4.13 SKK/EUR Period 2 GARCH(2,1) model

Series: Standardized Residuals	
Sample 5/02/2004 12/30/2006	
Observations 973	
Mean	-0.020452
Median	-0.002212
Maximum	4.074438
Minimum	-7.454935
Std. Dev.	1.003921
Skewness	-0.385933
Kurtosis	6.860935
Jarque-Bera	628.5010
Probability	0.000000
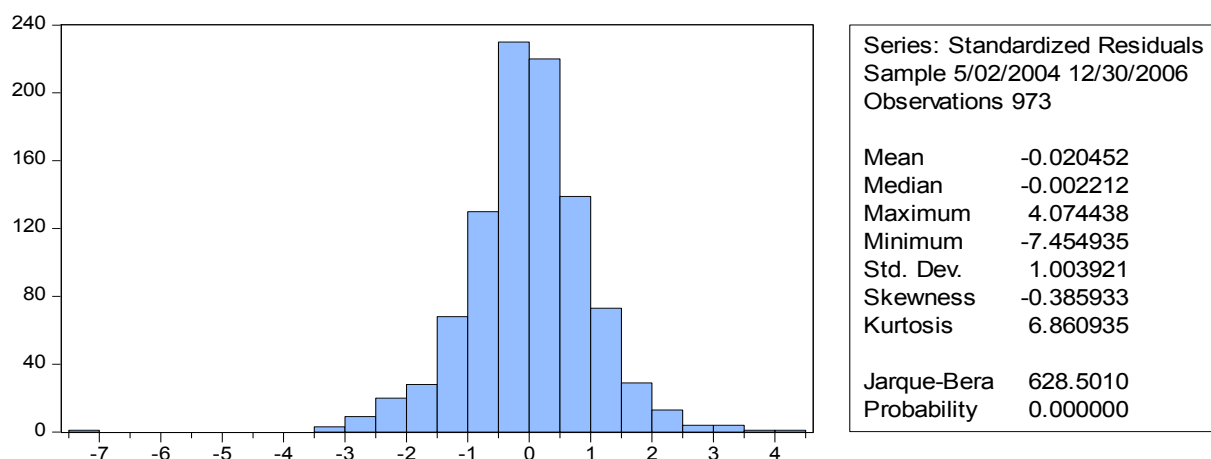
Figure 4.14 SKK/EUR Period 3 ARCH(1) model

|	
Series: Standardized Residuals	
Sample 1/04/2007 12/31/2008	
Observations 728	
Mean	-0.096167
Median	0.006319
Maximum	2.478484
Minimum	-8.794804
Std. Dev.	1.017976
Skewness	-1.527519
Kurtosis	10.97810
Jarque-Bera	2213.828
Probability	0.000000
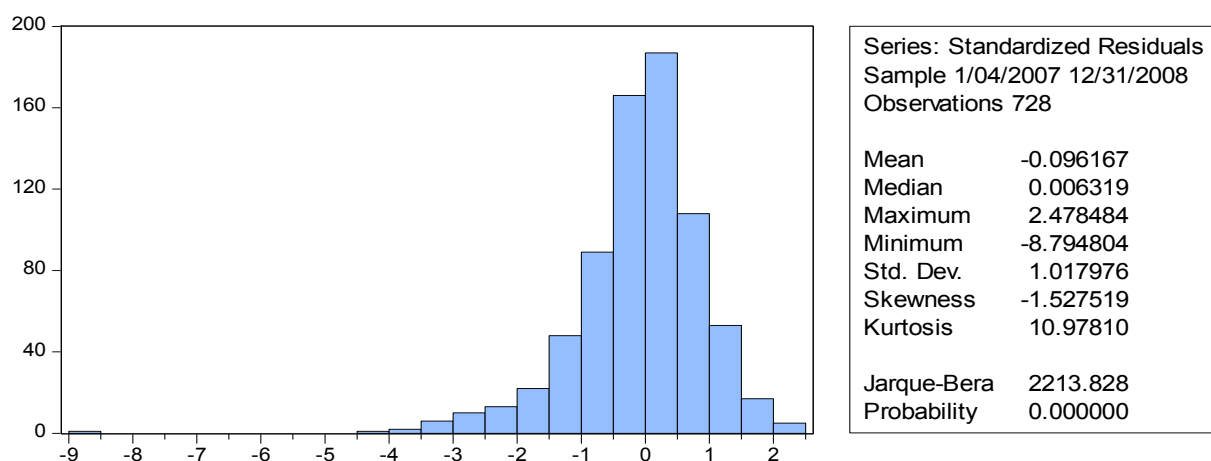
LVL/EUR

Following Figures 4.15, 4.16 and 4.17 are showing a Jarque-Bera normality test, descriptive statistics and a histogram of standardized residuals of LVL/EUR exchange rate for all three observed periods.

The 4.15, 4.16 and 4.17 are pointed more around the center with high values of kurtosis which leads to refusal of null hypothesis confirming that estimated models do not have normal distribution in any of the observed periods. Standardized residuals are distributed around the middle in all of the observed periods. The Figures 4.15 and 4.16 are skewed on the opposite directions.
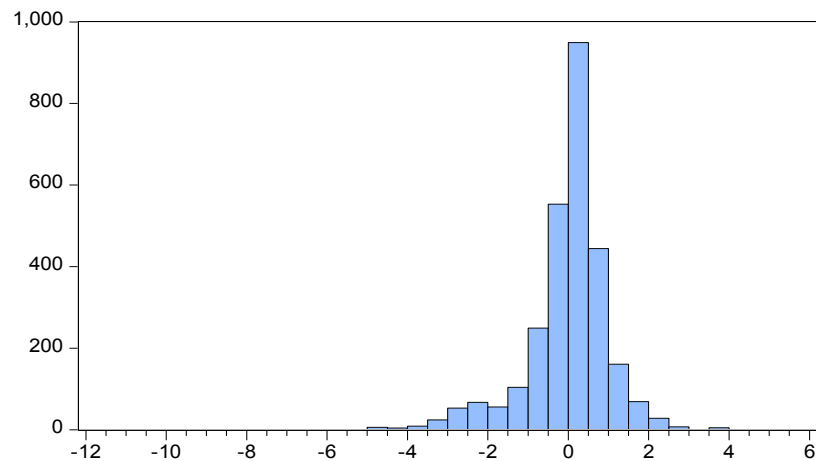
Figure 4.15 LVL/EUR Period 1 GARCH(2,1) model
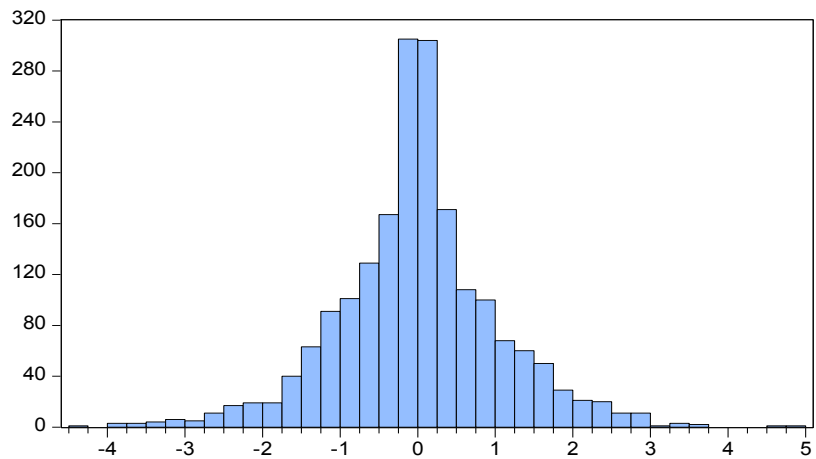
Series: Standardized Residuals	
Sample 1/02/1999 4/29/2004	
Observations 1945	
Mean	-0.004530
Median	-0.007326
Maximum	4.827225
Minimum	-4.264707
Std. Dev.	1.044588
Skewness	0.010699
Kurtosis	4.521339
Jarque-Bera	187.6058
Probability	0.000000

Figure 4.16 LVL/EUR Period 2 GARCH(1,2) model

Series: Standardized Residuals	
Sample 5/05/2004 12/31/2011	
Observations 2797	
Mean	-0.004824
Median	0.129208
Maximum	5.728822
Minimum	-11.95465
Std. Dev.	1.064553
Skewness	-1.388330
Kurtosis	12.43736
Jarque-Bera	11278.15
Probability	0.000000

Figure 4.17 LVL/EUR Period 3 GARCH(3,1) model
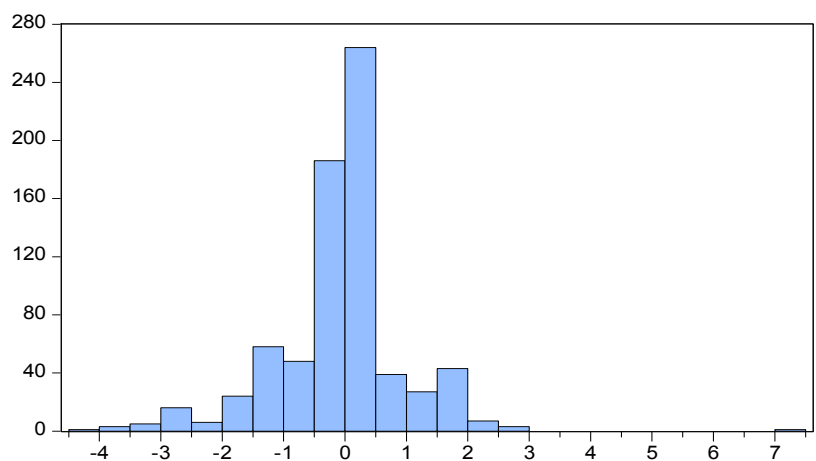
Series: Standardized Residuals	
Sample 1/01/2012 12/31/2013	
Observations 731	
Mean	-0.091675
Median	0.000000
Maximum	7.060651
Minimum	-4.332010
Std. Dev.	1.024660
Skewness	-0.116933
Kurtosis	7.779216
Jarque-Bera	697.3618
Probability	0.000000

4.5.2 ARCH Test Heteroskedasticity of Unsystematic Component

The test used to verify the presence of heteroskedasticity of standardized residuals is ARCH test described in the subchapter 3.6.1. The Table 4.3 ARCH test of conditional heteroskedasticity proved that the time series of daily logarithmic returns are heteroskedastic.This heteroskedasticity should be removed by appropriately estimated model, so the requisite results are opposite to the results in the Table 4.3.

In the Table 4.20 we can see, that the residuals are again tested up to three day lag. Every exchange rate and every period is tested individually on the 5% significance level of accepting the null hypothesis of homoskedasticity of the residuals. In the Table 4.20 we can see that the null hypothesis was refused for all observed exchange rates in all observed periods and we can conclude that all estimated models removed the unsystematic component. Estimated models can be used to model and forecast volatility.

Table 4.20 ARCH test of heteroskedasticity of standardized residuals

	RESID^2(-1)	Prob.	RESID^2(-2)	Prob.	RESID^2(-3)	Prob.
SIT/EUR1 GARCH(4,1)	-0.7883	0.4306	1.1214	0.2623	-0.7615	0.4465
SIT/EUR2 EGARCH(1,3)	-0.3629	0.7170	0.4754	0.6349	1.1650	0.2452
SIT/EUR3 GJR-GARCH(1,1)	-0.0718	0.9428	-0.0866	0.9310	-0.0751	0.9402
CYP/EUR1 GARCH(2,2)	0.4690	0.6391	-0.1137	0.9095	2.2786	0.0228
CYP/EUR2 GARCH(1,1)	-0.1720	0.8635	0.9673	0.3338	-1.2361	0.2169
CYP/EUR3 GARCH(1,1)	0.1472	0.8830	0.6075	0.5437	-0.2886	0.7730
SKK/EUR1 GJR-GARCH(1,2)	-0.1128	0.9102	-0.2647	0.7913	-0.2027	0.8394
SKK/EUR2 GARCH(2,1)	-0.5889	0.5560	1.3665	0.1721	-0.4968	0.6194
SKK/EUR3 ARCH(1)	-0.3707	0.7110	-0.5765	0.5644	-0.6445	0.5195
LVL/EUR1 GARCH(3,1)	-0.5087	0.6110	1.6008	0.1096	-1.0704	0.2846
LVL/EUR2 GARCH(1,2)	0.5020	0.6157	-0.1574	0.8750	-1.9581	0.0503
LVL/EUR3 GARCH(3,1)	-0.3517	0.7252	0.2045	0.8380	0.1541	0.8776

4.5.3 Autocorrelation Test of Unsystematic Component

This subchapter is dedicated to test whether the standardized residuals are autocorrelated. We are using the portmanteau test showing the value of autocorrelation and partial autocorrelation together with Ljung-Box Q statistics described in the subchapter 3.7.2. Squared residuals are tested against the null hypothesis of autocorrelation. The time series of

standardized residuals are lagged up to fourth order in all observed exchange rates and all observed periods.

SIT/EUR

We can see the autocorrelation test, partial autocorrelation test and Q-statistic in the Table 4.21. The results prove that standardized residuals of each of the estimated models are not correlated on the significance level of 5% in any of the observed periods.

Table 4.21 Autocorrelation test of standardized residuals SIT/EUR

		AC	PAC	Q-Stat.	Prob.
SIT/EUR1	1	-0.019	-0.019	0.701	0.403
	2	0.026	0.026	2.033	0.362
	3	-0.018	-0.017	2.683	0.443
	4	-0.010	-0.011	2.860	0.582
SIT/EUR2	1	-0.023	-0.023	0.127	0.722
	2	0.028	0.027	0.316	0.854
	3	0.074	0.075	1.673	0.643
	4	-0.018	-0.016	1.756	0.781
SIT/EUR3	1	-0.003	-0.003	0.005	0.943
	2	-0.003	-0.003	0.013	0.994
	3	-0.003	-0.003	0.018	0.999
	4	-0.003	-0.003	0.025	1.000

CYP/EUR

We can see the autocorrelation test, partial autocorrelation test and Q-statistic in the Table 4.22. The results prove that standardized residuals of each of the estimated models are not correlated on the significance level of 5% in any of the observed periods.

Table 4.22 Autocorrelation test of standardized residuals CYP/EUR

		AC	PAC	Q-Stat.	Prob.
CYP/EUR1	1	0.011	0.011	0.215	0.643
	2	-0.002	-0.002	0.223	0.895
	3	0.052	0.052	5.423	0.143
	4	-0.023	-0.024	6.478	0.166

		AC	PAC	Q-Stat.	Prob.
CYP/EUR2	1	-0.009	-0.009	0.005	0.816
	2	0.040	0.040	1.025	0.599
	3	-0.051	-0.050	2.617	0.455
	4	-0.037	-0.040	3.458	0.484

		AC	PAC	Q-Stat.	Prob.
CYP/EUR3	1	0.005	0.005	0.021	0.885
	2	0.022	0.022	0.386	0.825
	3	-0.011	-0.011	0.467	0.926
	4	-0.006	-0.006	0.492	0.974

SKK/EUR

We can see the autocorrelation test, partial autocorrelation test and Q-statistic in the Table 4.23. The results prove that standardized residuals of each of the estimated models are not correlated on the significance level of 5% in any of the observed periods.

Table 4.23 Autocorrelation test of standardized residuals SKK/EUR

		AC	PAC	Q-Stat.	Prob.
SKK/EUR1	1	-0.003	-0.003	0.013	0.910
	2	-0.006	-0.006	0.083	0.960
	3	-0.005	-0.005	0.123	0.989
	4	0.001	0.001	0.125	0.998

SKK/EUR2	1	-0.020	-0.020	0.406	0.524
	2	0.044	0.044	2.336	0.311
	3	-0.018	-0.016	2.643	0.450
	4	-0.025	-0.027	3.238	0.519

SKK/EUR3	1	-0.013	-0.013	0.123	0.725
	2	-0.021	-0.021	0.446	0.800
	3	-0.023	-0.024	0.848	0.838
	4	-0.004	-0.005	0.859	0.930

LVL/EUR

We can see the autocorrelation test, partial autocorrelation test and Q-statistic in the Table 4.24. The results prove that standardized residuals of each of the estimated models are not correlated on the significance level of 5% in any of the observed periods.

Table 4.24 Autocorrelation test of standardized residuals LVL/EUR

		AC	PAC	Q-Stat.	Prob.
LVL/EUR1	1	0.009	0.009	0.157	0.692
	2	-0.008	-0.008	0.276	0.871
	3	-0.011	-0.011	0.505	0.918
	4	-0.047	-0.046	4.733	0.316
LVL/EUR2	1	0.01	0.01	0.2546	0.614
	2	-0.003	-0.003	0.2839	0.868
	3	-0.037	-0.037	4.1413	0.247
	4	-0.038	-0.037	8.1181	0.087
LVL/EUR3	1	-0.013	-0.013	0.120	0.729
	2	0.008	0.008	0.166	0.921
	3	0.006	0.006	0.188	0.980
	4	-0.023	-0.023	0.572	0.966

4.6 Volatility of Estimated Models

This chapter shows the best estimations of linear and nonlinear models of conditional heteroskedasticity for all of the observed exchange rates in all observed periods. Volatility is expressed in a graphic form as a chart of conditional variance.

SIT/EUR

Volatility of period 1 showed in the Figure 4.18 is connected with steady incremental depreciation of SIT to EUR. This period shows very low volatility in the first two observed years with one exception in the middle of the year 2000. This sudden growth might be connected with the parliamentary elections held in that year. Volatility continues to grow since the third fourth of 2001 because many fundamentally important events were happening in Slovenia at that time, i.e. presidential elections in 2002, referendums about joining EU and NATO etc. Period 2 showed in the Figure 4.19 analyses only the rest of the year 2004 after Slovenia joined the European Union and shows very low volatility throughout the whole observed period. Period 3 in the Figure 4.20 shows constant, very low volatility with values close to zero. This can be explained by the presence of Slovenia in ERM II and thus obligation to keep the exchange rate SIT/EUR stabilized. There is one violation in the middle of year 2006 which might be fundamentally connected with the next parliamentary elections

and a very short period of uncertainty. Development of volatility is generally worse to observe because of the value of outliers.
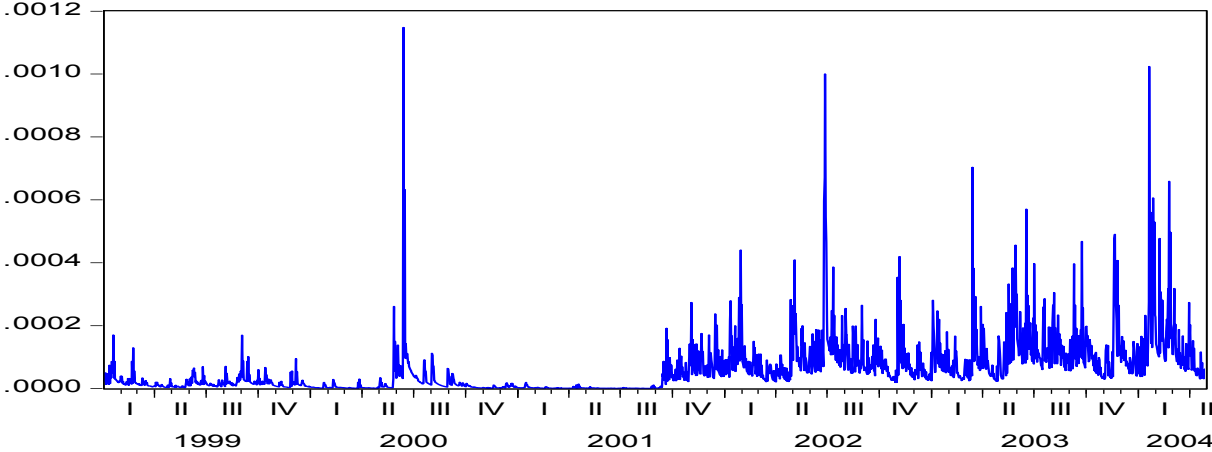
Figure 4.18 SIT/EUR Period 1 volatility
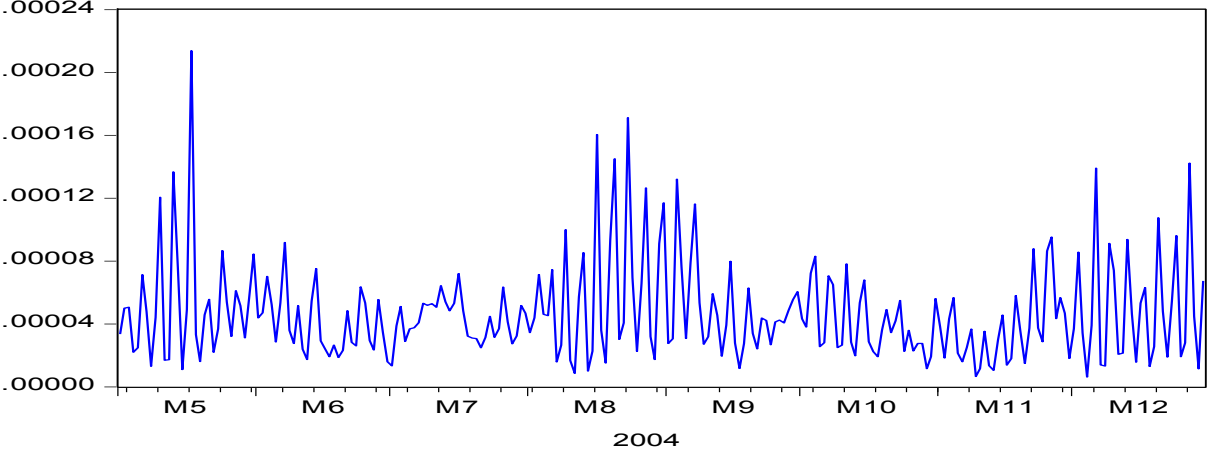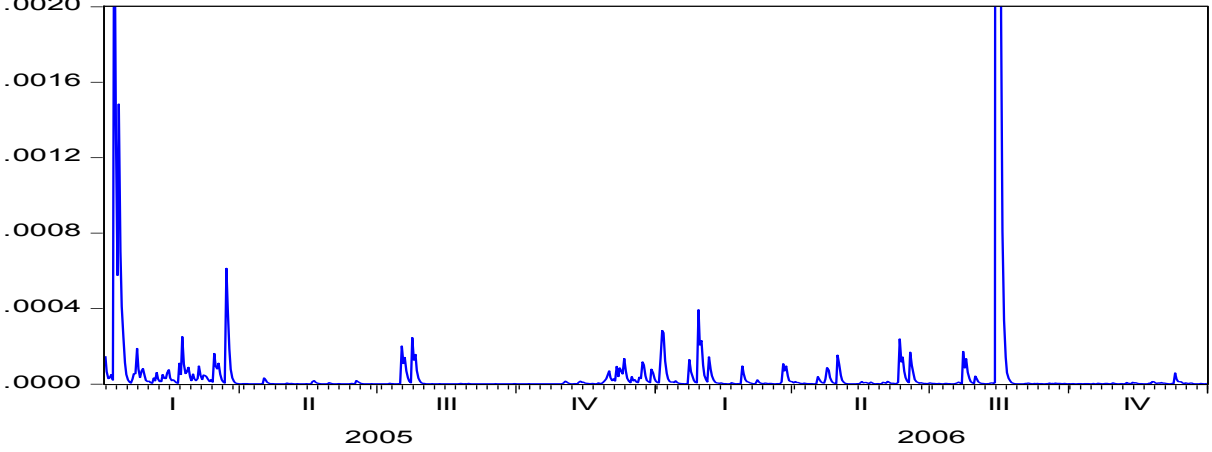

Figure 4.19 SIT/EUR Period 2 volatility

Figure 4.20 SIT/EUR Period 3 volatility

CYP/EUR

The development of the CYP/EUR exchange rate is going generally sideways except short period in the end of 2001. In the Figure 4.21 we can see that the volatility in the Period 1 is very low with one exception in the beginning of the second quarter 1999 and one in the end of 2001 when the European Court of Human Rights found Turkey guilty of continuing human rights violations against Cyprus citizens which affected the perception of stability in the region. Period 2 showed in the Figure 4.22 is characterized with very low volatility with stable development in the whole observed period. Period 3 showed in the Figure 4.23 shows similarly to Period 2 very low values of volatility unlike the SIT/EUR or SKK/EUR.. Even though lower political stability of Cyprus needed the presence of United Nations Peacekeeping Force in Cyprus (UNFICYP), the currency was quite stable with low values of volatility during all observed periods.

Figure 4.21 CYP/EUR Period 1volatility
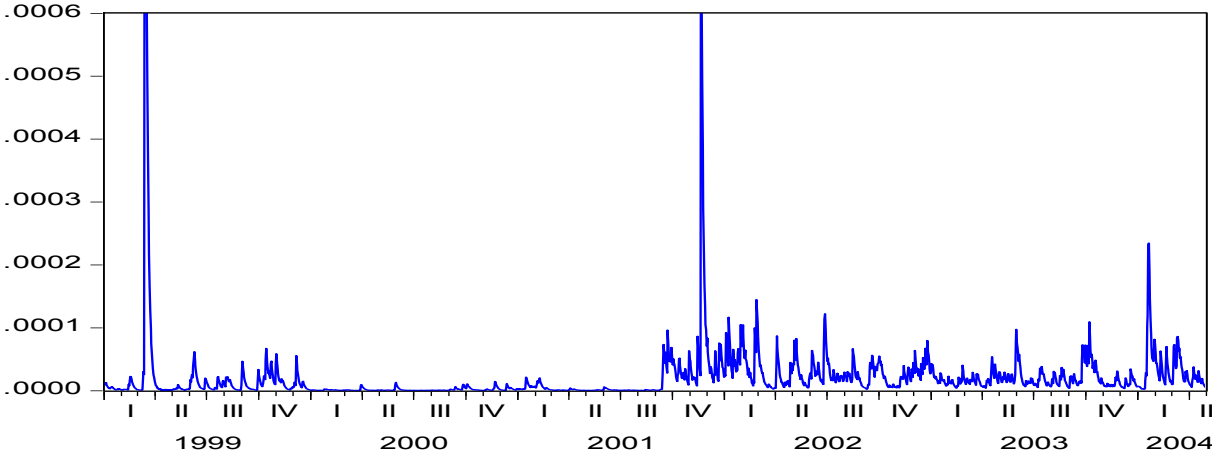
Figure 4.22 CYP/EUR Period 2 volatility
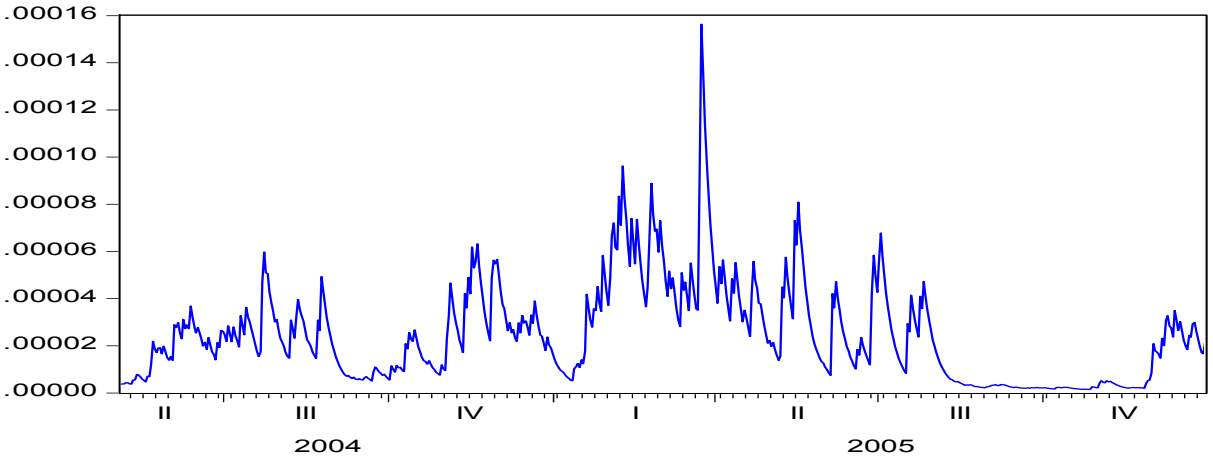
Figure 4.23 CYP/EUR Period 3 volatility
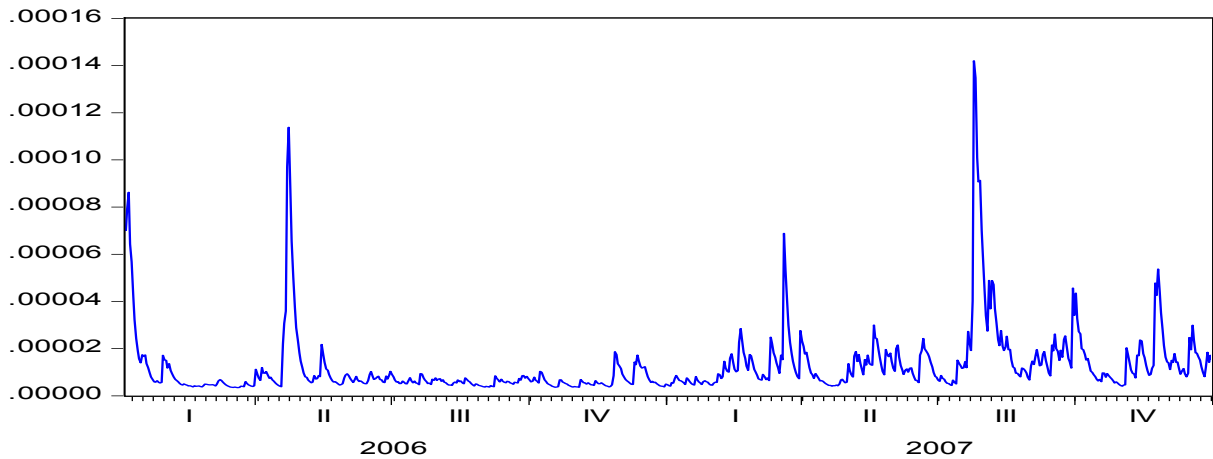
SKK/EUR

Slovakia showed the most stable level of volatility in all three observed periods compared to other countries. The exchange rate SKK/EUR was gradually appreciating since the existence of Euro as a currency. Period 1 which is shown in the Figure 4.24 has very low values of volatility with one exception in the first half of 2000 which influenced the scale values. This exception reflects the fact that during the year 2000 Slovakia joined Organization for economic co-operation and development (OECD) with ambiguous expectations. Period 2 in the Figure 4.25 shows a little higher volatility in the second half of the year 2005 which is the year when many of economic reforms came into force in Slovakia which might affect the exchange rate. Period 3 showed in the Figure 4.26 has very low volatility similarly to SIT/EUR Period 3 with one exception the first half of 2007 because from the Figure 4.3 we can see that SKK/EUR was appreciating till the second half of 2008.
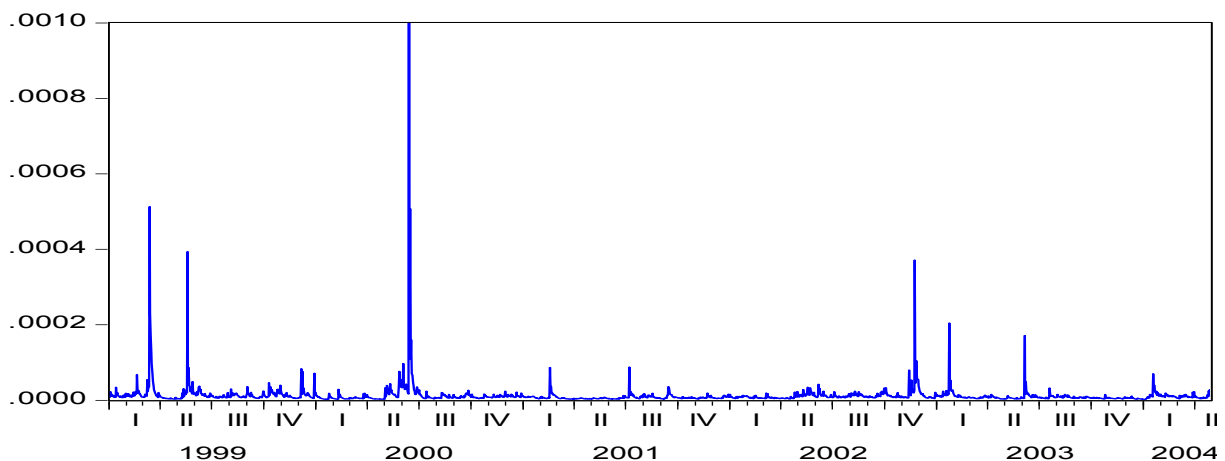
Figure 4.24 SKK/EUR Period 1 volatility

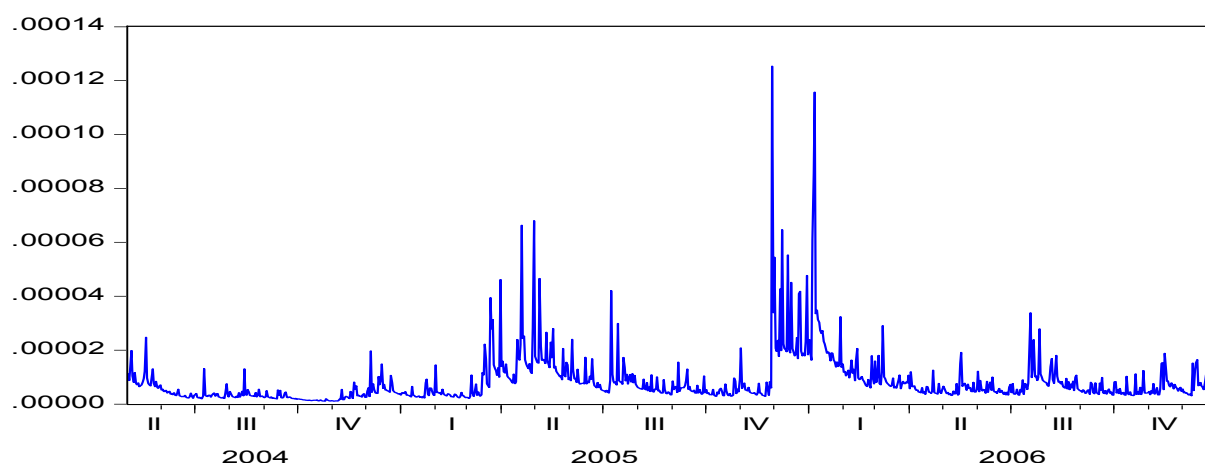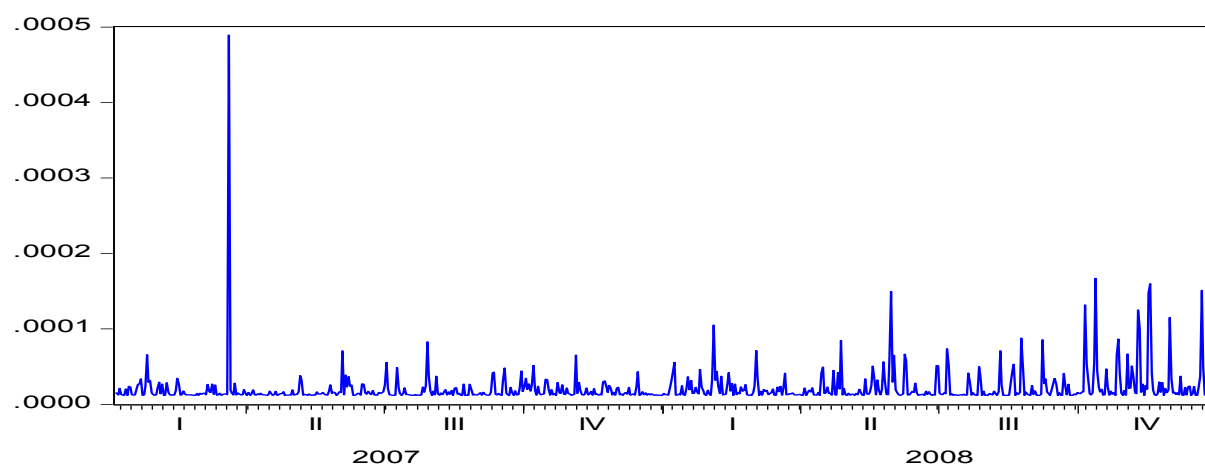Figure 4.25 SKK/EUR Period 2 volatility

Figure 4.26 SKK/EUR Period 3 volatility

LVL/EUR

The exchange rate LVL/EUR was firstly appreciating since the beginning of 1999 roughly till the middle of 2001 and since that time depreciating till the Latvia joined European Union. After that the exchange rate was influenced by the Latvian central monetary authority. Period 1 showed in the 4.27 has low volatility which is changing very fast unlike all other observed exchange rates. In the Figure 4.28 we can see that the volatility in the Period 2 was quite variable with slightly higher values especially around the years 2008 and 2009 when the economic crisis struck Latvia. Figure 4.29 showing the Period 3 looks different than other charts showing Period 3, because of frequent small interventions of Latvian central bank.
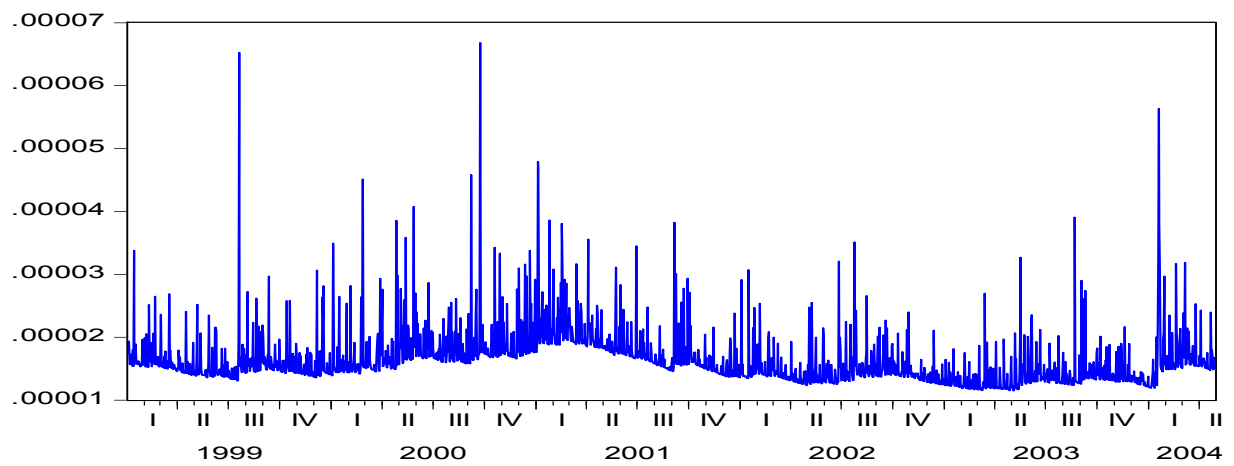
Figure 4.27 LVL/EUR Period 1volatility
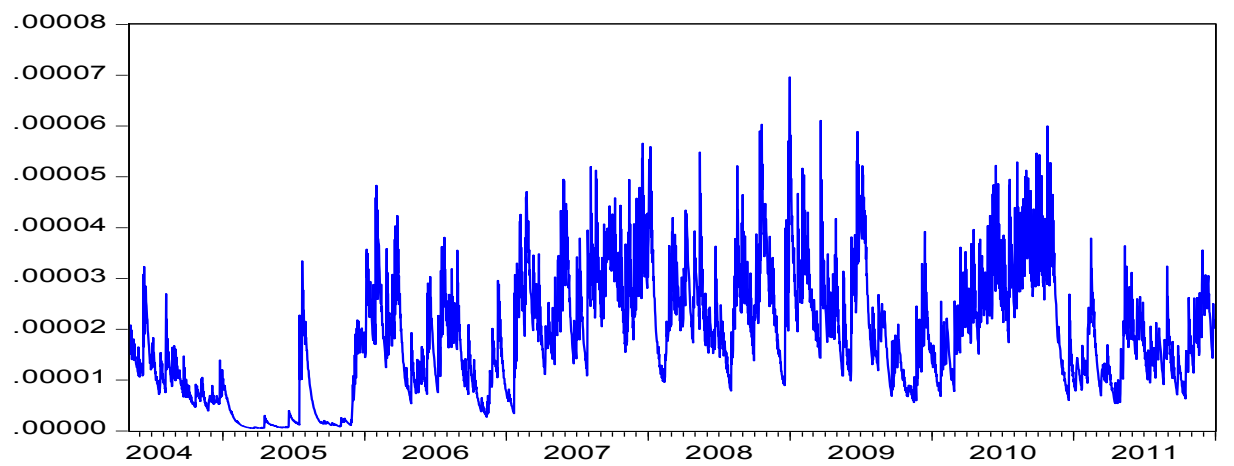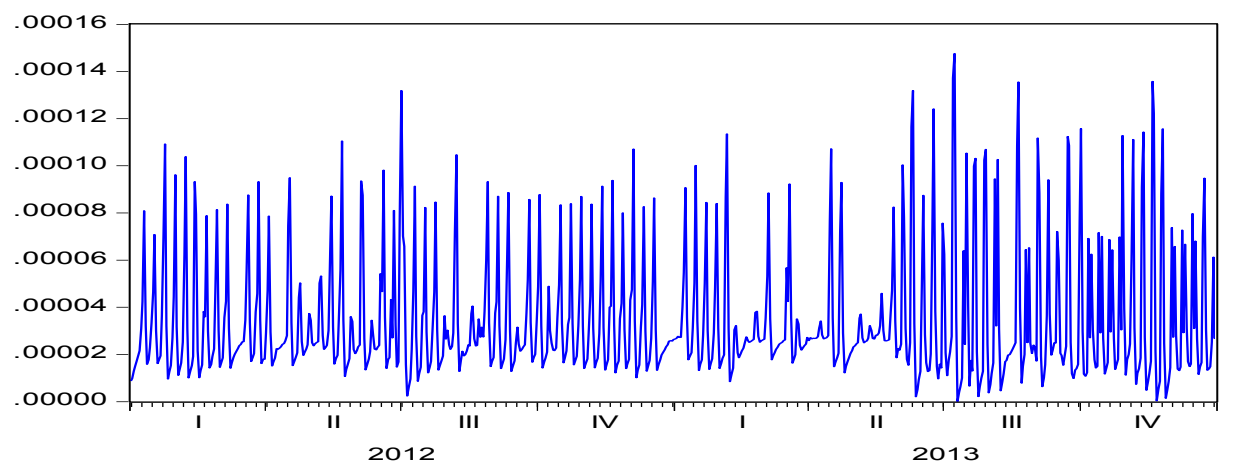

Figure 4.28 LVL/EUR Period 2volatility

Figure 4.29 LVL/EUR Period 3volatility

4.7 Forecasting of Estimated Conditional Heteroskedasticity Models

This chapter is dedicated to determine the ability of estimated models to forecast volatility. To analyze the forecasting ability of the models we will use loss functions RMSE, MAE and Theil inequality coefficient described in the chapter 3.5. The forecasting ability is analyzed for each observed exchange rate and each observed period. The lower is the value of loss functions the higher is the predictive ability of the estimated model. The results are showed in the Table 4.25.

Table 4.25 Forecasting ability of estimated models to predict volatility

		RMSE	MAE	Theil
SIT/EUR	SIT/EUR1 GARCH(4,1)	0.000129	0.000063	0.398596
	SIT/EUR2 EGARCH(1,3)	0.000089	0.000053	0.105541
	SIT/EUR3 GJR-GARCH(1,1)	0.000210	0.000078	0.482085
CYP/EUR	CYP/EUR1 GARCH(2,2)	0.000117	0.000086	0.591813
	CYP/EUR2 GARCH(1,1)	0.000042	0.000027	0.108581
	CYP/EUR3 GARCH(1,1)	0.000036	0.000014	0.129175
SKK/EUR	SKK/EUR1 GJR-EGARCH(1,2)	0.000226	0.000024	0.235499
	SKK/EUR2 GARCH(2,1)	0.000021	0.000029	0.113569
	SKK/EUR3 ARCH(1)	0.000058	0.000025	0.169619
LVL/EUR	LVL/EUR1 GARCH(2,1)	0.000036	0.000020	0.190547
	LVL/EUR2 GARCH(1,2)	0.000047	0.000027	0.272865
	LVL/EUR3 GARCH(3,1)	0.000062	0.000046	0.148751

The prediction is in-sample prediction and we can see the charts showing the comparation of estimated and predicted volatility for all three Periods of SIT/EUR exchange rate in Figures 4.30, 4.31 and 4.32. The results for other exchange rates are available in the Annexes. We can see the red line representing real volatility and blue line representing volatility calculated by estimated models.

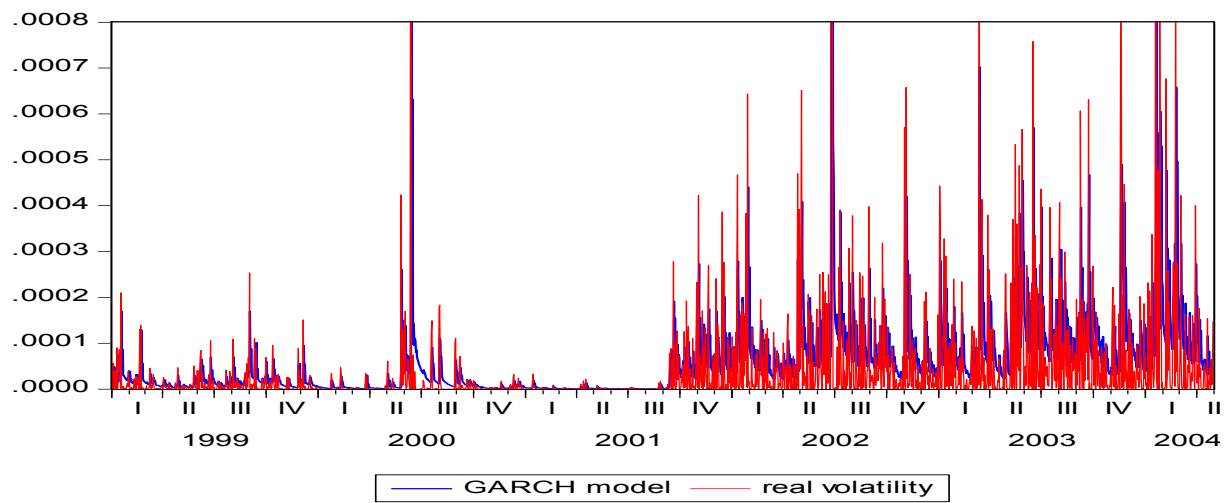Figure 5.1 SIT/EUR Period 1 GARCH(4,1) forecasted volatility

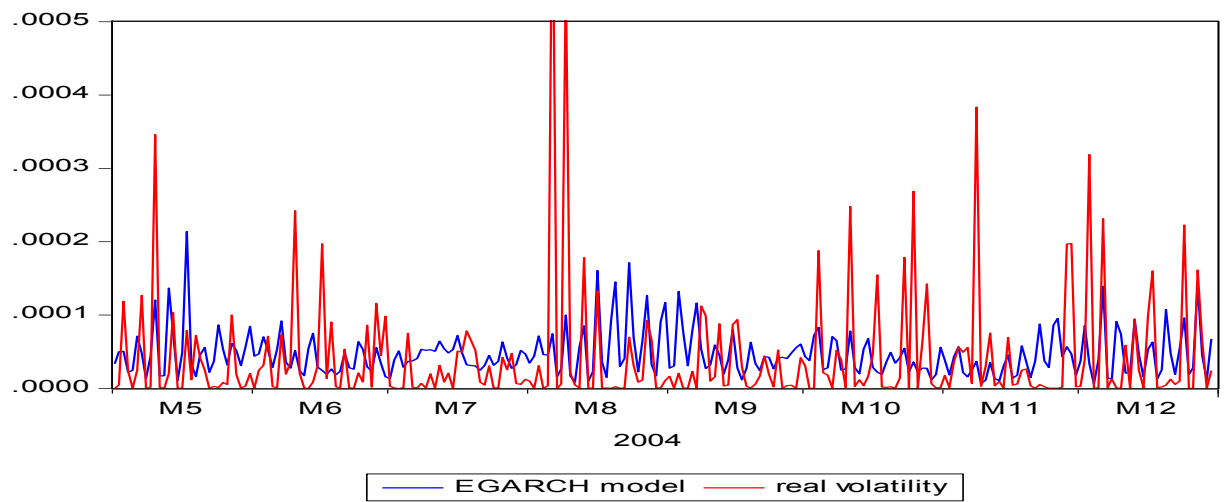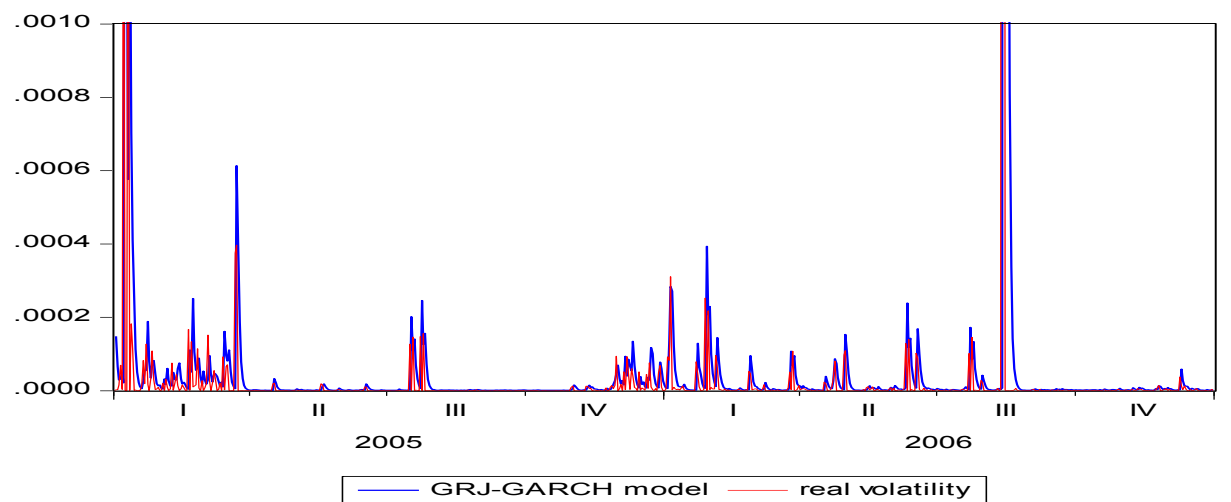Figure 5.2 SIT/EUR Period 2 EGARCH(1,3) forecasted volatility

Figure 5.3 SIT/EUR Period 3 GJR-GARCH(1,1) forecasted volatility

5 SUMMARY OF THE RESULTS

The fourth chapter used theoretical background of the third chapter to analyze the data used for this thesis. The fourth chapter was dedicated to the best possible estimation of conditional heteroskedasticity models for each of the observed exchange rates in all observed period. Estimated models have undergone the diagnostic tests and were checked about their forecasting abilities. This, fifth, chapter sums up the information received in the previous chapter and comments on achieved results and diagnostic tests.

First part of the fifth chapter introduced the data which were used for the following calculations and divided the time series of daily returns into three periods. First period, Period 1, begins 1/01/1999 for each of the observed exchange rate as it is the year when Euro started to exist in non-physical form (cheques, electronic transfers, banking etc.). This period ends with the entering of the observed country into EU, which was on 1/05/2004. Second period, Period 2, is different for each exchange rate and begins 1/05/2004 and finishes always two years before the country joined EMU. Third period represents two years before the country joins EMU, because in this period all countries have to be a part of ERM II and stabilize their exchange rate toward Euro.

These time series of daily returns were non-stationary and thus adjusted to time series of daily logarithmic returns in the second part which were proved to be stationary in the following chapter and were used for the following calculations.

The third part analyzed the series of daily logarithmic returns by the descriptive statistics. Using the Jarque-Bera test we proved that none of the observed exchange rates in any period have normal distribution. This part included also ADF test of stationarity and ARCH test of heteroskedasticity. All tested time series were proved to be stationary and heteroskedastic and thus appropriate for the estimation and construction of conditional heteroskedasticity models.

Fourth part as the most important one presented us the results of conditional heteroskedasticity models estimation. A model was individually estimated for every exchange rate in each of the observed periods. The models were estimated using the statistical software EViews and were estimated to have the best possible features according to the AIC and SBC criteria. As a dependent variable stood the time series of daily logarithmic returns and the independent variable was a constant and in some cases the time series of daily logarithmic

returns lagged from on up to three days. These independent variables were added just if they were statistically significant and positively affected the estimated model.

We can see the summary of the diagnostic tests which were run in the fifth part for each estimated model in the Table 5.1. The fourth chapter shows the descriptive statistics and distribution of standardized residuals of estimated models. Normality, heteroskedasticity and autocorrelation were proved not to be present in any of the observed exchange rates in any observed periods. The test used to analyze the presence of normal distribution was Jarque-Bera test, for heteroskedasticity was used ARCH test and for autocorrelation was used portmanteau test together with Ljung-Box Q statistics.

Table 5.1 Summary of the diagnostic tests of standardized residuals of estimated models

		Normality	Het.	AC
SIT/EUR	SIT/EUR1 GARCH(4,1)	No	No	No
	SIT/EUR2 EGARCH(1,3)	No	No	No
	SIT/EUR3 GJR-GARCH(1,1)	No	No	No
CYP/EUR	CYP/EUR1 GARCH(2,2)	No	No	No
	CYP/EUR2 GARCH(1,1)	No	No	No
	CYP/EUR3 GARCH(1,1)	No	No	No
SKK/EUR	SKK/EUR1 GJR-EGARCH(1,2)	No	No	No
	SKK/EUR2 GARCH(2,1)	No	No	No
	SKK/EUR3 ARCH(1)	No	No	No
LVL/EUR	LVL/EUR1 GARCH(2,1)	No	No	No
	LVL/EUR2 GARCH(1,2)	No	No	No
	LVL/EUR3 GARCH(3,1)	No	No	No

Sixth part presented graphs of volatility or in other words GARCH graphical expression of conditional variance. The graphs are showed in the Figures 4.18-4.29 and extraordinary development is commented and explained by fundamental events in each country.

Volatility of the observed exchange rates had similar process for Slovenia and Slovakia only with the difference in the Period 3 when the volatility remained higher in SIT/EUR. It can be explained by the relatively similar location of the country. The volatility of other two observed exchange rates is different. CYP/EUR shows very higher volatility only

during the second period maybe also because exchange rate was 0.5853 CYP/EUR and Cyprus pound was quite strong currency in the south region. LVL/EUR volatility is changing quite fast in short periods during each of the observed periods. It may be related with the fact that the Latvian lat was partly fixed to Euro even earlier than the country joined ERM II and economic crisis in 2008 hit Latvia quite hard so many monetary interventions were needed.

Seventh part analyzed the forecasting ability of each estimated model according to the loss functions criteria RMSE, MAE and Theil inequality coefficient and includes the Figures showing development of real volatility compared to the development of volatility according to estimated models.

The first partial aim of the thesis compares whether linear or nonlinear model are more optimal for conditional heteroskedasticity models. We can see the comparison in the Table 5.2. The best model of each exchange rate is bolded. In total there were nine linear models versus three nonlinear models used in this thesis. In few cases the nonlinear models showed better conditions for estimating volatility but in the same time included one or more than one insignificant values and thus the linear model was preferred. Generally we can conclude that linear models were more optimal for modelling the volatility than nonlinear models used for the purpose of this thesis.

Table 5.2 Comparison of linear and nonlinear models estimation efficiency

		Period 1	Period 2	Period 3
		GARCH(4,1)	EGARCH(1,3)	GJR-GARCH(1,1)
SIT/EUR	AIC	-8.4744	-7.4663	**-9.6157**
	SBC	-8.4486	-7.3803	**-9.5779**
		GARCH(2,2)	GARCH(1,1)	GARCH(1,1)
CYP/EUR	AIC	**-9.4409**	-8.5431	-8.7834
	SBC	**-9.4209**	-8.4922	-8.7392
		GJR-EGARCH(1,2)	GARCH(2,1)	ARCH(1)
SKK/EUR	AIC	-9.1559	**-9.2359**	-8.1864
	SBC	-9.1387	**-9.2007**	-8.1486
		GARCH(2,1)	GARCH(1,2)	GARCH(3,1)
LVL/EUR	AIC	-8.2010	**-8.3589**	-8.2733
	SBC	-8.1838	**-8.3398**	-8.2356

The second partial aim is focused on the ability of the estimated models to predict volatility. The results are shown in the Table 4.25 and include three loss functions criteria showing quite low values which signs great predictive abilities. We can see that model

EGARCH(1,3) in Period 2 of exchange rate SIT/EUR has the best forecasting ability from all estimated models and the worst forecasting ability showed GARCH(2,2) model in Period 1 of exchange rate CYP/EUR. Generally, we cannot confirm whether linear or nonlinear models have better forecasting ability. Vertically, the best forecasting abilities were showed for Period 2 except LVL/EUR exchange rate. Figures 4.30, 4.31 and 4.32 are showing the comparison of real volatility and in-sample forecast of volatility of estimated models. We can conclude that all of the estimated models are optimal for predicting volatility.

6 CONCLUSION

Diploma thesis was focused on modelling and forecasting the volatility of exchange rates using conditional heteroskedasticity models. The whole thesis was divided into six parts including Introduction and Conclusion.

Introduction shortly briefed what is the focus of the thesis and summed up the information included in each chapter.

Second and third chapter were theoretical and methodological. Second chapter led us through the foreign exchange market and the way of how the currencies are traded. We got information about quotation, various exchange rates systems and introduction about the volatility and its properties.

Third chapter was focused on conditional heteroskedasticity models and began with assumptions and features of financial time series. After the univariate linear models we got information about the first conditional heteroskedasticity model which is ARCH, then followed by information about GARCH model and their modifications, i.e. nonlinear EGARCH or GJR-GARCH models. The forecast construction included the maximum likelihood method used in the practical chapter. The instructions of how to build a volatility model continued and in the end of the third chapter we were explained how to run a diagnostic tests to verify the estimated models and which criteria are used for optimal model selection.

Fourth chapter was practical and empirical and used the information given in the theoretical and methodological chapters. Data used for this thesis were time series of daily returns of exchange rates of Slovenian tolar to Euro SIT/EUR, Cyprus pound to Euro CYP/EUR, Slovakian koruna to Euro SKK/EUR and Latvian lat to Euro LVL/EUR. Observed period began 1/01/1999 and finished by the entry of each country to EMU. It was furtherly divided into three periods with expected differences in volatility which were confirmed just partly. Time series were adjusted to daily logarithmic returns and diagnosed with normality, stationarity and heteroskedasticity tests which proved the time series to be suitable for conditional heteroskedasticity models estimation. The best possible model was estimated for each of the observed exchange rates in each observed periods. Estimated models were diagnosed by the normality, autocorrelation and heteroskedasticity tests. Descriptive statistics and GARCH graphs of conditional variance are presented for each observed exchange rate

and each observed period. The last part of the fourth chapter is testing the forecasting ability of the estimated models by the loss criteria RMSE, MAE and Theil inequality coefficient.

Fifth chapter included commented summary of the results of the fourth chapter with Tables summarizing the adequacy of the models, explaining fundamentally some of the volatility development and comparing the efficiency of linear and nonlinear models for estimating the volatility. Compare the linear and nonlinear models was the partial goal of this thesis and we can conclude that the models which are more suitable for modelling the volatility are linear models.

We can conclude that the aim of the Diploma thesis was fulfilled and that the volatility of the exchange rates was quite different for each of the observed exchange rates except small similarity between SIT/EUR and SKK/EUR. It is probably caused by the different location and political and economic situation of each observed country. Nevertheless all of the estimated models showed very low volatility during Period 3 for all of the exchange rates only without LVL/EUR with almost flat development. LVL/EUR is an exception because Latvia was partly fixing their currency to Euro and joined ERM II earlier than required two years before joining EMU. The assumption that volatility is the lowest during Period 3 is confirmed.

The first partial aim focused on comparison of linear and nonlinear models proved that linear models are better for modelling exchange rates volatility. The reason might be that the ability of nonlinear models to reflect different impact of negative and positive shocks is not a great advantage because it is not very significant for high frequency data used in this thesis. The second partial aim proved that all estimated models are optimal for predicting volatility.

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LIST OF ABBREVIATONS

ADF test – augmented Dickey-Fuller test

AR model – autoregressive model

ARCH – autoregressive conditional heteroskedasticity

ARMA – autoregressive moving average

CYP – Cyprus pound

DF test – Dickey-Fuller test

DS – differential stationary

EGARCH – exponential GARCH

EMA – exponential moving average

ERM II – European exchange rate mechanism II

EU – European Union

EUR – euro currency

EWMA - exponentially weighted moving average

FX – foreign exchange

FR – forward rate

GARCH – generalized autoregressive conditional heteroskedasticity

GJR-GARCH – Glosten, Jagannathan and Runkl GARCH

IGARCH – integrated GARCH

IR – interest rate

JB – Jaque-Bera

JPY – Japanese yen

KU – kurtosis

LM – Lagrange's multiplications

LVL – Latvian Lat

LWMA - linear weighted moving average

MA – moving average

MAE – mean absolute error

MLE – maximum likelihood estimation

MSE – mean square error

OLS – ordinary least square

OTC – over the counter

RMSE – root mean square error

SIT – Slovenian tolar

SK – skewness

SKK – Slovakian koruna

SMA – simple moving average

SMMA – smoothed moving average

SR – spot rate

SSE – sum of squared errors of prediction

SWIFT – society for worldwide interbank financial telecommunications

TS – trend stationary

USD – United States dollar

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Ondřej Mikulec

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