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Published in:
Proceedings 2017 11th Asian Control Conference (ASCC)

DOI (link to publication from Publisher):
[10.1109/ASCC.2017.8287618](https://doi.org/10.1109/ASCC.2017.8287618)

Publication date:
2017

Document Version
Accepted author manuscript, peer reviewed version

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Zhu, J., Ma, K., N. Soltani, M., Hajizadeh, A., & Chen, Z. (2017). Comparison of loads for wind turbine down-regulation strategies. In *Proceedings 2017 11th Asian Control Conference (ASCC)* IEEE Press.
<https://doi.org/10.1109/ASCC.2017.8287618>

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Comparison of Loads for Wind Turbine Down-regulation Strategies

Jiangsheng Zhu¹, Student Member, IEEE, Kuichao Ma¹, Student Member, IEEE, Mohsen Soltani¹, Senior Member, IEEE, Amin Hajizadeh¹, Senior Member, IEEE and Zhe Chen², Senior Member, IEEE

Abstract—For wind farm active power setpoint tracking, both farm level and turbine level down-regulation strategies should be optimized considering turbine health condition. Several down-regulation strategies are chosen to analyse the wind turbine load performance according to different wind speed and power reference. In this paper we suggest appropriate down-regulation strategy to control wind turbine for active power reference tracking, we compare four different down-regulation strategies, namely Const- Ω , Const- λ , Max- Ω and Min- C_t and discuss the loads on main components and downstream wind speed by presenting analysis of several wind scenarios and simulation in full-range operation.

I. INTRODUCTION

Wind energy has become the fastest growing renewable energy. Meanwhile, the installed capacity of wind turbines increased rapidly during the last decade. Historically, wind turbine can not provided APC (Active Power Control, APC) support, because most modern wind turbines are decoupled from the grid by the converter, so they do not inherently respond to fluctuations in grid frequency. However, with the increasing penetration of wind energy on grids, large-scale wind turbines are required to be able to participate in the control tasks as conventional power plants. This means that to fulfill the requirements of the power system regulation at any wind speed condition, the wind power production should follow the command from Transmission System Operator (TSO) [1]. A schematic consisting of the connection of the utility grid, TSO, farm controller, and wind turbines is shown in Fig. 1. The turbine controller need to adjust power production following the power reference dispatched by the wind farm controller. In normal conditions, the wind turbine has to produce maximum power in low wind speed region and remain rated power output in high wind speed region. In down-regulation conditions, wind turbines have to track the derated power reference which is normally below the available power.

The wind turbine down-regulation is carried out to meet the power demand from TSO: maximizing the total wind farm power production and following a limited power demand [2], eg. turbine down-regulation is used to increase the total power production for the entire wind farm by curtailing upwind turbine. The total power production can be achieved when the loss of power from down-regulating upstream wind turbines is smaller than the gain in power

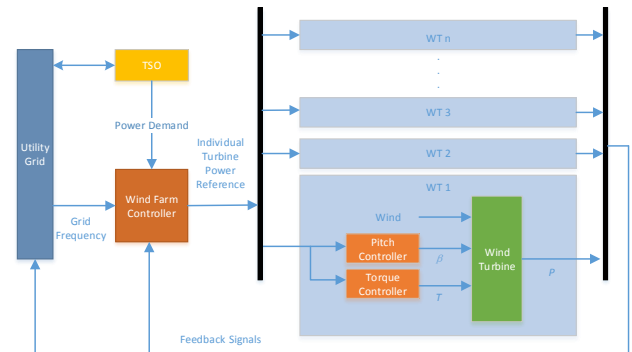


Fig. 1. A schematica showing the interconnection between grid, TSO, wind farm controller and the individual turbine.

from the downwind turbines [3]. On the other hand, turbine down-regulation is carried out to reduce loads on the turbine during abnormal conditions (e.g. when the blade system has some fault which is not serious to shutdown, the appropriate down-regulation strategy should be chosen with lower blade load). For some minor faults, the shutdown of wind turbine will cause an unnecessary downtime and power loss. In some cases, down regulated is better choice to protect damaged components. However, two conditions must be met. Firstly, further damage of fault can be prevented by power down-regulation. Secondly, the severity of the fault mode should be not high. Because the sustained operation with a severe fault will result in the damage of the component itself or in the higher levels of the system.. Therefore, an appropriate down-regulation strategy that can improve the power production of downstream wind turbine is valuable in a faulty condition of turbines.

Down-regulation strategies used to reserve power as ancillary service to support grid frequency. The research in [1] focuses on grid requested down-regulation and gives four objectives of control design. The author suggests that the down-regulation strategy design should avoid stalling and non-monotonic behaviour. Three down-regulation strategies were described and compared in [3]. Those are maximum rotor speed, constant rotor speed and constant tip speed ratio (TSR) respectively. Their benefits and drawbacks are discussed in the article. The research work in [4] gives priority to torque control than pitch control in medium and low wind speed. This strategy can decrease the frequency and amplitude of the pitch system. The author compares the optimization results of these three down-regulation strategies in wind farm power in [5]. The result shows that constant

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rotor speed strategy can produce more power. Because its steady state operating point has a smaller thrust coefficient C_t . In [6], both a centralized and distributed controllers are designed to decrease the fatigue of turbines by varying power reference at the down-regulation mode. In practical applications, the maximum rotor speed is the most used method. The benefit is that it can store kinetic energy in the rotor and respond the grid demand rapidly.

Based on the comparison of four wind turbine down-regulation strategies in different operation region and power reference, appropriate down-regulation strategy is suggested to control wind turbine for active power reference tracking. On one hand, The operation setpoint is chosen mainly considering the fatigue load on key components for down regulated turbine according to turbine health condition. The load on the damaged component should be the smallest. On the other hand, the power output of downstream wind turbine is also improved according to the command from wind farm controller (eg. power maximization).

This paper is organized as follows. Section II describes the normal control strategy with five operation regions. Four different down-regulation strategies will be discussed in section III, where different modes of power reference tracking, and the operation of the control system throughout the various regions of operation will be analyzed. Section IV presents selected simulation results. Loads of the main components based on different strategies are compared. Section V gives conclusions.

II. NORMAL CONTROL STRATEGY

Normally, the main objective of turbine control is to decreasing CoE (Cost of Energy), it means that wind turbines can produce highest possible energy at the lowest possible cost. Based on blade element theory, the wind power through the sweep area of rotor blade P_w is calculated as (1):

$$P_w = \frac{1}{2} \rho \pi R^2 v^3 \quad (1)$$

where ρ is the air density, R is the radius of blade and v is the wind speed. The mechanical power P_m extracted by turbine is calculated as (2):

$$P_m = P_w * C_p(\lambda, \beta) \quad (2)$$

As different turbines have different specifications. The turbine used in this paper is the 5 WM reference turbine (NREL/TP-500-38060), which is not only widely known and used in all kinds of literature, but also available for detailed data for this turbine [7]. The specifications for the turbine can be seen in Table I. The peak of the power coefficient as a function of the tip-speed ratio and blade-pitch angle is found. The aim of wind turbine control is to change the C_p in a different region, by choosing the values of TSR and β . But due to limitations of the rotor speed and power output, it might not always be possible to choose the optimal C_p value. E.g if the wind speed is above the rated wind speed, then the rated power output becomes constant and the power coefficient needs to be

reduced. Due to the limitations, the power curve for the turbine has been split into five regions, the start, low, optimal, high, and rated region according to different wind speed [8]. In each region, different limitations decide how the optimal power coefficient for that region should be found.

A. Region 1: Start Region

Region 1 is a control region before the wind speed reaches the cut-in wind speed. So no power is produced from the wind in this region. Meanwhile, the wind speed increases to accelerate the rotor for turbine start-up.

B. Region 1.5: Low Region

Region 1.5 is a start-up region which is a linear transition between Regions 1 and 2. The low region starts when the wind speed is higher than the cut-in wind speed. The wind speed is so low, that it is very hard to have the optimal TSR. Sometimes the rotational speed would be smaller than the cut-in value. Hence, the TSR must then be chosen in such a way that rotational speed is kept at the minimum value. The pitch is chosen to maximize the power coefficient for the given TSR, which causes the power coefficient to increase for higher wind speeds. The low region ends when the wind speed is high enough for TSR to be set at the value giving the optimal power coefficient.

C. Region 2: Optimal Region

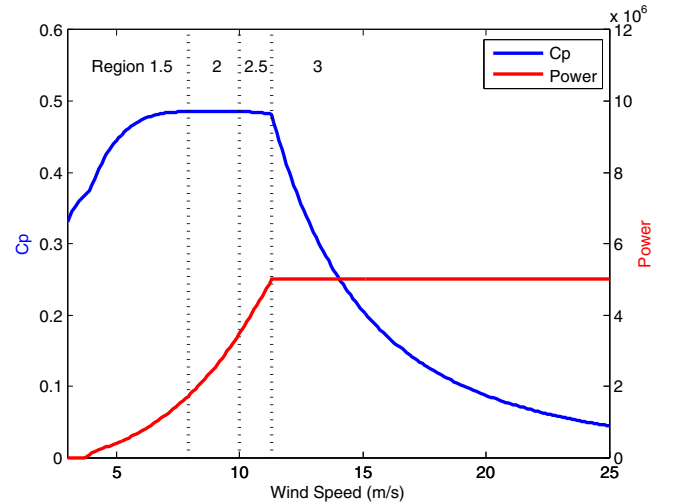


Fig. 2. Power output and C_p for the NREL 5MW turbine in normal case

Region 2 is maximum power point tracking (MPPT) region, meaning there is no limitation on TSR to extract maximum energy from the wind. The peak power coefficient of 0.482 occurred at a tip-speed ratio of 7.55 and pitch angle of 0.0. The optimal region starts when the optimal C_p value can be reached, then it should be maintained where the generator torque is the square of the generator speed. In this region, the power coefficient is kept constant at the peak value, which can be seen in Fig. 2. The pitch and TSR are

constant when the rated rotational speed is reached the TSR need reduce, which also can be seen in Fig. 3. The optimal region ends and the high region starts at this wind speed.

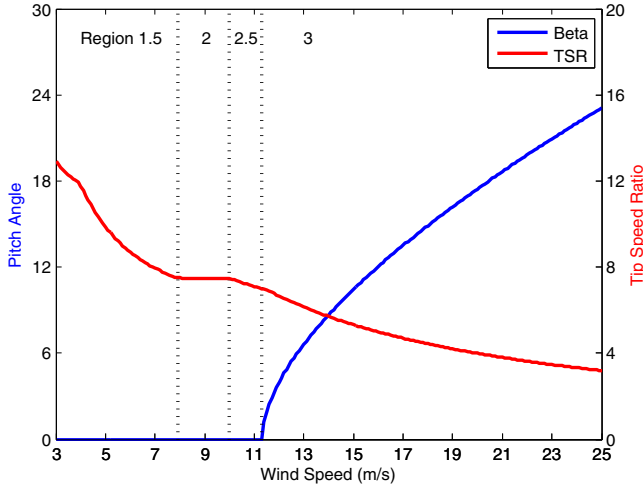


Fig. 3. Pitch angle and TSR for the NREL 5MW turbine in normal case

D. Region 2.5: High Region

Region 2.5 is a transition with a torque slope. Region 2.5 is typically needed to limit tip speed at rated power. Due to the rated rotational speed is limiting the TSR, the optimal C_p can't be reached in this region. Figure 4 shows that the pitch angle is constant and TSR decreased to ensure the best possible power coefficient.

E. Region 3: Rated region

Region 3 is a control region the generator power is held constant at the rated value so that the generator torque is inverse to the generator speed. The rated region starts when the wind speed is high enough for the turbine to reach rated power, the power coefficient has to be reduced in Fig.2. C_p couldn't reach its optimum in previous regions due to the constraint on the TSR, but both the pitch and the TSR can be used to lower C_p in this region. This introduces a degree of freedom on how to choose the pitch and TSR. However, the rotational speed has reached its rated speed in this region and cannot be increased further.

All the four regions define how the turbine operates in different wind speed in the nominal case. The C_p is seen to be kept constant at its optimal value in the region 2, C_p is high in the first three regions, but decreases fast in the region 3 which was obviously due to changes in the TSR and pitch angle. In Fig. 4 the power coefficient is plotted as a function of pitch angle and TSR, when the turbine is operated in the nominal setting without down-regulation for wind speeds between 0 to 25 m/s. The left plot shows how the turbine moves on the C_p curve. For lower wind speeds the TSR is high, due to the cut-in rotational speed. As the wind speed increases the turbine moves along the ridge of the C_p curve to the optimal value which is in the 1.5 region.

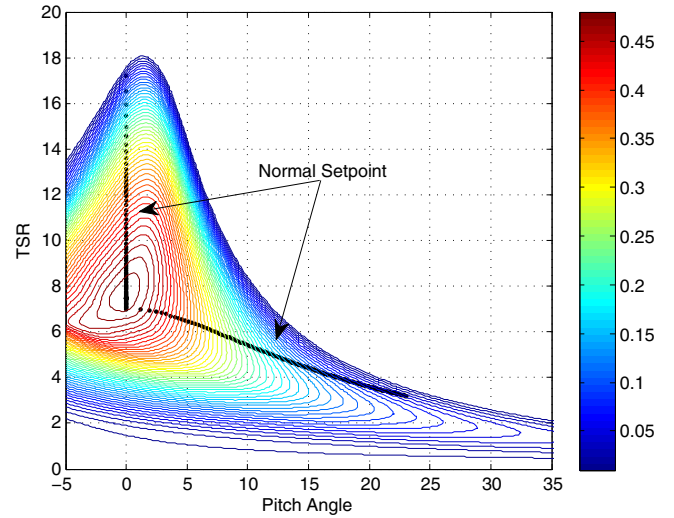


Fig. 4. Normal operation λ and β setpoint without power down-regulation

In region 2 the C_p is kept constant since it is also possible to maintain the peak value. In region 2.5 and region 3 the C_p value is decreased by decreasing the TSR further and increasing the pitch angle.

III. DOWN-REGULATION CONTROL STRATEGIES

In the down-regulation operation, there are cases where the power reference P_{ref} is below the available power P_m . Obviously, there are many choice for wind turbine controller, there is a degree of freedom on how to choose the set point. An suitable C_p value can be obtained by choosing pitch angle and TSR. The TSR is determined by rotational speed and wind speed: $\lambda = R * \omega / v_w$, in which R is the rotor radius.

C_p can be recalculated according to:

$$C_p = \frac{P_{ref}}{P_w} \quad (3)$$

The C_p is decided by the P_{ref} from the wind farm controller,

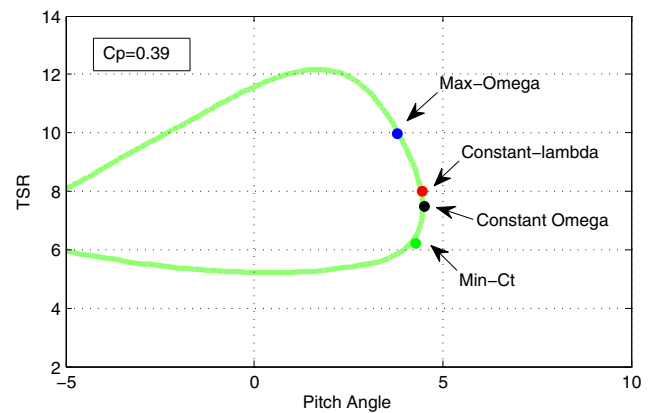


Fig. 5. A contour curve for $C_p = 0.4$, different setpoints show the different down-regulation strategies

but the different method on how to change the pitch angle and rotational speed on the C_p contour determines the different down-regulation strategy.

A. Constant rotational speed (Const- Ω)

In the Const- Ω control strategy, the rotational speed should keep constant after the generated power reached P_{ref} . Therefore λ becomes a unique function of wind speed as $\lambda = R * \Omega / v$. The pitch angle can be found on the C_p contour curve according to the TSR. Normally There are two values for the pitch, but only the one which is in the normal operating side can be chosen. Because the other one is in the stall region. In Fig. 5 the black point shows the setpoint of the TSR and pitch angle [3].

B. Constant tip speed ratio (Const- λ)

In the Const- λ control strategy, the TSR should keep constant after the power reached P_{ref} . So the rotational speed can be calculated by $\Omega = v / R * \lambda$. The pitch angle can be found on the C_p curve in figure 5 according to the TSR. There are also two values for the pitch and only the one which is on the normal operation side can be chosen for turbine controller. In Fig. 5 the red point shows the setpoint of the TSR and the pitch angle of this strategy [3].

C. Maximizing the rotational speed (Max- Ω)

In the Max- Ω control strategy, The rotational speed is always maximized for all operation region and it is only bounded by the rated rotational speed value. In figure 5 the blue point shows the setpoints of the TSR and the pitch angle [3].

D. Minimizing the wake deficit (Min- C_t)

In the Min- C_t control strategy C_t is minimized on all C_p contour curve. the objective of the down-regulation strategy is to decrease the conversion efficiency and get a smaller C_p value than that at normal condition. Although there are numerous operating points at the same C_p value, there must be an operating point make the C_t value minimum. In Fig. 6, the dark blue line is C_p curve at 0.4. Other color lines are C_t curves at different operating points. It can be seen that different operating points correspond to different C_t values. The green-circle point is the minimum C_t operating point. If the down-regulation wind turbine operates at this point, the wind speed deficit will be the smallest.

IV. CASE STUDIES

In this section we present numerical simulation under several critical scenarios. Simulation results for comparing Constant- ω , Constant- λ , Max- ω and Min- C_t strategies are presented. The controller s are implemented in MATLAB based on the NREL 5MW reference turbine [7]. The parameters of wind turbine are shown in Table I.

To choose the appropriate down-regulation strategy for active power reference tracking with different turbine faulty condition. We need to compare four different control strategies and result on fatigue load of main components (eg. Blade, Tower and Drivetrain system) and downwind speed through the typical scenario and simulation in full-range wind operation. The only difference among simulations is the down-regulation strategy. The downwind turbine keeps operating at normal operation.

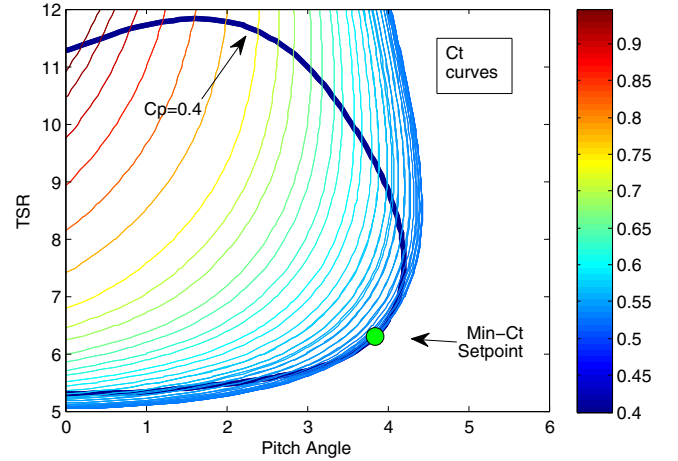


Fig. 6. A contour curve for $C_p = 0.4$ and Min- C_t values

TABLE I
TABLE I. PARAMETERS OF NREL 5MW WIND TURBINE

Parameter	Value
Rated Power	5MW
Rotor Diameter	126m
Min. and Max. Rotor Speed	6.9rpm, 12.1rpm
Cut-in, Rated, Cut-out Wind Speed	3m/s, 11.4m/s, 25m/s
Gearbox Ratio	97:1
Synchronous Frequency	50Hz
Electrical Generator Efficiency	94.4%
Number of Pole-pairs	3

A. Scenario 1 (WF power reference P_{ref} varies with constant wind speed $v=8m/s$ in Region 2)

The upwind turbine is derated from 1.79 to 1.43MW (20% down-regulation degree) and 1MW (42% down-regulation degree). In region 2, The simulation result is presented in Tab. II, and the downwind speed and fatigue load results of four strategies are plotted in Fig. 7.

1) For down-regulation turbine: In Max- ω strategy, the fatigue load on the drivetrain system is the smallest, but the fatigue load on blade and tower are the largest. In Min- C_t strategy, the fatigue load on blade and tower are the smallest, because wind turbine works at the Min- C_t setpoint and has the smallest C_t value than all the other operating points. However the fatigue load on drivetrain system is larger than others.

2) For downwind turbine : Compared with other strategies, the downwind wind speed increases. Meanwhile the power of downwind turbine also increases in Min- C_t strategy.

B. Scenario 2 (Wind speed varies with WF power reference constant $P_{ref}=1.43 MW$ in Regin 2)

This Scenario is also in low wind speed in region 2, the upwind turbine is derated from 1.79 to 1.43 MW (20% down-regulation degree) and the wind speed changes from 8 m/s to 9 m/s. The simulation result is presented in Tab. III. The results of four strategies are plotted in Fig. 8.

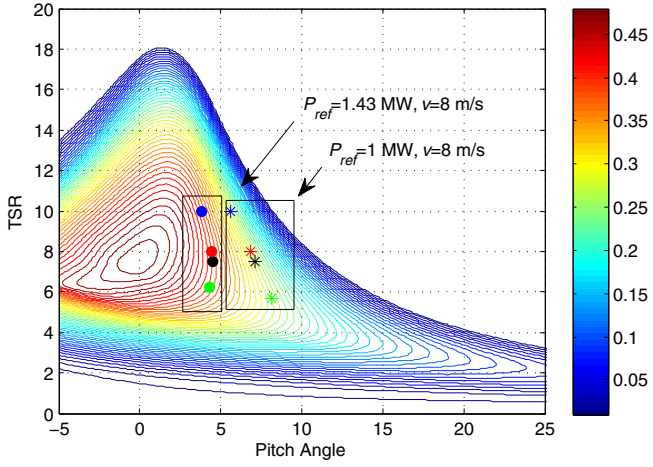


Fig. 7. C_p curve and setpoints as power reference changes for different down-regulation strategies. red is Max- Ω strategy, black is Constant- Ω strategy, blue is Constant- λ strategy, green is Min- C_t strategy.

TABLE II

TABLE II. SIMULATIN RESULTS IN SCENARIO 1

$P_{ref}=1.43\text{MW}, v=8\text{ m/s}$				
Variables	Max- Ω	Constant- Ω	Constant- λ	Min- C_t
V_{down}	6.7134	6.8891	6.8705	6.9179
M_{blade}	1.19e+07	1.06e+07	1.07e+07	1.03e+07
M_{tower}	2.83e+05	2.52e+05	2.55e+05	2.46e+05
M_{shaft}	1.19e+06	1.59e+06	1.49e+06	1.92e+06
$P_{ref}=1\text{ MW}, v=8\text{ m/s}$				
Variables	Max- Ω	Constant- Ω	Constant- λ	Min- C_t
V_{down}	7.2050	7.3089	7.2936	7.3384
M_{blade}	7.95e+06	7.02e+06	7.16e+06	6.75e+06
M_{tower}	1.89e+05	1.67e+05	1.70e+05	1.60e+05
M_{shaft}	8.35e+05	1.11e+06	1.04e+06	1.46e+06

1) *For down-regulation turbine:* The fatigue load on drivetrain is almost the same in Max- ω and Constant- ω strategies. The fatigue load on drivetrain system is the smallest in Max- ω strategy, and the fatigue load on blade and tower are the smallest in Min- C_t strategy. The result is almost the same with Scenario 1.

2) *For downwind turbine :* Although the wind speed changes, it also can be seen that the wake effect from upwind down-regulation turbine is reduced, and the power of downwind turbine is improved in Min- C_t strategy compared with other strategies. The effect of power improvement is related to the degree of increased wind speed.

TABLE III

TABLE IV. SIMULATIN RESULTS IN SCENARIO 2

$P_{ref}=1.43\text{ MW}, v=9\text{ m/s}$				
Variables	Max- Ω	Constant- Ω	Constant- λ	Min- C_t
V_{down}	8.1634	8.2392	8.2016	8.2578
M_{blade}	9.48e+06	8.71e+06	9.10e+06	8.52e+06
M_{tower}	2.25e+05	2.07e+05	2.16e+05	2.03e+05
M_{shaft}	1.19e+06	1.59e+06	1.32e+06	2.03e+06

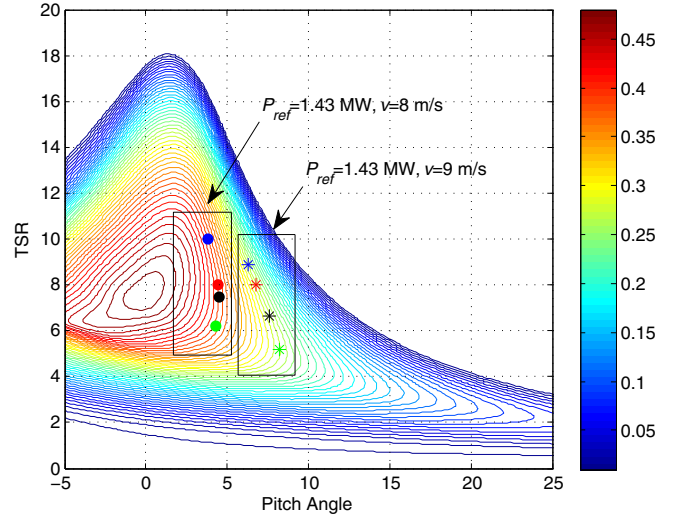


Fig. 8. C_p curve and setpoints as wind speed changes for different down-regulation strategies. red is Max- Ω strategy, black is Constant- Ω strategy, blue is Constant- λ strategy, green is Min- C_t strategy.

C. Scenario 3 (WF power reference P_{ref} varies with varied wind speed in Region 3)

In Region 3, normally, the generator power is held constant at rated value, the wind speed is high enough for the turbine to reach rated power. C_p couldn't reach its optimum in previous regions due to the constraint on the TSR, but the pitch system can be used to lower C_p in this region. Constant- λ strategy gives the same results as the Max- ω strategy in region 3. In Scenario 3, both power reference and wind speed are changing, the upwind turbine is derated from 4 to 3MW and 3.747 MW and wind speed from 13 m/s to 14 m/s. Two setpoints with $P_{ref}=3.747\text{MW}, v=14\text{m/s}$ and $P_{ref}=3\text{MW}, v=13\text{m/s}$ have the same C_p value, so the setpoints are on the same C_p contour which is equal to 0.1896. The simulation results are presented in Tab. IV, and plotted in Fig. 9.

From Fig. 9, we can see that: The setpoints are different although they are in the same C_p contour, the setpoint of Constant- Ω strategy is almost the same with Min- C_t strategy, but the setpoint of Max- Ω strategy is far from them.

1) *For down-regulation turbine:* The fatigue load on drivetrain system is the smallest in Max- ω strategy which is the same result of previous scenario, but the fatigue load on blade and tower are almost the same in other two strategies.

2) *For downwind turbine :* the downwind speed in Min- C_t strategy is similar with the others, so the effect of downwind power improvement is effective in low wind speed region, (eg. region 2) not in high wind speed region (eg. region 3).

V. CONCLUSIONS

In section III we outlined three traditional down-regulation strategies and a novel Min- C_t down-regulation strategy which is more focus on the wake deficit. Three typical scenarios are in the different region and different power reference. The comparison among them is presented in section IV. Max-strategy setpoints are always the farthest from the stall region

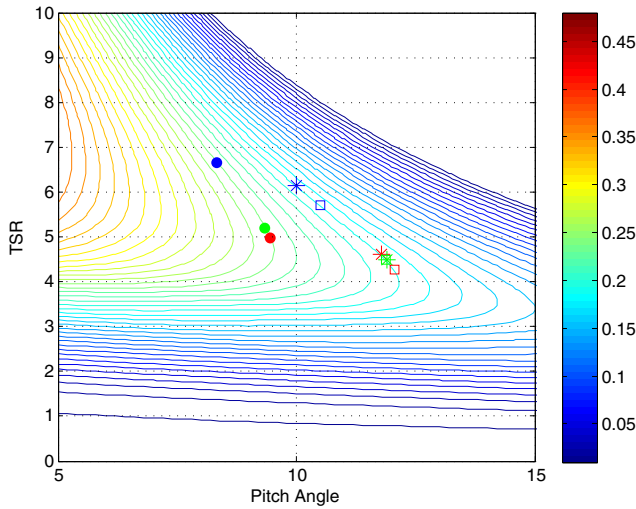


Fig. 9. C_p curve and setpoints for different down-regulation strategies. red is Max- Ω strategy, black is Constant- Ω strategy, blue is Constant- λ strategy, green is Min- C_t strategy. The point is $P_{ref}=4\text{MW}$, $v=13\text{m/s}$; the star is $P_{ref}=3\text{MW}$, $v=13\text{m/s}$; the square is $P_{ref}=3.747\text{MW}$, $v=14\text{m/s}$

TABLE IV

TABLE II. SIMULATIN RESULTS IN SCENARIO 3

$P_{ref}=4\text{MW}$, $v=13\text{ m/s}$			
Variables	Max- Ω / Constant- λ	Constant- Ω	Min- C_t
V_{down}	12.0037	12.0112	12.0161
M_{blade}	1.6647e+07	1.6533e+07	1.6458e+07
M_{tower}	3.9650e+05	3.9376e+05	3.9203e+05
M_{shaft}	3.3446e+06	4.4610e+06	3.9478e+06
$P_{ref}=3\text{ MW}$, $v=13\text{ m/s}$			
Variables	Max- Ω / Constant- λ	Constant- Ω	Min- C_t
V_{down}	12.2751	12.2938	12.2942
M_{blade}	1.2415e+07	1.2114e+07	1.2108e+07
M_{tower}	2.9565e+05	2.8862e+05	2.8845e+05
M_{shaft}	2.5077e+06	3.3452e+06	3.4229e+06
$P_{ref}=3.747\text{ MW}$, $v=14\text{ m/s}$			
Variables	Max- Ω / Constant- λ	Constant- Ω	Min- C_t
V_{down}	13.2278	13.2384	13.2393
M_{blade}	1.4251e+07	1.4068e+07	1.4053e+07
M_{tower}	3.3947e+05	3.3483e+05	3.3450e+05
M_{shaft}	3.1326e+06	4.1787e+06	3.9701e+06

and also the fact that the rotational speed is at its rated value is favorable for inertial response. But it always accompanied by high blade and fatigue load and low drivetrain load, so it is a better choice for the turbine with wake or faulty drivetrain componnets. Otherwise, if the blade or tower are under faulty condtion, this strategy should be prevented. The Min- C_t strategy has high efficiency on the low wind speed region when the power demand from TSO is maximizing the total wind farm power production. However, the effect for downwind speed increasing is almost the same for all down-regulation strategies. On the other hand, the Min- C_t strategy always accompanied by low blade and tower load, so it is effective when the turbine with faulty blades or tower.

This paper only focus on the turbine level down-regulation strategies following the power reference from wind farm controller, the farm level dispatch is also important considering wake effect and turbine health condition.

REFERENCES

- [1] A. S. Deshpande , R. R. Peters. Wind turbine controller design considerations for improved wind farm level curtailment tracking, in Power and Energy Society General Meeting, California, 2012, pp. 1-6.
- [2] T. Knudsen, T. Bak, M. Svenstrup. Survey of wind farm control power and fatigue optimization, Wind Energy, vol. 8, pp. 1333-1351, 2015
- [3] M. Mirzaei, M. Soltani, N. K. Poulsen, et al. Model based active power control of a wind turbine, in American Control Conference (ACC), Portland, 2014, pp. 5037-5042.
- [4] Z. Zhou, C. Wang, Output power curtailment control of variable-speed variable-pitch wind turbine generators, in Power and Energy Engineering Conference (APPEEC), 2014, pp. 1-7.
- [5] M. Mirzaei, T. Gmen, G. Giebel, et al. Turbine control strategies for wind farm power optimization, in American Control Conference (ACC), Chicago, 2015, pp. 1709-1714.
- [6] B Biegel, D Madjidian, V Spudic, et al. Distributed low-complexity controller for wind power plant in derated operation, Control Applications (CCA), in 2013 IEEE International Conference, 2013, pp. 146-151.
- [7] J. Jonkman, S. Butterfield, W. Musial, and G. Scott. Definition of a 5MW reference wind turbine for offshore system development. Technical report, National Renewable Energy Laboratory,, 1617 Cole Boulevard, Golden, Colorado 80401-3393 303-275-3000, 2009.
- [8] S. K. Wessel, Power optimization of wind farms by curtailment of upwind turbines, M.S. thesis, Department of Applied Mathematics and Computer Science, DTU, Denmark, 2015