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Optimizing Control of Wave Energy Converter with Losses and Fatigue in Power Take off

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Abstract: In this work we formulate a control strategy for the control of a Wave Energy Converter (WEC) aiming to maximize power take off taking into account losses in the conversion from mechanical to electrical power. The analysis is based on a point absorber. Maximizing electrical power however results in large variations in the forces or torques in the structure implying large fatigue burden giving reduced life time or requirements for increased dimensions of the structure; therefore there is a tradeoff between harvested energy and demands to the construction. We suggest analysis of this involving a Model Predictive Control (MPC) strategy. Fatigue is usually assessed using the method of rainflow counting and Miner's rule. This model is difficult to include in an MPC formulation, instead we chose to give torque in the shaft of the power take off a quadratic weight in the performance function and evaluate the fatigue from simulated results. The optimization of power take off relies on a model of losses in the power conversion. For the control we apply an approximated friction model. Simulations are performed using time-series of wave forms representing sea states typical for the intended location of the WEC. A Pareto front illustrates obtained mean power versus necessary dimensions due to fatigue. The results are compared with standard resistive and reactive controllers. The results show that the MPC produces more than 25 % more harvested energy than the reactive control for the same requirements for the dimensions.

Keywords: Wave Energy, Model Predictive Control, Convex Approximation, Fatigue.

1. INTRODUCTION

Renewable energy sources in Denmark meet more than 40% of the electricity demand and are predominantly from wind turbines, Danish Energy Agency (2016). Wind turbine power productions are varying and not always in balance with the consumption; to diminish this problem other renewables such as wave energy and solar energy, which are not so strongly correlated to the instantaneous wind speed, are investigated. Several concepts for using wave energy have been tested, Ringwood et al. (2014). In a Danish project* optimization strategies for a wave energy converter (WEC) from Wavestar A/S are developed.

The Wavestar WEC is a multiple point absorber concept, consisting of a number of hemisphere shaped floats attached to a single platform, Wavestar A/S (2016). A prototype with two point absorbers each with a diameter of 5 m, was placed in the North Sea outside Hanstholm, see Fig. 1. Practical experiences were obtained from this large scale prototype. In this work a 6 m point absorber to be used in larger waves is investigated using a simulation

model. This paper focuses on improving control algorithms for maximizing the power take-off taking the losses in the power conversion and the fatigue into consideration. However we aim to build in an element in the controller which tend to reduce fatigue. This may be seen as an element in the effort to make a trade off between lifetime energy production and the building cost of the WEC. A

Fig. 1. Point absorber prototype in Hanstholm, Denmark.

classic control strategy (reactive control) has been used at the 5 m absorber test-site. It is possible to improve

the power take-off using Model Predictive Control (MPC) algorithms as described in Vidal et al. (2012); Ringwood et al. (2014); Hals et al. (2011). In the present work the losses from the point absorber to the grid are included in the control design. Inclusion of losses in the reactive control has been investigated in Strager et al. (2014), where reactive control parameters are optimized without taking constraints into consideration. For PTO's with constraints experimental parameter tuning using simulations has been described in Vidal et al. (2012). Optimal control has been suggested by Nielsen et al. (2013). Handling constraints is a part of the MPC concept which has been suggested by Brekken (2011) and Hals et al. (2011), Richter et al. (2013a) and Richter et al. (2013b) suggests nonlinear MPC. Inclusion of PTO losses has been investigated in Andrade et al. (2014) where generator losses have been included. The total PTO loss (generator, hydraulic) has been investigated by Wavestar giving a loss function. This loss function is difficult to include in a MPC formulation as it is non-convex as also suggested in Andersen et al. (2015). In this work the loss function is approximated by introducing a linear friction term in the model, which makes the MPC optimization problem convex.

The trade off between lifetime harvested energy and plant investments has been treated in Tedeschi and Molinas (2012), where conventional control strategies and sizing of electrical equipment was considered. A recent work, O'Sullivan and Lightbody. (2017) use MPC and the effect of design choices versus energy harvest is incorporated by the use of constraints. Fatigue is an important topic in mechanical systems which are exposed to cyclic loading as the point absorber. Fatigue is normally investigated using rainflow counting and Miners rule, ASTM International (2005). Combining rainflow counting and MPC is not an easy topic, Barradas-Berglind et al. (2015). In the present work rainflow counting is performed for torques on the absorber shaft; in the MPC design it is decided to weight the torques in the performance function and afterwards evaluate the fatigue using rainflow counting on the simulated sequences. Results of the fatigue versus the absorbed energy are presented as a Pareto front.

Section 2 presents the models of the point absorber, the wave climate, the PTO losses and fatigue/lifetime, in section 3 control concepts are developed and in section 4 the results are presented. Finally section 5 is the conclusion.

2. MODEL FOR CONTROL

The prerequisites for design and evaluation of control strategies are the wave climate model, the model of the single point absorber, the model of losses in the power take off and finally how lifetime is affected by control torques.

2.1 Wave climate model

The wave climate is characterized by the significant wave height, the mean wave period and the wave direction. In this investigation we consider the most often appearing waves occurring where a Wave Star WEC could be located in the future, see Table 1, characterized by three typical operational sea states termed S1, S2, S3, and an extreme operational sea state termed S4.

Sea state	Wave height m	Wave period sec	Probability %
S1	0.5	3.5	30
S2	1.5	5.0	45
S3	2.5	6.5	10
S4(extreme)	3.5	8.5	-

Table 1. Average wave climate at the WEC location.

2.2 Model of single point absorber

The dynamic model for a point absorber is well described in the literature see i.e. Falnes (2002) and Ringwood et al. (2014).

Newton's second law for rotation is applied on the float with shaft, see Fig. 2; θ refers to the angle, $\omega = \dot{\theta}$ is angular velocity, $\dot{\omega} = \ddot{\theta}$ is angular acceleration.
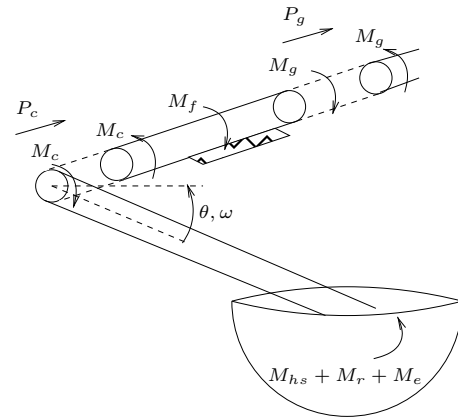

Fig. 2. Torques acting on the single point absorber shaft including torques in the torque model.

$$J_a\ddot{\theta} = M_{hs} + M_r + M_e - M_c, \qquad (1)$$

where J_a is the moment of inertia of the rotating body, M_e is the moment of the external force from waves, M_c is the control moment, M_{hs} is the moment of the hydro-static force, which is a non-linear function of θ. In this work we will use a linear approximation

$$M_{hs} = -k_h\theta; \qquad (2)$$

M_r is the moment of the radiation force, which has been investigated by Cummins (1962), who gave an often used linear description

$$M_r = -J_{h\infty}\dot{\omega} - \int_{-\infty}^{t} h_r(t-\tau)\omega(\tau)d\tau \qquad (3)$$

The first term on the right side of the equation can be combined with the left side of (1) resulting in a total moment of inertia $J = J_a + J_{h\infty}$. In the second term the impulse response $h_r(t)$ is typically calculated numerically using boundary-element potential methods such as done using software by Wamit Inc. (2002).

The radiation moment (3) can be represented in the Laplace domain as a transfer function

$$\mathcal{L}\{\int_{-\infty}^{t} h_r(t-\tau)\omega(\tau)d\tau\} = H_r(s)\omega(s) \qquad (4)$$

Inserting the equations (2) and (4) in (1) gives in the Laplace domain

$$(Js + H_r(s) + \frac{k_h}{s})\omega(s) = Z_i(s)\omega(s)$$
$$= J\omega(0) + M_e(s) - M_c(s) \quad (5)$$

where $Z_i(s)$ is termed the intrinsic impedance of the float and arm system.

A state space model will be used; the convolution integral part of the radiation force moment can be expressed as

$$\dot{x}_r = A_r x_r + B_r \dot{\theta} \quad (6)$$
$$M_r = -J_{h\infty}\ddot{\theta} + C_r x_r + D_r \dot{\theta}$$

The exitation force moment is described using a stochastic model

$$\dot{x}_e = A_e x_e + B_e w_e \quad (7)$$
$$M_e = C_e x_e + v_e$$

where w_e and v_e are white noise processes. This model is assumed to describe M_e over a relatively short time-scale. The order and the parameters of this model describe a certain sea state and are assumed to vary slowly compared to the time distance between two wave peaks. The model of the excitation torque is a combination of a description of the stochastic properties of waves associated with a certain sea state and the hydraulic properties of the float. The former part is generated using a Pierson-Moskowitz spectrum for each of the four sea states used in the simulation test. The properties of the float are determined using Wamit Inc. (2002) and used to convert wave sequences to sequences of torque M_e. This combines to the following state space model

$$\begin{bmatrix} \dot{\omega} \\ \dot{\theta} \\ \dot{x}_r \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} -\frac{D_r}{J} & -\frac{k_h}{J} & -\frac{C_r}{J} & \frac{C_e}{J} \\ 1 & 0 & 0 & 0 \\ B_r & 0 & A_r & 0 \\ 0 & 0 & 0 & A_e \end{bmatrix} \begin{bmatrix} \omega \\ \theta \\ x_r \\ x_e \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \\ 0 \end{bmatrix} M_c + \begin{bmatrix} \frac{1}{J} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_e \end{bmatrix} \begin{bmatrix} v_e \\ w_e \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \omega \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ \theta \\ x_r \\ x_e \end{bmatrix}$$

where 0 in the matrices should be read as zero matrices of appropriate dimensions. In short notation this reads

$$\dot{x} = A_c x + B_{cc} M_c + B_{ce} w$$
$$y = Cx + v \quad (9)$$

2.3 Models of PTO losses

Losses in the PTO have been studied extensively by Wavestar, who have considered losses in the hydraulic system as well as generator and inverter, see Hansen et al. (2013). A control strategy which optimizes harvested energy implies that the direction of power is from the grid to the PTO in some parts of every wave period. In Fig. 2 the grid power P_g and the absorber power P_c are shown and losses are illustrated via a mechanical analogy as the power to a fictitious friction torque M_f.

The loss model which has been developed in Vidal et al. (2015) is given by

$$P_g = \begin{cases} \eta P_c & \text{if } P_c \geq 0, \\ \frac{1}{\eta} P_c & \text{if } P_c < 0. \end{cases} \quad (10)$$

This can also be expressed like

$$P_g = \eta P_c^+ + \frac{1}{\eta} P_c^- \quad (11)$$

where η is the efficiency.

2.4 Models of fatigue and lifetime

When considering fatigue in the design process, lifetime plays an important role. As safety factor for the design for fatigue the so-called Fatigue Design Factor (FDF) value can be used:

$$FDF = \frac{T_F}{T_L} \quad (12)$$

where T_L is the expected lifetime of a component and T_F the lifetime for the design. For offshore wind turbines applications, an FDF equal to 3 is often used, Det Norske Veritas (2013); this value is also used for controller design here. We consider the torsional moment on the shaft which transports the energy to produce electricity but also gives the fatigue loads onto the shaft.

For a certain calculated control torque time series over the expected lifetime of the shaft, T_L, a model of fatigue should be able to calculate the necessary radius of the shaft. Different control algorithms give different time series of control torque, and consequently different necessary shaft radius. When using an aggressive controller which maximizes the harvested power without constraints, the load cycles might be large leading to large needed radius and consequently to high costs as more material is needed.

The resulting maximum shear stress for a cylindrical shaft given a certain torsional moment can be calculated as:

$$\sigma = \frac{M_T r}{I_r} = \frac{M_T r}{\frac{\pi r^4}{2}} = \frac{2M_T}{\pi r^3} \quad (13)$$

where I_r is the moment of inertia of a cylindrical shaft with radius r. It is expected here that the maximum shear stresses appear on the surface of the shaft.

When considering fatigue the number of cycles given a certain stress amplitude are of importance. Therefore the stress time series are transferred into stress amplitudes. Rainflow counting, as described in ASTM International (2005), is used to discretize the load time series into groups/intervals of load amplitudes.

For estimating fatigue of a structural component, so-called SN-curves together with Miners rule are considered. Miners rule uses sequence independent linearized damage accumulation and assumes that fatigue failure occurs when Miner (1945):

$$\sum_{i=1}^{N_i} \frac{n_i}{N_i} = 1 \quad (14)$$

where N_i is the total number of cycles of the i'th stress range leading to fatigue failure and n_i the expected number of cycles at the same stress range during the lifetime

of the device. The design is performed by assuming that at the end of T_F the component breaks due to fatigue.

For fatigue designs SN-curves are often also considered. An SN-curve gives the number of load cycles N leading to failure for a given stress cycle amplitude σ. In this example we consider a so-called linear SN-curve, where the number of load cycles N can be calculated as: $N = K\sigma^{-m}$, where K is the crack intensity factor and m the crack growth parameter. According to Det Norske Veritas (2010) the considered problem belongs to a so-called 'B1' detail (torsional loads on shafts) with $\log K = 12.436$ and $m = 3$.

The limit state equation which defines when structural failure occurs is expressed like

$$1 - \sum_j \sum_k \frac{FDF \cdot T_L \cdot n_{jk}}{K} \sigma_{jk}^m P(S_j) = 0 \qquad (15)$$

This can be used in combination with (13) to find the needed radius of the shaft. n_{jk} is the annual number of load cycles of size σ_{jk} and $P(S_j)$ is the probability of sea state S_j.

3. CONTROL

Reactive control is commonly used for point absorbers. The linear model (5) allows the derivation of conditions for optimal energy absorption for sinusoidal waves and the design of an energy maximizing controller in the frequency domain, Falnes (2002). This controller expresses the control torque as a function of angular velocity

$$M_c(s) = Z_{PTO}(s)\omega(s) \qquad (16)$$

where

$$Z_{PTO}(s) = Z_i^*(s) \qquad (17)$$

Z_{PTO} is the impedance of the load and $Z_i(s)^*$ denotes the complex conjugate of $Z_i(s)$. As shown in Strager et al. (2014) it is also possible to optimize the reactive control scheme taking losses in the PTO, as described in (11), into consideration. With realistic sea states with irregular waves and occasionally with high amplitudes, where the control torque will be constrained, better values of the impedance of the controller can be found by tuning two coefficients of the controller for each sea state.

The two control parameters to specify in the reactive control strategy are B_{reac} (damping coefficient) and K_{reac} (spring coefficient), which correspond respectively to the real and imaginary part of $Z_{PTO}(j\upsilon)$. Sometimes only the real part is used called Resistive control. The value of B_{reac} and K_{reac} used in section 4 are found with a simplex algorithm that maximizes the harvested energy for a given sea state as described in Vidal et al. (2012).

In this paper we will compare the reactive control optimized for the loss described in (11) with a Model Predictive Controller considering the same loss description. The model predictive controller takes constraints in the control torque. Constraints in the deflection of the PTO arm are not considered in this work since they are not experienced to be active. A problem arises in this context because the description results in non-convex optimization problems. To work around this problem we use an alternative PTO loss function described by

$$P_g = Pc - M_f\omega = Pc - b_f\omega^2 \qquad (18)$$

where the losses are introduced via a fictitious friction torque $M_f = b_f\omega$, see Fig. 2. The grid power described by (11) is rewritten here in terms of M_c and ω

$$P_g = \begin{cases} \eta M_c\omega & \text{if } M_c\omega \geq 0, \\ \dfrac{1}{\eta} M_c\omega & \text{if } M_c\omega < 0. \end{cases} \qquad (19)$$

For given efficiency η the friction factor b_f can be optimized such that the two functions are as close as possible within given ranges of ω and M_c. As seen in Fig. 3 the two graphs have the same characteristics and can be brought close together.
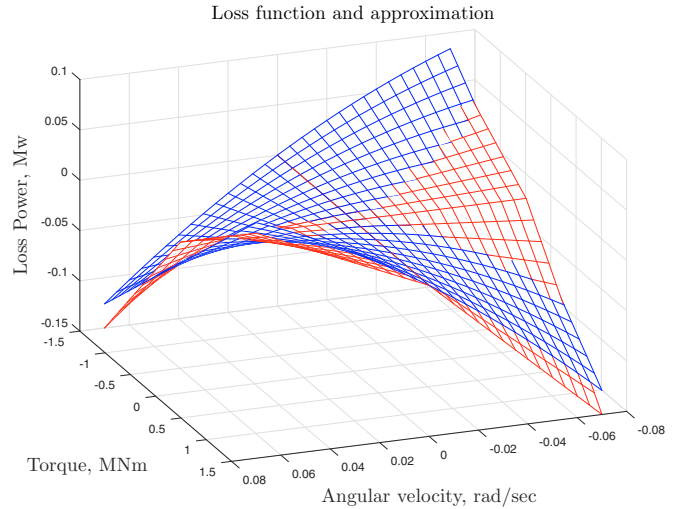

Fig. 3. Comparison of the considered loss model (red) and the used approximation (blue).

The values of the friction factor b_f used in the section 4 have been found iteratively for each sea state.

The performance of the control may be improved by taking the future wave torques into consideration; therefore MPC is investigated.

With the approximate model the power to the grid is given by

$$P_g = M_g\omega = (M_c - b_f\omega)\omega \qquad (20)$$

We now aim to formulate a control algorithm to maximize the harvested energy given as

$$V(t) = \int_t^\infty P_g(\tau)d\tau = \int_t^\infty \omega(\tau)((M_c(\tau) - b_f\omega(\tau))d\tau \qquad (21)$$

In a practical discrete time controller we must approximate this and consider at each sample instant a finite horizon. A first approximation could be

$$V_k = \sum_{i=k}^{k+N} (M_{c,i} - b_f\omega_{i+1})\omega_{i+1} \qquad (22)$$

As pointed out in Åstrøm et al. (1990) a better discrete time approximation of a continuous time performance can be found by integrating the harvested energy over a sampling period. This may lead to a performance function of the form

$$\mathcal{V}_k = \sum_{i=k}^{k+N} (M_{c,i} - b_f\omega_{i+1})\omega_{i+1} - Q\omega_{i+1}^2 - RM_{c,i}^2, \qquad (23)$$

where the term $RM_{c,i}^2$ originates from the discretization approximation. This term also facilitates a possibility to obtain a controller which gives a longer life time by avoiding large torques which would give reason to fatigue. The term $Q\omega_{i+1}^2$ can give the possibility to obtain a controller which avoids large velocities that could also give reason to wear. It could be noticed that the friction factor b_f and the weight factor Q of course can be combined to one factor; in standard quadratic optimization Q is included so in this case the part written into b_f can be interpreted as mechanical friction.

Since the excitation force has a stochastic input, the best we can do is to maximize stochastic expectation for the harvested energy. We will rely on separation of the optimization into

- optimal estimation of the current state including states connected to the external torque,
- optimization of future control torques given mean values of external torques within a prediction horizon as they are predicted from current state.

This separation gives optimal results with linear models; we will use the procedure even when constraints are active. The system described by A, B, Be is used for a Kalman observer where the gain L is calculated using the stochastic properties describing of white process noise affecting the states, w and measurements v. L is calculated from Matlab's kalman function. The observer equations are

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + B\hat{u}_{k-1|k-1} \qquad (24)$$
$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + L(y_{m,k} - C\hat{x}_{k|k-1})$$

y_m is a vector containing the measured angular velocity and angle.

Prediction of future states, Maciejowski (2002):

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + B\hat{u}_{k|k} \qquad (25)$$
$$\hat{x}_{k+2|k} = A^2\hat{x}_{k|k} + AB\hat{u}_{k|k} + B\hat{u}_{k+1|k}$$
$$\vdots$$
$$\hat{x}_{k+N|k} = A^N\hat{x}_{k|k} + A^{N-1}B\hat{u}_{k|k} + \cdots + B\hat{u}_{k+N-1|k}$$

To express the problem in a shorter notation we introduce stacked vectors and matrices

$$\mathcal{X}_k = \begin{bmatrix} \hat{x}_{k+1|k} \\ \vdots \\ \hat{x}_{k+N|k} \end{bmatrix}, \; \mathcal{U}_k = \begin{bmatrix} \hat{u}_{k|k} \\ \vdots \\ \hat{u}_{k+N-1|k} \end{bmatrix}$$

$$\mathcal{X}_k = \mathcal{A}\hat{x}_{k|k} + \mathcal{B}\mathcal{U}_k \qquad (26)$$

where

$$\mathcal{A} = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix} \; \mathcal{B} = \begin{bmatrix} B & 0 & 0 & \cdots & 0 \\ AB & B & 0 & \cdots & 0 \\ \vdots & & & & \\ A^{N-1}B & A^{N-2}B & \cdots & \cdots & B \end{bmatrix} \qquad (27)$$

In the model predictive approach we want to maximize a performance as formulated in (23). We will reformulate this using vectors with stacked values of torques and angular velocities

$$\omega_k = C_\omega x_k, \; C_\omega = [1\ 0\ 0\ 0] \qquad (28)$$

and the stacked vector of velocities

$$\Omega_k = \begin{bmatrix} \hat{\omega}_{k+1|k} \\ \vdots \\ \hat{\omega}_{k+N|k} \end{bmatrix} \qquad (29)$$

With $\mathcal{C} = diag(C_\omega)$ and $\Phi = \mathcal{C}\mathcal{A}$ and $\Gamma = \mathcal{C}\mathcal{B}$ we find

$$\Omega_k = \mathcal{C}(\mathcal{A}\hat{x}_{k|k} + \mathcal{B}\mathcal{U}_k) = \Phi\hat{x}_{k|k} + \Gamma\mathcal{U}_k \qquad (30)$$

We can now rewrite the performance in (23) like

$$\mathcal{V}_k = (\mathcal{U}_k^T - (b_f + Q)\Omega_k^T)\Omega_k - \mathcal{U}_k^T R \mathcal{U}_k \qquad (31)$$

and formulate the predictive control problem like

$$\max_{\mathcal{U}_k}(\mathcal{U}_k^T - (b_f+Q)(\Phi\hat{x}_{k|k}+\Gamma\mathcal{U}_k)^T)(\Phi\hat{x}_{k|k}+\Gamma\mathcal{U}_k)+R\mathcal{U}_k^T\mathcal{U}_k \qquad (32)$$

subject to

$$\mathcal{U}_{min} \leq \mathcal{U}_k \leq \mathcal{U}_{max} \qquad (33)$$

This is a constrained optimization problem with linear and quadratic terms in $\mathcal{U}_k$, which can be solved with standard tools. In the simulations we have used MatLab's quadprog function.

4. RESULTS

In Table 2 results from simulations with a controller aiming to optimize harvested energy are shown. The reactive controller is tuned to optimize for mean power at different sea states taking the PTO losses into account. The power rating MPC/Reactive shows that the MPC is superior with respect to energy production with improvements ranging from 17% to 35%. The last row is a weighted sum of produced energy according to the sea state probability, here it is assumed that only the three dominating sea states S1, S2 and S3 occurs during the year.

Sea State	Reactive Mean Power kW	MPC Mean Power kW	Power Rating MPC/Reactive
S1	1.04	1.35	1.30
S2	15.10	19.59	1.30
S3	33.80	43.66	1.35
S4	50.15	58.48	1.17
Average Year	12.34	15.98	1.30

Table 2. Simulation results from reactive control and model predictive control.

To investigate the controllers influence on fatigue a number of MPC simulations with different weight factors Q and R in (23) are performed. For each simulation the necessary shaft diameter is calculated. The corresponding values for shaft diameter and power are illustrated in Fig. 4. It is seen that there exist a Pareto front where it is impossible to improve one of the parameters keeping the other at a constant value. The hatched area is MPC implementable. Results from the two commonly used control strategies Resistive and Reactive control are indicated in the plot.

5. CONCLUSION

In this work we have formulated a control strategy for control of a WEC aiming to maximize power take off taking
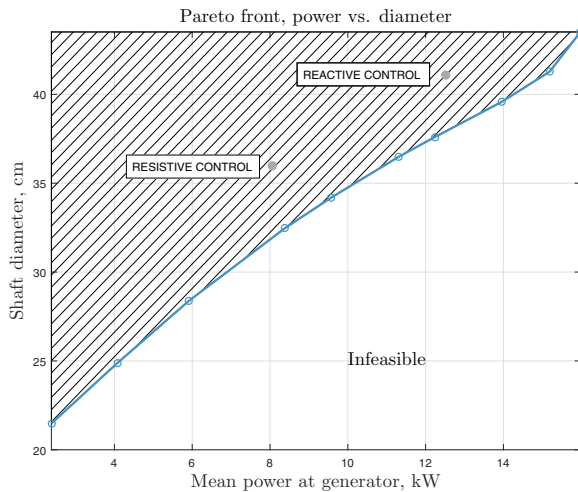
Fig. 4. The calculated MPC-Pareto front with points for
Resistive and Reactive control.

into account losses in the conversion from mechanical to
electrical power. The analysis is based on a WEC from
Wave Star A/S designed as a point absorber. Maximiz-
ing electrical power however results in large variations in
torques in the construction implying a fatigue burden giv-
ing extra requirements for dimensions of the construction.
We suggest an MPC strategy which increases the power
take off with approximately 30 %. To assess fatigue the
method of rainflow counting and Miner's rule is used. To
reduce fatigue load an MPC formulation with quadratic
weight of the shaft torque is used. The resulting fatigue has
been evaluated in the form of demands to shaft dimensions
for a given life time using rainflow counting and Miner's
rule. For the control we have applied a friction model in
the controller, which can be seen as an approximation of
the evaluation model assuming equal efficiency for both
directions of the power. A Pareto front illustrates obtained
mean power versus necessary shaft dimension when torque
is given different weight in the performance function. The
results show that the MPC solution give results with ap-
proximately 30 % more harvested energy than the reactive
controller for the same requirements for dimensions.

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