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Superstars and Renaissance Men: Specialization, Market Size and the Income Distribution

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Abstract
A general equilibrium model of individual specialization is presented in which agents trade off the productivity and price implications of producing a narrower range of goods. Agents with highly specific skills turn out to benefit most from large markets. The model is able to replicate features of the long-term evolution of the US income distribution, with specialization-biased technical change and the increase in employed population playing key roles. Among the results is that, at least along one dimension of ability, the skill premium is increasing in the relative supply of skills.

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1 Introduction

Between 1960 and 2000 income inequality in the United States soared, to levels not seen since the Depression era[1]. The employed population doubled over the same period [Figure 1]. This paper argues that the two phenomena are linked, with the widening income distribution a consequence of greater division of labor. This in turn was facilitated by the increase in employment, as well as by specialization-biased technical change. The theoretical mechanism suggested is novel, and moreover is consistent with another striking fact regarding the US income distribution: its collapse over the first half of the twentieth century[2].

Agents are assumed to trade off the productivity and price implications of specialization, with the result that market size, appropriately defined, is crucial for the return to ability. In thin markets each agent perceives insufficient demand for the output that would flow from a narrow production focus; as a result general ability is most valuable. In larger markets such demand is perceived, agents focus accordingly and the return to specialist skills is high. Market growth, due to increased population or to technical change, will alter the reward structure of the economy in favour of superstars with highly specific skills, while the relative earnings of multi-talented renaissance men will fall.

A feature of the model is the presence of two distinct aspects of ability. Agents j may differ in their fundamental productivity α_j and in their return to specialization δ_j. The former is their total output when producing the maximal range of varieties of good; the latter is the rate at which this output rises as they narrow their range of activity[3]. Having two dimensions of skill allows a parsimonious explanation of the observed U-shaped timepath of earnings inequality. Indeed, one of the forces driving the recent spread in the income distribution - namely specialization-biased technical change, or an increase in average δ_j - may be responsible for the initial fall in inequality.

Most existing work on the US income distribution focuses on the recent rise in wage differentials. This development has attracted understandable attention, coinciding as it has with a broad increase in educational attainment and the demographic bulge caused by the baby boomers. Authors using a supply-and-demand [S-D] labor market framework have therefore inferred a significant increase in the relative demand for characteristics such as education and experience[4]. This skill-biased technical change is thought by some to have its roots in a

[1] In this paper income is synonymous with earnings. See Atkinson (2003) for a discussion of the importance of income sources outside of wages.

[2] Consistent data series covering the whole century are not available. However, Goldin and Katz (1999) conclude from a range of sources that earnings inequality in 1900 exceeded that in 1940, *i.e.* before the 'Great Compression' of mid-century, described by Goldin and Margo (1992) and detailed in Figure 1. Empirical work in this area is discussed further in Section 4.

[3] The 'superstar' and 'renaissance man' assignations are for the high-δ_j and high-α_j agents respectively. Note that I do *not* use labels such as 'specialist' and 'generalist' to distinguish between agents with different skill endowments, as these refer to actions rather than abilities. As will be seen, superstars may in equilibrium produce a broader range of goods, despite having more specialised skills.

[4] See Katz & Autor (2000) and Acemoglu (2002) for comprehensive reviews of the empirical and theoretical literatures that use the S-D framework.

microprocessor-led technological revolution [Autor et al. (1998), Caselli (1999), Dunne at al. (2000)], while others have emphasized more general capital-skill complementarity [Stokey (1996), Krusell et al. (1997)] and changes in organizational structure [Snower (1999)].

Rather than examine industry- or economy-wide labor demand curves, I consider individual specialization decisions. A prior of the model is that each agent's total output increases as he produces a narrower range of the symmetric varieties of good. This may be thought either to be due to pure diseconomies of scope or to arise as individuals focus on tasks at which they are naturally gifted. The other basic assumption is that agents consider their local impact on prices. They play Cournot in their individual production markets but take aggregate variables as given, a general equilibrium approach to modelling imperfect competition also taken by Neary (2003). The consequent tension between high productivity and high price per unit of output is at the core of the model.

Two key results with respect to the income distribution are generated. The first regards the importance of market size, defined as total output deflated by average fundamental productivity. The larger the market, the more important is return to specialization δ_j for one's equilibrium earnings. A corollary is that a rise in the proportion of high-δ_j agents - which will increase market size - will tend to raise such agents' relative income. This is obviously different to the conclusion reached using the S-D approach, and affords a powerful explanation for the recent behavior of the US income distribution.

The second key result concerns technical change. While fundamental productivity progress is distribution-neutral, an increase in the average return to specialization δ has two separate implications. It will lower the cross-sectional return to fundamental productivity α_j, holding market size constant. It will also increase market size and by implication the salience of heterogeneity along the δ_j dimension. This provides the mechanism by which the U-shaped timepath for US inequality may be generated. Secular increase in δ can imply an initial collapse, then a spread in the income distribution as the market grows, with the latter process accelerated by any increase in employed population[5].

[5] One could generate the observed timepath for income inequality simply by assuming that technical change somehow favours those who are initially poor. Take a simple example with two dimensions of skill called 'Brains' and 'Smarts'. Assume that all agents high in Brains are necessarily low in Smarts, and vice versa; assume further that Smarts are initially more important for productivity, but technical progress favours Brains. Then we would observe falling, then rising inequality as the Brainiacs first catch up with, then outstrip the Smarties.

The hypothesis advanced below is more subtle. It is instead that the distributional impact of technical progress will depend on which skill dimension is currently most important; and, moreover, that such progress will alter the relative salience of the two dimensions. Put another way, the simple Brains/Smarts story relies on the negative correlation between Brains and Smarts across agents. I need no such correlation to generate the U-shaped timepath for inequality, and in the simulations below assume the agent-specific α_j and δ_j skill parameters are independently distributed.

1.1 Related Literature

An obvious predecessor is Rosen's (1981) model of superstars, which provides a different mechanism by which some individuals benefit disproportionately from access to large markets. There is also the vast literature on the division of labor beginning with Smith (1776). The emphasis there, in a wide range of contexts, has been on the return to specialization as some sort of Marshallian externality - see for example Romer (1987). Here in contrast the pure productivity benefits of specialization are fully internalized[6]. Closer in spirit is the research programme of Xiaokai Yang and coauthors [Borland and Yang (1991,1992), Yang and Ng (1993), Yang (2001)], who consider the trade off between internal scope diseconomies and transaction costs associated with exchange[7]. The most direct antecedent is Baumgardner (1988a), who as here focuses on the tension between the productivity benefits and price implications of product focus. His is a partial equilibrium framework in which the demand curves facing the individual agents are given, which leads to the result that an increase in the number of producers tends to reduce the degree of specialization. This is opposite to the result in general equilibrium below. Also, producers are identical in Baumgardner's model, while here the focus is on agent heterogeneity and the income distribution.

Mitchell (2001) does consider the distributional impact of specialization. He argues that capital flexibility - its ability to engage in multiple tasks - was U-shaped over the twentieth century. In the first half it declined, which led to increased optimal plant size. This in turn implied greater division of labor, which in his model reduces the skill premium because high-skilled workers are those with a lower fixed cost per task. In the second half of the century, he maintains, capital flexibility increased to opposite effect. He does not explicitly consider the role of market size and population growth. Moreover, in his model increased labor specialization always reduces inequality, whereas a central message here is that the opposite may be true. Finally, the extra degree of freedom afforded here by two dimensions of ability renders unnecessary an assumption of a mid-century reversal in the nature of technical progress. A U-shaped timepath for the spread of earnings will naturally be generated as an economy changes from one where general ability is rewarded to one where specialist skills are most valuable[8].

[6]No new mechanism by which diseconomies of scope arise is offered here. Adam Smith (1776) provided three rationales for their existence: learning-by-doing, time saved switching between tasks and greater ability to invent specialised machinery. Becker and Murphy (1992) suggest that firms are devices to coordinate the actions of workers endowed with disparate, specialised bodies of knowledge; implicit is that workers have limited cognitive ability and are best suited to a narrow range of tasks. Alfred Marshall, closely associated with external economies, also argued for the existence of internal returns to specialisation. Marshall (1910) writes extensively on internal economies of scale, but those he identifies are isomorphic to diseconomies of scope: large-scale production permits specialised machinery to be introduced and individual workers to specialise in their tasks. In a similar vein Young (1928) argues that 'it would be wasteful to make a hammer to drive a single nail'. Increasing returns in a single task, for example due to a fixed cost - as Marshall and Young seem to have had in mind - will imply diseconomies of scope for a given amount of labour. See Edwards and Starr (1987).

[7]See also Kim (1989) and Weitzman (1994).

[8]In Mitchell (2001) population growth would presumably lead to more specialisation and

The paper is organized as follows. Section 2 provides the general model. Section 3 introduces the specific dimensions of ability discussed above, and considers in turn the implications of heterogeneity along each. Section 4 considers empirical evidence and shows how, with heterogeneity along both dimensions at once, the model is able to mimic the long-run timepath of the US income distribution. Section 5 concludes. Derivations of key results are in the Appendix.

2 The model

Consider the following repeated one-period economy. There is a continuum of symmetric varieties of good arranged around the unit circle, and a mass of size n of agents. One may think of each good as a separate task that contributes towards production of a composite good for final consumption. Each agent j is endowed with a single unit of labor, derives utility only from consumption and has homothetic preferences of the form $U_j = \int_{i \in G} u\left(c_j^i\right) di$, where c_j^i is agent j's consumption of variety i, G the variety space and $u(.)$ a homogeneous felicity function with the usual properties.

There are two dimensions to agent j's production decision. He must choose the set $R_j \subset G$ of varieties he will produce, where R_j is a segment of the variety circle, and how much of each variety $i \in R_j$ to bring to market. Agents are distributed evenly around the circle such that there is a mass n at each point. An agent's location indicates his central competence in production, with technology such that R_j must be centred on this variety.

The core assumption of the model is that j's labor productivity $a(r_j)$ is decreasing in r_j, the length of the segment R_j:

$$\begin{aligned} \int_{i \in R_j} x_j^i di &= a_j(r_j) \qquad (1) \\ & \quad a_j(.) > 0, \ a_j{}'(.) < 0 \end{aligned}$$

where x_j^i is j's output of variety i. Denote the absolute elasticity of labor productivity with respect to the product range as $-\frac{r_j}{a_j(r_j)} \frac{da_j(r_j)}{dr_j} = \varepsilon_j^a(r_j) > 0$. I assume throughout that $\varepsilon_j^a(.)$ is less than one and non-decreasing in r_j.

Note that $\varepsilon_j^a(.)$ is indexed by j. I allow the labor productivity function to vary across agents, but require that the distribution of agent types be identical across locations on the variety circle. There will be no tendency for high-ability agents to produce different goods to low-ability ones.

lower the skill premium, the opposite conclusion to that reached here. Mitchell and Holmes (2003) develop the ideas regarding fixed costs and plant size, but without any role for labour specialisation. They argue that capital is to unskilled labour as unskilled labour is to skilled: a substitute with higher setup costs per task. They then argue that increased plant size has an ambiguous effect on the skill premium. While it leads firms to substitute unskilled workers for skilled, it also leads them to replace the unskilled with capital; demand for skilled relative to unskilled labour could then rise or fall with market size. Both Mitchell (2001) and Mitchell and Holmes (2003) focus on plant-based production in manufacturing, while the model here is perhaps more generally applicable across sectors. Neither captures the idea that individuals with highly specific skills may flourish in large markets.

Agents ignore their effects on the aggregate variables but play Cournot in their production markets. When acting as consumers, though, they take all prices as given. Since preferences are homothetic, aggregate consumption behavior can be modelled as resulting from the decisions of a representative agent. The inverse demand function for variety k is then:

$$p^k = \frac{u'\left(x^k\right)}{\int_{i \in G} x^i u'\left(x^i\right) di} Y \tag{2}$$

where Y is aggregate income and $x^i = \int_0^n x_j^i dj$ is total output of variety i, with $x_j^i = 0$ if $i \notin R_j$. Since producers ignore the aggregate impact of placing more output of a variety on the market, both Y and the denominator in (2) are held constant when computing the perceived elasticity of inverse demand:

$$-\frac{x^k}{p^k}\frac{\delta p^k}{\delta x^k} = -\frac{x^k u''\left(x^k\right)}{u'\left(x^k\right)} = \varepsilon^p. \tag{3}$$

Homotheticity of preferences guarantees that ε^p is constant, with standard assumptions on $u(.)$ guaranteeing that $0 < \varepsilon^p \leq 1$.

2.1 The individual production decision given r_j

Agent j seeks to maximize his utility from consumption subject to the production constraint (1). The implied maximand is:

$$\Gamma_j = V\left(P, Y_j\right) - \lambda \left[\int_{i \in R_j} x_j^i di - a_j\left(r_j\right)\right] \tag{4}$$

with $V\left(P, Y_j\right)$ his indirect utility function, P the vector of all variety prices and $Y_j = \int_{i \in R_j} p^i x_j^i di$ his nominal income. Differentiating with respect to x_j^i and using Roy's identity yields the first order conditions given a choice of production range r_j:

$$\begin{aligned} \frac{\delta \Gamma}{\delta x_j^i} &= \frac{\delta V\left(P, Y_j\right)}{\delta Y_j}\left[p^i + \left[x_j^i - c_j^i\right]\frac{\delta p^i}{\delta x^i}\right] - \lambda \\ &= 0 \qquad \forall i \in R_j. \end{aligned} \tag{5}$$

Agents do not simply equate marginal revenue across markets - and thereby maximize income - but take account of the price implications of their behavior for their budget set; hence the appearance of c_j^i on the right-hand-side of (5). Note the use of the Cournot conjecture that $\frac{\delta p^i}{\delta x_j^i} = \frac{\delta p^i}{\delta x^i}$.

2.2 Choice of r_j

I consider only symmetric equilibria, where identical agents make the same choices regarding product range and output per variety. Given identical distributions of agent types across variety locations, this restricts attention to equilibria where total output of each variety is the same: $x^i = x$, $p^i = p$, $\forall i \in G$. The first-order conditions (5) above then imply that in any equilibrium each agent places the same amount of output on each of his markets: $x_j^i = x_j$, $\forall i \in R_j$. This implies the following first-order condition for optimal choice of r_j:

$$\lambda \frac{da(r_j)}{dr_j} - x_j \left[x_j \frac{\delta p}{\delta x} \right] \frac{\delta V(P, Y_j)}{\delta Y_j} = 0 \tag{6}$$

The basic trade-off facing each agent is captured in (6). Recall that λ is the shadow value of an extra unit of output. The first term above is the utility impact of the fall in labor productivity that accompanies an increase in product range. The second term measures the gain, again in terms of utility, of spreading a given amount of output across a greater number of markets and raising the revenue obtained. This trade-off, between the productivity and price effects of specialization, is at the heart of the model.

Finally, use (5) to substitute for λ in (6) to get the condition characterizing each agent's optimal market share. This is given in Result 1, which uses the fact that $c_j^i = c_j = r_j x_j$ in symmetric equilibrium[9].

Result 1 An aggregate equilibrium is characterized by total output per variety of x and each agent j producing the same amount x_j of each variety in his production set R_j:

$$\frac{x_j}{x} = \frac{1}{\varepsilon^p} \cdot \frac{\varepsilon_j^a(r_j)}{1 + \varepsilon_j^a(r_j)[1 - r_j]} \qquad \forall j. \tag{7}$$

Equation (7) implicitly defines agent j's optimal market share and product range, given the output of others. ■

It is clear that, since optimality involves a target for j's market share, j's specialization decision depends on specialization decisions elsewhere. The division of labor is limited by the extent of the market [Smith (1776)], which is limited in turn by the division of labor [Young (1928)]. The following corollary is immediate.

Corollary 1.1 Suppose all agents are identical so that $a_j(r_j) = a(r_j)$, $\forall j$, and that the elasticity of productivity with respect to the product range is constant: $\varepsilon_j^a(r_j) = \varepsilon^a$. Then each agent chooses $r^j = r$, where:

$$r = \frac{\varepsilon^p}{n + \varepsilon^p} \cdot \frac{1 + \varepsilon^a}{\varepsilon^a}. \tag{8}$$

Per capita output is given by $\frac{x}{n} = a(r)$, with each agent's market share $\frac{x_j}{x} = \frac{1}{rn}$. □

[9] Note that Γ_j is in fact discontinuous at equilibrium r_j,since if j increases his product range any further he goes from markets with output of $(x - x_j)$ to ones with output of x in his absence. There may therefore be other equilibria in the neighbourhood of that defined by Result 1. I focus throughout on equilibria where (6) is satisfied exactly for all agents.

The situation with identical agents and constant ε^a reveals some more general predictions of the model. The more sensitive are prices to output changes or the less sensitive is productivity to increases in product range, the less specialized will agents be in equilibrium. The greater the mass of agents n, the greater the degree of specialization and the higher the per capita output level - the higher is n, the smaller is each agent relative to the aggregate economy and the closer he comes to being a price-taker. As a result he perceives that he can concentrate his productive efforts on a few markets without overly depressing prices.

Note that aggregate increasing returns arise in this economy in the absence of any explicit externalities such as technological spillovers. Agents interact only through aggregate demand. The equilibrium is inefficient, as each agent ignores the effects of his own specialization decision on those of others; efficiency would require each agent specializing completely, with $r_j \to 0$. The model exhibits strategic complementarities as defined by Cooper and John (1988).

3 Market size, technical change and the return to ability

I introduce potential heterogeneity along two dimensions and assume agent j's labor productivity function takes the following form:

$$a_j(r_j) = \begin{cases} f_j\left(\frac{r_j}{1-r_j}\right) & r_j \in (0, \frac{1}{2}] \\ 0 & r_j > \frac{1}{2} \end{cases} \tag{9}$$

where the agent-specific function $f_j(.)$ is increasing and homogeneous of degree $-d_j$ in its argument, with $d_j < 1$. Let the fundamental productivity parameter $\alpha_j = f_j(1)$ denote j's total output at the maximal product range. To simplify notation define $\delta_j = \frac{d_j}{1+d_j} \in \left(0, \frac{1}{2}\right)$; the return to specialization δ_j thus indexes the rate at which output increases as j narrows his production locus. There is potential cross-sectional variation along both dimensions of ability: $\alpha_j = \lambda_j \alpha$ and $\delta_j = \mu_j \delta$, where α and δ are the respective economy-wide averages and λ_j and μ_j are agent-specific constants.

For the rest of the paper I assume that preferences are Cobb-Douglas, so that $\varepsilon^p = 1$. Substituting $\varepsilon_j^a(r_j) = \frac{\delta_j}{1-\delta_j} \cdot \frac{1}{1-r_j}$ into (7) we have an equation that implicitly defines agent j's optimal market share, given the output of other agents:

$$\frac{x_j}{x} = \delta_j \frac{1}{1-r_j}. \tag{10}$$

The following result characterizes aggregate equilibrium when agents' market shares satisfy (10).

Result 2 Assume $\delta n > \frac{\lambda_j}{\mu_j}, \forall j$. With technology given by (9) total equilibrium output x is implicitly characterized by:

$$1 = \int_0^n \delta_j \left[\frac{\delta_j x}{\alpha_j}\right]^{\delta_j - 1} dj = \int_0^n \mu_j \delta \left[\frac{\mu_j \delta}{\lambda_j} \cdot \frac{x}{\alpha}\right]^{\mu_j \delta - 1} dj \tag{11}$$

with each agent's total output and choice of product range in turn given by:

$$a_j(r_j) = \lambda_j \alpha \left[\frac{\mu_j \delta}{\lambda_j} \cdot \frac{x}{\alpha}\right]^{\mu_j \delta} \tag{12}$$

$$\frac{1}{r_j} = 1 + \left[\frac{\mu_j \delta}{\lambda_j} \cdot \frac{x}{\alpha}\right]^{1-\mu_j \delta}. \tag{13}$$

The assumption on δn guarantees that all agents choose an internal product range $r_j \in \left(0, \frac{1}{2}\right)$. ■

Without further assumptions regarding the distributions of the λ_j and μ_j, (11) does not readily yield an explicit solution for $\frac{x}{n}$. It is simple nonetheless to confirm, absent any such assumptions, that per capita output is increasing in α, δ and n. Once again there are increasing returns in the aggregate: in the absence of heterogeneity - with $\lambda_j = \mu_j = 1, \forall j$ - equilibrium output per head is $\frac{x}{n} = \alpha [\delta n]^{\frac{\delta}{1-\delta}}$.

The following corollary to Result 2 provides an expression for the skill premium.

Corollary 2.1 Let $\tau(h,l) = \frac{r_h x_h}{r_l x_l}$ be the equilibrium income ratio of two agents h and l. Then:

$$\tau(h,l) = \left[\frac{\lambda_h}{\lambda_l}\right]^{1-\mu_l \delta} \left[\frac{\mu_h}{\mu_l}\right]^{\mu_l \delta} \left[\frac{\mu_h \delta}{\lambda_h} \cdot \frac{x}{\alpha}\right]^{[\mu_h - \mu_l]\delta} \tag{14}$$

In the rest of the paper I typically assume that h is the more productive agent in equilibrium, so that $\tau(h,l) > 1$. □

Of particular relevance for $\tau(h,l)$ is $\frac{x}{\alpha}$, total economy output deflated by average fundamental productivity. One may think of this as a market size term. Again from Result 2 we have the following:

Corollary 2.2 Equilibrium market size $\frac{x}{\alpha}$ is increasing in the number of agents n and the return to specialization δ, but orthogonal to fundamental productivity α. Total output x is linearly increasing in α. □

The orthogonality of equilibrium $\frac{x}{\alpha}$ to α can be seen from (11), which implicitly gives a unique solution for $\frac{x}{\alpha}$ in terms of the δ, λ_j and μ_j parameters only. As a result the specialization decision and therefore $\tau(h,l)$ are also independent of α. Holding constant choices of product range, a change in α does not alter the ratio of an individual agent's productive capacity to that of the economy as a whole. With homothetic preferences it is this ratio that is key for the specialization decision, since under such preferences the elasticity of inverse demand is independent of the scale of the economy.

We therefore have the first result of interest with respect to the skill premium:

Corollary 2.3 Changes in economy-wide fundamental productivity α do not affect $\tau(h,l)$. □

I postpone further analysis of (14), as it is instructive first to consider the determinants of $\tau(h,l)$ with each dimension of heterogeneity shut down in turn.

3.1 Renaissance men

Suppose agents differ only in their fundamental productivity, with $\mu_j = 1$, $\forall j$, so that δ is the common return to specialization. Explicit solutions for output per head, $a(r_j)$ and r_j may then be retrieved from Result 2:

$$\frac{x}{n} = \widetilde{\lambda}\alpha [\delta n]^{\frac{\delta}{1-\delta}} \tag{15}$$

$$a(r_j) = \lambda_j^{1-\delta}\widetilde{\lambda}^{\delta}\alpha [\delta n]^{\frac{\delta}{1-\delta}} \tag{16}$$

$$\frac{1}{r_j} = 1 + \left[\frac{\lambda_j}{\widetilde{\lambda}}\right]^{\delta-1}\delta n \tag{17}$$

where $\widetilde{\lambda} = \left[\frac{1}{n}\int_o^n \lambda_j^{1-\delta}dj\right]^{\frac{1}{1-\delta}}$. The skill premium is therefore:

$$\tau(h,l) = \left[\frac{\lambda_h}{\lambda_l}\right]^{1-\delta} > 1 \Leftrightarrow \lambda_h > \lambda_l. \tag{18}$$

Unsurprisingly, high-λj agents produce more in equilibrium. This is despite being less specialized than low-λ_j agents, as can be seen from (17). The greater diversification of highly productive agents is easily understood: if they were to focus their output on only a few markets, the downward pressure on prices would be larger than for the less productive. High-λ_j agents therefore spread their talents more thinly. They are equilibrium generalists, despite sharing the return to specialization δ with low-λ_j agents.

It is clear from (14) that, with δ common, $\tau(j,k)$ is independent of market size $\frac{x}{\alpha}$. This is reflected in the absence of n from (18) above, the intuition for which is as follows. Each agent has a target market share given by (10). As the mass of agents n rises, the output that an individual places on a given market in order to hit this target market share also rises. When agents share the same return to specialization, this move towards greater output on fewer markets favours neither disproportionately. To see this, note that the homogeneity of $f_j(.)$ means one may write $r_j\varepsilon_j^a(r_j) = g_j(a_j(r_j))$, where $g_j(.)$ is a function homogeneous of degree $\frac{1-\mu_j\delta}{\mu_j\delta}$. Then use (10) to write the income ratio for two agents h and l as:

$$\tau(h,l) = \frac{r_h x_h}{r_l x_l} = \frac{1-\mu_h\delta}{1-\mu_l\delta}\cdot\frac{g_h(a_h(r_h))}{g_l(a_l(r_l))} \tag{19}$$

$$\Longleftrightarrow \frac{g_h(\tau(h,l))}{\tau(h,l)} = g_l(1) \qquad iff\ \mu_h = \mu_l. \tag{20}$$

The second line (20) holds only if $\mu_h = \mu_l$, in which case it implicitly defines a unique $\tau(h,l)$ independent of the scale of $a_h(r_h)$ and $a_l(r_l)$. If market size rises due to an increase in n, implying greater specialization and output per head, the skill premium is unaffected so long as $g_h(.)$ and $g_l(.)$ are homogeneous of the same degree[10].

The next two subsections consider factors that do influence the skill premium when $\mu_h = \mu_l = 1$.

3.1.1 Geographical sorting and inequality

In Appendix 6.3 I consider the effect of costly migration between distinct local economies. Agents prefer economies characterized either by higher n or greater average productivity, as there they are closer to being price-takers and more able to specialize. The key assumption made is of a fixed cost of migration. This leads to high-λ_j agents being more likely to migrate; while their relative income is unchanged by an increase in market size, their absolute gain is greater.

This sorting of high-λ_j agents into large markets has two implications. First, it increases the incomes both of the high-λ_j migrants and of the (high- or low-λ_j) agents already incumbent in migrants' destinations. Second, it reduces the productivity of the low-λ_j agents left behind, as their markets become smaller and they specialize less - the 'ghost-town' effect. Absent strong assumptions on the initial location patterns of agent types, there is thus a tendency towards increased income dispersion as an economy urbanizes, as the US continued to do over the course of the twentieth century.

This broad hypothesis has empirical support. Glaeser and Mare (2001) find that cities tend to be inhabited by the relatively highly-skilled, and that urban

[10] This may seem a knife-edge result, dependent on the particular form taken by $a_j(r_j)$ in (9). If one assumed that agents ignored the price implications of diversification for their budget set, and instead simply maximised income, the equilibrium condition (7) would not have the correction $[1-r_j]$ for consumption on the right-hand-side, and would instead read: $\frac{x_j}{x} = \frac{1}{\varepsilon^p}\frac{\varepsilon^a_j(r_j)}{1+\varepsilon^a_j(r_j)}$. Then, were (9) instead a constant-elasticity function of the form $a_j(r_j) = \alpha_j[r_j]^{-\frac{\widehat{\delta}_j}{1-\widehat{\delta}_j}}$, one could again write $\tau(h,l) = \frac{1-\widehat{\delta}_h}{1-\widehat{\delta}_l}\cdot\frac{r_h\varepsilon^a_h(r_h)}{r_l\varepsilon^a_l(r_l)} = \frac{\widehat{g}_h(a_h(r_h))}{\widehat{g}_l(a_l(r_l))}$. Similar reasoning would apply: with $\widehat{\delta}_h = \widehat{\delta}_l$, the functions $\widehat{g}_h(.)$ and $\widehat{g}_l(.)$ would be homogeneous of the same degree and the skill premium would be independent of the size of the market. In general, one can regard the specification in (9) as 'correcting' for the consumption-price impact of diversification. The results generated are analogous to those when $\varepsilon^a_j(r_j)$ is a constant and agents are naive income-maximisers.

productivity remains higher than non-urban even when skill levels are controlled for. Garicano and Hubbard (2004) report that lawyers are more likely to work in hierarchical firms when the local market is larger; they argue that this is because such firm structures facilitate specialization. Baumgardner (1988b) finds that individual physicians tend to offer a narrower range of services in larger local markets. Duranton and Jayet (2004) provide evidence from France that scarce occupations are over-represented in large cities. Finally, a feature of the widening income distribution in the US over the late twentieth century was the falling real wages of those in the lower tail, consistent with the ghost-town effect mentioned above.

3.1.2 Change in δ and inequality

Consider again a unitary economy. Apart from explicit skill-biased technical progress - change in $\frac{\lambda_j}{\lambda_k}$ - the remaining source of variation in $\tau(j,k)$ is variation in δ. From (18) it is clear that a falling return to specialization would imply a gradual spread in the income distribution, one with the fractal quality observed in US data. We saw from (17) that relatively low-skilled, low-λ_j agents are more specialized in equilibrium - the sole 'advantage' of being low-skilled. A fall in δ erodes this advantage relative to the high-λ_j renaissance men[11],as then in equilibrium all agents are less specialized and the fundamental productivity parameters relatively more important. This result mirrors that in Mitchell (2001), where inequality also varied inversely with the division of labor.

3.2 Superstars

Now consider cross-sectional variation only in the return to specialization $\mu_j\delta$, with fundamental productivity identical across agents so that $\lambda_j = 1$, $\alpha_j = \alpha$, $\forall j$. Equilibrium $\frac{xg}{\alpha}$ is implicitly defined by:

$$1 = \int_0^n \mu_j\delta \left[\mu_j\delta \cdot \frac{x}{\alpha}\right]^{\mu_j\delta-1} dj. \tag{21}$$

The income ratio is now:

$$\tau(h,l) = \left[\frac{\mu_h}{\mu_l}\right]^{\mu_l\delta} \left[\mu_h\delta \cdot \frac{x}{\alpha}\right]^{[\mu_h-\mu_l]\delta} > 1 \Leftrightarrow \mu_h > \mu_l. \tag{22}$$

The skill premium afforded high-μ_j agents will unambiguously rise with market size $\frac{x}{\alpha}$. This is the superstars phenomenon. Agents with a particularly high return to specialization are the most restricted by thin, low-$\frac{x}{\alpha}$ markets, where there is insufficient demand for the output that would result if they were to focus on their core competence. In large markets there is such demand and

[11] It is worth emphasising that a fall in δ in no way implies technological regress. As long as $a^j\left(r^j\right)$ does not fall for any given r_j, it is reasonable to consider the possibility that δ falls over time as α rises. The parameter indexes agents' productivity when focused relative to their productivity when diversified, and conjectures regarding δ's evolution are conjectures about the *nature* of technical progress.

they focus accordingly. High-μ_j superstars have the most to gain from a move towards greater output of fewer varieties[12]. In the terms of the discussion in Section 3.1, the $g_h(.)$ and $g_l(.)$ functions are no longer homogeneous of the same degree.

If agents differ only in their returns to specialization, then upward trends in either n or in δ, each of which will raise $\frac{x}{\alpha}$, will imply a spread of the income distribution[13]. This is quite different to the case outlined in Section 3.1, where the sole source of cross-sectional variation was the fundamental productivity parameter λ_j. Below I consider the implications for the income distribution of heterogeneity along both dimensions simultaneously. The results of the current section are first summarized.

Summary In a unitary economy the effects of n and δ on the skill premium depend crucially on the nature of heterogeneity:

- If agents vary only in their fundamental productivity parameter $\lambda_j\alpha$, then $\tau(h,l)$ is unaffected by n, decreasing in δ and independent of α.
- If agents vary only in their return to specialization $\mu_j\delta$, then $\tau(h,l)$ is increasing in both n and δ but remains independent of α.

Meanwhile sorting may raise income inequality, as high-ability agents move to urban areas to the detriment of those left behind.

4 Multidimensional heterogeneity and the US income distribution

There is much indirect evidence for the ideas contained above. In terms of a causal relationship between productivity and specialization, it seems incontrovertible that individuals have particular natural talents and are better at some activities than others; as for pure diseconomies of scope, Gollop (1997) finds that decreasing product heterogeneity at plant level is second only to technical change in accounting for productivity growth in manufacturing[14]. With respect to a link between market size and the extent of individual specialization, there

[12] The arguments made in Section regarding the effect of geographical sorting can be made here. With fixed migration costs one would expect high-μ_j agents to be most likely to cluster in urban areas, thus increasing their productivity advantage in equilibrium.

[13] Inspection of (22) reveals that higher δ in fact raises $\tau(h,l)$ for a given market size. One might therefore think that using the specification $\delta_j = \mu_j\delta$ and consider rising δ is rather loading the dice in favour of the high-μ_j agents. It is easy to show that $\tau(h,l)$ remains an increasing function of δ under the alternative specification $\delta_j = \delta + \varepsilon_j$, with ε_j the idiosyncratic ability term.

[14] This is likely to understate the role of specialisation, as Gollop includes economies of scale as a determinant of plant productivity. While diseconomies of scope are conceptually distinct from scale economies, the two may be closely related - see footnote 6. An argument could also be made that technical change is induced by specialisation, for example due to learning-by-doing [Arrow (1962)].

is the evidence of Baumgardner (1988b), Duranton and Jayet (2004) and Garicano and Hubbard (2004) cited above. At a more aggregate level, Baldwin et al. (2001) document a shift towards greater firm- and plant-level specialization in Canadian manufacturing since the 1970s, and report that the shift was greatest in plants which moved most strongly into export markets following the 1989 Free Trade Agreement with the United States.

Evidence for a link between market size and wage differentials is perhaps less compelling, if only due to empirical difficulties. There is the developing country phenomenon of increased inequality following trade liberalization[15], but there the water is muddied by the fact that there are often concurrent labor market reforms. Factor content studies of the impact of increased trade on the wages of the low-skilled in the United States typically conclude that there is a most a small effect [Freeman (1995)], although these have been criticized on the grounds that they do not capture price effects unrelated to trade volumes [Lawrence (1994), Deardorff and Hakura (1994)]. In this paper the market size mechanism works entirely through prices, and lower trade barriers would potentially affect the earnings distribution even if actual trade flows were negligible. Moreover, most studies of the distributional impact of trade work in the S-D framework discussed in the Introduction, and as such make implicit identifying assumptions that would be inappropriate here[16].

Direct empirical tests of the results above are made difficult by the fact that it is not clear to which observable variables the separate ability dimensions correspond[17]. The range of tasks r_j in which each agent is engaged is also difficult to measure. Even if rough estimates of r_j were available, inferring the agent-specific λ_j and μ_j parameters would not be straightforward, as it is not always the case that high-μ_j agents are the most specialized in equilibrium - see the discussion in Appendix 6.2.2.

I instead turn to simulation, and argue that the model above is able to provide a parsimonious explanation for the long-term behavior of the US earnings distribution. In particular it is able to reproduce the distribution's initial collapse and subsequent expansion over the twentieth century, as reported by Goldin and Margo (1992) and Goldin and Katz (1999). The former take as their starting point the 'Great Compression' of the US wage structure over the 1940s, during which the log-wage-differential between the 90th and 10th percentiles fell from 1.45 to 1.18 and the variance of log wages fell from 0.325 to 0.259. The following passage is reproduced from Goldin and Margo:

> The wage structure...has been on a long-run roller-coaster ride since

[15] Attanasio et al. (2004) compare the distributional impact of the Colombian trade reforms of the late 1980s and early 1990s with the earlier experience in Mexico. Pavcnik et al. (2002) consider the case of Brazil.

[16] For example, Pavcnik et al. (2002) find that the rise in the skill premium in Brazil following trade liberalisation was not due to Hecksher-Ohlin-type adjustments. They necessarily conclude that the trade reforms must have induced skill-biased technical change.

[17] Both broad-based and narrow ability are presumably useful for educational attainment, although one could argue that the latter becomes more important as an individual proceeds into higher education. Similarly, it is not clear *a priori* the proportions in which δ_j and α_j rise with experience.

> 1940 - with inequality falling precipitously during the 1940s, rising slightly during the 1950s and 1960s, and finally increasing sharply from the 1970s. The statistical properties of the initial fall and recent rise are, in many ways, mirror images of each other. Not only did the between-group variance change in comparable but opposite ways, but the decrease in the within-group variance in the 1940s was of similar magnitude to the increase in the post-1960s period. [Goldin and Margo (1992), p.3]

Goldin and Katz focus on the pre-1940s period, and use a variety of sources to show that US wage dispersion at the beginning of the century was even greater than at the end of the 1930s. They infer a narrowing of the wage structure that probably pre-dated the early 1920s, and that was more than twice as large as the 1940s compression in terms of the 90-10 log wage differential. The overall picture then, is of two significant compressions of the wage distribution that occurred around the two World Wars, followed by a gradual increase in inequality that accelerated when the baby-boomers approached their prime.

How to explain this in the current framework? Section 3.1.1 above suggested that geographical sorting could exacerbate inequality. However, such sorting cannot explain the collapse in the income distribution over the first half of the twentieth century, while the US urbanization process was in full flow. An alternative explanation provided by Sections 3.1 and 3.1.2 is the existence of heterogeneity along a single dimension, coupled with a mid-century reversal in the nature of technical change, as in Mitchell (2001). In the model here, assuming a hump-shaped path for δ would deliver the observed U-shaped path for inequality if the (relative) fundamental productivity parameters λ_j were the sole source of difference. One could equally suppose heterogeneity lay only in the μ_j parameters governing individual (relative) returns to specialization, in which case a U-shaped path for δ would generate the same result.

It is not necessary to invoke any such qualitative break in the path for δ in order to explain falling and then rising inequality. Assume instead cross-sectional variation both in fundamental productivity $\lambda_j\alpha$ and in the return to specialization $\mu_j\delta$, with the expression for the income ratio of two agents h and l reproduced here for convenience:

$$\tau(h,l) = \left[\frac{\lambda_h}{\lambda_l}\right]^{1-\mu_l\delta} \left[\frac{\mu_h}{\mu_l}\right]^{\mu_l\delta} \left[\frac{\mu_h\delta}{\lambda_h}\cdot\frac{x}{\alpha}\right]^{[\mu_h-\mu_l]\delta}. \tag{23}$$

Now consider the impact of secular, specialization-biased technical change, bearing in mind the conclusions of the previous section. The implications for the income distribution depend on the source, at any point in time, of income inequality. Suppose that initially the upper tiers of the distribution are comprised of high-λ_j renaissance men; this could be because market size $\frac{x}{\alpha}$ is small, agents are not specialized and so variation along the μ_j dimension is less important for relative productivity. Then by the results of Section 3.1.2 an increase in δ should narrow the income distribution, favouring as it does the low-λ_j agents. There is a second effect: as δ rises so will $\frac{x}{\alpha}$, and the μ_j dimension becomes the more salient. As this continues the high-μ_j superstars will eventually constitute the rich, with further increases in δ serving to widen their productivity advantage in equilibrium. It is easy to see how this could generate the required timepath for inequality.

I also investigate the effect of varying n, which will influence the wage distribution via its impact on market size. It is notable that the two periods of wage compression were also periods of conflict when many working-age males were not in the labor force and global trade was disrupted; and that the fastest rise in wage inequality came after the baby boom[18]. These discrete demographic events occurred against a background of consistent US urbanization[19] and a gradual dismantling of global trade barriers, both of which may be thought of as raising n and in turn $\frac{x}{\alpha}$.

4.1 Simulations

Unless otherwise stated, all simulations assume that the idiosyncratic productivity parameters μ_j and λ_j are uniformly and independently distributed, over $[0.75, 1.25]$ and $[0.2, 1.8]$ respectively. Aggregate fundamental productivity α is normalized to one. Figure 2 shows how per capita output increases in δ and in n. Figure 3 graphs the log-wage ratio of the 75th and 25th percentiles against the same variables; inequality is increasing in n and U-shaped in δ as predicted. The same is true when other measures of dispersion such as the coefficient of variation and alternative percentile ranges are used.

The same pattern is displayed in Figure 4, which simulates the log-wage ratio of the 90th and 10th percentiles from 1900 to 2000. Secular specialization-biased technical progress is assumed, with δ increasing by 0.03 each decade. Two timepaths for the dispersion measure are simulated: one assuming $n = 100$ throughout, one with growth in n calibrated to match the historical growth in US civilian employment. Each replicates the narrowing of the income distribution over the first half of the century, but the latter does a significantly better job when it comes to the recent rise in inequality, confirming the potential explanatory role for market size. While neither matches the historical dispersion measure's dramatic fall in the 1940s, this is perhaps unsurprising given the abstract nature of the model and the simplistic assumptions on technical change and skill distributions. Neither do the simulations do capture the market size

[18]Macunovich (1998) has argued that the macroeconomic effects of the baby boom have been underestimated, although she emphasises the impact of the changing age distribution on the nature of demand.

[19]See Kim (1999) for an overview of urban development in the US. While urbanisation was at its fastest in the first half of the twentieth century, and the proportion of the population living in large cities peaked around 1960, the proportion living in metropolitan areas continued to rise throughout. Lang and Dhavale (2004) report that,as of the 2000 Census, 53 per cent of all the land area in the continental United States comprise either metropolitan or the newly-defined 'micropolitan' areas, so that rural areas constitute a minority share for the first time.

effects of urbanization, global conflict and a changing trade environment[20].

Figure 5 provides three snapshots of the simulated income distribution at 1900, 1950 and 2000, assuming growth in n. Consistent with the discussion above, high-λ_j agents have the highest relative incomes in 1900; as δ and n grow, μ_j becomes more important until by 2000 the high-μ_j agents constitute the rich. Another view of the decline in importance in general ability is provided in Figure 6, which graphs the evolution of the relative incomes of selected individuals. Figure 7 repeats the exercise for individual choices of r_j and confirms that it is not necessarily the high-μ_j agents who specialize the most in equilibrium; after about mid-century the product range of a high-μ_j agent exceeds that of his low-μ_j counterpart, holding λ_j fixed. As δ rises, the high-μ_j superstars are so productive at low r_j that they become the equilibrium generalists, for the same reason that high-λ_j renaissance men were at low δ: to do otherwise would drive prices down too far.

The simulations reported so far assume an unchanging distribution of the μ_j and λ_j parameters. However, an undoubted source of variation in US inequality is change in relative skill endowments. The 'high school movement' saw a significant increase in secondary school enrolment and graduation between 1910 and 1940 [Goldin and Katz (1999b)]. The post-baby boom era saw college enrolment rates similarly rise. As discussed above, the latter phenomenon has motivated the literature on skill-biased technical change, as the increase in the relative supply of graduates was accompanied by a rise in the return to a college education. In the current model a greater relative supply of high-μ_j workers naturally implies a rise in such workers' relative earnings, even absent any change in δ. High-μ_j superstars are more productive, all else equal, and so a larger proportion of them will raise market size $\frac{x}{\alpha}$. As we have seen, superstars are also those who gain most from larger markets.

To confirm this intuition, I simulate an economy with two distinct groups of workers, endowed with returns to specialization of 0.05 and 0.35 respectively. Heterogeneity along the λ_j skill dimension is shut down for simplicity. Figure 8 graphs the relative earnings of the high-δ_j group - the skill premium - against the fraction of the population the group constitutes. At least along the δ_j dimension, the skill premium is increasing in the relative supply of skill.

[20] For low levels of δ the earnings distribution is determined largely by the λ_j parameters; the $[0.2, 1.8]$ spread for the λ_j was chosen to generate an equilibrium log-wage ratio of about 1.5 at the start of the (simulated) sample period. The choices of uniform distribution, normalisation of initial n to 100 and constant growth in δ were deliberately arbitrary. Given these choices, the $[0.75, 1.25]$ spread for the μ_j implied the best approximation for the observed log-wage ratio, although I found that the U-shaped timepath for inequality was robust across a wide selection of such spreads. Allowing for alternative distributions of the λ_j and μ_j, normalisations of n and, especially, varying growth rates for δ would presumably result in better approximations still.

5 Concluding Remarks

In the model above the evolution of the earnings distribution is not governed by variations in a single skill premium. Instead it is the relative return to different aspects of ability that change over time. As market size rises the reward structure shifts in favour of specialist skills, in such a way as to mimic the timepath of US inequality. The approach taken here has at its core the individual specialization decision in general equilibrium. I conjecture that the results relating to market size, technical change and the income distribution would carry over to a setting where firms existed to partially coordinate the actions of dispersed agents.

Making the model here explicitly dynamic, for example by introducing capital accumulation, would be useful insofar as it endogenized technology and related the skill parameters here to observable variables. It was shown that fundamental productivity α is distribution-neutral, but that a secular increase in the aggregate return to specialization δ could help explain the evolution of US earnings inequality. This is a hypothesis about the qualitative nature of technical progress, and as such is open to debate. To the extent that increasing division of labor is a natural consequence of economic progress - thanks to population growth, urbanization, lower transportation costs and greater international trade - one might expect technical change to be directed in such a way as to most improve the productivity of specialists and increase δ. In the spirit of Acemoglu (1998), increasing specialization thanks to increased market size may stimulate technical change that is biased towards such specialization[21].

There are a couple of significant features of the recent evolution of the US income distribution that the simulations reported here do not replicate. The first is the increase in earnings instability, with the transitory variance of wages rising significantly since the 1970s [Katz & Autor (2000), Section 2.5]. Augmented by good-specific demand shocks, the model here would presumably predict such higher wage volatility as the flip-side of increased specialization.

The second unexplained feature is the fall in earnings in the lower tail over the last 30 years or so. The importance of market size for the specialization decision suggests a possible explanation. It was shown how, if agents sort across space, the ghost-town effect leads the output of low-ability agents actually to fall when those of high ability move away. This could be extended to a setting where agents sort along other dimensions such as product quality, with economic growth leaving the low-skilled isolated in thin markets for low-quality goods. Embedding the ideas here in a dynamic model of vertical product differentiation seems a logical next step.

[21] Acemoglu provides a model where an increase in the supply of skilled labour induces skilled-bias technical change in such a manner.

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6 Appendix

6.1 Proof of first-order condition (6)

Differentiate agent j's objective function (4) with respect to x_j^i:

$$\frac{\delta\Gamma}{\delta x_j^i} = \frac{\delta V(P, Y_j)}{\delta Y_j}\frac{\delta Y_j}{\delta x_j^i} + \frac{\delta V(P, Y_j)}{\delta p^i}\frac{\delta p^i}{\delta x^i} - \lambda. \tag{24}$$

One may then use Roy's Identity to get (5) in the text. To obtain (6), recall that any equilibrium is characterised by the same total output of each variety. As a result the behaviour of others that j takes as given is summarised by identical output - in his absence - in each surrounding market. Given such behaviour, as j expands his range of production, (5) implies that he will produce the same amount $x_j^i = \frac{a(r_j)}{r_j}$ of each good. Recognising that output-per-variety will thus depend on the product range, differentiate (4) with respect to r^j to obtain the following optimality requirement:

$$\begin{aligned}\frac{\delta\Gamma}{\delta r_j} \approx \frac{\delta V(P, Y_j)}{\delta Y_j}&\left[p^m x_j^m + \int_{i\in R_j}\frac{\delta Y_j}{\delta x_j^i}\cdot\frac{\delta x_j^i}{\delta r_j}di\right] \\ &+ x_j^m\frac{\delta V(P, Y_j)}{\delta p^m}\cdot\frac{\delta p^m}{\delta x^m} + \int_{i\in R_j}\frac{\delta V(P, Y_j)}{\delta p^i}\cdot\frac{\delta p^i}{\delta x^i}\cdot\frac{\delta x_j^i}{\delta r_j}di \\ &- \lambda\left[x_j^m + \int_{i\in R_j}\frac{\delta x_j^i}{\delta r_j}di - \frac{da(r_j)}{dr_j}\right] = 0\end{aligned} \tag{25}$$

where m is the marginal market into which agent j enters as his production locus expands[22]. Note that the beneficial implications for j's budget set, holding Y_j constant, of entering a new market m and thereby driving down p_m are approximated as $x_j^m\frac{\delta V(P, Y_j)}{\delta p^m}\frac{\delta p^m}{\delta x_j^m}$. This simplifies the analysis considerably and will hold almost exactly for $\frac{x_j^m}{x^m}$ sufficiently small; when preferences are Cobb-Douglas, as assumed in the bulk of the paper, it is equivalent to a first-order Taylor approximation. Using (5) and once again the fact that in equilibrium $x_j^i = x_j = \frac{a(r_j)}{r_j}$, $x^i = x$, $\forall i, j$, one can rearrange (25) to obtain (6). The assumptions on $\varepsilon_j^a(.)$ - that it is less than one and non-decreasing in r_j - guarantee that second-order conditions hold.

Note that Γ_j is in fact discontinuous at equilibrium r_j,since if j increases his product range any further he goes from markets with output of $(x - x_j)$ to ones with output of x in his absence. There may therefore be other equilibria in the neighbourhood of that defined by Result 1. I focus throughout on equilibria where (6) is satisfied exactly for all agents.

[22] Strictly speaking, as r_j rises j enters two new markets, since R_j must be centred on j's location on the variety circle. With symmetric varieties and constant output across markets, though, (25) delivers the correct result.

6.2 Proof of Result 2

Using (10), individual output levels can be characterised as functions of total output x:

$$a_j(r_j) = r_j x_j = \frac{r_j}{1-r_j}\delta_j x = f_j^{-1}(a_j(r_j))\,\delta_j x \tag{26}$$

where $f_j^{-1}(.)$ is the inverse of $f_j(.)$. Define a new set of functions $q_j(z) = \frac{z}{f_j^{-1}(z)}$, so that:

$$x = \int_0^n a_j(r_j)\,dj = \int_0^n q_j^{-1}(\delta_j x)\,dj. \tag{27}$$

Recognising that the homogeneity of the $f_j(.)$ in turn implies $q_j(z) = \frac{1}{f_j^{-1}(\alpha_j)}\alpha_j^{-\frac{1-\delta_j}{\delta_j}} z^{\frac{1}{\delta_j}}$, homogeneous of degree $\frac{1}{\delta_j}$, then delivers (11) in the text. Equations (12) and (13) are similarly obtained. The assumption $\delta n > \frac{\lambda_j}{\mu_j}$ guarantees that $\frac{\mu_j\delta}{\lambda_j}\cdot\frac{x}{\alpha} > 1$ and therefore that $r_j \in \left(0, \frac{1}{2}\right)$, $\forall j$.

6.2.1 per capita output, market size and α, δ, n

Once it is noted from (11) that equilibrium $\frac{x}{\alpha}$ is independent of α, (12) confirms that per capita output $\frac{x}{n}$ is increasing in α. For the aggregate increasing returns, note that the arrival of an additional, representative agent with $\lambda_j = \mu_j = 1$ must lead to an increase in $\frac{x}{\alpha}$ by (11). This will increase equilibrium productivity of other agents by (12), and so $\frac{x}{n}$ must rise with n. Finally, for per capita output to be increasing in δ it is enough that $\frac{x}{\alpha}$ is so increasing, again from (12). This must be the case because for $1 = \int_0^n \delta_j \left[\frac{\delta_j x}{\alpha_j}\right]^{\delta_j - 1} dj$ to hold as the δ_j rise the $\frac{\delta_j x}{\alpha_j}$ terms under the exponent must be rising at a greater propotional rate than are the δ_j.

6.2.2 equilibrium product range

From (13) it is clear that r_j rises with λ_j, so that in cross-section the high-λ_j agents are most diversified, all else equal - the renaissance man result.

The effect of μ_j in cross section is not so straightforward, since:

$$sign\left\{\frac{d\left[\frac{1}{r_j}\right]}{d\mu_j}\right\} = sign\left\{-\delta\ln\left[\frac{\mu_j\delta}{\lambda_j}\cdot\frac{x}{\alpha}\right] + \frac{1-\mu_j\delta}{\mu_j}\right\}. \tag{28}$$

For low μ_j and low δ and $\frac{x}{\alpha}$, then, increases in μ_j are associated with greater specialisation. However, at high enough μ_j the converse may hold: especially if δ and $\frac{x}{\alpha}$ are large, further increases in μ_j may reduce specialisation, for much the same reason that high-productivity renaissance men are less specialised in equilibrium. High-μ_j superstars may be so productive at low r_j that they flood their markets, and so must diversify a little.

A similar effect is present when one moves away from the cross-section and instead considers an increase in δ. Specialisation must increase in some average sense, given that we may rewrite (13) as:

$$1 = \int_0^n \mu_j \delta \frac{r_j}{1 - r_j} dj. \tag{29}$$

However, it is possible that for some particularly high-μ_j agents the increase in productivity is great enough to induce them to diversify.

This, coupled with the cross-sectional analysis, suggests that two things will occur as δ rises: specialisation will generally increase, and the economy will change from one where the high-μ_j agents are specialists to one where they are the more diversified in their product ranges. This is confirmed in Figure 7, and would make inference of the λ_j and μ_j parameters problematic even if time-series data on r_j were available.

6.3 Migration

Consider a simplistic economic geography model with two locations, Metropolis and Backwater. Their initial populations are n^M and n^B respectively, with $n^M > n^B$. The return to specialisation is δ for all agents, but fundamental productivity varies; in each location the α_j are uniformly distributed between 0 and 1 over the respective populations. Geography is such that the two cities constitute distinct economies. There is no flow of goods between them - a convenient way of capturing transport costs - so agents must produce and consume entirely within their home market.

Now introduce the possibility of migration from Backwater to Metropolis only; this rules out expectations-driven equilibria. Backwater residents are able to move permanently to Metropolis at cost c in terms of consumption goods foregone, there to take advantage of the larger market by narrowing their product range. Time is continuous and agents are infinitely-lived, discount the future at rate ρ and compare lifetime real incomes in the two cities when making their independent migration decisions.

The most productive in Backwater, with $\alpha_j = 1$, will be the most likely to move to the larger market: while their relative income increase is identical to that of their less-productive fellow residents, their absolute gain is greater. Assume that c is low enough that at least some leave for Metropolis. Using (15) and the assumption that the α_j are uniformly distributed, one can show that all j such that $\alpha_j > \widehat{\alpha}$ will migrate, with $\widehat{\alpha}$ defined by:

$$\widehat{\alpha}^{1-\delta} \left[\left[1 + \frac{n^M}{n^B} - \widehat{\alpha}^{2-\delta} \right]^{\frac{\delta}{1-\delta}} - \left[\widehat{\alpha}^{2-\delta} \right]^{\frac{\delta}{1-\delta}} \right] = \rho c \left[n^B \frac{\delta}{2-\delta} \right]^{-\frac{\delta}{2-\delta}}. \tag{30}$$

Implicit in (30) is that potential migrants anticipate real income flow of the form (16) but do not internalise their effects on the size of the Metropolis economy.

Since $\widehat{\alpha} > 0$, the least productive inhabitants of Backwater will remain. As noted in the text, their productivity will fall as their home market has shrunk, while the productivity both of the migrants and the Metropolitan incumbents will rise.

7 Figures

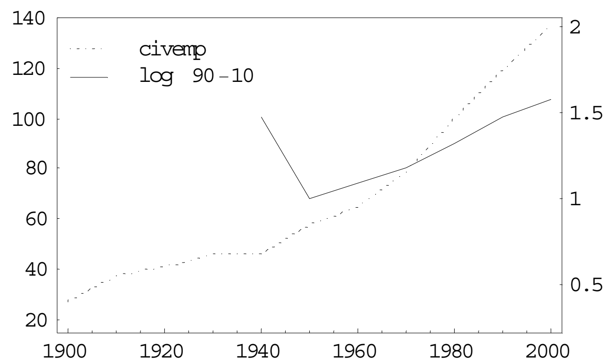

Figure 1: Inequality and Employment in the US, 1900-2000

civemp = US civilian employment (left-hand axis, millions) [source: NBER historical database, series 08171a and 18171b (1900-1940), and Bureau of Labor Statistics series LNS12000000 (1950-2000)]; log 90-10 = log weekly wage of full-time, full-year, non-agricultural workers, with bottom 1% omitted; ratio of 90th to 10th percentiles (right-hand axis) [source: Katz & Autor, Table 8].

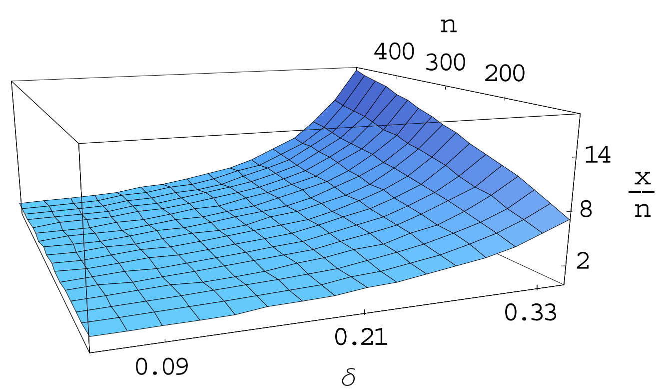

Figure 2 - Per capita output, δ and n

$\delta \in$ [0.05, 0.35], $n \in$[100, 500].

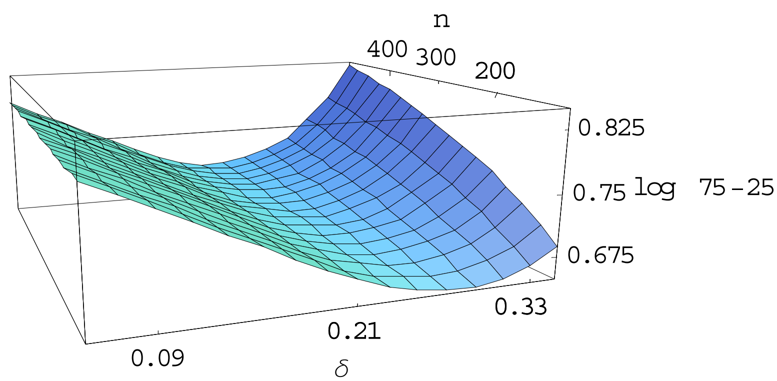

Figure 3 - log 75-25, δ and n

$\delta \in$ [0.05, 0.35], $n \in$[100, 500]; log 75-25 = log-wage ratio of the 75th and 25th percentiles.

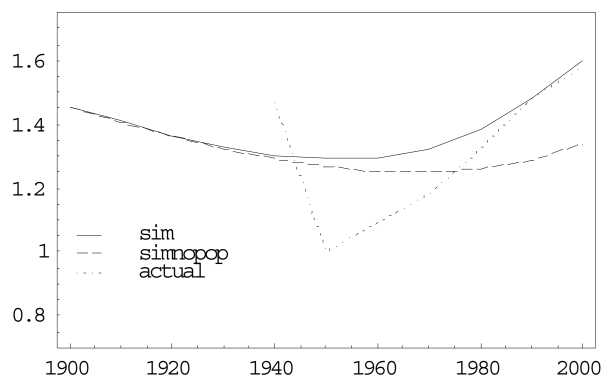

Figure 4 - Simulated log 90-10 with population growth and specialisation-biased technical change

log 90-10 = log-wage ratio of the 90th and 10th percentiles; sim = simulations of log 90-10 assuming growth in δ (= 0.05 in 1900, increases by 0.03 every decade) and n (= 100 in 1900, growth calibrated to match civemp in Figure 1); simnopop = as sim, but with n = 100 throughout; actual = data series from Figure 1.

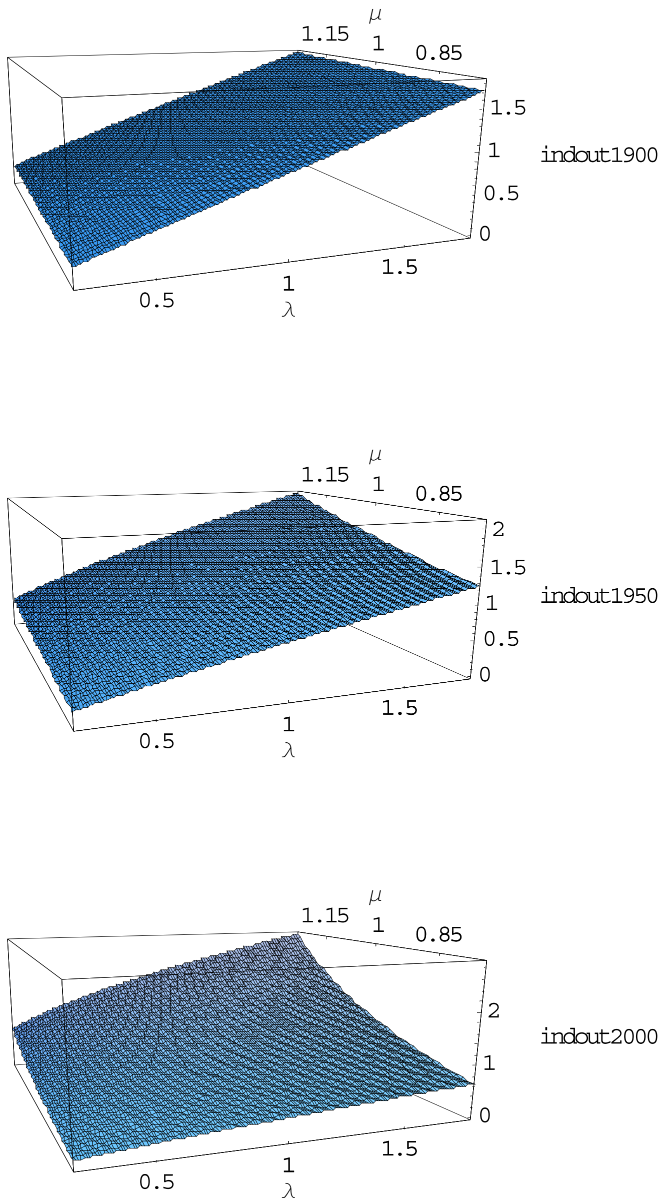

Figure 5 - Snapshots of simulated income distribution in 1900, 1950 and 2000

indout = individual output relative to average in 1900, 1950 and 2000, based on sim from Figure 4; $\lambda \in [0.2, 1.8]$, $\mu \in [0.75, 1.25]$.

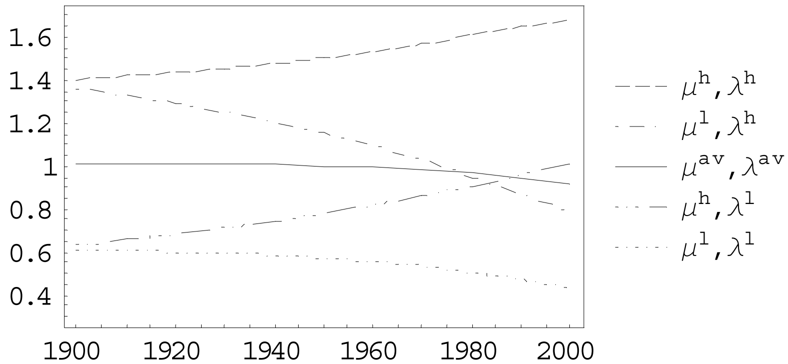

Figure 6 - Simulated relative incomes of selected agents, 1900-2000

timepaths for indout for selected agents, 1900-2000, based on sim from Figure 4; $\mu^h = 1.125$, $\mu^{av} = 1$, $\mu^l = 0.875$, $\lambda^h = 1.4$, $\lambda^{av} = 1$, $\lambda^l = 0.6$.

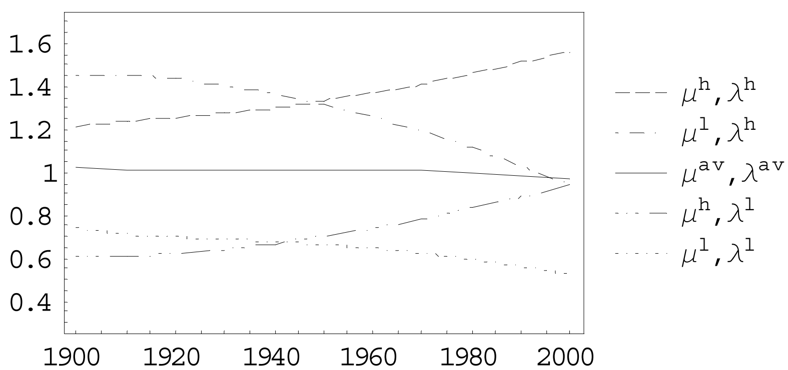

Figure 7 - Simulated choice of product range for selected agents, 1900-2000

timepaths for r_j for selected agents, 1900-2000, based on sim from Figure 4; $\mu^h = 1.125$, $\mu^{av} = 1$, $\mu^l = 0.875$, $\lambda^h = 1.4$, $\lambda^{av} = 1$, $\lambda^l = 0.6$; note that absolute r_j falls for all agents over the period, with mean product range falling from 0.164 in 1900 to 0.005 in 2000.

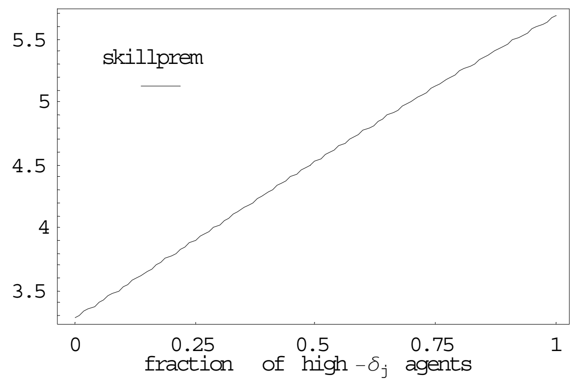

Figure 8 - The skill premium is increasing in the relative supply of skills

based on simulated economy with $n = 100$ and two groups of workers, with $\delta_j =$ 0.05 and 0.35 respectively and $\alpha_j = 1$ for all agents; skillprem = equilibrium output of high-δ_j group relative to low-δ_j group.

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