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SORANZO, Alessandro and AGOSTINI, Tiziano

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Impossible shadows and lightness constancy

Alessandro Soranzo, Tiziano Agostini

Abstract. The intersection between an illumination and a reflectance edge is characterised by the

'ratio-invariant' property, that is the luminance ratio of the regions under different illumination remains the same.

In a CRT experiment, we shaped two areas, one surrounding the other, and simulated an illumination edge dividing them in two frames of illumination. The portion of the illumination edge standing on the surrounding area (labelled contextual background) was the contextual edge, while the portion standing on the enclosed area (labelled mediating background) was the mediating edge. On the mediating background, there were two patches, one per illumination frame. Observers were asked to adjust the luminance of the patch in bright illumination to equate the lightness of the other. We compared conditions in which the luminance ratio at the contextual edge could be (i) equal (possible shadow), or (ii) larger (impossible shadow) than that at the mediating edge. In addition, we manipulated the reflectance of the backgrounds. It could be higher for the contextual than for the mediating background; or, vice versa, lower for the contextual than for the mediating background. Results reveal that lightness constancy significantly increases when: (i) the luminance ratio at the contextual edge is larger than that at the mediating edge creating an impossible shadow, and (ii) the reflectance of the contextual background is lower than that of the mediating one. We interpret our results according to the albedo hypothesis, and suggest that the scission process is facilitated when the luminance ratio at the contextual edge is larger than that at the mediating edge and/or the reflectance of the including area is lower than that of the included one. This occurs even if the ratio-invariant property is violated.

1 Introduction

The visual system exhibits two types of constancy: one with respect to changes in the illumination, the other with respect to changes in the reflectance of the background. We will refer to the first type as illumination-independent constancy, and to the second type as background-independent constancy. In order to achieve both types of constancy, the luminance edges in the stimuli produced by illumination changes must not be confused with the luminance edges produced by reflectance changes. Therefore, one of the main problems in lightness perception is the differentiation between illumination and reflectance edges.

A systematic investigation of this problem was conducted by Gilchrist (1988). He devised a paradigm, named 'edge substitution', in which the same luminance edge could be seen as an illumination or as a reflectance edge. Gilchrist asked the observers to choose, from a Munsell scale arranged on a white background in bright illumination, a sample matching the lightness of one patch, the standard, simultaneously presented in a shadowed region of the same white background. The border dividing the two illuminated sides of the white background, labelled 'mediating edge', was made to appear as either a reflectance edge (contrast condition) or an illumination edge (constancy condition) by either hiding or exposing the larger context. Despite the local stimulation being the same for both conditions, observers' matches differed greatly. In the constancy condition the observers chose the Munsell patch that, approximately, shared with the standard the same reflectance, that is, they performed an equal-ratio

match (the standard and the Munsell patch selected by observers approximately shared the same luminance ratio with the respective backgrounds). In the contrast condition, the observers chose, instead, the Munsell patch which shared with the standard almost the same luminance (equal-luminance match). Gilchrist argues that when the observers approximated the equal-luminance-ratio match, the mediating edge was seen as an illumination edge, while when they approximated the equal-luminance match, the mediating edge was seen as a reflectance edge. Therefore, the visual system classifies the luminance edges in two different categories: illumination and reflectance edges.

In order to explain these outcomes, Gilchrist suggested that the critical factor for edge classification is the nature of the intersection where an illumination edge crosses a reflectance edge. This intersection has the property that may be called 'ratio-invariance'; that is, the luminance ratio between the regions under different illumination remains the same when an illumination edge crosses one or more reflectance edges. In other words, in the constancy condition the mediating edge was seen as an illumination edge because the context revealed another luminance edge sharing the same ratio.

However, it should be noted that in the contrast condition observers chose a patch that only approximated the luminance of the standard, and in the constancy condition their matches did not exactly coincide with the equal-luminance ratio. This implies that the edge classification is not a categorical process.

This outcome is even more evident in Bruno's (1994) work. Bruno simulated the edge substitution paradigm on a CRT monitor. He shaped an enclosed (mediating) and a surrounding (contextual) background and simulated an illumination edge dividing them in two frames of illumination. He replayed the conditions employed by Gilchrist (constancy and contrast) plus some additional conditions named 'intermediate cases'. In these new conditions, the ratio-invariant property was violated because, compared to the constancy condition, the luminance ratio at the contextual edge (the portion of the illumination edge standing on the contextual background) was systematically lowered.

Bruno substantially replicated Gilchrist's results for the constancy and contrast conditions. However, he found that the observers' mean ratings for the intermediate cases fell in an intermediate position between the matches obtained in the contrast and in the constancy condition, and concluded: "The visual system will perform qualitatively different integrations of the edges present in a scene as a function of the context" (page 2213).

Another investigation on this topic was conducted by Agostini et al (1999). They used the same paradigm as that used by Bruno, but manipulated the luminance profile of both the mediating edge and the contextual edge. They found that observers' matches approximate the ratio match more when the luminance profile of both edges was gradual rather than sharp. However, if the luminance profile of the mediating edge was different from that of the contextual edge (the first gradual and the second sharp, or vice versa) the observers' lightness match tended toward the luminance match. Agostini et al concluded that the congruence of the luminance profile between the mediating and the contextual edge is a fundamental factor for edge classification, and consequently for illumination-independent lightness constancy.

From these studies, it emerged that the relation between the contextual and the mediating edge plays a crucial role in lightness perception. The aim of the present work was to investigate the nature of this relation. One important question to answer was: how does the visual system treat the mediating edge when the luminance ratio at the contextual edge is larger than that at the mediating edge? Of course, this occurrence is very unusual in nature. Real shadow cannot produce such a pattern of luminances. In nature there is a physical limit: the luminance ratio at the contextual edge can only be lower (when a filter crosses a reflectance edge) or equal (when a real shadow crosses

a reflectance edge) than the luminance ratio at the mediating edge. Therefore, the question we need to answer is: will a lightness match still approximate the luminance-ratio match when the ratio-invariance property of the illumination edges is violated creating an impossible shadow?

Second, in previous experiments the edge classification process was investigated by putting a standard patch on a background (mediating background) having a reflectance value (or simulated reflectance value in Bruno's and Agostini et al's works) higher than that of the contextual background. In this way, the relation between the contextual and the mediating edge followed a precise scheme: the two luminances forming the contextual edge were always lower than those forming the mediating edge. Therefore, the second question is: will the classification of the mediating edge remain unchanged if the luminance values forming the mediating background are lower than those forming the contextual background?

In order to answer these questions, we ran an experiment simulating the edgesubstitution paradigm on a CRT monitor. We manipulated the relation between the luminance ratios at the mediating and contextual edges, and the simulated reflectance relationship between the mediating and the contextual background.

2 Experiment

- 2.1 Method
- 2.1.1 Observers. Ten volunteer observers participated in this experiment. All had normal or corrected-to-normal acuity and were nanve with regard to the experimental design.
- 2.1.2 Apparatus and stimuli. The stimuli were all generated with a Pentium computer and were presented on a carefully calibrated 18-inch 523X Daewoo monitor (944×648 pixels). We simulated the edge-substitution paradigm on the CRT monitor. We shaped an inner (mediating) and an outer (contextual) background in a way that they formed the mediating and the contextual edge.

Both the mediating and the contextual backgrounds were vertically divided into two halves. The two halves (left and right) of the contextual background were a 10 deg×14 deg visual angle each; those of the mediating background were a 6 deg 17 min of arc×7 deg 20 min of arc each (see figure 1).

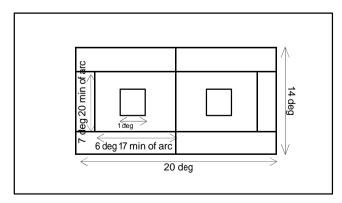


Figure 1. Size of the stimuli.

Furthermore, two squares (1 deg×1 deg each), the standard on the left and the target on the right, were placed in the middle of the two halves of the mediating background: We had two experimental variables:

- (i) Luminance ratio at the contextual edge, with four levels (contrast, constancy, impossible shadow 1, and impossible shadow 2);
- (ii) Reflectance relationship between the backgrounds, with two levels (mediating > contextual and mediating < contextual).

Thus, there were eight stimuli that are depicted in figure 2.

The luminance ratio at the contextual edge was:

- 1:1 (contrast condition):
- 1:6 (constancy condition possible shadow);
- 1:12 (impossible shadow 1);
- 1:18 (impossible shadow 2).

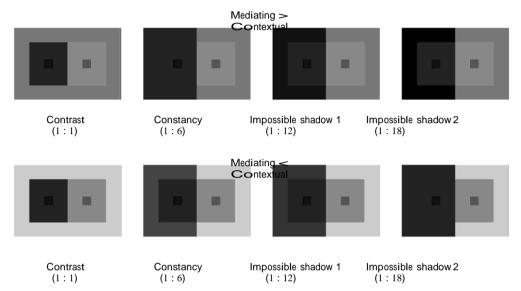


Figure 2. Experimental displays. They are grouped in two rows according to the level of the reflectance relationship between the backgrounds.

The luminances of both sides of the mediating background and that of the standard were the same for all the trials: the luminances of the mediating background were 4.40 cd m⁻² for the left and 26.44 cd m⁻² for the right side. In this way, the luminance ratio at the mediating edge was always (almost) equal to 1:6 (approximations are due to the conversion from luminances into R, G, B values). The luminance of the standard was 2.21 cd m⁻². The luminances of the two sides of the contextual background varied as a function of the experimental condition. They are listed in table 1. The first column indicates the level of the reflectance relationship between the backgrounds variable; the second column indicates the level of the luminance ratio at the contextual edge variable; the third and fourth columns indicate the luminance values for the left and right sides of the contextual background.

Table 1. Luminances of the left (third column) and right (fourth column) side of the contextual background. The first column indicates the level of the reflectance relationship between the backgrounds variable; the second indicates the level of the luminance ratio at the contextual edge variable (between brackets the luminance ratio at the contextual edge).

Reflectance relationship	Luminance ratio at the contextual edge	Contextual backgroundjcd m ⁻²	
between the backgrounds		left	right
Mediating > Contextual	contrast (1:1)	21.95	21.85
	constancy (1:6)	3.64	21.85
	impossible shadow 1 (1:12)	1.82	21.85
	impossible shadow 2 (1:18)	1.21	21.85
Mediating < Contextual	contrast (1:1)	82.80	82.80
	constancy (1:6)	13.80	82.80
	impossible shadow 1 (1:12)	6.90	82.80

In the condition mediating > contextual the simulated reflectance of the mediating background was higher than that of the contextual background, while in the condition mediating < contextual it was lower.

Summarising, there were eight stimuli that differed in the luminance ratio at the contextual edge, and in the reflectance relationship between the backgrounds.

2.1.3 Procedure. Observers viewed the stimuli, presented in random order, in a darkened room from a distance of 80 cm from the monitor. They were instructed to match the lightness of the target patch on the right side to the corresponding standard patch on the left side using the plus and minus keys of the keyboard. Pressing another button signalled that a satisfactory match was achieved, and at that point the target luminance was recorded and the next trial began. The luminance of the target was set to a random value at the beginning of each trial. First, we asked the observers to describe the different displays. Then, in order to achieve a lightness match, we asked them to make the target patch "look as if it were cut from the same piece of paper as the standard". The observers performed four matches for each of the eight stimuli, so they provided thirty-two adjustments. Each display was left on the screen as long as needed to produce the match. The whole session lasted about 30 min.

2.2 Results and discussion

Observers' matches were transformed with the following formula:

$$I = \frac{\text{log}(L_{\text{T}} \mathbf{j} L_{\text{S}})}{\text{log}(R_{\text{MF}})} \; , \label{eq:energy_logical_logical}$$

where I is the index, L_T is the luminance value assigned by the observers to the target, L_S is the luminance of the standard (ie 2.21 cd m⁻²), and R_{ME} is the luminance ratio at the mediating edge (ie 6).

Using this transformation, we obtained an index I that is independent from the absolute luminances. This measure is expressed in a proportion ranging from zero (equal-luminance match) to one (equal-luminance-ratio match).

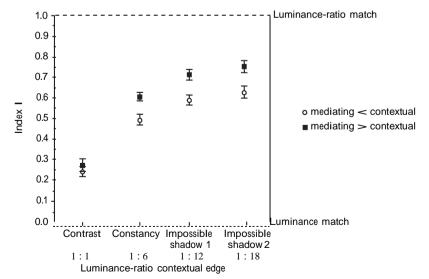


Figure 3. Results of the experiment. Observers' mean ratings are expressed with an index of constancy (see text for details), where zero and one correspond, respectively, to the equal-luminance match and the equal-luminance-ratio match. Bars indicate standard errors.

The transformed observers' mean ratings, together with the standard errors, are shown in figure 3. The upper dashed line indicates the equal-luminance-ratio match and the lower dashed line indicates the equal-luminance match.

A two-way repeated-measures ANOVA reveals a significant effect of both the luminance ratio at the contextual edge ($F_{3,\,9}=61.07,\ p<0.001$) and the reflectance relationship between the backgrounds ($F_{1,\,9}=41.59,\ p<0.001$) variables. The interaction between the two variables was not significant.

As can be seen on the graph, we replicated the results of both Gilchrist (1988) and Bruno (1994) concerning the comparison between the contrast and constancy conditions. Furthermore, observers' lightness matches approximate more the ratio match when the (simulated) reflectance of the mediating background was higher than that of the contextual background.

Within the same level of reflectance relationship between the backgrounds, a least-square means analysis reveals a significant difference between the constancy condition and both the impossible shadow 1 and 2 conditions with a p value lower than 0.01. Thus, in both impossible shadow 1 and 2 conditions observers' matches approximate significantly more the luminance-ratio match than in the constancy condition (possible shadow). This suggests that the degree of illumination-independent lightness constancy improves as the luminance ratio at the contextual edge is increased, independently of the ratio-invariance property of the illumination edges. However, the difference between the impossible shadow 1 and 2 conditions was not statistically significant for both the levels of the reflectance relationship between the backgrounds. It seems, therefore, that the relation between the increase of the luminance ratio at the contextual edge and the improvement of the degree of illumination-independent lightness constancy is not linear.

3 Discussion

Our work was aimed at answering the following questions:

- (a) How does the visual system classify the mediating edge when the ratio-invariance property of the illumination edges is violated originating impossible shadows?
- (b) Does the reflectance relationship between the backgrounds within the same illumination frame affect the edge classification process?

In order to examine these issues we simulated on a CRT monitor the edge-substitution paradigm and manipulated both the luminance ratio at the contextual edge (creating an impossible shadow) and the reflectance relationship between the backgrounds. Below we discuss the relative outcomes.

3.1 Impossible shadows

In order to perceive lightness constancy under different illumination intensities, the luminance edges produced by reflectance borders must not be confused with luminance edges produced by illumination borders. With reference to this, Gilchrist (1988) suggested that the visual system classifies the luminance edges in two perceptual categories: illumination edges and reflectance edges. However, these perceptual categories do not necessarily correspond to physical categories. For example, in the edge-substitution paradigm, if the contextual edge is hidden from the view of the observer, a physical illumination edge is seen as a reflectance edge. Therefore, a physical illumination edge belongs to the perceptual category of illumination edges only when the contextual edge is visible. Gilchrist suggests that this is due to the 'ratio-invariant' property of the illumination edges. When an illumination edge crosses a reflectance one, it creates, indeed, an intersection that possesses the following property: the luminance ratios along the illumination edge remain the same. In the edge-substitution paradigm, hiding the contextual edge obscures the effectiveness of the ratio-invariant property since only one luminance edge remains visible.

Thus, the ratio-invariant property is generated by physical illumination edges. But, is there some correspondence, at the perceptual level, of the ratio-invariant property? In other words, the question is: is the visual system constrained to the physical features of the stimulus in such a way that, when an illumination edge crosses a reflectance one, the luminance ratio at the contextual edge cannot be larger than that at the mediating edge?

To answer this question, we compared possible and impossible shadows (see figure 2).

The possible shadow was produced by making the luminance ratio at the contextual edge equal to that at the mediating edge. (1) Conversely, in the impossible-shadow conditions the luminance ratio at the contextual edge was larger than that at the mediating edge, but preserving the same luminance polarity. In this way we simulated one shadow in which the luminance of the outer background is reduced more than that of the inner background. This is impossible at the physical level; (2) or, better, such an arrangement of luminances can be obtained but not by using only one illumination and one reflectance edge. This situation occurs, for example, when the border of a shadow exactly coincides with a reflectance border or with the border of a hole. However, these settings are very rare in everyday life. The impossible-shadow displays used in our experiment simulated exactly these types of situations. Nevertheless, none of our observers described the impossible-shadow conditions by saying that there were two different causes reducing the luminance of the outer background. All of them reported seeing one single shadow (or filter) that covered the left side of the display.

We found a better degree of lightness constancy in the impossible-shadow conditions rather than in the possible ones. This fact seems to demonstrate that the visual system, in order to achieve lightness constancy, is not constrained to the physical features stating that a shadow cannot alter differentially the luminances of the underlying surfaces on which it is cast.

According to Gilchrist (1988), an interpretation of this effect based on the simultaneous lightness contrast could be discarded. Indeed, Gilchrist pointed out "the larger context [...] exerted its effect on the data not by the addition or subtraction of luminances per se (with an attendant increase or decrease in inhibition), but rather by producing a perceptual reorganization of the visual field" (page 416).

On these premises, we maintain that the principle based on the concept of scission can explain our results (Bergstrom 1977; Barrow and Tenenbaum 1978; Gilchrist 1977, 1979; Gilchrist et al 1983; Todd and Mingolla 1983; Mingolla and Todd 1986; Bulthoff and Mallot 1987, 1988; Adelson and Pentland 1990). Scission theories share the idea that the visual system decomposes the pattern of light intensities that reach the eyes into separate contributions: reflectance, illumination, depth, and so on. In our displays, the visual system would be able to decompose the luminance of both the standard and the target into their reflectance and illumination components. However, in both the possible-shadow and impossible-shadow conditions the lightness match performed by the observers did not exactly equate the luminance-ratio match. This means that the standard and the target looked equal in lightness when the (simulated) reflectance

⁽¹⁾ The sharpness of the illumination edge might favour the impression of a filter rather than that of a shadow. However, according to Metelli (1975) shadows are indistinguishable from filters of virtually no reflectance (0% reflectance). Since only virtual, rather than real, filters have the ratio-invariant property, we may consider virtual filters in the same way as shadows.

⁽²⁾ It is interesting to note that an opposite arrangement of the luminances is physically possible. When one filter, rather than one shadow, covers the backgrounds of the edge-substitution display, the luminance ratio at the contextual edge can be smaller than that at the mediating edge. The intermediate cases of Bruno (1994) simulated this situation (see section 1).

of the target was lower than that of the standard. Therefore, there was an error in the luminance decomposition process. According to the albedo hypothesis (Kozaki 1965; Oyama 1968; Beck 1972; Kozaki and Noguchi 1976; Noguchi and Kozaki 1985; Logvinenko and Menshikova 1994; Agostini and Galmonte 1997a, 1997b, 2002), this error is due to misattribution of luminances in the reflectance and in the illumination components. In our displays, this misattribution can occur in two different ways: (a) part of the standard luminance that should have been attributed to its reflectance is attributed to the illumination; and/or (b) part of the target luminance that should have been attributed to the illumination is attributed to its reflectance. We assume that the luminance misattribution is lower in the impossible-shadow conditions than in the possible-shadow ones. In other words, only when the luminance ratio at the contextual edge is larger than that at the mediating edge, the luminance misattribution will decrease. Conversely, the luminance misattribution will increase if the relationship between the two types of edges is inverted. Indeed, as cited in section 1, Bruno (1994) demonstrated that decreasing the luminance ratio at the contextual edge and keeping constant that at the mediating edge, results in a loss of lightness constancy. Therefore, it seems that given a contextual and a mediating edge sharing the same luminance polarity, increasing the value of the luminance ratio only at the contextual edge results in a facilitation of the scission process leading to a higher degree of lightness constancy.

It remains to explain why the scission process is facilitated only when the context has a higher luminance ratio than that occurring in the enclosed area. We suggest that, since the amount of luminance that the enclosed area must yield to the total apparent illumination is lower when the luminance ratio at the contextual edge is higher, then, in this condition, the amount of luminance remaining to the standard in order to constitute its lightness must be higher.

3.2 Reflectance relationship between the backgrounds

The reflectance relationship between the backgrounds within the same illumination frame strongly influences the edge classification process. When the reflectance of the mediating background was higher than that of the contextual one, the approximation to the luminance ratio was always higher compared to the reverse condition. This leads to the following argument: it is known that, when the luminance of the standard patch is higher than that of its surround (increment), illumination-independent lightness constancy is weaker than in the situation in which the luminance relations are reversed (decrement) (Helson 1943; Leibowitz et al 1955; Kozaki 1963, 1965; Bruno 1994). However, in our experiment the standard was a decrement in both the mediating > contextual and the mediating < contextual conditions (actually, the luminance of the standard and that of both sides of the mediating background was kept constant in all conditions). Therefore, our results suggest that the loss of constancy for increments can be extended also to the cases in which the increment/decrement relationship concerns the luminance of the backgrounds rather than the luminance of the surfaces that have to be equated in lightness.

Summarising, the results we obtained in the impossible-shadow conditions reveal the fundamental importance of the contextual edge. To achieve good levels of illumination-independent lightness constancy, the ratio-invariant property of the illumination edges can be violated but only in a specific way: the luminance ratio at the contextual edge has to be larger than the luminance ratio at the mediating edge.

Furthermore, lightness constancy depends also on the reflectance relationship between the backgrounds within the same frame of illumination; that is, illumination-independent lightness constancy improves when the including area is darker than the included one.

Maybe it is possible to suggest a practical application of our results. A painter or a computer designer desiring to paint a good (for the viewer) shadow does not need

to pay attention to preserving the luminance ratios among the surfaces in light and in shadow. She/he must only be sure to paint the including area darker than the included one and the shadowed part of it must be as dark as possible.

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