

A STUDY OF AMERICAN ELEMENTARY ARITHMETIC TEXTBOOKS FOR  
GRADES ONE AND TWO WRITTEN DURING THE PERIOD, 1821 TO  
1938, TO TRACE THE ATTEMPTS OF THE AUTHORS TO  
CONFORM TO THE CHANGING AIMS OF EDUCATION

by

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## TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION . . . . .	1
The problem . . . . .	2
Statement of the problem . . . . .	2
Importance of the study . . . . .	2
Limitations of the study . . . . .	2
Previous studies relative to this subject . .	3
Organization of the remainder of the thesis .	5
II. REVIEW OF OLD AND NEW AIMS OF EDUCATION . . . .	6
The old aims of education . . . . .	7
New aims of education . . . . .	34
Summary of chapter . . . . .	42
III. REVIEW OF OLD AND NEW AIMS OF ARITHMETIC . . . .	47
The old aims of arithmetic . . . . .	47
The new aims of arithmetic . . . . .	80
Summary of chapter . . . . .	83
IV. ANALYSES OF ELEVEN ARITHMETIC TEXTBOOKS . . . .	86
An Arithmetic on the Plan of Pestalozzi with Some Improvements . . . . .	88
Practical and Mental Arithmetic on a New Plan . . . . .	117
Ray's Arithmetic, First Book . . . . .	124
French's First Lessons in Numbers . . . . .	145





## CHAPTER I

### INTRODUCTION

Publication of American arithmetic textbooks for the primary grades began in the early part of the nineteenth century.<sup>1</sup> For many years there seemed to be little question regarding the suitability of the content of these textbooks for young children. Teachers attempted to teach all the material included in the book. But beginning about 1889<sup>2</sup> there was evidenced among educators a critical attitude relative to the content of arithmetic textbooks. Many theories have been advanced regarding the unsuitability of the content, and various investigations have been made to establish the validity or falsity of these theories. There has been one reason ventured, mostly by classroom teachers, that has not been listed in previous researches, and which has seemed to deserve attention. Some teachers have felt that authors of arithmetic textbooks have not conformed to the changing aims of education.

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<sup>1</sup> Walter S. Monroe, Development of Arithmetic as a School Subject (Washington: Government Printing Office, Bulletin No. 10, 1917), p. 39.

<sup>2</sup> Ibid., pp. 128-129.

## I. THE PROBLEM

Statement of the problem. It was the purpose of this study to trace, in American primary arithmetic textbooks of the period, 1821 to 1938, inclusive, the attempts of the authors to conform to the changing aims of education, as revealed by the analysis of the preface, the table of contents, teacher aids, and problems of these textbooks.

Importance of the study. It was thought that this study would emphasize the following: that, in previous studies relative to the materials of arithmetic textbooks, attention to authors' attempts to conform to the changing aims of education had been lacking; that, in analyzing and evaluating the content of arithmetic textbooks it is important to consider whether the author had conformed to the changing aims of education; that it would be a valuable point to add to the items of analysis and evaluation of textbooks; and, that the study would reveal the past practices of authors of the arithmetic textbooks relative to the changing aims of education during the period, 1821 to 1938, inclusive; and perhaps it might point out future trends.

Limitations of the study. The limitations of the study were: first, the primary books for the period, 1821 to about 1880, did not state for what grades the texts were

intended (usually, only that they were for young learners); and, second, the present trend toward the use of workbooks for the first and second grades, and no formal arithmetic in the first grade, lessened the number of books available for the first grade in the last five years of the period.

## II. PREVIOUS STUDIES RELATIVE TO THIS SUBJECT

A careful survey was made of the "Summary of Arithmetic Investigations"<sup>3</sup> in the volumes of the Elementary School Journal of May 26, 1926 to July 1, 1937, inclusive, the Twenty-ninth Yearbook,<sup>4</sup> the report of the Committee of Ten,<sup>5</sup> the report of the Committee of Fifteen,<sup>6</sup> the Jessup

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<sup>3</sup> Guy T. Buswell, "Summary of Arithmetic Investigations," Elementary School Journals, 26:692-703, 745-758; 27:685-694, 731-744; 28:702-709; 29:691-698, 737-747; 30:766-775; 31:756-766; 32:766-773; 34:209-213; 35:212-214; 36:211-213; 37:209-212; 38:210-214, May, 1926 to July 1, 1937.

<sup>4</sup> "Research in Arithmetic," The Twenty-ninth Yearbook of the National Society for the Study of Education (Bloomington, Illinois: Public School Publishing Company, 1930), Part II.

<sup>5</sup> Report of the Committee of Ten (Chicago: American Book Company, Published for the National Education Association, 1894), pp. 105-108.

<sup>6</sup> Report of the Committee of Fifteen (Chicago: The American Book Company, Published for the National Education Association, 1895), pp. 53-58.

study of 1911,<sup>7</sup> and a study of "Arithmetic One Hundred Years Ago," by E. R. Breslich,<sup>8</sup> and the report on "Trends in Arithmetic since 1860" by Metter.<sup>9</sup>

None of these studies and articles had direct bearing upon this study. However, one paragraph of the report by Metter seemed to have a slight relationship. The paragraph follows:

Philosophers, psychologists, and educators have from time to time shifted their points of view with respect to the proper emphasis in securing what is termed an "education," and the type of learning exercises have varied accordingly. However, it must not be forgotten that when, a certain type of subject matter has once been added to the curriculum, tradition tends to keep it there, and the elimination of such matter is secured much more slowly than is the addition to the curriculum. Because of this fact and others, practice lags behind theory, and the content of textbooks and of courses of study do not always reflect that which leading educators desire at a given time.<sup>10</sup>

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<sup>7</sup> Walter A. Jessup, "Current Practices and Standards in Arithmetic," Fourteenth Yearbook of the National Society for the Study of Education, Part I (Chicago: University of Chicago Press, 1915), p. 118.

<sup>8</sup> E. R. Breslich, "Arithmetic One Hundred Years Ago," Elementary School Journal, 25:644-674, September, 1924 to June, 1925.

<sup>9</sup> Harry L. Metter, "Trends in Arithmetic since 1860," Elementary School Journal, 34:767-775, September, 1933 to June, 1934.

<sup>10</sup> Ibid., p. 767.

### III. ORGANIZATION OF THE REMAINDER OF THE THESIS

Old and new aims of education will be reviewed in Chapter II. Old and new aims of arithmetic will be discussed in Chapter III. Chapter IV will include (1) an introduction concerning the arithmetic textbooks used in the study, and (2) the report of the study of each of the textbooks for grades one and two written during the period, 1821 to 1938, inclusive. The report of the study will include the following points:

Name of the textbook

Name of the author

Name of the publishing company

Date of the copyright

Preface of the textbook

Teacher aids relative to use of the textbook

Table of contents

Problems as illustrations of content material relative to the prevailing aims of education and of arithmetic

Summary statement of the study of each textbook

Chapter V will include the general summary and conclusions of the study and recommendations or suggestions for further study of the problem or problems discovered in this study. The bibliography concludes the thesis.

## CHAPTER II

### REVIEW OF OLD AND NEW AIMS OF EDUCATION

Modern American education with its schools and equipment has been an outgrowth of earlier forms of organizations set up in various ancient nations to meet the needs of the special types of civilization. Likewise, the aims of education and the materials and methods of instruction of the present time have been handed down as traditions from the earlier times. Along with the development of the educational institutions of all civilizations, there have been great educational writers and thinkers, who have influenced changes in the aims of education and changes in educational institutions. In some instances, these educators have set in operation movements that have reached far beyond their immediate teachings or writings.

Therefore, as a basis for understanding the materials found in the arithmetic textbooks to be included in this study, it seemed logical to review the old and new aims of education and include in it some of the educators and theorists who have effected changes in the aims of education, the organization of schools, and the content of instructional materials for use in the classroom.

The review will be discussed in the following order

of topics: (1) the old aims of education, and (2) the new aims of education. A summary concludes the chapter.

### I. THE OLD AIMS OF EDUCATION

The discussion of old aims of education will begin with the primitive times and extend through the medieval to the early part of the twentieth century.

Primitive people did not have a formal education, such as that offered in schools. Their society was static, therefore, education was simple. In this society, the child learned to do by doing, thus having actual participation in real, lifelike situations. Education was living. The girl learned by helping her mother and the boy learned by assisting his father in his fishing, hunting, and cultivation of the soil. In religion and social customs, education was imitation of impressive ceremonies of their elders. It may be said that the aim of education was to train the individual to work and worship according to fixed ways of doing things determined by the social group.

Education as an organized institution dates back to the ancient systems of education. Underlying all organized systems of education are the aims. The following quotations give a clear idea of the ancient systems of education and the aims on which they were based:

of ancient systems of education

Ancient systems of education must be understood as parts of the religious and governmental organization of the nations to which they belonged. Numerous glimpses into the educational system of Egypt are given by passages in the Old Testament. It is recorded that Moses was trained in all the wisdom of the Pharaohs. This means that Moses, as a member of the royal family, was given all the benefit of such knowledge as the governmental officials and priests of the day were able to accumulate and transmit.

The educational system of Egypt was thus strictly a religious and royal institution carried on through the priests. The records show that schools for the young noblemen were organized in the palaces. Here discipline was severe and flogging was common. Instruction dealt largely with good manners, with ethical and political principles, and with such sciences as priests and other teachers could command. The measurement of land was one subject taught. This was especially important in view of the necessity of relocating property after the inundations of the Nile. The boys were also taught swimming.<sup>1</sup>

From the preceding quotation, the aims of education seemed to be moral, political, physical strength for the boys, and knowledge of calculations in order to carry on the sciences of the practical every day activities of life.

Greek education was of a somewhat different type. The Greek states had a form of government which required participation for all members of the aristocracy. All those wishing to become leaders in the state must learn to speak well. This required the teaching of rhetoric and the stressing of mental development.

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<sup>1</sup> "Educational Institutions," The Lincoln Library of Essential Information, Vol. II, p. 1617.



Greek education of all periods fell into two divisions,--mental education and physical education, and, that development of the mental and physical in due proportion and harmony was the aim of the Greeks. But, in the course of time, the Greeks discovered that individuals have different mental and physical powers, not only undeveloped but also "unharmonious." They realized that to meet this situation varied training must be given. So to some, those of lesser mental ability, was given training in art and music.<sup>2</sup>

Among the educational theorists, and teachers of the ancient Greeks, who were outstanding were Pythagoras, Pericles, Socrates, Plato, and Aristotle.

Pythagoras seems to have been the first person in Greece, in fact in the western world, who attempted to establish an ethical institution separate from the state. His aim was stated by Davidson as "harmony."<sup>3</sup> Pythagoras wanted men to strive to lead perfect lives. Harmony was to be attained under religious sanction. Pythagoras believed that only a limited number of persons were capable of such harmony, and consequently he selected his pupils with much care. He

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<sup>2</sup> Thomas Davidson, Aristotle and the Ancient Educational Ideals (New York: Charles Scribner's Sons, 1912), p. 7.

<sup>3</sup> Ibid., p. 55.

subjected them to a long period of education in which silence, self-examination, and absolute obedience predominated. He believed this program would enable pupils "to overcome impulse, concentrate attention and develop reverence, reflection and thoughtfulness--the first conditions of all moral and intellectual excellence."<sup>4</sup>

Pythagoras also emphasized the physical by giving attention to proper food, clothing, and exercise. Although his teachings roused bitter resentment to the point that he was persecuted, his teachings left a deep and lasting influence on subsequent thought.

With the close of the Persian Wars came changes. Foreign men, some of whom were private teachers, "sophists,"<sup>5</sup> as they termed themselves, came into the country and began to introduce foreign ideals. They taught the spirit of individualism as against that of the old political spirit in all its institutions--in religion, in politics, and in education. These sophists encouraged the youth to seek their end in their own pleasure and regard the state as a means to their end. To this end young men were taught the art of convincing and showy oratory. By this instruction, the

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<sup>4</sup> Ibid., p. 59.

<sup>5</sup> Ibid., p. 100.

sophists laid the foundation of the art of rhetoric and the science of grammar. But there was a harmful result of their teaching. The young men began to imagine for themselves a care free, indolent, private life and began to look with contempt upon the ideals and duties of the old citizenship.

Efforts were made by Pericles to restore the moral stamina by establishing Athens as the capital and building the great Lyceum and music hall, but he failed.

Next came Socrates who held the belief that men fell into error because what they called thoughts were only opinions, "mere fragments of thoughts."<sup>6</sup> Socrates concluded that in order to remedy this error, both intellectual and moral, men must be required to think whole thoughts. With this in mind, Socrates gathered together the ordinary questions, arranged them into a series of well directed questions, and by use of them attempted to bring out "the wholes of which they were parts."<sup>7</sup> Thus the conversational or "Socratic"<sup>8</sup> method of classroom procedure of today had its origin with Socrates. However, Socrates failed somewhat in his teaching because he was blinded by his belief that if men knew the right they would be ready to follow it.

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<sup>6</sup> Ibid., p. 108.

<sup>7</sup> Ibid., p. 109.

<sup>8</sup> Ibid., p. 110.

He failed to stress right thinking, right feeling, and right doing.

Plato, who was a follower of Socrates and a philosopher, attempted to carry out Socrates' idea of "thinking whole thoughts." But he was faced with three other questions: "How can whole truths be reached? What do they prove when they are reached? How can they be applied to the moral re-organization of human life?"<sup>9</sup> Plato's answers to these were:

"Dialectics, including Logic and the Theory of Knowledge, Theoretics, including Metaphysics and Physics, and Practis including Ethics and Politics."<sup>10</sup>

Plato's theory of education partially failed because it did not take into account the evolution of society and did not include all classes.

Aristotle, a pupil of Plato for twenty years, differed with Plato. He held that truths reached by the "dialectic" process are formal and therefore useless. He repeatedly said that we must draw our general principles from the particular facts but he omitted the verification. His method was that of "induction," although he limited his work to the analysis of deductive reasonings. Davidson

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<sup>9</sup> Ibid., p. 134.

<sup>10</sup> Ibid., p. 109.

quotes from Wilhelm Oncken regarding Aristotle:

Aristotle is the Father of the Inductive Method, and he is so for two reasons: First, he theoretically recognized its essential principles with a clearness, and exhibited them with a conviction which strike the modern man with amazement; and then he made the first comprehensive attempt to apply them to all the science of the Greeks.<sup>11</sup>

Davidson further quotes:

Three principles Aristotle lays down as valid for all education: (1) that the training of the body ought to take precedence in time over that of the mind; (2) that pupils should be taught to do things before they are taught the principles of them; (3) that learning is never playing, or for the sake of playing.<sup>12</sup>

Aristotle also classified intellectual branches as:

"Letters, Music, and Drawing; and the literary subjects as: Grammar, Rhetoric, Dialectic, Arithmetic, Geometry, and Astronomy."<sup>13</sup>

Aristotle did not associate science with the idea of the inductive method of accumulating knowledge, but he thought of it as a deductive and expository method and identified it with "teaching." His view of science is clearly set forth in his statement, "All teaching and all intellectual learning arises out of previously existing knowledge."<sup>14</sup>

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<sup>11</sup> Ibid., p. 154.

<sup>12</sup> Ibid., p. 183.

<sup>13</sup> Ibid., p. 240.

<sup>14</sup> Sir Alexander Grant, Aristotle (Edinburgh and London: William Beackwood and Sons, 1910), p. 69.

A review<sup>15</sup> of the education of China shows that it was a cross between that of the education found in Egypt and Greece. Confucius was the outstanding educator and teacher who laid the foundation for the political system of China. The young men who wished to become educated set themselves to the task of memorizing all the maxims laid down by Confucius. Because of the complexity of the Chinese vocabulary, learning was extremely formalized, and because of the isolation China was content to continue the formalized art of reading and of memory.

A glance at Roman education<sup>16</sup> shows that previous to Rome's ascendancy education was carried on by the father in the home. After about 300 B. C., the Roman boy's education was under the supervision of educated Greeks, who, in many instances had been brought into Rome to serve as slaves of rich Roman families. Often groups of families would employ a Greek to teach their boys. The education of the Romans did not differ materially from that of the Greeks due to the fact that it was an "imported" education. Therefore, the aim of Roman education was practically the same as that of the Greeks.

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<sup>15</sup> "Chinese Education," The Lincoln Library of Essential Information, Vol. II, p. 1618.

<sup>16</sup> "Roman Education," The Lincoln Library of Essential Information, Vol. II, pp. 1618-1619.

except that the Romans stressed training in physical strength to become warriors instead of citizens of the state.

In summarizing the aims of education of the Greeks and the Romans, the former emphasized training for leadership in the state which included intellectual and moral thinking, physical education for strength and beauty of body, and art expressed in music, sculpture and other forms; and the latter emphasized the physical development of powers of resistance needed as future warriors.

Beginning with the seventeenth century, Francis Bacon publicized the use of the scientific method of "induction." According to Graves<sup>17</sup> Bacon did not agree with Aristotle's view of scientific method of deduction and in 1620 published his treatise, Novum Organum, in which he formulated the scientific method of induction. Regarding the value of Bacon's method, Graves says:

In endeavoring to create a method whereby any one could attain all the knowledge of which human mind was capable, he undertook far too much. His efforts to put all men on a level in reaching truth resulted in a most mechanical mode of procedure and neglected the part played by scientific imagination in the framing of hypotheses. But he did largely put an end to the existing a priori reasoning, and did call attention to the necessity of careful experimentation and induction.

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<sup>17</sup> Frank Pierrepont Graves, Great Educators of Three Centuries (New York: The Macmillan Company, 1912), p. 12.

As far as education is concerned, Bacon influenced profoundly the writing and practice of many, and has done much to shape the spirit of modern education. His method was first applied directly to education by a German, known as Ratich, and, in a more effective way by Comenius, a Moravian.<sup>18</sup>

Comenius published many remarkable textbooks on the method of the study of Latin. But the most important work was his Didactia Magna.

According to Graves,<sup>19</sup> Comenius believed that education had three aims, knowledge, morality, and piety. He believed education should enable one to become pious through the forming of moral habits, which are in turn to be formed by adequate knowledge.

Comenius further believed in one system of schools for both sexes. He made it quite evident that education should be natural and not artificial and traditional. He outlined a regular system of schools and described their grading. He was the first to suggest a training for very young children. He emphasized sense training, physical training, enrichment and correlation of subjects. Comenius had little influence upon the schools of his time except through his language textbooks, but his principles influenced Rousseau, Pestalozzi, Herbart, and Froebel and became the

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<sup>18</sup> Ibid., p. 18.

<sup>19</sup> Ibid., pp. 18-19.



basis of modern education. Of the importance of Comenius' contribution to education Graves states:

Hence it happened, when the works of Comenius were once more brought to light by German investigators, it was discovered that the old realist of the seventeenth century had been the first to deal with education in a scientific spirit, and work out its problems practically in the schools. His evidently was the clearest of visions and broadest of intellect. While it is easy to criticize him now, in the light of history Comenius is a most important individual in the development of modern education.<sup>20</sup>

John Milton has been classed among reformers of education, who attempted to introduce the broader study of the classics for their meaning instead of the mere repetition of words. Of Milton, his ideas of education and the curriculum, and his contribution to education Graves says:

Even in his recommendation of a most encyclopaedic program of studies, which is usually one of the marks of the sense realist, he (Milton) seems to imply the "humanistic" rather than the "sense" realism, although he wrote half a century after Bacon and was a younger contemporary of Comenius.

As with some of the other humanistic realists, notably Montaigne, Milton would also have considerable time given at the end of the course, to social sciences, such as history, ethics, politics, economics, and theology, and to such practical training as would bring one in touch with life. He likewise advocates the experience and knowledge that would come from travel in England and abroad. Thus, in the place of the usual restricted conception of humanistic education, Milton would substitute a genuine study and understanding of the classical authors and a real preparation for life. While at first

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<sup>20</sup> Ibid., pp. 49-50.

he piously declares that the aim of learning is "to repair the ruins of our first parents by regaining to know God aright," he is more specific later when he frames his famous definition:--

"I call therefore a complete and generous education that which fits a man to perform skillfully, and magnanimously all the offices both private and public of peace and war."

The school in which Milton would carry out his ideal education, he calls an Academy, and states that it should be held in "a spacious house and ground about it, big enough to lodge one hundred and fifty persons." This institution should keep the boys from the ages of twelve to twenty-one, and should provide both secondary and higher education, "not heeding a remove to any other house of scholarship, except it be some peculiar college of Law or Physic." And he adds: "After this pattern as many edifices may be converted to this use as shall be needful in every city throughout this land."

Strangely enough, this educational curriculum and organization of Milton's, exaggerated as they were, found a partial embodiment and function in a new educational institution that became of importance in England and the United States. "Academies" based upon this general plan were organized to meet certain exigencies of the English nonconformists, that arose toward the end of Milton's life.

So, in America . . . the first suggestion of an "academy" was made in 1743 by Benjamin Franklin. He wished to inaugurate an education that would prepare for life, and not merely for college.<sup>21</sup>

Concerning Locke and his educational aim and contribution to education Graves<sup>22</sup> says that Locke's educational position is usually misunderstood; that the general estimate of his theory is taken from his work entitled

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<sup>21</sup> Ibid., pp. 2, 3, and 5.

<sup>22</sup> Ibid., p. 52.

Some Thoughts Concerning Education, and that his underlying principles are at variance with those as given in his Essay Concerning the Human Understanding, and with the intellectual training recommended in his Conduct of the Understanding. Graves says that if Thoughts alone is read, Locke's aims of education are "Virtue, Wisdom (i. e., worldly wisdom), Breeding and Learning."<sup>23</sup>

Locke gave character the first place in importance in education in his works, Some Thoughts Concerning Education, as may be seen from the quotation found in Quick's book, Locke on Education:

A Sound Mind in a sound Body, is a short but full Description of a Happy State in this World. He that has these two, has little more to wish for; and he that wants either of them will be but little better for anything else. Men's happiness or Misery is most part of their own making. He, whose Mind directs not wisely, will never take the right way; and he whose Body is crazy and feeble will never be able to advance in it. . . . I think I may say, that of all the men we meet with, nine Parts of ten are what they are, good or evil, useful or not, by their Education. 'Tis that which makes the great Difference in Mankind.<sup>24</sup>

Locke stressed in this same treatise recreation in the form of play, aesthetic dancing, music, riding, wrestling, education by travel, and care in speaking well the mother

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<sup>23</sup> R. H. Quick, Locke on Education (Cambridge: The University Press, 1902), pp. 1-2.

<sup>24</sup> Loc. cit.

tongue. He advocated learning through sense perception. He emphasized the lasting effect of impressions in infancy, the proper care of the health and development of the mind by discipline in their earliest years. Of the importance of this last point, early mental discipline, Locke says:

The Great mistake I have observed in People's breeding their Children, has been, that this has not been taken Care of in its due Season; that the Mind has not been made obedient to Discipline and pliant to Reason when at first it was most tender, most easy to be bow'd.<sup>25</sup>

Graves<sup>26</sup> says that Locke's real attitude in education must be taken chiefly from the Conduct, and read in the light of his philosophy as expressed in his Essay Concerning Human Understanding. He says that in the former works, Locke holds that the mind like the body grows through exercise, and that the best practice for reasoning is found in mathematics; while in his work Thoughts he maintains that moral training is to be obtained by control of reason; and that physical training is to be obtained by that of formal discipline, that has since been known as the "hardening process."

Because of the disciplinary conception of education held by Locke, Graves said that Locke would seem to be the

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<sup>25</sup> Ibid., p. 21.

<sup>26</sup> Frank Pierrepont Graves, op. cit., pp. 58-66.

first writer to advocate "formal discipline." And as to the effects of the principle of formal discipline upon education, it may be said to have had tremendous influence upon each stage of education in practically every country and during every period up to the last decade when a reaction began.

Jean Rousseau's position among the great educators, who effected changes in educational thinking and organization and methods of procedure cannot be slighted.

Rousseau's publication of Emile made him famous. Emile, it seems, is a product of Rousseau's childhood experiences. His mother having died at his birth, he was unrestrained in the home of his indulgent aunt when he indulged in stealing, lying, or other moral offences of childhood. His careless father, when Rousseau was only six, sat up night after night until daylight reading the silliest and most sensational of romances from the large collection of Jean's mother. When this source of reading could be had no more, Jean turned to his grandfather's library, where he found such works as the Parallel Lives by Plutarch and standard histories of the day. Later Jean went to the country, where his already existing love for nature was greatly enhanced. He remained here until he received a severe punishment for a boyish offence, which Rousseau himself said

caused him to begin to evolve the theory that restraint and discipline of the impulses and the departure from nature has always corrupted and ruined humanity.

After this experience Rousseau spent two years in idleness, four years as an apprentice, in which time he was influenced by bad companions. Eventually he ran away and wandered from place to place. At last he settled in Paris and there began to assume a sense of responsibility.

Emile was directed against the artificial education of the day. It aimed to replace it with a training that should be natural and spontaneous. Emile was divided into five books. The first book starts with Rousseau's basic principle that "everything is good as it comes from the hands of the Author of Nature; but everything degenerates in the hands of man."<sup>27</sup> Therefore Rousseau thought that a child should be removed from society when an infant and be given a natural and physical training, but no moral training, because Rousseau held that in infancy a child's instincts are good by nature, and free from vice, and his intelligence is free from error. This negative type of training is continued in the second book. The child from his fifth to his twelfth year is to learn only through the consequences of his own actions. Development of body versus mind is stressed.

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<sup>27</sup> Ibid., p. 86.

The third book deals with the boy from his twelfth to his fifteenth year, which Rousseau calls "boyhood." During this time he is to learn useful studies without books, except Robinson Crusoe, and to take up cabinet making. This book shows Rousseau's belief in the appeal to curiosity and investigation.

The fourth book deals with the boy from fifteen on, when his mind is to be prepared morally, and religiously by his visits to people of unfortunate conditions, and to be exposed to all classes of literature.

The fifth book deals with the education of the woman, whom Emile is to marry. In it Rousseau advised physical training (but more to the purpose of beauty than that of strength), domestic arts, obedience and industry, singing, dancing and other accomplishments, and instruction in religion at an early age. As to ethical manners the woman was to guide herself largely by public opinion.

Graves holds that the fifth book is the weakest, because Rousseau abandoned the natural, individualistic training which he advocated for man and recommended the passive, repressive training for woman.

But with all the criticism that Rousseau's theory had to undergo, many of its important principles are seen in practice in the schools of today. To Rousseau may be

credited the laying of the foundation of the present social aim of education, the study of the child as a basis for the type material suitable for his mental level, the modern regard for the psychological freedom of the child in his thinking, feeling and acting, and the gradual disappearance of the old educational idea that a task is of no value except to the degree that it is difficult and distasteful.<sup>28</sup>

The first outstanding educator after Rousseau was Pestalozzi. Of him Graves says:

The happiest educational results of Rousseau came through Pestalozzi. It was Pestalozzi that developed the negative and inconsistent naturalism of the Emile into a positive attempt to reform corrupt society by proper education and a new method of teaching.<sup>29</sup>

Pestalozzi's early training by his mother influenced his ideals of education and his grandfather's example inspired him to elevate the peasantry through the ministry and law. But in this he was not successful. Then Pestalozzi became interested in demonstrating to the peasants the improved methods of agriculture, but this, too, was a failure. In the meanwhile, a son had been born to him. Pestalozzi followed the suggestions of Emile in rearing him, and recorded the results. These gave him new ideas and educational

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<sup>28</sup> Ibid., pp. 77-110.

<sup>29</sup> Ibid., p. 122.



principles. He held that education did not consist merely in books and knowledge, and that children could be taught to earn a living and at the same time develop mentally and morally. This venture he tried out in his home, where he taught twenty of the most needy children, feeding, clothing and treating them as his own. To the boys he gave practice in farming and to the girls domestic duties and needlework. In bad weather he taught both sexes to spin and weave. They were trained in proper speaking and given an opportunity to practice it in conversing and memorizing the Bible before learning to read and write. Most of the training in this type of subject matter was done while the children were working. The results of Pestalozzi's experiment encouraged him to the extent that he increased the enrollment of pupils, and as a result was forced to bankruptcy.

However, Pestalozzi later took charge of orphan children in the Ursaline Convent at Stantz where he taught them through observation rather than by use of books. But, again, Pestalozzi had to abandon his project because the convent at Stantz had to be used for a hospital by French soldiers. After some time, influential friends secured a position at Burgdorf, where he continued to develop his method, and attempted to reduce the elements of reading and arithmetic to their simplest forms and group them psychologically so

that the child could progress from the first step to the second and so on in succession. He devised for arithmetic, boards divided into squares upon which he placed dots or lines representing each unit up to one hundred. This table was used to enable the pupil to get a clear idea of the meaning of the digits and the process of addition.

To explain his method in detail, Pestalozzi wrote How Gertrude Teaches Her Children in which he states his educational creed. Graves quotes these as summarized by Pestalozzi's biographer, Morf. The summary follows:

1. Observation is the foundation of instruction.
2. Language must be connected with observation.
3. The time for learning is not the time for judgment and criticism.
4. In each branch, instruction must begin with the simplest elements and proceed gradually by following the child's development; that is by a series of steps which are psychologically connected.
5. A pause must be made at each stage of the instruction sufficiently long to get the new matter thoroughly into his grasp and under his control.
6. Teaching must follow the path of development, and not that of dogmatic exposition.
7. The individuality of the pupil must be sacred for the teacher.
8. The chief aim of elementary education is not to furnish the child with knowledge and talents, but to develop and increase the powers of his mind.
9. To knowledge must be joined power; to what is known, the ability to turn it to account.
10. The relation between master and pupil, especially so far as discipline is concerned, must be established and regulated by love.
11. Instruction must be subordinated to the higher end of education.<sup>30</sup>

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<sup>30</sup> Ibid., p. 136.

Pestalozzianism was introduced in the United States early in the nineteenth century but at first did not receive much attention. A little later various articles concerning it appeared in the American educational journals describing the Pestalozzian principles and methods. Educators and travelers, who had returned from Germany, began to suggest the new principles as remedy for the educational defects in the United States. Warren Colburn was attracted by the methods and applied them to his "mental arithmetic," First Lessons, in which he even printed Pestalozzi's "table of units." This arithmetic spread "mental arithmetic" throughout the country.

The most influential movement was brought about by Horace Mann, who spoke most enthusiastically about the Pestalozzian methods and recommended them. This resulted in the establishment of the "Oswego methods" in the Oswego schools, where teachers were trained for teaching. As a consequence the Pestalozzian methods were, during the last quarter of the nineteenth century, the prevailing methods in the elementary schools of the United States.

The industrial phases of the Pestalozzian influence came later into the United States. Such schools as Carlisle, Hampton, and Tuskegee are examples of industrial schools.<sup>31</sup>

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<sup>31</sup> Ibid., p. 122.

The next influential educator following Pestalozzi was Johann Herbart, who stressed Pestalozzian principles from the standpoint of the teacher. Graves says of Herbart: "He is the first example of the philosopher and psychologist in education."<sup>32</sup>

Herbart believed that any idea once formed struggles to preserve itself, and that each new idea or group of ideas is retained, modified, or rejected according to the degree of harmony or conflict it sets up with the already existing idea. This is Herbart's principle of apperception and is the central doctrine in his whole educational system.

Herbart's aim of education is that of establishing a moral and religious life. To accomplish this end Herbart thought instruction was necessary. He held that this instruction must be so selected and arranged as to appeal to the previous experiences of pupils, and reveal all the relations existing. He would have many varied interests so that the pupils would experience a wide range of ideas.

Herbart emphasized correlation of subjects. In order to have this broad range of materials and to unify them, Herbart held that the two-fold mental process called absorption and reflection was necessary. Absorption meant

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<sup>32</sup> Ibid., p. 167.

the acquisition of facts or ideas and reflection meant unifying or assimilating related facts or ideas previously acquired. From this process of absorption and reflection, Herbart worked out the four steps in his method of instruction. The first step is clearness, the presentation of the material to be learned; the second is association, the uniting of related facts previously acquired; the third, the system, the coherent and logical arrangement of the united-related facts; and the fourth, applying the system to new data.

This method formulated by Herbart was only in principle, but it has since been modified and developed by his followers into what are now known as formal steps of instruction. These steps usually are stated as: preparation, presentation, comparison and abstraction, generalization, and application.<sup>33</sup>

The United States was attracted to the Herbartian doctrine to the extent that an Herbartian Society was formed to extend the principles and adopt them. Besides, individuals utilized these principles in textbooks. Among these textbooks were: Essentials of Method by Charles De Garro; General Method by Charles McMurry; and The Method of Recitation by Frank McMurry and Charles McMurry.

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<sup>33</sup> Ibid., pp. 167-193.

Furthermore, when the reaction against the formalized period set in, Herbartian principles began to gain favor and committees were formed to investigate the instruction in the schools. The Committee of Fifteen appointed by the National Education Association show the effect of the Herbartian influence.<sup>34</sup>

It seems to be generally agreed by writers on Herbartian methods of education, that no system of pedagogy has had as extended an influence in the United States, to date, on educational thinking and classroom teaching.<sup>35</sup>

Another great educationalist, who was also a follower of Pestalozzi, was Friedrich Froebel. He concerned himself with the child's development and its activities. According to various writings, Froebel had difficulty in finding his life work. Finally, however, he began to teach. But he realized he needed a broader training and began his study under Pestalozzi. In 1835 Froebel was invited to come to the castle of Burgdorf, where Pestalozzi had formerly been, and establish a school for the training of teachers. It was while here, that Froebel began to devise playthings, games, and songs with bodily movements to accompany them, because

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<sup>34</sup> "Report of the Committee of Fifteen," Proceedings of the National Education Association (Chicago: American Book Company, 1895), pp. 53-58.

<sup>35</sup> Frank Pierrepont Graves, op. cit., p. 191.

he believed these would be valuable in the young child's development.

Later Froebel opened a school in Blankenburg, Germany, and named the school Kindergarten, which means a "garden" in which "children" are the unfolding plants. Here Froebel put into practice his ideas, principles, and materials which he had invented earlier.

Froebel's principles were based on his conception of "organic unity" of the universe. Out of this grew his educational aim: that education should lead one to see the continuity of all things in the universe and to recognize the unity of all things in the eternal unity, "God."

Froebel applied this idea of unity or connectedness between the home and the school in training the child. His method of education was "self-activity" through which the child was to carry out his own plans. Thus he was led to the act of "creativity," by which new ideas were formed and expressed.

Froebel's educational principles are not generally followed, but in the United States they have had a lasting influence.<sup>36</sup>

Herbert Spencer is included in this review for the reason that he gathered together the educational principles

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<sup>36</sup> Ibid., pp. 194-234.

of Rousseau, Pestalozzi, and other educational reformers and stated these principles in a logical order, and defined the aim of education.

Spencer says: "The function of education is to prepare for complete living."<sup>37</sup>

Spencer's principles are as follows:

1. In education we should proceed from the simple to the complex.
2. Our lessons ought to start from the concrete and end in the abstract.
3. The education of the child must accord both in mode and arrangement with the education of man considered historically.
4. In each branch of instruction we should proceed from the empirical to the rational.
5. The process of self-development should be encouraged to the fullest extent.
6. There is always a method productive of interest, and this is the method proved by all other tests to be the right one.<sup>38</sup>

With the "common school revival" in education in the United States, during the latter half of the nineteenth century, Horace Mann was outstanding, because of his interest

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<sup>37</sup> Orlie M. Clem, A Work Book Syllabus in Principles of Education (Baltimore: Warwick and York, Inc., 1928), p. 60.

<sup>38</sup> Herbert Spencer, Education (New York: D. Appleton and Company, 1909), pp. 115-125, cited by Graves, op. cit., p. 282.



and direct leadership in education in Massachusetts and his influence in general on education in all the United States. He was elected to the Board of Education in Massachusetts. In his attempts to reform the schools he had to meet the conservatism and bitter prejudices even of the schoolmasters. Among the greatest of his accomplishments was the establishment in Massachusetts of the first public state normal schools in this country, one at Lexington, one at Barre, and one at Bridgewater.

Graves discusses Mann's view of the aims of education as follows:

His foremost principle was that education should be universal. Girls should be trained as well as boys and the poor should have the same opportunities as the rich. This universal education, however, should have as its chief aim moral character and social efficiency, and not mere erudition, culture, and accomplishments. But while the public school should cultivate a moral and religious spirit, this could not be accomplished, he felt, by inculcating sectarian doctrines.<sup>39</sup>

Graves further says:

But Mann was mainly a practical, rather than a theoretical reformer. To the material side of education, he gave serious attention. He declared that school buildings should be well constructed and sanitary. . . . As to methods he maintained that instruction should be based upon scientific principles, and not upon authority and tradition. Pestalozzi's inductive method of teaching received his approval, for he felt that the pupils

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<sup>39</sup> Frank Pierrepont Graves, A History of Education in Modern Times (New York: The Macmillan Company, 1913), p. 165.

should be introduced at first to the facts of the humanities and sciences. . . .

Mann rendered practical and brought into use many of the contributions made to the educational theory by others, and thereby anticipated many of the features of later educational practices. The word method of reading took the place of the uneconomical, artificial, and ineffective method of the alphabet, and the Pestalozzian object methods and oral instruction were introduced. The connection between physical and mental health became better understood. Thus during the educational awakening the people of Massachusetts renewed their faith in the common schools.<sup>40</sup>

## II. NEW AIMS OF EDUCATION

The aims of education of the first quarter of the present century have been concerned with child development in terms of unfoldment and adjustment in preparation for future living in society. Textbooks on the aims, principles, and methods in education and on psychology have been written by many educators.

Bolton states the aim of education as follows: "Education is a process of development and of modification or adjustment to environment and to the ideals of perfection conceived by society and the individual."<sup>41</sup>

"Education may be tentatively defined, then, as a

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<sup>40</sup> Ibid., pp. 174-179.

<sup>41</sup> Fredrick Elmer Bolton, Principles of Education (New York: Charles Scribner's Sons, 1910), p. 11.

process by means of which the individual acquires experiences that will function in rendering more efficient his further action,"<sup>42</sup> says Bagley. He next explains the forces or kinds of education to which the child is exposed. He says there are two: informal education, which embodies the modifying influences or learnings by actual experiences, and formal education, which embodies the modifying influences of some persons, or institution, such as home, church, and school. Bagley defines the aim of the school in terms of formal education as follows: "The school, then, is a specialized agency of formal education which aims to control in a measure the experiences of the child during the plastic period of infancy."<sup>43</sup>

Bagley deduces a proposition as to the ultimate aim of education and states it as follows: "Social efficiency has been proposed as the ultimate aim of education."<sup>44</sup> Then in turn he defines social efficiency to mean the "bread-and-butter" aim, "the knowledge" aim, the "harmonious development of all the powers and faculties" aim, and the development

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<sup>42</sup> William Bagley, The Educative Process (New York: The Macmillan Company, 1914), p. 22.

<sup>43</sup> Ibid., pp. 23 and 32.

<sup>44</sup> Ibid., p. 61.

of "moral character" aim.<sup>45</sup>

Klapper states the following relative to education: "Education is the organization of acquired habits of action such as will fit the individual to his physical and social environment."<sup>46</sup>

Hopkins says: "There are those who believe in education as preparation for life, usually adult life."<sup>47</sup>

Edward L. Thorndike, leader of the scientific movement in the twentieth century began to question the emphasis upon the unfolding of mental processes in education and preparation to live in the future. His emphasis is upon the child and the achievement of the fullest satisfaction of the wants of mankind.

The following paragraphs show some of Thorndike's criticisms and views on education:

Education is concerned with any change which influences the interaction of man and his world. The chief aim of education is to realize the fullest satisfaction of human wants. Human wants are given this position of supreme importance for the reason that anything, act, or event in life has importance, value, interest or

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<sup>45</sup> Ibid., pp. 44-64.

<sup>46</sup> Paul Klapper, Principles of Educational Practices (New York: D. Appleton and Company, 1912), p. 16.

<sup>47</sup> L. Thomas Hopkins, Curriculum Principles and Practices (Chicago: Benjamin H. Sandborn and Company, 1934), p. 50.

significance only as it tends to affect--to satisfy or thwart--man's cravings. Human cravings become the central concern of the process of education because they are the primary and essential factors in initiating and sustaining actions of all kinds. Thinking, imagination, feeling, acting, forming and breaking habits are subordinate to dynamic forces--which may be termed wants, urges, cravings, impulses, interests--which generate and maintain them. To change a want is to make the most fundamental of possible changes. Once a want is changed, all sorts of subordinate changes in thought, feeling, and action occur as a result. . . . To secure for each person the fullest satisfaction of his wants, we must seek to effect those changes both in man and nature which add to the satisfaction not of any particular person, family, nation, race, or other group, but of humanity in general. Each individual will secure the fullest realization of his wants when they harmonize with and facilitate the fulfillment of the wants of mankind as a whole.<sup>48</sup>

J. Stanley Gray says the following regarding the social aims:

The function of education is to train individuals so that they will become production specialists and actively co-operative with other specialists in the common problem of society--that of reducing the limitations of: disease and death, restricted sensory range, faulty inheritance, food and shelter activities and competition. This is a dichotomous function and implies both technical and liberal education. Specialization can be developed only by liberal training. Whether they should be given separately or together is a problem for educators to solve, but no individual should be without either type of training if he is to contribute to the reduction of the limitations mentioned above. Specialization and consequent division of labor are impossible without co-operation, and co-operation is unnecessary without

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<sup>48</sup> Edward L. Thorndike and Arthur I. Gates, Elementary Principles of Education (New York: The Macmillan Company, 1929), pp. 9-31.

specilization.<sup>49</sup>

Dewey says:

Since growth is the characteristic of life, education is all in one with growing; it has no end beyond itself. The criterion of the value of school education is the extent in which it creates a desire for continued growth and supplies means for making the desire effective in fact.<sup>50</sup>

Kilpatrick states:

Mere school education cannot possibly suffice for the whole of life. To think otherwise is to misconceive and belie the very meaning of education in relation to life. Education goes on as life goes on. Life is a novelly developing process. It does not repeat itself. . . . Education is most truly conceived as being life itself creatively facing its novelly emerging problems. Under such circumstances education must continue all through life.<sup>51</sup>

During the past century tradition has largely governed the educational content and method. But today educators are questioning: (1) what education may be best for society, (2) why may that education be best, and (3) how may that education function?

Draper gives the following regarding the principles of life and of education:

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<sup>49</sup> J. Stanley Gray, Pathological Foundations of Education (Chicago: American Book Company, 1935), p. 35.

<sup>50</sup> John Dewey, Democracy and Education (New York: The Macmillan Company, 1916), p. 62.

<sup>51</sup> William Heard Kilpatrick, Education and the Social Crisis (New York: Liveright 'Inc' Publishers, 1932), p. 50.

Education will always present problems since the social life of any group is constantly changing. . . . Education can establish no final goal although the starting point and the general guide lines can be more or less definitely determined since they represent the social outcomes and purposes which are the essence of the life of the society. Education must be considered as a process of experience and type of modification through which the learner can adjust himself to the society in which he lives, and in turn, modify and advance that society. Modern education is growth in terms of the native endowment of the individual and also in terms of the social purposes of the nation or race to which he belongs.

Further it should be pointed out that these goals are not static in the affairs of men, but are constantly changing in terms of the evolution of society. They are dynamic in the sense that the aims have different connotations from age to age and from generation to generation. . . .

Educators cannot look to the past for guidance in the solution of the problems of today and tomorrow. The economic adjustment that is being made in American life and the resulting social adjustments find no counterpart in history. However, it will be significant for the reader to think of education as having a geographic and cultural location as being a function of a nation or a state at a particular time in the development of western civilization.<sup>52</sup>

Percival Symonds expresses the present day aim as follows:

Today we are concerned not only with the academic child--but with the whole child, which will include his attitudes and ideals, his likes and his dislikes, his fears and worries, his conflicts and inhibitions, his unified and integrated outlook on life, and many

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<sup>52</sup> Edgar Marion Draper, Principles and Technique of Curriculum Making (New York: D. Appleton-Century Company, 1936), pp. 76-83.

little habits and skills of social adaptation.<sup>53</sup>

Most educators agree today that in order to understand the child the teacher must give much attention to his social background. The teacher of today is conscious of the fact that two social groups are in the child's social background. One is the family or home group in which he has experience of some kind, wholesome or unwholesome; the other is his play group, or gang, which may be either good or bad. The problem of the teacher and of the school is that of attempting to substitute for the unwholesome activities those activities which will aid the child to live well socially both in school, in his play, and in his home. All of this led to, and is continuing to lead to, a change away from set textbook and formal class recitation procedure to life-like situations, in so far as it is possible with the equipment that the school administration has supplied.

Bode describes the progressive school as follows:

"The progressive school is a place where children go, not primarily to learn, but to carry on a way of life."<sup>54</sup> Bode continues his comments on the progressive schools as to the

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<sup>53</sup> Percival M. Symonds, "The Contribution of Research to the Mental Hygiene Program for Schools," School and Society, 34:40, July 11, 1931.

<sup>54</sup> Boyd H. Bode, Progressive Education at the Cross Roads (Chicago: Newsch and Company, 1938), p. 9.



interest appeal of school activities contrasted to real life activities. He says:

The progressive school is an artificial situation, not in any invidious sense, but in the sense that it is a substitute for the life outside of school. . . . What is substituted is not some implied way of life, but a series of discrete activities. Hence there is not the same practical reason for doing things as there is outside the school. There is not the incentive that comes from direct participation in the activities of adults. There is in short, no adequate continuity between the school and what may by contrast, be called "real life." Yet incentives must be presented in order to prevent school work from degenerating into meaningless routine. Consequently appeal has been made to immediate interests, which provided an escape from the difficulty but which could hardly be expected to go the whole way. To interpret the doctrine of interest as meaning that all activity must be motivated by immediate and spontaneous interests is to misrepresent it. There is no warrant for such interpretation in the facts of everyday life. We have this doctrine of interest because we have "progressive" schools.<sup>55</sup>

The following summarizes progressive education of today:

1. Concern for the individual, the development of his personal traits and his ever growing personality versus acquisition of subject matter.
2. Interest, not mere entertainment or enjoyment, but that which involves purpose and appreciation of the activity.
3. Activity, or learning to do by doing. Construction

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<sup>55</sup> Ibid., p. 52.

or planning performance, accomplishment, and application are stressed.

4. Special provisions are made for creativeness. Hard and fast rules for perfect performance in the production are not in evidence, but appreciation for the effort of necessary skills, however imperfect they may at first be.

5. Stress is given to thinking, by placing the child in problematic situations and by helping him to find a way to solve them.

6. Special effort is made to promote social-mindedness, by having situations in which children and teacher participate in group enterprises and learn to live in the give and take conditions of a little democratic society.

### III. SUMMARY OF CHAPTER

The aim of education of the primitive was to train the individual in fixed ways of doing things and in fixed ways of religion. In other words, education was imitation of the ways of life in a static social group.

The aim of the ancient Greek education was provision, except in case of Sparta, for the development of personalities, with emphasis on intellectual, aesthetic, moral and physical development of the individual. The aim tended toward the idealistic in that the harmonious development of the individual

intellectually, morally, and physically was stressed.

The aim of the Romans was an imported aim, but education for service as a warrior was uppermost and education became formal and useless for other purposes.

The sixteenth and seventeenth centuries were periods of scientific awakening when educationalists began to advocate elements that later were found in modern society and education, such as the technique of science both in application to physical health, to the good of society, and to the mental training of individuals. Sense realists began to advocate the scientific, practical, and social studies. Comenius was the outstanding sense realist, far in advance of others of this period. He recommended education through sense training and according to the natural, gradual development of the child. He advocated education for the good of the child, and not for the good of the institution. He was a believer in education in early childhood.

The eighteenth century had as its outstanding contributor to points of view in modern education, Rousseau. He believed in education as the natural, unfoldment method beginning with the young child apart from society until about fifteen years of age. He stressed interest and education through sense impression and physical development through physical activity.

Pestalozzi endeavored to make practical application of Rousseau's workable theories to school practice, thus formulating principles of education on the basis of experimentation. His aim was to develop the child mentally, physically, and religiously. Learning was to take place through sense training or object teaching. He believed so strongly in this method that he made tables for use in arithmetic. This method later in the century became that of mere formal drill for the exercise of the mind.

Herbart followed Pestalozzi. He gained his distinction of being the first to formulate a program of education based on psychology. He rejected the theory that the mind consists of separate faculties. He held the mind to be a unit and place where ideas are stored, and that mental life begins with presentations which are the sensations and perceptions of objects. He believed that when these objects are presented the mind reorganizes them by analysis, and regrouping. From this theory Herbart worked out the steps for teaching, which have, with somewhat different terms given by Herbart's followers, come down to modern education. Today they are known as the five formal steps of teaching--preparation, presentation, association, generalization and application.

Froebel, Herbart's contemporary, was the promoter of education as self-activity and the founder of kindergarten

training which became very important in the United States during the latter half of the nineteenth century. The importance of kindergarten training has continued to the present day.

The nineteenth century marked the establishment of the public schools of the United States and the writing of American textbooks. Religious training was emphasized in the first half of the century and faculty psychology and mental training were important to about the last quarter of the century. During the latter quarter of the century there began to be a dissatisfaction with the static systems, followed by growth in education in the next century and a swing away from formal education.

The present century is concerned with two important movements in education: the scientific and the progressive.

The scientific has led educators to look to experimentation instead of tradition and custom. Research studies, involving the study of the child in order to find out more about his ability to learn, and the study of the types of materials suitable for the various levels of the child's mental ability, are constantly being carried on. The name of Edward L. Thorndike is connected with this movement.

The progressive movement has connected with it the names of Dewey, the leader, and Kilpatrick and Bode as outstanding supporters. Progressive education is concerned

with the development of the whole child in relation to the group. This concept is the foundation of the school curriculum which is based on the nature of the community. Progressive educators believe, (1) that education is not preparation for living, but life itself; (2) that education is continuous throughout life; (3) that education is an active process; and (4) that education grows out of human experience rather than out of the mere learning of materials included in textbooks. Progressive education stresses first hand experiences, interest, thinking, social-mindedness, personality, physical health, and mental hygiene.

## CHAPTER III

### REVIEW OF OLD AND NEW AIMS OF ARITHMETIC

Modern aims of arithmetic, like those of education, had their basis in primitive society, when primitive man experienced his first need for arithmetical meaning and expression. Arithmetic has thus been important in the lives of all people from the earliest to the present time. The aims of arithmetic have changed from time to time. One is able to trace reasonably well these aims during this period of time by the study of the history of arithmetic, and the educational influences effecting changes in arithmetic, and by the educational influences effecting changes in arithmetic materials and methods.

Therefore, as a basis for understanding the materials in the arithmetic textbooks, a review of the old and new aims of arithmetic will follow.

The review will be discussed in the following order of topics: (1) the old aims of arithmetic, and (2) the new aims of arithmetic.

#### I. THE OLD AIMS OF ARITHMETIC

All writers of the history of arithmetic seem to be agreed that as a science arithmetic dates from the earliest days of the primitive people, but as a school subject

arithmetic is relatively young.

Of arithmetic as a science, Smith says:

Of all the sciences, of all the subjects generally taught in the common schools, arithmetic is by far the oldest. Long before man had found himself an alphabet, long before he first made rude ideographs upon wood or stone, he counted, he kept tallies upon notched sticks, and he computed in some simple way by his fingers or by pebbles on the ground.<sup>1</sup>

Susan Cunnington discusses arithmetic of the primitive times as follows:

The earliest efforts of man in counting were made so long ago that it is impossible to trace them. But as some idea of number must have followed closely upon the recognition of things we may conclude that counting is as old as speech, and much older than writing. In the childhood of the world, when the wants of men were few, his words for expressing them were few; and we find that primitive languages have hardly any other parts of speech than nouns and verbs, and no connected account or description is possible in them. Similarly, ancient counting consisted merely in enumerating the numbers of things actually seen, and not in the combining of quantities by means of calculation. Before writing was known there was no way of registering numbers, and so a system of notation was unknown.

The first calculation was no doubt that of 1 and 1; and though now the term "calculation" applied to such a process sounds absurd, yet once upon a time real calculation was needed. The earliest primitive man who grasped the idea that as 1 goat together with 1 goat comprised 2 goats, and 1 ox together with 1 ox comprised 2 oxen, so any one thing together with a similar thing comprised 2 such things, had made a most important advance in mental perception. Many centuries passed, however, before familiarity with this truth led to the

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<sup>1</sup> David Eugene Smith, The Teaching of Arithmetic (Chicago: Ginn and Company, 1913), p. 1.



grasping of the abstract idea that one and one are two.

The above earliest calculations would be made by the help of those natural aids to counting, the two hands. Later on the calculations became possible by the help of the less obvious counters, the ten fingers. We have permanent witness to this in simplest method of grouping, the pair; in the denary scale in which we count; and in the term digit, from L. digitus, finger. . . .

It is evident that though the finger method was both convenient and adequate for a primitive state of society, it is entirely insufficient for large calculations, however ingeniously adapted and extended. Hence we find the next counters employed were shells or pebbles of the beach.<sup>2</sup>

As primitive groups mingled with other groups, counting became more complicated. The finger method and the use of shells and pebbles no longer sufficed. It is thought that the abacus was invented for the purpose of making counting less difficult. According to Cunningham<sup>3</sup> the abacus was used by nations widely separated in time and place.

The aim of the arithmetic of the primitives was not difficult to define, because its use defined the aim. Since all the arithmetic was the process of counting in some form, either by the cutting of notches, laying together the amount denoting a quantity, or by using the fingers, there appears to be only one aim--the purely practical aim, that ability to

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<sup>2</sup> Susan Cunningham, The Story of Arithmetic (London: Swan Sonnenschein and Company, Lim., 1904), pp. 1, 2, and 11.

<sup>3</sup> Ibid., p. 14.

indicate numerical quantity.

According to Smith<sup>4</sup> the aim of arithmetic remained purely practical until philosophy began to develop in ancient civilization. Then numbers began to be differentiated and arithmetic became a subject taught. The arithmetic of the ancient Greeks had two distinct phases: (1) the "logistic," which dealt with numerical calculations in business and was taught to those who would engage in trade or commerce; and (2) the "arithmetic," which was the science of number such as factoring, power, and root of numbers, was taught to the man of "liberal education" or to the "philosopher" to make his mind more philosophic.

The philosophers attitude toward the "logistic," taught to those engaged in business, was that "logistic" was a "vulgar and childish art." On the other hand these philosophers expressed belief in the value of "arithmetic" as being a "mover and guide to truth," and a means of improving the mind.<sup>5</sup>

The aims of arithmetic of the early Greeks, then, would seem to be: (1) the ability to use number calculations in business, and (2) the knowledge of the science of

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<sup>4</sup> David Eugene Smith, loc. cit.

<sup>5</sup> Susan Cunningham, op. cit., p. 16.

numbers for the purpose of establishing truth and improving the mind.

The earliest documents relating to mathematics are those of Babylon and Egypt. Clay tablets of Babylon indicate a well developed system of numbers and knowledge of land measure as early as 3000 B. C. There is in the Rind Collection at the British Museum a treatise written by Ahmes as early as 1700 B. C. But it is thought that the Egyptians studied the science of numbers at a much earlier date, because Ahmes stated that his treatise of 1700 B. C. was a copy of an earlier treatise.<sup>6</sup>

Susan Cunnington says the following regarding the work of Ahmes and other ancient mathematicians:

It (manuscript of Ahmes) is entitled, Directions for obtaining knowledge of all dark things; and consists of a collection of problems and results in arithmetic and geometry. . . .

This book of Ahmes consists of four parts: fractions, examples in subtraction and division, equations, and geometrical problems.

The earliest Greek mathematician was Thales (B. C., 600), but there are no written remains of his teaching.

The first Greek scholar to add anything to the sum of human knowledge was Pythagoras. . . .

The arithmetic of Pythagoras consisted of four principal parts: (1) The Discussion of Polygonal Numbers,

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<sup>6</sup> "Babylonia and Egypt," The Lincoln Library of Essential Information, Vol. 11, p. 1133.

(2) Ratio and Proportion, (3) Factors, (4) Series and Progressions. The treatment of ratio and proportion was perhaps the most useful and valuable branch of the Pythagorean arithmetic.<sup>7</sup>

Two other names are of importance, Euclid, for his work in arithmetical and geometric progressions, and Erastoshenes for the correction of the calendar. His results were used by later astronomers.<sup>8</sup>

The next period, the mediaeval period, revealed more extensive developments of arithmetic and arithmetic textbooks. These contained, chiefly, materials relating to the ratio and properties of numbers. Nichomachus was the first great arithmetician of the Christian era. His works were similar to those of Pythagoras. Nichomachus's works were studied for about four centuries and his fame was carried on by Boethius, whose works formed the basis of nearly all the mediaeval arithmetic.

Among the Hindus, only the names of Brahmagupta and Bhaskara are well known. The former compiled a book containing all the mathematical science then known to the Hindu people; the latter wrote a fuller and more valuable arithmetic, as a part of astronomy. His book dealt with many of the topics of later arithmetic such as weights and measures,

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<sup>7</sup> Susan Cunnington, op. cit., pp. 103, 106, and 109.

<sup>8</sup> Ibid., pp. 114 and 116.

numeration and notation, square and cube root, fractions, simple and compound proportion, interest, discount, and barter.

Although the Hindus had only these two outstanding mathematicians, scholars from Arabia were instrumental in maintaining mathematical knowledge. It was through the Arabs that the Greek and Hindu learning were united and brought to Europe. The Arab's contribution to Europe was a practical system of notation and calculation. The Hindu and Arabic system of numerals and the Roman system are made practical for computational purposes by means of a new method of symbol-writing. It was soon adopted by merchants and accountants and spread by means of trade to other parts of Europe and thence to America.<sup>9</sup>

Before the middle of the sixteenth century there were some works of merit by English writers, one of whom was Cuthbert Tunstall, Bishop of London. His works De Arte Supputanda formed the basis of later works on arithmetic. His book was written in Latin. About the same time Richarde Benese wrote a kind of ready reckoner, in which one table supplied the relation between amount of land and the payment for labor on it, and another table supplied information regarding wool, a principal source of wealth at the time.

<sup>9</sup> Ibid., pp. 118, 123, 124, and 128.

The most outstanding works of the century were those of Robert Recorde. They are considered to be the foundation of American textbooks. Recorde was the first author who wrote his works in English.

Names of persons, important in arithmetic development during the seventeenth century, were Stevius, Robert Norton, William Oughtred, and Noah Bridges. Stevius, a Flemish mathematician, extended the denary numeration and notation downward and Norton, an Englishman, translated it into English. Oughtred published "Clavis Mathematica." It contained all the then known arithmetical truth. It was the principal textbook in elementary use at Oxford and Cambridge. Fifteen years later it was translated into English as "The Key of the Mathematicks new forged and filed." Oughtred is said to have been the greatest mathematician in the seventeenth century because of his wide reading and power of systematic arrangement. Noah Bridges' book was "Vulgur Arithmetique, Explayning the Secrets of that Art." In it, appeared for the first time, the ordinary method of long division.

The nineteenth century books, although more numerous, did not add to the study of the subject of arithmetic. Processes were taught as tricks, rules were learned without any basic reason or need, and the acquiring of arithmetical

knowledge was only mechanical.<sup>10</sup>

This type of book with its trick problems as well as that which had justified its existence by its usefulness in business transaction, found its way into America during the colonization by Europe.

When the colonists left England they brought with them the things they deemed essential in their new civilization. Among these things were the Bible, Psalter, and Testament, together with books of their childhood days,--the Horn Book, A.B.C., and Primer Book of Civilitie. These formed the basis of the education and religion of the colonists.<sup>11</sup>

Not only did the colonists bring with them their personal possessions, but also the traditions and ideals which influenced their plan of education. In the first part of the seventeenth century, during the period of colonization of America, the European attitude toward arithmetic was that it was essential to a boy's education only if he were to enter commercial life or engage in certain trades. A boy was often given instruction in arithmetic in a separate school known as a writing or reckoning school. When it was taught in the

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<sup>10</sup> Ibid., pp. 133, 138, 141, and 145.

<sup>11</sup> Edwin Grant Dexter, A History of Education in the United States (New York: The Macmillan Company, 1914), p. 207.

grammar school it was very rudimentary. Among the educated aristocracy arithmetic was considered "common" or "vile," because it was used by tradesmen and those persons connected with the common computation of business transactions. Therefore arithmetic as a subject was not taught to the aristocratic boy who was capable of learning the science of numbers. In case he could not learn the science, he was taught arithmetic so that he could enter a trade or business. Therefore the colonists, who had grown up and been educated in this European environment, brought with them either the attitude of the nobility or that of the tradesman regarding arithmetic.

The Dutch colonists, who had experienced the commercial prominence of Europe, brought with them to their settlement in New York in 1621 this attitude.<sup>12</sup>

Regarding the importance of arithmetic to the Dutch colonists Kilpatrick states the following:

What might be called the official Dutch program for the colonists was that promulgated by the classics in 1636 in the instruction "for schoolmasters going to the East or West Indies."

He is to instruct the youth in reading, writing, and arithmetic, with all the zeal and diligence.<sup>13</sup>

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<sup>12</sup> Walter S. Monroe, Development of Arithmetic as a School Subject (Washington: Government Printing Office, Bulletin No. 10, 1917), p. 5.

<sup>13</sup> William Heard Kilpatrick, The Dutch Schools of New Netherlands and Colonial New York (Washington: Government Printing Office, Bulletin, No. 12, 1912), p. 220.



Kilpatrick further said that, although the official curriculum was not carried out uniformly, according to all the available records in New Amsterdam, now New York, arithmetic was always included in the curriculum. Albany was always an important commercial center and arithmetic was therefore one of the chief subjects of the schools. Only in outlying villages of Dutch New York was arithmetic omitted.<sup>14</sup>

The preceding remarks seem to indicate that the Dutch colonists regarded arithmetic valuable as a practical subject for those who would engage in trade but not as a cultural subject.

Monroe gives the following regarding his study of the place of arithmetic in the early colonial schools:

This survey of the early schools of the American colonies shows that, whether arithmetic was explicitly mentioned along with reading and writing in the official acts of the colonial governments, as in New York, or was omitted, as in case of Massachusetts and Pennsylvania, arithmetic was taught in the public schools in many towns, probably from the beginning. The activities of trade and commerce, which were centered in these towns created a demand for arithmetic, and instruction was given in the subject either in public schools or in private institutions. In these schools arithmetic was primarily a tool of commerce.<sup>15</sup>

Monroe further states:

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<sup>14</sup> Ibid., pp. 220-221.

<sup>15</sup> Walter S. Monroe, op. cit., p. 12.

The aim of arithmetical instruction in this period was not well defined. In a general way the practical needs of trade and commerce were to be satisfied, and this was the principal aim.

The immediate end sought which also represents the standard of instruction was a knowledge of the rules and their application.<sup>16</sup>

Interest in arithmetic as a school subject began to grow during the latter half of the eighteenth century. Yale and Harvard required it as an entrance to their courses. This necessitated the teaching of arithmetic in the elementary schools. Massachusetts and New Hampshire made arithmetic compulsory. English and American writers began publishing textbooks.

The first arithmetic textbook by an American author was, Arithmetick, Vulgar and Decimal: with Applications thereof, to a Variety of Cases in Trade and Commerce, written by Isaac Greenwood, 1729. But it gained no importance in the schools. This was true of all the texts previous to one written by Pike in 1788, entitled A New and Complete System of Arithmetic, Composed for Use of the Citizens in the United States. Pike's book was a very large volume of 511 pages and not a text for young children. It was used in academies and colleges.<sup>17</sup>

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<sup>16</sup> Ibid., p. 15.

<sup>17</sup> Ibid., pp. 17-18.

The table of contents of Pike's book included a large number of varied topics, a few of which were as follows:

Pensions in arrears at simple interest  
 Extraction of the biquadrate root  
 Barter  
 Alligation medial  
 Of pendulums  
 To find the time of the moon's southing  
 Table of Dominical letters according to the cycle of the sun  
 To find the year of indiction  
 Table to find the date of Easter from 1753 to 4199  
 To measure a rhombus  
 To gauge a mash tub  
 The proportions and tonnage of Noah's Ark<sup>18</sup>

Other arithmetic texts followed Pike's. By 1800 at least twenty had been published, but not many of these texts were in the hands of the pupil until about 1821. The plan of teaching was that of the schoolmaster giving the problem to the pupil from a ciphering book, which he had made when he had learned to cipher. He copied the problem on a blankbook which the pupil had made of paper and had sewed together. Some pupils had books of better quality of paper and bound with board or leather covers. The pupil worked the problem on a scrap of paper or a slate. When he had finished he took his work to the master for his approval. If the answer was identical with the master's, the pupil then copied it in

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<sup>18</sup> Sister M. Marguerite, "Arithmetic as a School Subject," The Catholic Educational Review (Washington, D. C.: The Catholic Press, Vol. 34, 1936), p. 542.

his ciphering book. If it was not identical, the pupil was told to "do it all over again" even though his work was correct.

Later when a pupil had a text of his own, the method of instruction was the same. The pupil's ciphering book had the same topics as in the text. The rules were copied and memorized verbatim without any knowledge of their reasoning.<sup>19</sup>

The Ciphering Book Period, which extended up to 1821, seems to have had no aim, except on the part of the pupil, to excel in ability to do difficult sums, learn the rules, and have a ciphering book of nearly copied rules and problems.

The next important change in the aims and teaching of arithmetic was due to the publication, in 1821, of a textbook by Cummings and Hilliard Company in Boston. The author was Warren Colburn, a teacher in one of the select schools of Boston. His book was entitled First Lessons in Arithmetic, on the Plan of Pestalozzi, with some Improvements.<sup>20</sup>

Colburn had become interested in the Pestalozzian

<sup>19</sup> Walter S. Monroe, op. cit., pp. 44, 45, 46, 51, and 52.

<sup>20</sup> Sister M. Marguerite, "The Influence of Pestalozzi on Early American Texts," The Catholic Educational Review (Washington, D. C.: The Catholic Education Press, Vol. 34, 1936), p. 577.

plan of teaching. Some critics even suggested that Colburn had merely copied what Pestalozzi had already done. But, in the preface of his book in editions 1822 and 1826, Monroe said that Colburn acknowledged his indebtedness to Pestalozzi for the tables he had copied.<sup>21</sup>

Sister M. Marguerite states the following regarding the principles or aims of Pestalozzi and Colburn relative to arithmetic:

Pestalozzian arithmetic had no reference to the business world; it dealt with neither weights, moneys, nor measures. Its interests consisted entirely in the mental exercises which it involved; its benefit, in the increase of strength and acuteness of mind, derived from that exercise. Its aim, then, was neither practical nor useful. Arithmetical study according to Pestalozzi, was one of the greatest means of mental discipline. It led, he claimed, to the purpose of all education, the development of intelligent ideas, and was, therefore, to be regarded as the most important subject in the curriculum. His fundamental contention was that the mental processes of the pupil are the most important factors in the study of arithmetic. These mental processes were to be developed gradually and in a method similar to that used by nature itself. All things reach the mind through the senses; hence, in the teaching of arithmetic, as well as in all other subjects, Pestalozzi attempted to make the subject evident to the pupils through vivid sense impression. Objects and illustrative materials of all sorts were used for their purpose. In order to eliminate the old emphasis upon "ciphering" according to rule, all written arithmetic was postponed in the Pestalozzian school until the child had made considerable progress in forming clear and correct number concepts. Thus originated the "oral" or "mental" arithmetic of the nineteenth century. Number concepts were developed in connection with the arranging

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<sup>21</sup> Walter S. Monroe, op. cit., p. 61.

and the manipulation of material objects, such as lines, charts, and other things. Oral addition, subtraction, multiplication, and division followed next. Since Pestalozzi believed all these operations to be merely a matter of combination and separation of units, these were developed by giving the child an impression of the relationship between numbers. After the child had obtained clear ideas in regard to the value of various numbers and had mastered, by means of concrete objects, the addition and subtraction facts below ten, he was given a table in which similar sequence of ideas was shown by use of dots and lines. These tables, like concrete objects, were used as guides in counting. Later on in the learning process, a table of regular figures without any sense impressions were introduced. All this work, as was mentioned before, was oral; written exercises were not introduced until the child had completely mastered the fundamentals.<sup>22</sup>

Sister M. Marguerite says that Colburn's opinions regarding the aims of arithmetic agreed with those of Pestalozzi. As evidence of this agreement she cited the following excerpt from Colburn's address before the American Institute of Instruction at Boston in 1830:

Arithmetic when properly taught is acknowledged by all to be very important as a discipline of the mind; so much so that even if it had no practical application which should render it valuable on its own account, it would still be worth while to bestow a considerable portion of time on it for this purpose alone. This is a very important consideration, though a secondary one compared with its practical utility.<sup>23</sup>

Sister M. Marguerite further writes:

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<sup>22</sup> Sister M. Marguerite, op. cit., p. 578.

<sup>23</sup> Ibid., p. 579, citing Jacob W. Keller, "Warren Colburn on the Teaching of Arithmetic," Pedagogical Seminary, 30, 1923, pp. 162-167.

A study of Colburn's writings show that some of his underlying principles are very similar to those of Pestalozzi. In the two essays entitled "Juvenile Studies," published in The Prize Book of the Latin School, are contained some of the principles: (1) Arithmetic must be taught by example. (2) The examples should be simple and familiar, involving only such terms and numbers as those with which he (the beginner) is already acquainted. (3) He should be led from the easiest to the less easy and finally, by just gradation, to the most difficult. (4) Examples should be repeated and varied until the learner can invent them for himself and explain them to others. He will then deduce the rule and proceed with ease and satisfaction through the most complex operations and reasonings. (5) In most treatises on arithmetic, this method so natural and pleasing is completely reversed.<sup>24</sup>

Next it was pointed out that the similarity between the principles of Pestalozzi and those of Colburn is more noticeable in the preface to the third edition of The First Lessons. Quotations from the preface follow:

As soon as the child begins to use his senses, nature presents to his eyes a variety of objects, and one of the first properties he discovers is the relation of number. He intuitively fixes upon unity as a measure, and from this he forms an idea of more and less, which is the idea of quantity. . . . The idea of number is first acquired by sensible objects. Having observed that this quality is common to all things with which we are acquainted, we obtain an abstract idea of number. We first make calculations about sensible objects and we soon observe that these same calculations will apply to things very dissimilar; and finally that they may be made without reference to particular things. . . . Examples of this kind are of very little use until the learner discovers the principles from practical examples.<sup>25</sup>

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<sup>24</sup> Loc. cit.

<sup>25</sup> Ibid., p. 580, citing Warren Colburn, First Lessons. Intellectual Arithmetic upon the Inductive Method of Instruction (Boston: Cummings and Hilliard Company, 1826), "Preface."

The opinion regarding the agreement of the content material with the basic principles, stated by Colburn in First Lessons, is expressed in the following paragraph:

Colburn's entire text is based upon the very principles (as stated in the preface). Every combination begins with a practical example well illustrated. Abstract examples are, in many instances, placed immediately after the practical, so that the pupil can see the relation between the two.<sup>26</sup>

From the discussion of the preceding paragraphs emphasis upon mental drill and faculty psychology is clearly evident. But Pestalozzi should be credited for having introduced the idea of using concrete objects in teaching. That both Pestalozzi and Colburn believed in the utilitarian value of arithmetic is also evident. But both failed in the application of their theory because their emphasis on mental drill overshadowed the utilitarian value of meaningful learning.

It seems justifiable, next, to include in this review of the old aims of arithmetic, the main points of Monroe's description of Colburn's First Lessons of 1826 for the reasons: (1) that Colburn's works had important influence on the development of arithmetic as a school subject and on the introduction of the Pestalozzian theory of mental discipline; (2) that the description will give a clearer

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<sup>26</sup> Loc. cit.



idea of Colburn's application of his principles to the materials of his books.

Monroe's description of the 1826 edition follows:

The arithmetic which became known as Colburn's First Lessons was first published at Boston in 1821, with the title, First Lessons in Arithmetic, on the plan of Pestalozzi. With some improvements. In 1826 it had the title, Colburn's First Lessons. Intellectual Arithmetic upon the Inductive Method of Instruction, which it still retains. . . .

The First Lessons was intended to be begun at the age of 5 or 6, and studied for three or four years. . . .

The book itself is divided into two parts. The first contains the examples, the tables of the common denominate numbers, the system of notation up to 100, and a few explanatory notes. Part II is called a Key, and is primarily for the use of the teacher. . . .

The primary purposes of the book were to furnish the child with practical examples which required arithmetical operations and to provide exercises for drill upon the combinations which the child discovers are needed to solve the examples proposed. With few exceptions the practical examples are taken from situations in the life of children or from situations which children easily understand. The examples are about buying oranges, dividing apples among playmates, buying family provisions, counting marbles, etc. There are few examples from situations in commerce, but on the whole the problems of the text stand out in marked contrast to the commercial problems with which the texts of the previous period were filled. In addition to the practical examples, there are well-graded lists of abstract exercises for drill. They stand to the practical examples about the ratio of three to one.

Section I, which covers 19 pages is concerned with addition and subtraction. . . . The first article consists of very "simple practical questions," and in the second article the addition facts are called for in regular order by questions such as: "Two and one are how many?"; "Two and two are how many?"; etc. In the third article the same questions are repeated, but the

order is varied. The answers are not given in the book. Colburn assumes that the pupil has grasped the idea of addition from the practical questions of the first article. Knowing the meaning of such questions as "Three and two are how many?" the pupil can easily find the answer for himself. In the process of discovery he is to use sensible objects such as beans, nuts, etc. or the plates. (Pestalozzian tables)

The next article has to do with larger numbers, and in some instance there are three or more numbers to be added together. The numbers from 1 to 10 are to be added to the numbers from 10 to 20. In the fifth article subtraction is treated briefly, and in the next (the sixth article) the numbers 1 to 10 are added to the numbers 20 to 100. All the preceding are then combined together and the section closes with a list of "practical" questions which show the application of all the preceding articles. . . .

In the Key directions are given for using the Pestalozzian tables and other objective materials. . . . Certain it is that Colburn was the first author in the United States to introduce objective materials in an arithmetic text. The plates represent just one type of objects which he used. Beans, grains of corn, pieces of crayon, marks, etc., are recommended for use and even preferred. . . .

The examples are to be solved without the use of pencil and slate or paper.<sup>27</sup>

The above description of Colburn's First Lessons revealed significant features of Colburn's thinking and methods, which made his contributions of first importance in the development of arithmetic. According to Monroe, Colburn's First Lessons of 1821 marks the beginning of an active period of the production of arithmetics, which extended over a period

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<sup>27</sup> Walter S. Monroe, op. cit., pp. 63, 65, and 69.

of more than thirty-five years. Some of the texts were revised to keep pace with the expanding ideas of the time. But by 1860 these revisions and new publications were of no great importance. Monroe considered the active period as ending with 1857, because (1) it marked the date of the last revision of a series of Ray's arithmetics until 1877, and (2) the widespread and continued use of the Ray's arithmetic beyond the close of the nineteenth century and extending as far as the first few years into the twentieth century.<sup>28</sup>

A quotation by Monroe, relative to the Ray arithmetic, will show the widespread use and importance of Ray's textbooks. The quotation follows:

Of all the texts of this period (1821 to 1892), the series by Joseph Ray has enjoyed the most popular and extended use. Ray's arithmetics became popular soon after their first publication in 1834, and it seems that their popularity increased rapidly for a number of years. Until within the last quarter of a century, no arithmetics were published which supplanted them except locally. Even now (1913), after more than a decade which has been characterized by texts of another type, they are still a widely used series of arithmetics. The average yearly sale for the last ten years has been approximately 250,000 copies.<sup>29</sup>

The period from 1857 to 1890 or 1892 was the inactive period also known as the formalized period of arithmetic. During this period there were no textbook revisions or new

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<sup>28</sup> Ibid., pp. 89 and 90.

<sup>29</sup> Ibid., p. 97.

publications worthy of note. The textbooks of the active period remained in use. Arithmetic teaching became formalized drill with no meaning attached. But in about 1890 or 1892 new interest in arithmetic was aroused. The period 1892 was chosen by Monroe as the end of the period of the formalized arithmetic while 1890 has been chosen by others.<sup>30</sup>

Sister Marguerite chose the period 1860 to 1890 as the period of formalized arithmetic. She said that by 1860 the content of the arithmetic textbooks and the method of teaching was beginning to be very formalized, due to the influence that Pestalozzi and Colburn had upon those persons who were inclined to be extreme in their thinking. Of the formalization of arithmetic in the United States from 1860 to 1890, Sister M. Marguerite writes:

The influence of Pestalozzi and Colburn in regard to the teaching of arithmetic bore fruit which was not altogether desirable nor fruitful. Pestalozzi had begun as a reactionary against the formalism of the eighteenth century, but his own method, falling into the hands of misunderstanding enthusiasts, became before the middle of the nineteenth century the very essence of formalism. Pestalozzi had suggested that the child be taught to think in number work. Colburn had used the word "intellectual" in his book. The extremists, therefore, hastened to act upon the principle that, if it did the child good to think a little, it would profit him a great deal to think much more. Accordingly, the idea of mental discipline came to dominate the entire field of arithmetic for nearly half a century. . . .

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<sup>30</sup> Ibid., p. 90.

The importance attached to the disciplinary function of arithmetic soon caused it to rank first in the school curriculum. . . . Promotions were, in many cases, based solely on arithmetic achievement. Subject matter was retained which changes in conditions, scientific progress, and social and business practice had rendered obsolete. Combined with this was the fact that textbooks were very formal. During this period few new texts appeared. The only ones of any considerable importance were Edward Brooks' Normal Elementary Arithmetic (1863), Joseph Ray's New Practical Arithmetic (1877), and Daniel Fish's Arithmetic Series Number One and Number Two (1883). In these texts, definitions are, for the most part, of a philosophical nature, rules are numerous and are inserted at the beginning instead of at the close of a section, as Pestalozzi and Colburn had suggested. Explanation and solutions are stated more dogmatically; deduction supplants induction as a method of development; there is a gradual decrease in the use of objective and illustrative materials and of concrete problems. Many of the problems are of the puzzle type which were at that time thought to possess great disciplinary value. . . .

Methods of teaching naturally followed the same course as did the changing aims and content. In opposition to the Ciphering Book period, where little or no attention was paid to drill, this new period made skill, thoroughness, and mastery the outstanding objectives of arithmetic instruction. Teachers and authors were evidently much concerned with making pupils proficient in the mechanical operations of arithmetic. Little attention was paid, however, to making practical the applications of what was learned. Mechanical repetition became the one supreme method of learning. Rules were first memorized, then applied. Colburn's method of induction was soon abandoned, and the teaching of arithmetic as well as the teaching of all school subjects followed the method of deduction. Much of the work in the classroom was done orally and in concert recitation. . . . Pupils were required to recite the tables backward and forward.

Part of the new emphasis on thoroughness in arithmetic was undoubtedly due to the Grube Method which was introduced in this country between 1840 and 1870. The method was named after its author, a German, who first published his ideas on teaching arithmetic. . . . Grube attempted

to improve the teaching of arithmetic by applying it to the Pestalozzian principle of reducing each subject to its elements and then making a thorough study of each element, mastering it completely before progressing to the next. The noteworthy feature of the method was its simultaneous presentation of all four fundamental processes at once; for example, in taking up the number four, the pupil was taught addition, subtraction, multiplication, and division of four. The same was done with each number below ten.

The Grube Method was introduced into this country largely through the work of Louis Soldan of St. Louis, who in 1870 read a paper on the method explaining fully all of its principles. Much time was wasted in adhering to the method, for it was characterized by a thoroughness carried to extremes. . . . In the United States the plan was not long-lived. It reached its heyday around 1885, and by 1890 it was discarded.

The theory of mental discipline which dominated the field of psychology and education caused arithmetic to grow in importance as a school subject. Pupils began in many cases to take up the study of numbers before their fourth birthday. It was not, therefore, until about the middle of the nineteenth century that arithmetic really became one of the "three R's."<sup>31</sup>

The disciplinary aim of arithmetic was regarded as important throughout the nineteenth century, but by the year 1880 the value of arithmetic as a mental "trainer" began to be questioned by leading educators and psychologists. At a meeting of the National Association in 1880, Edgar Singer<sup>32</sup>

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<sup>31</sup> Sister M. Marguerite, "Formalization of Arithmetic in the United States, 1860-1890," The Catholic Educational Review (Washington, D. C.: The Catholic Education Press, Vol. 35, 1937), pp. 28-31.

<sup>32</sup> Ibid., p. 32, citing Edgar A. Singer, "What Constitutes a Practical Course of Study?" Addresses and Proceedings of the N. E. A., 1880, pp. 108-128.

said that too much time was being devoted to the study of arithmetic to the neglect of reading and other subjects.

The following year, at the next meeting of the same association, Andrew J. Rickoff<sup>33</sup> protested the predominance of arithmetic over that of other school subjects. He contended that the school's duty was to train all people for the common life, and that only the mastery of elements should be required of the pupils of elementary grades.

In 1887, Francis Walker<sup>34</sup> pointed out to the School Committee of Boston the unfair amount of time that was being spent in the study of arithmetic. He stressed the fact that its methods were still based on the mental discipline aim instead of training for practical affairs of life. The result of Walker's attack was that the Boston School Board passed several regulations relating to the elimination of some of the topics, the confining of fractions to small numbers, the regulation of speed in oral arithmetic, and the reducing of the time devoted to arithmetic in the grammar and primary grades.

This reaction against mental discipline and faculty

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<sup>33</sup> ibid., p. 32, citing Andrew J. Rickoff, "The Common School Studies," Addresses and Proceedings of the N. E. A., 1881, pp. 21-28.

<sup>34</sup> ibid., p. 33, citing Francis Walker, Discussion in Education (New York: Henry Holt and Company, 1899), p. 342.

psychology in arithmetic grew stronger until in the early part of the twentieth century the reaction ended in most vigorous attacks. This was due largely to two educational movements, based on psychological ideas and educational principles, coupled with the growing interest and criticism of educators and the general public in the subject matter, aims, and methods of the public schools. William James,<sup>35</sup> the leading psychologist of the time published his two volumes on the Principles of Psychology in 1890, and in it he maintained that no amount of exercise was capable of modifying a person's general ability to retain, but that remembering could only be improved by the day by day recurring of various related experiences.

The other movement was that relative to Herbart's<sup>36</sup> educational theories, which were directly opposed to mental discipline in that they placed stress on enrichment of the content of a subject by correlation with interests in various topics.

In 1892 the Committee of Ten,<sup>37</sup> appointed by the

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<sup>35</sup> William James, The Principles of Psychology (New York: Henry Holt and Company, Vol. I, 1890), p. 667.

<sup>36</sup> Frank Pierrepont Graves, Great Educators of Three Centuries (New York: The Macmillan Company, 1912), pp. 167-193.

<sup>37</sup> Committee of Ten, Report on Secondary Studies, with Reports of Conferences arranged by the Committee (New York: American Book Company, 1894), p. 108.



National Education Association, recognized that the value of formal discipline was much inferior to what might be obtained by the use of a different type of class exercises.

The Committee of Fifteen<sup>38</sup> in their report of 1895 stated that they were fully convinced that arithmetic was receiving too much emphasis in the schools and that much of the impractical and obsolete topics should be eliminated.

That the social environment in which man lives presents problems which he solves in some form of number and number relation was the idea expressed by Dewey<sup>39</sup> in his Psychology of Number published in 1895 in collaboration with James McLellan. To this idea Dewey added his general principle that the process of education is more efficiently carried on when the child is placed in the physical and social environment which demands that the child use his mind in solving these number relations.<sup>40</sup>

The changing concepts regarding the function of the teaching of arithmetic from that of acquiring skill and power toward that of growth and development of the whole

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<sup>38</sup> Committee of Ten, Report on Elementary Education with Reports of Sub-Committees (Chicago: American Book Company, 1895), pp. 53-58.

<sup>39</sup> John Dewey and James McLellan, The Psychology of Number (New York: Macmillan Company, 1895), p. 32.

<sup>40</sup> Ibid., p. 23.

child was revealed by the examination of many courses of study as reported in the Twenty-Ninth Yearbook.<sup>41</sup> In an 1858<sup>42</sup> course of study arithmetic was a prominent study for mental training.

The Cincinnati course of study for 1862<sup>43</sup> showed the belief that the power gained in the study of arithmetic would function in all other subjects.

The Suggestions accompanying the 1878 course of study of Boston<sup>44</sup> stated two aims, "practical utility" and "mental discipline," but suggested that neither be sacrificed for the other.

The Saginaw course of study of 1890-1897<sup>45</sup> placed emphasis on the practical and disciplinary value of arithmetic.

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41 Twenty-Ninth Yearbook of the National Society for the Study of Education (Bloomington, Illinois: Public School Publishing Company, 1930), pp. 71-73.

42 Ibid., p. 71, citing New Haven, Connecticut, Report of the Superintendent of Common Schools to the General Assembly, May Session, 1858.

43 Loc. cit., citing Cincinnati, Ohio--Superintendent's Report--33rd Annual Report, 1862, p. 22.

44 Ibid., p. 72, citing Boston, Massachusetts--School Document No. 17, Suggestions Accompanying the Course of Study for Grammar Grades and Primary Schools, 1878, p. 17.

45 Loc. cit.

The Pittsburgh course of 1893<sup>46</sup> seemed to question the study of arithmetic for its value in mental development and expressed disfavor of the method of memorizing rules and definitions to be used simply as aids in mechanical operation in which there was no meaning.

That McMurry believed in the practical side of arithmetic is seen in two quotations from his book of 1905, Special Method in Arithmetic. The quotations follow:

The chief aim in arithmetic is the mastery of the world on the quantitative side through number concept.

...

The thing we aim at, therefore, is a completely practical and accurate mastery of our material surroundings from the narrow point of view of number. It is not mathematical processes and discipline for their own sake.<sup>47</sup>

The course of study of Chicago, 1904, contains the following statement: "Concrete problems should be drawn from the field of the child's interests and experiences."<sup>48</sup>

This indicates that the aim of arithmetic was to give practical meaning through experiences with which the child was familiar.

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<sup>46</sup> Loc. cit., citing Pittsburgh, Pennsylvania--Grade and Course of Study for the Pittsburgh Public Schools, Edition of 1893, p. 6.

<sup>47</sup> Charles A. McMurry, Special Method in Arithmetic (New York: Macmillan Company, 1905), pp. 16 and 17.

<sup>48</sup> Twenty-Ninth Yearbook, op. cit., p. 88, citing Chicago--Chicago Public Schools, Course of Study for the Elementary Schools Adopted August 31, 1904, p. 11.

The arithmetic of 1910 to 1920 was largely based on the practical arithmetic needs in order to get along in the various future vocations. The teacher was to attempt to make the learning situation as real as she could by providing situations in which the children could dramatize, or by bringing in concrete things from the business world, or by planning actual school room situations in which the children could actually carry on the number situation. The following excerpt from the 1912 course of study for Michigan schools shows this trend:

Upper grades--Make the work real in teaching percentage, interest, proportion, discount, taxes, insurance, stocks, mensuration. Let the children dramatize the work, keep store, buy and sell, act as insurance agents, brokers, bankers, commission merchants. Get copies of all forms of business paper, copy and use in actual transactions. Let them measure, estimate, approximate, and plan for problems by actual measurements.<sup>49</sup>

The following quotation by Suzzallo shows the trend of arithmetic toward the practical aim relating to the business world:

The utilitarianism that first attacked the older course of study and its methods was the utility of the business world. The arithmetic of business life became the standard. The practices of the market determined what matter, skill and accuracy should be demanded of the elementary school pupil. Recently it became the habit to call upon the business man to give his opinion as to what constitutes good arithmetic teaching. Committees on courses of studies have investigated the

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<sup>49</sup> Ibid., p. 87, citing Michigan--District Schools of Michigan, Tenth Edition, 1912, pp. 142 ff.

relative frequency and importance of specific arithmetical processes in the business world with the idea of utilizing the results as a basis of change in the mathematics curriculum.

This aim of business utility, coming at a time when the elementary school course was felt to be overcrowded, met with hearty reception. It operated for the time being to eliminate materials not actually in the business world.<sup>50</sup>

The following excerpt from a course of study of 1910 shows the trend which Suzzallo discussed in the above paragraphs:

The problems should also be as practical in their nature as possible. They should represent the operations of real life, and not some abstract or fancied view of what these operations might be. In denominate numbers the measures should be applied to the actual use of the store, the shop, the market, the household, etc. The problems in percentage should not merely illustrate the theoretical principle, but represent the actual business of the store or office.<sup>51</sup>

The course of study for the state of Indiana, 1914-1915, states the following aims:

1. To train the pupil to a high degree of accuracy and facility in reckoning.
2. To provide him with sound knowledge of facts and affairs associated with arithmetical work as an intelligent citizen is expected to use.
3. To train him to think clearly, to reason accurately, and thus gain such power as will enable him to

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<sup>50</sup> Henry Suzzallo, The Teaching of Primary Arithmetic (Chicago: Houghton Mifflin Company, 1911), p. 15.

<sup>51</sup> Twenty-Ninth Yearbook, op. cit., p. 98, citing Philadelphia, Pennsylvania--The Course of Study in Arithmetic for the Public Schools of Philadelphia, 1910, p. 44.

apply the machinery and logic of arithmetic to the solutions of everyday problems that he may meet in the various callings and situations for life.

4. To train him in habits of order and neatness, and to persevere along the line of sustained effort.<sup>52</sup>

Thorndike's<sup>53</sup> educational psychology influenced the drill side of arithmetic. Thorndike emphasized the importance of concrete drill in the teaching of the fundamental processes that the child would need in computation at the present time or in the future. He emphasized that short drill repeated at frequent intervals would establish habit formation more easily than longer drills at less frequent intervals.

In 1917 Thorndike published the Thorndike Arithmetics which were copyrighted again in 1924. A statement from the preface of Book One stated the basis upon which the books were written. Thorndike said: "These books apply the principles discovered by the psychology of learning, by experimental education, and by observation of successful school practice, to the teaching of arithmetic."<sup>54</sup>

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<sup>52</sup> Indiana, Uniform Course of Study for the Elementary Schools of Indiana, 1914-1915, p. 66.

<sup>53</sup> John W. Studebaker, "New Trends in the Teaching of Arithmetic," The Mathematics Teacher, Vol. XXXII, May, 1939, p. 196.

<sup>54</sup> Edward Lee Thorndike, The Thorndike Arithmetic, Book One (Chicago: Rand McNally and Company, 1917, 1924), "Preface," p. v.

The Indiana State Course of Study for 1924<sup>55</sup> advised the teachers to study carefully Thorndike's viewpoint regarding the teaching of arithmetic.

The years 1928 and 1929 were marked as periods in which units of arithmetic began to appear in a number of the courses of studies. These units consisted of activities centering around the keeping of a grocery store, buying in the lunch room, or in the building of a house. There were courses of study which showed two types of arithmetic. One type was that in which the number facts and fundamental processes were consciously planned for the activities previous to the teaching of the unit. This was the teacher's plan being executed. The second type was the unit in which the activities were planned by the pupils and teachers and the arithmetic content came naturally into the activity, the computation being employed in a meaningful way. Examples of these types of work are seen in the Kansas City course of study for 1928<sup>56</sup> and that of the Raleigh course of study for 1928.<sup>57</sup>

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<sup>55</sup> Indiana, Manual with Course of Study in Arithmetic for the Elementary Schools of Indiana, Bulletin No. 47A, 1924, p. 7.

<sup>56</sup> Twenty-Ninth Yearbook, op. cit., pp. 91-92, citing Kansas City, Missouri--Course of Study in Arithmetic for Grades I to VI, 1928, pp. 158ff.

<sup>57</sup> Ibid., p. 93, citing Raleigh, North Carolina--Teaching in Grades Two and Three, 1928, pp. 69 f.

The years 1928 and 1929 seem to mark clearly the close of the period of old aims in arithmetic, because of the trend away from that of formal discipline, and preparation for living in the future as a member of society. These last courses of study, mentioned above, point to the conception of social utility that is emphasized in the new or modern aims of arithmetic.

## II. THE NEW AIMS OF ARITHMETIC

The new aims of arithmetic from 1930 to the present time are defined in terms of social utility. The major aims have been well stated by Norton as follows:

. . . to bring children to a proper appreciation of the cultural value and social significance of arithmetic, and, to help them acquire the ability to perform with accuracy and understanding the arithmetic required by everyday life.<sup>58</sup>

David Eugene Smith has emphasized the social significance in the following quotation:

Arithmetic must be taught as a social subject; something that helps make the world better, something that everyone needs in order to get on in life, and something that helps to bind people to one another; and if it is not taught with this feeling and in this spirit, the teaching will fail of its purpose, the results will be disappointing, and the pupils will be deprived of a

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<sup>58</sup> John N. Norton and Margaret A. Norton, Foundations of Curriculum Building (Chicago: Ginn and Company, 1936), p. 381.



heritage which is their right and privilege.<sup>59</sup>

This doctrine of social utility has brought about changes in arithmetic textbooks and courses of study in the last few years and is continuing. Makers of courses of study and textbook writers have used research studies and materials as given in the Twenty-Ninth Yearbook,<sup>60</sup> Buswell's Summary of Investigations,<sup>61</sup> and articles in educational magazines as guides regarding the kind and amount of arithmetic material to be presented at the various grade levels.

As a result courses of study are showing changes, due to recent studies, as to when certain arithmetic processes are to be taught and the amount of material to be included in the different grades. Some of the topics and problems that have been proved to be obsolete are being eliminated from textbooks as well as from courses of studies. Formal arithmetic no longer has a place in the first grade. Concepts of number relationships are emphasized. The following quotation from the Indiana Course of Study illustrates this:

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<sup>59</sup> David Eugene Smith, "Arithmetic for Intermediate Grades," The Classroom Teacher (Chicago: The Classroom Teacher, Inc., 1927), Vol. 7, p. 394.

<sup>60</sup> Twenty-Ninth Yearbook, op. cit., p. 748.

<sup>61</sup> Guy T. Buswell, "Summary of Arithmetic Investigations," Elementary School Journal, 26:692-703, 745-758; 27:685-694, 731-744; 28:702-709, 730-742; 20:691-698, 737-747; 30:766-775; 31:756-766; 32:766-773; 34:209-213; 35:212-214; 36:211-213; 37:209-212; 38:210-214, May, 1926 to July 1, 1937.

Grade I. No formal arithmetic should be attempted in this grade; however the idea of number and number relationships should be introduced in genuine problems closely related to the experience of children. A vocabulary suited to the child's experiences and needs should be developed.<sup>62</sup>

The content, organization, and method of teaching arithmetic, based on the new aims of arithmetic, will depend upon the meaning that the term "social utility" has to curriculum makers and authors of textbooks and classroom teachers. Social utility in the narrow sense includes only that which is actually in use, while the broader meaning emphasizes the value to society in all its phases. Arithmetic has not just arithmetical value in what it is actually used, but it has informational value to the every-day activities of every individual. This is well illustrated by the following quotation:

Many have recommended the elimination of longitude and time. Yet in the operation of the radio and in traveling, a considerable part of the population needs to have at least a reasonable understanding of differences in time. Although this might be thought to be a geographical concept, there are mathematical elements involved, so that this topic can easily be considered as an application of aspects of number to practical life situations.<sup>63</sup>

There appears to be a trend toward the writing of

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<sup>62</sup>Indiana, Program of Studies and Digest of the State Course of Study for Indiana Schools, Elementary and Secondary Supplement to Bulletin 100 Rev., 1936, p. 46.

<sup>63</sup>Twenty-Ninth Yearbook, op. cit., p. 80.

supplementary materials such as workbooks and informational stories to aid in the understanding and appreciation of arithmetic as a social utility. Smith<sup>64</sup> has written a book for young children regarding the origin of number and number symbols of all races. The book is illustrated by pictures of the children of these races and of modern children in their particular environment. This is an indication that the term social utility has the broad meaning for at least a number of thinkers.

### III. SUMMARY OF CHAPTER

Aims of arithmetic had their origin with the first need for number conception and computation.

The aim of the primitive was the ability to count or indicate a measure or quantity in the carrying out of some activity. It was purely for practical purposes.

Arithmetic to the ancient peoples meant: (1) the ability to use number computations in business, and (2) the knowledge of the science of numbers to establish truths and strengthen the mind. (This applied to the philosopher and the man of liberal education.)

<sup>64</sup> David Eugene Smith, The Wonderful Wonders of One-Two-Three (New York: McFarlane, Warde, McFarlane, 1937), 47 pp.

The aims of the early colonial period were traditional to the extent that the attitude toward arithmetic was that of the European culture of which the colonists were a part. Arithmetic, for the purpose of the boy who would engage in business or commerce, dealt with the computations for practical application. Arithmetic, for the boy capable of learning, dealt with the science of numbers for the purpose of strengthening his mind.

During the eighteenth century the aim relating to arithmetic of the ciphering period was probably nothing more than the ability to do hard sums, to learn rules, and to have a book of neatly copied problems, definitions, and rules in his ciphering book.

The aims of arithmetic of the nineteenth century were influenced by the Pestalozzian theory of mental discipline and of concrete sense perception. Emphasis was placed upon oral arithmetic by the inductive method, and the introduction of a new process by a practical illustration. The aim, then, was to give mental training through the use of drill. This resulted in formalized arithmetic devoid of meaning.

The twentieth century has experienced a revolt against the disciplinary aim. The Herbartian theory of supplying meaningful content as opposed to mere formal drill and the James theory of the inability to improve the memory by any amount of exercise (pure repetition in the form of drill)

brought about change. The arithmetic aim began to be based on its use. The aim was to provide experiences or activities on which to base the abstract processes involved in arithmetic.

Next came the change due to investigations of the out-of-school-life usage of arithmetic. This brought about elimination of topics considered by the business man as obsolete. The aim of arithmetic was to prepare for future participation by learning the computational processes used in the business world.

The present aim is that of social utility, with emphasis on meaningful concepts and appreciation of the arithmetical values to society in all its phases. The two important aims for the teaching of arithmetic today are: (1) to guide the pupils to appreciate properly the cultural values and social significance of arithmetic; and (2) to help them acquire the ability to perform with accuracy and understanding the common computations in everyday living, by utilizing as far as possible real life situations for this learning.

## CHAPTER IV

### ANALYSES OF ELEVEN ARITHMETIC TEXTBOOKS

The following books are included in this study:

1. An Arithmetic on the Plan of Pestalozzi, with some Improvements by Warren Colburn (1821)
2. Practical and Mental Arithmetic on a New Plan by Roswell C. Smith (1835)
3. Ray's Arithmetic, First Book by Joseph Ray (1857)
4. French's First Lessons in Number by J. H. French (1866)
5. A Primary Arithmetic Uniting Oral and Written Exercises in a Natural System of Instruction by E. E. White (1868)
6. New Primary Arithmetic by John H. Walsh (1895)
7. School Arithmetics, Primary Book by George Wentworth and David Eugene Smith (1919)
8. A Child's Book of Number for First and Second Grades by John C. Stone (1924)
9. First Days with Numbers by Clifford Brewster Upton (1933)
10. Child-Life Number Book by Charles E. Garner and Livingstone McCartney (1938)
11. The Wonderful Wonders of One-Two-Three by David Eugene Smith (1937)

The first three books represent the active, formalized

Pestalozzian period of which Colburn was the first outstanding American author of arithmetic textbooks.

The next two books represent the static period in arithmetic textbook production.

The three books which follow represent the revival period in which there was a reaction against the formalized mental discipline theory and a trend toward the practical utility theory.

The remaining books represent the modern social utility theory. The last of these three books is not in reality an arithmetic, but is a supplementary book, which, if properly used, forms an excellent basis for the understanding of numbers as used by social groups of all nations.

The analysis includes a discussion of each book according to the following points: (1) the name of the book, (2) the author, (3) the publishers, (4) the date of copyright, (5) the preface, (6) the teacher aids, (7) the table of contents, (8) problems as illustrations, and (8) a summary statement. Interesting and peculiar features of the early textbooks are included.

The reader may find what appears to be incorrect spelling, poor English, and unusual punctuation in excerpts quoted from the early arithmetic textbooks. Particular attention has been given to verbatim copy from the various books.

The discussion of the study of the early textbooks is

extensive in some cases, but this seems justified for the reason that these textbooks are old and will soon be in such a state of deterioration that they cannot be used. Descriptions regarding the mechanical features of these books and reproductions of pictures, illustrations, tables, and other features are included. This method will not be adhered to for the later texts, except for one or two.

The order of analyses is the same as that in the foregoing list. Due to the fact that quotations from the eleven books included would require a large number of footnotes, only one footnote appears at the beginning of each analysis. Instead of footnote citations, specific page numbers are cited in the report.

#### I. AN ARITHMETIC ON THE PLAN OF PESTALOZZI WITH SOME IMPROVEMENTS

This book measures three and five-eighth inches by six inches and contains 143 pages. The cardboard back is covered with splotchy paper of grayish-green and reddish-orange. The bound edge is of light brown leather. There are neither pictures nor plates in the book. The title page gives the following information:

An Arithmetic on the Plan of Pestalozzi, with some Improvements by Warren Colburn, Boston: Published by Cummings and Hilliard, Hilliard and Metcalf printers,



1821.<sup>1</sup>

The next page gives information concerning the copy-right. It is as follows:

District of Massachusetts, To Wit: District Clerk's Office.

BE IT REMEMBERED, That on the fifteenth day of November, in the forty sixth-year of the Independence of the United States of America, CUMMINGS and HILLIARD, of the said District, have deposited in this Office, the title of a Book, the right whereof they claim as proprietors, in the words following, viz.,

An Arithmetic on the plan of Pestalozzi, with some improvements. By Warren Colburn.

In conformity to the Act of Congress of the United States, entitled "An Act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned;" and also, to an Act, entitled, "An Act, supplementary to an Act, entitled, an Act for the encouragement of learning by securing the copies of maps, charts, and books, to the authors and proprietors of such copies during the times therein mentioned, and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints."

J. W. Davis,  
Clerk of the District of Massachusetts.

The next two pages of the book include two recommendations regarding Colburn's text. They are as follows:

Boston, 15 November, 1821.

Sir,

I have made use of the Arithmetic and Tables, which you sometime since prepared, on the system of Pestalozzi;

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1 Warren Colburn, An Arithmetic on the Plan of Pestalozzi with some Improvements (Boston: Cummings and Hilliard, 1821).

and have been much gratified, with the improved edition of it, which you have shown me. I shall continue to use it; satisfied from experiment, that it is the most effectual and interesting mode of teaching the science of numbers with which I am acquainted.

Respectfully  
Your obedient servant  
Henry Colman

Mr. Warren Colburn

Having been made acquainted with Mr. Colburn's treatise on Arithmetic, and having attended an examination of his scholars who had been taught according to this system, I am well satisfied that it is the most easy, simple and natural way of introducing young persons to the first principles in the science of numbers. The method here proposed is the fruit of much study and reflection. The author has had considerable experience as a teacher, added to a strong interest in the subject, and a thorough knowledge not only of this but of many of the higher branches of mathematics. This little work is therefore earnestly recommended to the notice of those who are employed in this branch of early instruction, with the belief that it only requires a fair trial in order to be fully approved and adopted.

J. Farrar,  
Prof. Math. Harvard University

Cambridge, Nov. 16, 1821. (p. iv)

The preface includes the pages marked as b to xiv, inclusive. Parts of the preface follow:

As soon as a child begins to use his senses, nature continually presents to his eyes a variety of objects, and one of the first properties, which he discovers, is the relation of number. He intuitively fixes upon unity as a measure, and from this he forms the idea of more and less; which is the idea of quantity.

The names of a few of the first numbers are usually learned very early; and children frequently learn to count as far as a hundred before they learn their letters.

As soon as children have the idea of more and less, and the names of a few of the first numbers, they are able to make small calculations. And this we see them do every day about their playthings, and about the little affairs which they are called upon to attend to. The idea of more and less implies addition; hence they will often perform these operations without any previous instruction. If, for example, one child has three apples, and another five, they will readily tell how many one has more than the other. If a child be requested to bring three apples for each person in the room, he will calculate very readily how many to bring, if the number does not exceed those he has learnt. Again, if a child be requested to divide a number of apples among a certain number of persons, he will contrive a way to do it, and will tell how many each must have. The method which children take to do these things, though always correct, is not always the most expeditious.

The fondness which children usually manifest for these exercises, and the facility with which they perform them, seem to indicate that the science of numbers, to a certain extent, should be among the first lessons taught to them.

To succeed in this, however, it is necessary rather to furnish occasions for them to exercise their own skill in performing examples, than to give them rules. They should be allowed to pursue their own method first, and then they should be made to observe and explain it, and if it was not the best, some improvement should be suggested. By following this mode, and making the examples gradually increase in difficulty; experience proves, that, at an early age, children may be taught a great variety of the most useful combinations of numbers.

Few exercises strengthen and mature the mind so much as arithmetical calculations, if the examples are made sufficiently simple to be understood by the pupils; because a regular, though simple process of reasoning is requisite to perform them, and the results are attended with certainty.

The idea of number is first acquired by observing sensible objects. Having observed that this quality is common to all things with which we are acquainted, we obtain an abstract idea of number. We first make calculations about sensible objects; and we soon observe,

that the same calculations will apply to things very dissimilar; and finally, that they may be made without reference to any particular things. Hence from particulars, we establish general principles, which serve as the basis for our reasonings, and enable us to proceed step by step, from the most simple to the more complex operations. It appears, therefore, that mathematical reasoning proceeds as much upon the principle of analytical induction, as that of any science.

From the above observations, and from his own experience the author has been induced to publish this treatise; in which he has pursued the following plan, which seemed to him the most agreeable to the natural progress of the mind.

#### General View of the Plan

Practical examples are first proposed, in which addition and subtraction are involved. These are so simple, that almost any child of five or six years old will readily perform them in his mind, or by means of sensible objects such as beans, nuts, etc., or by means of the plate at the end of the book. The pupil should first perform the examples in his own way, and then be made to observe and tell how he did them, and why he did them so. The use of the plates is explained in the Key at the end of the book. Several examples in each section are performed in the Key, to show the method of solving them. No answers are given in the book, except where it is necessary to explain something to the pupil. Most of the explanations are given in the Key: because pupils generally will not understand any explanation given in a book, especially at so early an age. The instructor must, therefore, give the explanation viva voce. These, however, will occupy the instructor but a very short time.

Practical examples are given before the abstract in all cases where it could be done to advantage, because they are more easily understood, and they show the pupil the use of abstract numbers before he learns them.

After the practical examples, questions requiring the same kind of operations are proposed in an abstract form, but they are to be solved by means of sensible objects. (p. d)

The first part of the abstract questions contain the common addition table. The questions are proposed, and the pupil is to find the answers himself. After he can readily find the answers, he should commit them to memory. The remaining part of the abstract questions contains addition and subtraction in various forms.

After the abstract questions, practical questions are again proposed, of the same kind with which preceded them, but more difficult.

The second section contains multiplication. It commences with practical examples, which are very simple. Then follows the multiplication table, extended as far as the ten first numbers. The questions in this table are proposed in an abstract form, but are to be answered by means of sensible objects, or by plate. The answers are to be committed. After the table, practical questions are again proposed, which involve multiplication, and sometimes addition and subtraction.

The third section contains division. This commences also with simple practical questions. The division of abstract numbers commence in the most simple manner; that is by dividing two into two parts, and then finding how many times two is contained in some of the smaller numbers. Then three, four, &c. are divided and made divisors in the same manner. Here the pupil also learns the first principles of fractions, and the terms which are applied to them. (p. x) He is made to consider one as the half of two, the third part of three, the fourth part of four; &c. and two as two thirds of three, two fourths of four, two fifths of five, &c.

The fourth section commences with multiplication. It differs from the multiplication in the second section only in this, that the pupil is taught to repeat a number a certain number of times, and a part of another time. In the second part of this section the pupil is taught to change a certain number of twos into threes, threes into fours, &c.

In the fifth the pupil is taught to find  $1/2$ ,  $1/3$ ,  $1/4$ , &c. and  $2/3$ ,  $3/4$ ,  $4/5$ , &c. of numbers which are exactly divisible into these parts. This is only an extension of the principles of fractions, which is contained in the third section.

In the sixth section the pupil learns to tell of what number any number, as 2, 3, 4, &c. is one half, one third, one fourth, &c.; and also knowing  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{2}{5}$ , &c. of a number, to find that number.

In all these sections are a large number of practical examples, which shew the application of the principles contained in them.

These combinations contain all the most common and most useful operations of vulgar fractions. But being applied only to numbers which are exactly divisible into these fractional parts, the pupil will observe no principles but multiplication and division, unless he is told of it. In fact, fractions contain no other principle. The examples are so arranged, that almost any child of six or seven years old will readily comprehend them. And the questions are asked in such a manner, that, if the instructor (p. xi) pursues the method explained in the Key, it will be almost impossible for the pupil to perform any example without understanding the reason of it. Indeed, in every example which he performs, he is obliged to go through a complete demonstration of the principle by which he does it; and at the same time he does it in the simplest way possible. These observations apply to the remaining part of the book.

These principles are sufficient to enable the pupil to perform almost all kinds of examples that ever occur. He will not, however, be able to solve questions in which it is necessary to take fractional parts of unity, though the principles are the same.

After section sixth, there is a collection of miscellaneous examples, in which are contained almost all the kinds that usually occur. There are none however, which the principles explained are not sufficient to solve.

In section eighth and the following, fractions of unity are explained, and, it is believed, so simply as to be intelligible to most pupils of seven or eight years of age. The operations do not differ materially from those in the preceding sections. There are some operations, however, peculiar to fractions. The two last plates are used to illustrate fractions.

When the pupil is made familiar with all the principles contained in this book, he will be able to perform all examples, in which the numbers are so small, that the operations may be performed in the mind. Afterwards he has only to learn the application of figures to these operations, and his knowledge of arithmetic will be complete. (pp. b-xi)

This arithmetic is divided into two parts. The first part includes fourteen sections of problems, pages 1 to 108, inclusive. The second part is called the Key, pages 109 to 143, inclusive. This includes the plates which are used as concrete aids in the solution of the problems in the first part of the book. Plate I, on pages 110 and 111, is described as follows:

This plate, horizontally, presents ten rows of rectangles, and in each row ten rectangles.

In the first row, each rectangle contains one mark, each mark representing a unity or one.

In the second row each rectangle contains two marks, and in the third, three marks, &c.

The purpose of this plate is, first, to represent unity, or as making a part of a sum of units. Secondly, to represent a collection of units, either as forming an unit itself, or as making a part of another collection of units; and thus to compare unity and each collection of units with another collection, in order to ascertain their ratios. (p. 111)

All the examples, as far as the eight section, can be solved by this plate. The manner of using it, is explained in the key for each section in its proper place.

The pupil, if very young, should first be taught to count the units, and to name the different assemblages of units in the following manner:

The instructor showing him the first row which contains ten units insulated, requests the pupil to put

his finger on the first, and say, one; and then on the second and say, and one are two, and on the third, and say, and one are three, and so on to ten; then commencing the row again, let him continue and say, ten and one are eleven.

After adding them, let him begin with ten, and say ten less one are nine, nine less one are eight, &c. Then taking larger numbers, as twenty or thirty, let him subtract them in the same manner.

Next let him name the different assemblages, as twos, threes, &c. Afterward let him count the number in each row. (pp. 110 and 111)

Section I has sub-topics which the author calls articles. They are lettered A, B, C, D, E, and F. Page 1 of this section has a footnote which refers to the Key regarding the method of solving questions. Page 112 of the Key gives the following information regarding Article A of Section I:

A. The first examples may be solved by means of beans, peas, &c. or by plate I. The former method is preferable, if the pupil be very young, not only for the examples in the first part of this section, but for the first examples without any instruction.

1. This example is solved by the plate as follows:

Select one rectangle containing 2 marks, and another containing 5, and ascertain the number of marks contained in both together.

N. B. If the pupil be young, he will perform addition by adding 1 at a time.

4. Select a rectangle containing 7 marks and take away 3 of them. Or, keeping 7 in the mind, select a rectangle containing 3, and take them away, 1 at a time. (p. 112)

The preceding is the explanation of the solutions of



problems 1 and 4 of Article A of Section I, page 1. The problems are as follows:

A 1. James has two apples, and William has three; if James gives his apples to William, how many will William have?

4. David has seven nuts, and gave three of them to George; how many had he left? (p. 1)

No problem of this section contains easy combinations. Each succeeding problem is more complicated. A few examples on pages 1, 2, and 3 follow:

5. Three boys, Peter, John and Oliver, gave some money to a beggar, Peter gave seven cents; John four cents; and Oliver, three cents; how many did they all give him?

8. Dick had ten peaches, Harry twelve, and Charles thirteen; Dick gave three to Stephen, Harry gave him six, and Charles gave him five; how many had Stephen? and how many had each left?

11. A boy went to the confectioner's and bought three cakes of gingerbread, for which he gave a cent apiece; two buns for which he gave three cents apiece, one glass of soda water for four cents, and one orange for six cents; how many cents did he spend for the whole?

13. A boy bought a box for eighteen cents, and gave eight cents to have it painted, and then sold it for thirty-two cents; how much did he gain by the bargain?

14. A man bought a sleigh for seventeen dollars, and gave nine dollars to have it repaired and painted, and then sold it for twenty-three dollars; how much did he lose by the bargain? (pp. 1, 2, and 3)

Article B is explained in the Key, p. 113, as follows:

B. This article contains the common addition table. The questions may be answered by beans, peas, &c. or by use of the table. It will be best to let the pupil

answer them in both ways. When the pupil can find the answers to these questions readily, he should commit them to memory.

To solve the questions in this article by the plate, select two rectangles which contain the numbers proposed, and ascertain the number of marks contained in both together.

9. Select 1 rectangle containing 7, and another containing 3, and say, 7 and 1 are 8, and 1 are 9, and 1 are 10. (p. 113)

The 9 in the preceding paragraph refers to question 9, page 3, in Article B; "Seven and three are how many?" (p. 3)

There are eighty-one addition problems in Article B. The questions include the addition combinations, whose sum ranges from 2 to 20.

Article C, pages 6 and 7, contains such combinations as:

Eleven and three are how many? Nineteen and two are how many? Twenty-one and five are how many? Nineteen and eight are how many? Fifteen and nine are how many? and Eighteen and seven are how many? (pp. 6-7)

Article C is explained in the Key, page 113, as follows:

C. This article is only an extension of the last. In this, the numbers from 1 to 10 are added to the numbers from 10 to 20.

To solve these, keep the larger number in the mind, and select a rectangle containing the smaller, and add the latter, 1 by 1, to the former.

As soon as the pupil can answer them readily by the plate, it would be well to teach him to apply the last article to the solution of these.

2. 14 and 4 (4 and 4 are 8) are 18.

27. 17 and 8 (7 and 8 are 15) are 25. (p. 113)

Article D contains subtraction. The Key recommends that the pupil first answer the questions by beans, pebbles and the like. Then, in the next place, use the plate, page 114. Problems from the list on page 7 of Article C are explained by the Key as follows:

2. 7 less 3. Select a rectangle containing 7, and take away 3, 1 at a time.

24. 18 less 5. Keeping 18 in mind, select a rectangle containing 5, and take them away, 1 at a time. Lastly make use of Article A of this section, as follows:

7. 8 less 3. 3 and 5 are 8; Therefore 8 less 3 are 5.

18. 17 less 5 (5 and 2 are 7; therefore 5 and 12 are 17) are 12. (p. 114)

Article D is explained by the Key, page 114, using Question 14 as an illustration:

E. In this article all the preceding are combined together, and the numbers from 1 to 10 are added to all numbers from 20 to 100; and subtracted in the same manner.

14. 57 and 6 (6 and 7 and 13) are 63, and 3 (3 and 3 are 6) are 66, and 5 (6 and 5 are 11) are 71, and 2 are 73, less 8 (13 less 8 are 5) are 65. (p. 114)

The preceding question, page 8, which is used as an illustration is as follows:

14. Fifty-seven, and six, and three, and five and two, less eight, are how many? (p. 8)

Article F contains what the author cited in the

Key, page 114, as "practical questions, which show the application of all the preceding articles." (p. 114) Problem six, page 10, is used as an illustration. It is as follows:

6. A boy had thirty-seven apples; he gave five to one companion; and eight to another; and when he had given some to another, he had six left; how many did he give to the last? (p. 10)

The solution to the preceding question is given as follows in the Key, page 114:

6. 37 less 5 are 32, less 8 (12 less 8 are 4) are 24, less 6 (which he kept himself) (14 less 6 are 8) are 18; consequently he gave 18 to the third boy. (p. 114)

Other problems on pages 10 and 11 show the increased complication. They are as follows:

4. From Boston to Roxbury it is three miles; from Roxbury to Dedham, six miles; from Dedham to Walpole, eleven miles; from Walpole to Wrentham four miles; from Wrentham to Attleborough, four miles; from Attleborough to Pawtucket, nine miles; from Pawtucket to Providence, four miles; how many miles is it from Boston to Providence?

9. A man bought a horse for forty-five dollars, and paid fifteen dollars for keeping him; he let him enough to receive twenty dollars; and then sold him for forty-three dollars; did he gain or lose by the bargain? and how much? (pp. 10, 11)

Section II deals with multiplication. The author says in the Key, page 115, that at first "the pupil will see no difference between multiplication than in addition and that it is best, but that it may be well to explain it to him after a while." (p. 115)

Article A contains fifteen problems which the author says are practical questions. The following are examples from pages 11 and 12:

1. What cost three yard of tape, at two cents a yard?

11. If a man travel three miles an hour, how many miles will he travel in four hours?

15. If there are three feet in one yard, how many feet are there in four yards? (p. 11, 12)

The preceding problems are explained in the key, page 115, in the following manner:

1. Three yards will cost 3 times as much as 1 yard. This question is solved on the plate thus: in the second row, count 3 rectangles, and find their sum. 2 and 2 are 4 and 2 are 6.

11. A man will travel 4 times as far in 4 hours as he will in 1 hour. In the third row count 4 times 3, and ascertain their sum.

15. There are 4 times as many feet in 4 yards as in 1 yard, or 4 times three. (p. 115)

Article B, pages 11 to 15, deals with the multiplication table. The key, page 115, states the following:

B. This article contains the common multiplication table, as far as the product of the first ten numbers. The pupil should find the answers once or twice through, until he can find them readily, and then let him commit them to memory. (p. 115)

Pages 11 to 15 include eighty drill problems on the multiplication table. A few of them are as follows:

1. Two times two are how many?
2. Three times two are how many?
10. Two times three are how many?
11. Three times three are how many?

- 27. Eight times four are how many?
- 46. Six times nine are how many?
- 60. Four times seven are how many?
- 70. Seven times eight are how many?
- 80. Ten times nine are how many? (pp. 11-15)

Article C is described in the Key, page 116, as follows:

C. In this article multiplication is applied to practical problems. They are of the same kind as those in Article A of this section. (p. 116)

Examples of the foregoing problems, pages 16, 17, and 18 follow:

- 11. There is an orchard consisting of ten rows of trees, and nine trees in each row; how many trees are there in the orchard?
- 14. How many farthings are there in eight pence?
- 29. How many gills are there in one gallon?
- 32. A person bought two oranges at six cents apiece; and seven lemons, at four cents apiece; and five pears, at two cents apiece; how much did the whole come to?
- 37. Two men start from the same place and travel different ways; one travels two miles in an hour; the other travels three miles in an hour; how far apart will they be at the end of two hours? How far at the end of three hours? How far at the end of four hours? (pp. 16, 17, and 18)

Section III, Article A, is described in the Key, pages 117 and 118, as follows:

A. This section contains division. The pupil will scarcely distinguish it from multiplication. It is not important that he should at first.

Though the pupil will be able to answer these questions by the multiplication table, if he has committed it to memory thoroughly; yet it will be better to use the plate

for some time. (pp. 117-118)

The following are problems from Article A, Section III, pages 18 and 19:

1. How many apples, at one cent apiece, can you buy for four cents?

2. How many pears, at two cents apiece, can you buy for four cents?

5. How many pears, at nine cents apiece, can you buy for nine cents? How many for twelve cents?

7. If a man travel six miles in two hours, how many miles does he travel in an hour?

8. If a man travel three miles in an hour, how many hours will it take him to travel nine miles?

10. If you have eleven apples, and wish to give two apiece to your playmates; to how many boys could you give them? (pp. 18-19)

The preceding problem is explained in the Key, page 118, as follows:

10. If you give 2 apples to each boy, you can give them to as many boys as 11 contains 2. This is solved on the table almost in the same manner as multiplication. In the second row, say, 2 and 2 are 4, and 2 are 6, and 2 are 8, and 2 are 10, and 1 are 11; then count the twos; there will be found 5 times 2 and 1 over, which is 1 half of another two. (p. 118)

Of Article B, Section III, the author makes the following comment in the Key, pages 118 and 119:

B. In this article, in which division is applied to abstract numbers, the pupil obtains the first idea of fractions. The pupil has already been accustomed to look upon a collection of units, as forming a number, or as being itself a part of another number. He knows, therefore, that one is a part of every number, and that every number is a part of every number larger than itself. As

every number may have a variety of parts in order to distinguish of parts, it is necessary to give names to the different parts in order to distinguish them from each other. The parts receive their names, according to the number of parts which any number is divided into. If the number is divided into two equal parts, the parts are called halves; if it is divided into three equal parts, they are called thirds, if into four parts, fourths, &c.; and having divided a number into parts, we can take as many of the parts as we choose. If a number be divided into five equal parts, and three of the parts be taken, the fraction is called three fifths of the number. The name shows at once into how many parts the number is to be divided, and how many parts are taken.

The examples in this book are so arranged that the names will usually show the pupil how the operation is to be performed. In this section, although the pupil is taught to divide numbers into various parts, he is not taught to notice any fractions, except those where the numbers are divided into their simple units, which is the most simple kind.

It will be best to use beans, pebbles, &c, first; and then plate I. (pp. 118-119)

The following questions 1, 10, 12, and 23 on pages 19, 20, and 21 of Article B are explained in the Key. The questions are given first in the order given and the explanation follows in the same order:

B. 1. Into how many parts can two be divided?

Ans. Into two parts

Remark. When anything or any number is divided into two parts, one part is called one half of the thing or number.

10. Nine are how many times two?

12. Into how many parts can three be divided?

Remark. When anything, or any number is divided into three equal parts, one of those parts is called the third part of the thing or number. When it is divided into four parts, one part is called the fourth part, and so on.



23. Eleven are how many times three? (pp. 119-120)

The explanation in the Key, pages 119-120, of the foregoing questions is:

1. Show the pupil one of the rectangles in the second row, and ask him into how many parts 2 can be divided. Explain to him that 1 is 1 half of 2.

10. In the second row count 9. It will take all the marks in the four first rectangles and 1 in the fifth. Therefore 9 is 4 times 2 and 1 half of another.

12. Show the pupil a rectangle in the third row, and ask him the question, and explain to him that 1 is 1 third of 3.

23. In the third row count 11. It will take 3 rectangles and 2 marks in the fourth. Therefore 11 is 3 times 3, and 2 thirds of another 3. Or it may be answered 4 times 3, less 1 third of 3. (pp. 119-120)

The next article, Article C, page 25, introduces the use of figures as symbols for the names of numbers. The author explains that the figures are introduced and used as an abridged method of writing numbers, and not with any reference to their use in calculating.

The following method is used in introducing figures:

"One is written . . . . . 1

Two is written . . . . . 2

Three is written . . . . . 3"

and so on to ten. (p. 25)

The same type of division questions are given as in Article B, except that figures are used and there are more

parts included in each question. The following is an example on page 31:

"80. Ninety-seven are how many time 10? 9? 6? 7? 8?" (p. 31)

The questions in Article D are similar to those of Article A and are solved in the same manner. The problems increase in complexity from the first to the last in the section. Examples on pages 31, 32, 33, and 34 showing increase in complexity follow:

D. 1. How many yards of tape, at 2 cents a yard, can you buy for six cents?

22. In twenty seven quarts, how many gallons?

26. A labourer engaged to work 8 months for ninety-six dollars; how much did he receive for a month? how much a week, allowing 4 weeks to the month? how many shillings a day, allowing 6 working days to the week?

36. There is a vessel containing sixty-three gallons of wine; it has a pipe which discharges 7 gallons in an hour; how many hours will it take to empty the vessel? (pp. 31-34)

Article A of Section IV, page 35, contains multiplication, repeating the number of times a certain number is of another. Examples follow:

6. 5 times 5, and 2 fifths of 5 are how many?

14. 9 times 7, and 6 sevenths of 7 are how many? (p. 35)

Article B of Section IV is explained in the Key, page 122, as follows:

In this article the pupil is taught to change a certain number of twos into threes, threes into fives, &c. This article combines all the preceding operations. (p. 122)

An example of these questions, page 31, follows:

"20. 7 times 8, and 5 eighths of 8 are how many times 9? 6? 10? 4? 5?" (p. 37)

The explanation of the Key, page 122, follows:

"20. 7 times 8 are 56, and 5-8ths of 8 are 5, which added to 56 make 61. 61 are 6 times 9, and 7-9ths of 9." (p. 122)

The questions or problems of Article C of Section IV are review exercises of the four fundamental processes. Examples, pages 38, 39, and 40, follow:

1. Bought 4 bushels of apples, at 3 shillings a bushel, how many dollars did they come to?

16. How much wheat at 7 shillings a bushel, can be bought for 2 barrels of cider at 4 dollars and a half a barrel?

23. 5 men bought a horse for sixty-three dollars, and paid two dollars a week for keeping him; at the end of 8 weeks they sold him for fifty-four dollars; how much did each man lose by the bargain? (pp. 38-40)

Section V and Article A of the section is explained in the Key, pages 123 and 124, as follows:

In this section the principle of fractions is applied to larger numbers, but such as are divisible into the parts proposed to be taken. The pupil, who is familiar with what precedes, will easily understand the examples in this section. They require nothing but division and multiplication.

A. Let the pupil explain each example in the following manner. What is 1 sixth of 18? Ans. 3. Why? Because 6 times 3 are 18; therefore if you divide eighteen into 6 equal parts one of the parts will be 3.

To find the answer on the plate; on the 6th row, the pupil will find 3 times 6 make 18; this will direct him to the third row, where he will find 6 times 3 are 18. Consequently, he will see 18 divided in 6 equal parts. It will be well to let the pupil prove a large number of examples on the plate.

The pupil will be very likely to say 3 is the 6th part of 18, because 3 times 6 are 18. Be careful to make him say it the other way, viz. 6 times 3 are 18. (pp. 123-124)

Examples of the questions, pages 40-41, in Article A of Section V, are:

A. 1. What is the half part of 4?

14. What is 1 half of twenty four? 1 third? 2 thirds? 1 fourth? 3 fourths? 1 sixth? 5 sixth? 1 eighth? 3 eighths? 5 eighths? 7 eights? 1 twelfth? 5 twelfths? 7 twelfths? (pp. 40-41)

Article B has problems which are more complicated.

Illustrations of these, pages 42 and 43, follow:

"B. 1. 3 fourths of twelve are how many times 4?

16. 9 sevenths of sixty-three are how many times 7?" (pp. 42-43)

Article C includes the written problems involving the same method as the problems in Articles A and B. The following are examples of these problems on pages 43 to 45.

C. 1. Charles had 6 apples, and gave 1 third of them to John; how many did he give him?

8. James had twenty-four cents, and he gave  $\frac{4}{5}$  of them for apples? how much did he give for all the apples? how much apiece?

23. Three men setting out on a journey, purchased 5 loaves of bread apiece, but before they had eaten any of it, two other men joined them, and they agreed to share the bread equally among the whole? how many loaves did they have apiece? (pp. 43-45)

Section V is concluded by the presentation of the figures from ten to one hundred, page 45, as follows:

"Ten is written . . . . . 10

Eleven is written . . . . . 11" (p. 45)

and so on to one hundred.

Section VI, pages 46 to 55, is a continuation of the same type of work as that of Section V. At the end of the section a list of problems entitled Miscellaneous Examples appears. The problems increase in complexity. The following problems, pages 51 and 53, illustrate this fact.

1. If 1 yard of cloth cost 4 dollars, what will 5 yards cost?

6. A fox is 80 rods before a greyhound, and is running at the rate of 27 rods in a minute, the greyhound is following at the rate of 31 rods in a minute; in how many minutes will the greyhound overtake the fox?

18. If 47 gallons of water, in 1 hour, run into a cistern containing 108 gallons, and by a pipe 38 gallons run out in an hour, how much remains in the cistern in an hour? and in how many hours will the cistern be filled? (pp. 51-53)

Section VII, pages 55 to 57, is the continuation of drill on parts of numbers or fractions. Preceding the

section, page 55, the author states:

The following Section contains some combinations which are useful as exercises for the mind of the pupil, and may sometimes be useful in practice. The pupil may study them or not, as the instructor may think best. (p. 55)

Examples of the combinations, pages 55 and 57, are:

3. 2 thirds of 9 is 3 fourths of what number?
11. 5 sixths of 24 is 10 sevenths of how many times 5?
9. 2 eights of 72 is 3 tenths of how many fifths of 40? (pp. 55 and 57)

Section VIII, pages 57 to 63, introduces problems in division with the following note of explanation in the Key? page 129:

It will be best to apply these divisions first to sensible objects. The instructor will find it extremely useful to refer frequently to sensible objects in illustrating the lessons, which follow. (p. 129)

The author continues the explanation, pages 129 and 130:

Plate II presents the units as divisible objects, the different fractions of which form parts, and sums of parts of unity.

This plate is divided into ten rows of equal squares, and each row into ten squares.

The first row is composed of ten empty squares, which are to be represented to the pupils as entire units. The second row presents squares, each divided into two equal parts by a vertical line, each of these parts of course represents one half. In the third row, each square is divided into three equal parts, by two vertical lines, each part representing one third, &c. to the tenth row, which is divided into ten equal parts, each part representing one tenth of unity.

Be careful to make the pupil understand, 1st, that

each square on the plate is to be considered as an entire unit, or whole one. 2nd. Explain the divisions into two, three, four &c. parts. 3rd. Teach him to name the different parts. Make him observe that the name shows into how many parts one is divided, and how many parts are taken, in the same manner as it does when applied to larger numbers.  $4/7$ ths, for example, shows that one thing is divided into 7 equal parts, and 4 of those parts are to be taken. 4th. Make the pupil compare the different parts together, and observe which is the largest. Ask him such questions as the following: Which are the smallest, halves or thirds? Ans. Thirds. Why? Because, the more parts a thing is divided into, the smaller the parts must be. (pp. 129 and 130)

Sections IX, X, XI, pages 63 to 77, and the miscellaneous examples are continuations of Section VIII.

The following problems, pages 57 to 77, illustrate the type of questions in Sections VIII, IX, X, XI, and miscellaneous examples:

1. If you cut an apple into two equal parts, what is one of these parts called?

2. How many halves of an apple will make the whole apple?

19. How can you tell how many halves there are in any number?

Answer. Since there are 2 halves in one, there will be twice as many halves as there are whole ones.

2. In 3 halves how many times 1?

17. How can you tell how many whole ones there are in any number of fourths?

4. How much is 3 times 2 thirds?

10. 5 times 2 sevenths are how many sevenths? how many times 1?

9. How much is 3 times 2 & 2 thirds?

40. How much is 3 times 10 & 4 ninths?

8. If it take 1 yard and 1 fourth to make a pair of pantaloons, how many yards would it take to make 8 pair?

15. If it take 1 yard and 3 sevenths to make 1 pair of pantaloons, and 2 yards and 4 sevenths for a coat; how many yards would it take to make 3 pair of pantaloons and 3 coats?

60. What is 7 eighth of 40?

16. If 2 cocks will empty a cistern in 3 hours, in how long a time would 1 empty it? in how long a time would 7 cocks empty it?

11. 6 & 3 eighth is 1 eighth of what number?

12. 13 is 7 times what number?

25. If 4 is 7 ninths of some number, what is 1 ninth of the same number? 4 seventh is 1 ninth of what number? Then 4 is 7 ninths of what number?

6. A man sold a piece of cloth for 47 dollars, by which bargain he lost 2 ninths of what the cloth cost him; how much did it cost him, and how much did he lose?

16. If a piece of cloth, 5 quarters wide, be worth 37 dollars, what is the price of the same length 3 quarters wide, worth? (pp. 57, 59, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 75, and 76)

Sections XII, XIII, and XIV deal with fractions. The definition and the writing of the fraction is given. The author explains the terms numerator and denominator on pages 77 and 78. He emphasizes the fact that (1) the pupils should be made familiar with the mode of expressing fractions and be able to apply it to any familiar objects such as apples, oranges, and the like, and to the table before he is allowed to proceed; (2) that particular care should be given to making



the pupil understand the terms numerator and denominator; and (3) that the denominator should always be explained first. (pp. 77 and 78)

The following are types of division problems found in Sections XII, XIII and XIV, pages 79 to 84, inclusive.

1. In 2 how many times  $1/2$ ? Ans.  $4/2$ .

15. Reduce  $9-4/7$  to an improper fraction.

8.  $23/7$  are how many times 1?

5. How much is 5 times  $3/8$ ?

9. How much is 8 times  $9-4/9$ ?

19. What is  $4/7$  of 65?

8.  $7-6/7$  is  $1/8$  of what number?

20. 54 is  $10/9$  of what number?

6. If when the days are  $9-1/2$  hours long a man perform a journey in 10 days, in how many days would he perform it when the days are 12 hours long?

19.  $2/9$  are how many  $1/36$ ?

21. Reduce  $1/2$  to sixths and  $1/3$  to sixths.

45.  $1/4$  and  $1/2$ , and  $2/5$ , and  $1/10$ , and  $1/20$ , less  $7/8$  are how many  $1/40$ ? (pp. 79, 80, 81, 83, 84, 85)

After one hundred fifty-one problems in fractions have been given, the author defines common denominator, pages 85 to 87, inclusive. He explains that when fractions have common denominators they may be added or subtracted. Next, problems in which the pupils are to reduce the fractions to the lowest terms appear. At the end of these problems, the

author says in a note that if both the numerator and denominator be multiplied by the same number, the value of the fraction will not be changed, or if both are divided by the same number without a remainder, the fraction would not be altered.

Pages 90 to 92, inclusive, deal with the tables of coins, weights and measures, and miscellaneous examples. The tables are: Federal Money, Sterling Money, Troy Weight, Avoirdupois Weight, Cloth Measure, Wine Measure, Dry Measure, Measure of Time (hours, minutes, day, week, month, year). (pp. 90, 91, 92)

A unique table regarding Cloth Measure is found on page 91, as follows:

2-1/4 inches	make	1 nail	nl.
4 nails	"	1 quarter of a yard	qr.
4 quarters	"	1 yard	y.
3 quarters	"	1 ell Flemish	
5 quarters	"	1 ell English	
6 quarters	"	1 ell French	(p. 91)

Problems from the miscellaneous groups, pages 93 to 108, inclusive, are as follows:

1. In 2 pounds how many ounces?
62. In 16 nails how many quarters of a yard? how many yards?
98. If a man spend  $2\frac{3}{5}$  dollars in a day, how much would he spend in a week?
124. Three men bought a lottery ticket for 10 dollars; the first gave 3 dollars, the second 5 dollars, and the third 2 dollars. They drew a prize of 120 dollars. What

was each man's share?

157. Said Harry to Dick, my purse and money together are worth 16 dollars, but the money is worth 7 times as much as the purse; how much money was there in the purse? and what is the value of the purse?

164. A farmer being asked how many sheep he had, answered; that he had them in 4 pastures; in the first he had  $\frac{1}{3}$  of his flock; in the second  $\frac{1}{4}$ ; in the third  $\frac{1}{6}$ ; and in the fourth 15; how many had he?

165. A man, driving his geese to market was met by another, who said, good morrow, master, with your hundred geese; says he, I have not a hundred; but if I had half as many more as I now have, and two geese and a half, I should have a hundred; how many had he?

170. A man being asked his age, answered, that if its half and its third were added to it, it would be 77; what was his age?

171. What number is that, which being increased by its half, its fourth, and eighteen more, will be doubled?  
(pp. 93, 97, 98, 102, 106, 107, 108)

Summary. The author believes in the strengthening of the mind through exercise by first counting concrete objects or by using the Pestalozzian plates as counters. The author follows the plan in the majority of cases of presenting what he terms practical or concrete examples or questions first. These questions introduce names of objects or people. These questions are followed by those having only abstract numbers and requiring much mental drill. Practical but more complicated questions follow. This plan is used for the four fundamental processes. The author considers fractions simple

enough for children of six and seven years of age to understand. He states that throughout the book the problems become more complicated. This is true, as may be seen in the problems cited in the paragraph preceding this summary.

Both the concrete drill problems and written problems become mere puzzles to test the "wits." Examples of these are such questions as: (1) 2 thirds of 9 is 3 fourths of what number? and (2) A man driving his geese to market was met by another, who said, good morrow, master, with your hundred geese; says he, I have not a hundred; but if I had half as many more as I now have and two geese and a half, I should have a hundred, how many had he?

The author states that his book is based on the inductive method, which he calls "the natural method." It is true that he gives no definitions or rules before he presents the topics.

In conclusion it may be said that the book does conform to the educational aim of the period, which aim was to strengthen the mind through exercise or drill. While he seems to realize that the practical or concrete aim is important, the emphasis upon drill formalizes the material of the text; and the mental level of the first and second grade children is ignored.

## II. PRACTICAL AND MENTAL ARITHMETIC ON A NEW PLAN

This book was written by Roswell C. Smith. The book measures four inches by six and one-half inches. The cover is of brown cardboard. This text has neither pictures nor plates. The title and information on the cover and title page are the same. They follow:

Practical and Mental Arithmetic on a new Plan: In which Arithmetic is combined with the use of the Slate; containing A Complete System for all Practical Purposes; Being in Dollars and Cents. Stereotype Edition, Revised and Enlarged, with Exercises for the Slate. To which is added A Practical System of Bookkeeping. By Roswell C. Smith. New York: Published By Paine and Burgess, 62 John Street.<sup>2</sup>

The page following the title page gives the information that this book was "Entered according to Act of Congress, in the year 1835, by Carter, Hendricks and Co., in the Clerk's Office of the District Court of Massachusetts." (p. ii)

There is also on the second page an excerpt from the Journal of Education recommending the arithmetic. It is as follows:

A special examination of this valuable work will show that its author has compiled it, as all books for school use ought to be compiled, from the result of actual experiment and observation in the school-room. It is entirely a practical work, combining the merits

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<sup>2</sup> Roswell C. Smith, Practical and Mental Arithmetic on a New Plan (New York: Paine and Burgess, 1835).

of Colburn's system with copious practice on the slate.

There are several very valuable peculiarities in this work, for which we cannot, in a notice, find sufficient space. We would recommend a careful examination of the book to all teachers who are desirous of combining good theory with copious and rigid practice. (p. ii)

The preface on pages iii and iv, portions of which are as follows:

. . . The present edition professes to be strictly on the Pestalozzian, or inductive, plan of teaching. This, however, is not claimed as a novelty. In this respect it resembles many other systems. The novelty of this work will be found to consist in adhering more closely to the true spirit of the Pestalozzian plan; consequently, in differing from other systems, it differs less from the Pestalozzian. This similarity will now be shown.

1. The Pestalozzian professes to unite a complete system of Mental with Written Arithmetic. So does this.

2. That rejects no rules, but simply illustrates them by mental questions. So does this.

3. That commences with examples for children as simple as this, is as extensive, and ends with questions adapted to minds as mature. . . .

The following are a few of the prominent characteristics of this work, in which it is thought to differ from all others.

1. The interrogative system is generally adopted throughout this work.

2. The common rules of Arithmetic are exhibited so as to correspond with the occurrence in actual business. . . .

3. There is constant recapitulation of the subject attended to, styled "Questions on the foregoing." . . .

As in this work, the common rules of Arithmetic are retained, perhaps the reader is ready to propose a question frequently asked, "What is the use of so many rules?" "Why not proscribe them?" The reader must here be

reminded, that these rules are taught differently, in this system, from the common method. The pupil is first to satisfy himself of the truth of several distinct mathematical principles. These deductions, or truths, are then generalized; that is, briefly summed in the form of a rule, which, for convenience sake, is named. Is there any impropriety in this? On the contrary, is there not a great convenience in it? Should the pupil be left to form his own rules, it is more than probable he might mistake the most concise and practical one. Besides different minds view things differently, and draw different conclusions. Is there no benefit then in helping the pupil to the most concise and practical method of solving the various problems incident to a business life? . . .

In this work, the author has endeavored to make every part conform to this maxim, viz., that names should succeed ideas. This method of communicating knowledge is diametrically opposed to that which obtains, in many places, at the present day. The former, by first giving ideas, allures the pupil into a luminous comprehension of the subject. (pp. iii and iv)

This arithmetic has no table of contents, but is divided into sections. It contains ninety-eight sections and a practical system of bookkeeping for farmers and mechanics included at the end of the text. The author states that the questions in I and II are intended for the very young children; therefore, only these two sections will be discussed here.

Section I includes Addition. Questions are introduced first. There are twenty-four of these in succession. None of these has pictures accompanying them. A few of the following problems, on pages 1 and 2 of this section, are typical of the kinds of questions asked:

1. How many little fingers have you on your right hand? How many on your left? How many on both?

2. How many eyes have you?

7. If you have three pins in one hand, and James puts two more in, how many will you have in your hand? How many are three and two?

8. How many legs have two cats and a bird?

19. If you count all your fingers, thumbs, and nose, how many will they make?

24. You gave thirteen cents for a spelling-book, and three for an inkstand; how much do they come to? How many are thirteen and three? (pp. 1 and 2)

The twenty-fifth problem, page 2, gives the direction to count one hundred. This is illustrated by a table form.

The reproduction of a portion of it follows:

One . . . . .	1	Six . . . . .	6
Two . . . . .	2	Seven . . . . .	7
Three . . . . .	3	Eight . . . . .	8
Four . . . . .	4	Nine . . . . .	9
Five . . . . .	5	Ten . . . . .	10

The author's note at the end of the counting table, page 3, explains the kind of drill to be used and the purpose of succeeding questions. The note is as follows:

The pupil is to recite the above, with the written numbers covered over. The answers to the following questions are to be given by writing them down on the slate at recitation, to test the pupil's knowledge of numbers from one to one hundred. (p. 3)

The question which follows the table is:

26. Write down in proper figures, Four; Seven; Eight; Twelve; Eighteen; Twenty-two; Thirty-two; Forty-five; Forty-nine; Fifty-six; ~~Fifty-nine~~; Sixty-three; Seventy-five; Eighty-seven; Ninety-two; Ninety-seven; Ninety-nine. (p. 3)



The next two questions concern the size of numbers.

One of these questions is:

Thomas has fifty-nine dollars and William sixty-nine; which has the most? Which is the most, eighty-nine, or ninety-nine? Forty-seven, or seventy-four? (p. 3)

The Addition Tables of one to twelve, inclusive, conclude the section on Addition. Pages 3 to 5, inclusive, are drill questions on the tables. The following note, page 5, explains the purpose of the questions which are at the end of the tables:

The design of the foregoing and succeeding questions is to prevent the scholar from resting satisfied with saying his table merely by rote, which frequently happens. For if he can count, he will say it, without making a single addition in his mind. (p. 5)

Some of the questions illustrating the above point are:

How many are 2 and 5? 2 and 7? 2 and 10? 2 and 12?  
3 and 3? 3 and 9? 3 and 12? 4 and 2? 4 and 6? 4 and 8?  
4 and 10? 4 and 12? 5 and 3? 5 and 5? . . . 9 and 12?  
10 and 3? 10 and 4? 10 and 6? 10 and 8? 10 and 11?  
10 and 12? 11 and 3? 11 and 5? 11 and 6? 11 and 9?  
11 and 12? 12 and 3? 12 and 6? 12 and 9? 12 and 12?

You borrow 12 dollars at one time, and 2 at another? how much have you borrowed in all? How many are 12 and 2?

Your brother William gave you 19 cents, your brother John 10, and your cousin 2; how many did you have given you in all? How many are 19, 10, and 2? (pp. 4 and 5)

Section II, pages 6 to 9, inclusive, deals with Subtraction. Problems, which the author calls "practical" or "concrete," introduce this topic. The first problem is stated and

the answer and the reason for it are given. Each problem in this first group of subtraction problems, pages 6 and 7, ends with the question "Why?" The first problem showing the form of analysis and other problems of this group showing the increase in complexity follow:

If you should lose one finger from one hand, how many would you have left on that hand? How many are 4 less one? Why? Ans. Because 1 and 3 are 4.

5. If you have 9 cents in a box, and take out 4, how many will be left in the box? How many are 9 less 4, or 4 and 9? Why?

12. A man bought a barrel of molasses for 15 dollars, and sold it for 19; how much more than he gave for it did he sell it for? How many are 15 from 19 then? Why?

17. A man bought a barrel of beef for 20 dollars, and, being damaged, he is obliged to lose 12 dollars on the sale of it; how much did he sell it for? How many are 12 from 20 then? Why?

18. How many legs will 4 chairs have to stand on, if 1 have 3 broken legs? How many are 3 from 16? Why?

20. Suppose you and William lose a finger apiece, how many fingers will you both have then? How many are 2 from 16? Why?

22. A poor man had 16 bushels of rye given him; his eldest son gave him 10 bushels and the youngest the rest; how many bushels did the youngest give him? How many did the elder give him more than the younger? How many are 10 from 16? 6 from 10? Why?

23. 28 boys were sliding on the ice, which breaking, all but 4 fell in and perished; how many lost their lives? How many are 4 from 28? Why? (pp. 6 and 7)

The Subtraction Table of one to twelve, inclusive, Questions on the Table, and Practical Questions on the Table,

pages 7 to 10, inclusive, conclude the section on Subtraction.

Questions on the Table, page 9, contain abstract drill exercise, such as:

How many does 2 from 8 leave? 5 from 20? 11 from 19? 12 from 16? 12 from 24? (p. 9)

Examples of the Practical Questions, pages 9 and 10, are:

26. If you buy 15 cents worth of tape, and give the shopkeeper a pistareen, or seventeen cent bit, how many cents must you have in change? How many are 15 from 17? Why?

27. If you had 17 fingers how many would you have more than you have now? How many are 8 from 17? Why?

30. If you have 25 cents, and should give 10 cents for a ruler, and 10 cents for a top, how many cents will you have left? How many do 10 and 10 from 25 leave? Why?

32. A man bought a mirror for 12 dollars, for which he gave 6 bushels of corn, worth 5 dollars, 3 bushels of potatoes, worth 1 dollar, and the rest in money; how much did he pay? How many do 5 and 1 from 12 leave? Why?

33. The distance from Boston to Walpole is 20 miles; after you have arrived at Dedham, which is 11 miles from Boston, how many more miles will you have to travel to reach Walpole? How many are 11 from 20? Why? (pp. 9 and 10)

Summary. Smith follows the Pestalozzian plan of induction. The pupil is to first learn the principles from solving the written and oral questions. Then the rule is given to the pupil, because the author feels that the pupils might make the wrong deductions and draw different conclusions.

This view shows the tendency toward mechanical, formalized learning of rules and definitions. There is the following similarity of this book to that of Colburn's: (1) Written problems citing concrete things are presented first. They are called "practical problems;" (2) abstract questions for oral drill for the purpose of strengthening the mind follow the practical questions; (3) there is the rapid increase in complexity of situation, size of numbers in problems and difficulty of solution; (4) no attention is given to the learning ability of children of the first and second grade level.

It may be said in conclusion that Smith seems to have copied from both Pestalozzi and Colburn, and that his aim is in harmony with the educational aim of mental drill and formal discipline.

### III. RAY'S ARITHMETIC, FIRST BOOK

This little book, containing eighty pages, measures four and a quarter inches by six and five-eighths inches. There are some plates and concrete illustrations included in the texts. Reproductions of a few follow in the analyses.

The title page gives the following information:

Eclectic Educational Series. Ray's Arithmetic, First Book. Primary Lessons and Tables in Arithmetic for Young Learners. By Joseph Ray, M. D., Late Professor of Mathematics in Woodward College. Stereotype Edition.

Publishers: Cincinnati--Winthrop B. Smith and Co. New York--Clark, Austin, Maynard and Co.<sup>3</sup>

The following page includes the advertisement of the various mathematical publications of the author and the copyright of 1857.

The preface is found on page 3. In it the author discusses (1) one improvement in teaching, (2) the purpose of his book, (3) the value of mental arithmetic, and (4) the contributions of educators to whom he expresses acknowledgement. The preface follows:

One of the most important improvements in the art of instruction, is that by which the study of Arithmetic has been rendered interesting and attractive to children.

In the preparation of the following pages, extreme care has been used in making the lessons gradually and almost imperceptibly progressive, so that the little learner is all the while unconsciously, but thoroughly mastering the introductory principles of numbers.

The study of Mental Arithmetic by quite young learners can not be too highly appreciated; it is an exciting and profitable exercise of the juvenile mind, develops the faculties, and gives them valuable discipline for a more vigorous pursuit of other studies in which they may be engaged.

In presenting a remodded and greatly improved edition of this volume, widely known as Ray's Arithmetic, First Part, grateful acknowledgments are made to the numerous educators who have extended to this, as well as to the other mathematical works by the same author, a large approbation and a wide and increasing patronage.

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<sup>3</sup> Joseph Ray, Ray's Arithmetic, First Book (Cincinnati: Winthrop B. Smith and Co., 1857).

The method of giving the tables in reverse order, (as on pages 10 to 60), each table followed by simple questions, was presented in 1834, in "Calculations for the Head,"--a small tract on the oral method of instruction,--and has been complimented by adoption in the Primary Table Book of the distinguished Professor Davies.

Page 4 carries the title Directions to Teachers. It gives suggestions for the teaching of the content material of the book. These directions follow:

The following suggestions are respectfully submitted to the consideration of Teachers, believing they will be found especially valuable to those who are inexperienced in teaching young pupils this important branch of study.

Let the Pupil have his book before him until able to solve the questions readily; then thoroughly and repeatedly review without it, the Teacher reading the question distinctly, and the pupil answering it. The learner, in answering, should repeat the problem to be solved.

Teach one thing at a time, and teach it thoroughly. This is an important direction as regards all branches of study, but should be more particularly observed in acquiring a knowledge of the elementary principles of Arithmetic, since every successive step in the pupil's progress, depends, in a great degree, upon what has preceded it.

Under no circumstances permit the pupil to leave a lesson until he has entirely mastered it, and is able, not only to solve all the questions it contains, but to solve them readily and understandingly, so that he shall know he is right, and be able to tell why he knows it.

To awaken and fix the attention of the class, let the problems be assigned promiscuously, not in rotation: allow one pupil to read the question, another to answer it, and a third to give the reasons. A strict observance of these directions will break up that dull, listless manner of recitation so frequently seen, and give to the exercise interest, spirit and variety.

Let no considerations induce the Teacher to advance the pupils to "Ray's Intellectual Arithmetic," (Second Book) until he has proved by sustaining a final rigid review that he has completely mastered this work.

This book has no table of contents, but the topics, which are divided into lessons, are:

1. Numeration and Notation
2. Addition
3. Subtraction
4. Multiplication
5. Division
6. Review Exercises

The topic, Numeration and Notation, is discussed on pages 5 to 9, inclusive. At the beginning of this topic, page 5, there is a note to the teacher regarding the illustration of the unit in numbers on the same page.

A reproduction of this lesson follows:

## NUMERATION AND NOTATION

### LESSON I.

**To Teachers.**--The object of this lesson is to teach children to count in words: that is, to express in words the number denoting a unit, or a collection of units. Also, to show the FIGURE that represents each number, as high as TEN.

Here is a picture of some balls: the pupil must learn to count them by pointing to each, and saying, one: one, two: one, two, three, etc.

0	.	.	.	.	.	.	.	.	.	one,	.	1.
0	0	.	.	.	.	.	.	.	.	two,	.	2.
0	0	0	.	.	.	.	.	.	.	three,	.	3.
0	0	0	0	.	.	.	.	.	.	four,	.	4.
0	0	0	0	0	.	.	.	.	.	five,	.	5.
0	0	0	0	0	0	.	.	.	.	six,	.	6.
0	0	0	0	0	0	0	.	.	.	seven,	.	7.
0	0	0	0	0	0	0	0	.	.	eight,	.	8.
0	0	0	0	0	0	0	0	0	.	nine,	.	9.
0	0	0	0	0	0	0	0	0	0	ten,	.	10.

The object of the next four lessons, pages 6 and 7, is to teach the figures for the number names from one to one hundred, inclusive. In Lesson II, the numbers to be learned are one to thirty, inclusive. The next two lessons include the numbers from thirty-one to one hundred, inclusive, but these lessons may, according to the author, be omitted for very young children until progress has been made in Addition and Subtraction. Lesson V contains the figures from one to one hundred arranged in columns. These are to be read. However, the author suggests this lesson may be omitted until later by the young children.

Lesson VI, page 8, has questions about figures such as the following:

What figure stands for five? What stands for nine?  
 What for eighty-nine? What for one hundred?

Lesson VII, page 8, introduces an illustration explaining the size of numbers. A reproduction of this lesson is shown on the following page.

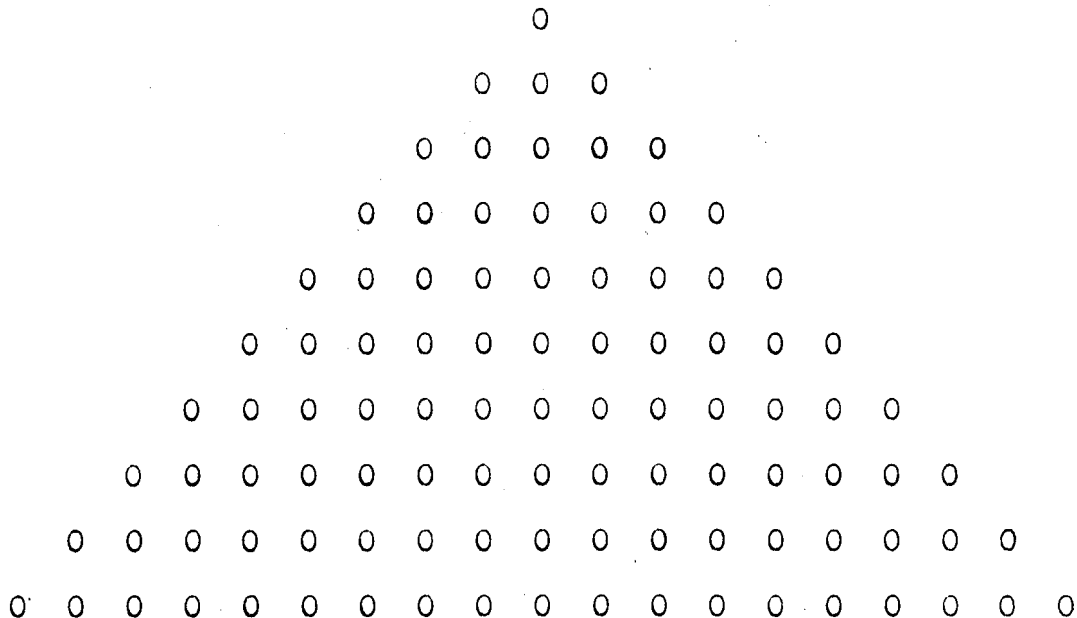
one is ten, ...

name of each ...



## LESSON VII

To Teachers.--This lesson is designed to make pupils familiar with the relative magnitude of numbers.



How many balls are in the first (top) line? How many in the first and second lines? How many in the first, second, and third lines? How many in the first four lines? How many in the first five lines? How many in the first six lines? How many in the first seven lines? How many in the first eight lines? How many in all?

The questions contained in this lesson do not refer to the relative increase in number, but only give practice in counting.

There are fifteen lessons in the Addition topic. These lessons include (1) one illustrated lesson, (2) the tables of one to ten, inclusive, in various lessons, (3) questions using names of concrete things, (4) questions using the abstract



9. Nine trees and one tree are how many?  
How many are 1 and 9?

0

000000000

It will be noted that the names of concrete things are used in the first statement of each question. This is followed by the reverse order of the first statement and the use of abstract numbers. The illustration is also arranged in the reverse order.

Lesson II, page 10, introduces the addition table of one, which gives both the direct and reverse form. An example follows which will be typical of the remaining addition tables:

1 and 1 are 2	1 and 1 are 2
2 and 1 are 3	1 and 2 are 3
3 and 1 are 4	1 and 3 are 4
4 and 1 are 5	1 and 4 are 5
5 and 1 are 6	1 and 5 are 6
6 and 1 are 7	1 and 6 are 7
7 and 1 are 8	1 and 7 are 8
8 and 1 are 9	1 and 8 are 9
9 and 1 are 10	1 and 9 are 10
10 and 1 are 11	1 and 10 are 11

The remainder of this lesson consists of written problems based on the addition table of one. The answer and reason, are given in problem 1. Examples of these questions from Lesson II follow:

1. Daniel had 1 apple, and his mother gave him 1 more: how many apples had he then? Ans. 2. Why?  
Because 1 and 1 are 2.

9. One peach and 9 peaches are how many?

The other eight lessons on the tables of two to ten,

inclusive, follow the same plan as that of Lesson II.

Lesson XII is the oral review of all the preceding lessons, Lesson XIII is the written review, using what the author terms "concrete practical problems," and Lesson XIV is a review of both the "abstract" and "practical" problems, which are more complicated.

Examples from each type of review questions, pages 20 to 22, inclusive, follow:

1. How many are 2 and 4? 3 and 6?

19. How many are 6 and 8? 7 and 3?

30. How many are 9 and 5? 10 and 10?

1. There are 6 trees in one row, and 5 trees in another: how many in both?

9. Susan is 9 years old, and Emma is 10 years older than Susan: How old is Emma?

11. A boy had a number of cents: after spending 10, he had 6 left: how many had he at first?

2. How many are 2 and 3 and 4? 4 and 2 and 3? 4 and 3 and 2? 1 and 2 and 3 and 4? 1 and 3 and 4 and 2? 1 and 2 and 4 and 3?

9. Seven peaches, and 2 peaches and 9 peaches, are how many peaches?

14. I owe to one man 4 dollars, to another 5 dollars; and to another 9 dollars: how much do I owe to all?

The author seems not to have observed the statement he made regarding the gradual progressive difficulty in the material of the text. In the preceding problem three numbers are introduced.

The same procedure is used in the Subtraction, Multiplication and Division topics as in the Addition topic with the following exceptions: (1) A review of addition and subtraction by the abstract drill method and by written problems called Promiscuous Questions follows the abstract drill in the topic of Subtraction. (2) The Multiplication topic is concluded with the Promiscuous Questions. These include the processes of addition, subtraction, and multiplication. Usually each of these questions requires the process of addition and multiplication, or subtraction and multiplication, or addition and subtraction, to obtain the answer.

Examples from the lessons in the Subtraction, Multiplication, and Division topics follow:

The examples in Subtraction include (1) a reproduction of Lesson I, page 23, the development of the subtraction table of one, (2) an example of the subtraction table, page 33, (3) some written problems, pages 24 to 34, (4) some abstract drill problems in review, pages 34 to 36, and (5) some problems from the Promiscuous Questions, page 36.

## SUBTRACTION.--LESSON I.

To Teachers.--The counters on the right represent the numbers to be subtracted; those on the left, what remains after performing the subtraction.

1. James had 1 apple, and he gave it to his brother: how many had he left?

One from 1 leaves how many?

0

2. Mary had 2 peaches: after giving her sister 1, how many had she left?

0

One from 2 leaves how many? Ans. 1.

Why? Because 1 and 1 are 2.

3. Daniel had 3 canary birds, and 1 of them flew away: how many had he left?

00

One from 3 leaves how many? Why?

0

4. Lucy had four oranges: after eating 1, how many had she left?

000

One from 4 leaves how many? Why?

0

5. Thomas had 5 pet rabbits, but 1 of them died: how many were left?

0000

One from 5 leaves how many? Why?

0

6. If you take 1 raisin from 6 raisins, how many raisins are left?

00000

One from 6 leaves how many? Why?

0

7. If you take 1 cent from 7 cents, how many cents are left?

000000

One from 7 leaves how many? Why?

0

The subtraction table, some written problems, abstract drill problems as review of preceding lessons in subtraction and of addition and subtraction, and problems from the Promiscuous Questions follow:

0 from 10 leaves 10	10 from 10 leaves 0
1 from 11 leaves 10	10 from 11 leaves 1
2 from 12 leaves 10	10 from 12 leaves 2
3 from 13 leaves 10	10 from 13 leaves 3
4 from 14 leaves 10	10 from 14 leaves 4
5 from 15 leaves 10	10 from 15 leaves 5
6 from 16 leaves 10	10 from 16 leaves 6
7 from 17 leaves 10	10 from 17 leaves 7
8 from 18 leaves 10	10 from 18 leaves 8
9 from 19 leaves 10	10 from 19 leaves 9

#### WRITTEN PROBLEMS ON SUBTRACTION TABLES

10. Ella has 10 plums, and she gave 1 to her sister: how many had she left? Why?

5. Cora had 7 pins and lost 4 of them: how many pins did she have then? Why?

10. A hen had 13 chickens and 4 of them died: how many were left? Why?

9. A boy had 15 cents; after spending part he had 8 cents left: how much did he spend? Why?

5. Anna had 13 birds, and 4 of them died: how many had she left? Why?

#### REVIEW OF THE PRECEDING

1. How many are 4 less 2? 7 less 3?
10. How many are 11 less 8? 7 less 4?
19. How many are 14 less 9? 13 less 7?
22. How many are 11 less 6? 18 less 9?
30. How many are 19 less 10? 17 less 7?

## ADDITION AND SUBTRACTION

1. One and 3, less 4, make how many?
10. Two and 8, less 6, make how many?
20. Three and 9, less 8, make how many?
25. Four and 10, less 9, make how many?
30. Six and 9 less 8, make how many?

## PROMISCUOUS QUESTIONS

1. John spent 3 cents for a top, and 5 cents for a kite: how many cents did he spend?
7. Two numbers added together make 11: one of them is 7: what is the other?
9. Frank had 8 cents: after spending 5, his mother gave him 4: how many had he then?
13. Emma paid 5 cents for thread, 2 cents for tape, and 3 cents for needles: she had 15 cents; how much had she left?
14. I have 10 cents in one hand, and 6 in the other: if I take 2 cents from each hand, how many cents will I have in both?

The increase on complexity of situation is especially noticeable in the last three problems in the preceding list of examples.

The order of examples as illustrations of the topic of Multiplication is: (1) reproduction of page 37, the development of the multiplication table 2; (2) a multiplication table, page 44; (3) written problems following the tables, pages 38 to 47, inclusive; (4) abstract, oral drill and written drill, pages 48 to 49; and (5) Promiscuous Questions, page 50.



## MULTIPLICATION.--LESSON I.

1. What cost 2 pencils, at 1 cent each?  

$$\begin{array}{r} 0 \\ 0 \end{array}$$
 How many are 1 and 1?
2. What cost 2 figs, at 2 cents each?  

$$\begin{array}{r} 00 \\ 00 \end{array}$$
 How many are 2 and 2?
3. What cost 2 plums, at 3 cents each?  

$$\begin{array}{r} 000 \\ 000 \end{array}$$
 How many are 3 and 3?
4. What cost 2 pears at 4 cents each?  

$$\begin{array}{r} 0000 \\ 0000 \end{array}$$
 How many are 4 and 4?
5. What cost 2 peaches at 5 cents each?  

$$\begin{array}{r} 00000 \\ 00000 \end{array}$$
 How many are 5 and 5?
6. What cost 2 balls, at 6 cents each?  

$$\begin{array}{r} 000000 \\ 000000 \end{array}$$
 How many are 6 and 6?
7. What cost 2 oranges, at 7 cents each?  

$$\begin{array}{r} 0000000 \\ 0000000 \end{array}$$
 How many are 7 and 7?
8. What cost 2 slates at 8 cents each?  

$$\begin{array}{r} 00000000 \\ 00000000 \end{array}$$
 How many are 8 and 8?
9. What cost 2 books, at 9 cents each?  

$$\begin{array}{r} 000000000 \\ 000000000 \end{array}$$
 How many are 9 and 9?
10. What cost 2 knives, at 10 cents each?

## EXAMPLE OF MULTIPLICATION TABLE OF SEVEN

once 7 is 7	7 times 1 is 7
2 times 7 are 14	7 times 2 are 14
3 times 7 are 21	7 times 3 are 21
4 times 7 are 28	7 times 4 are 28
5 times 7 are 35	7 times 5 are 35
6 times 7 are 42	7 times 6 are 42
7 times 7 are 49	7 times 7 are 49
8 times 7 are 56	7 times 8 are 56
9 times 7 are 63	7 times 9 are 63
10 times 7 are 70	7 times 10 are 70

## WRITTEN PROBLEMS ON THE MULTIPLICATION TABLES

1. When apples are selling at 1 cent each, James bought 1: how many cents did it cost? Ans. 1 cent. Why? Because one 1 is 1.

2. I bought 2 figs at 1 cent each: how much did they cost? Why? Because twice 1 are 2.

1. When peaches are selling at 2 cents each, how many cents will 2 cost? Ans. 4 cents. Why? Because 2 peaches will cost 2 times as much as 1 peach: if 1 peach cost 2 cents, 2 peaches will cost 2 times 2 cents, which are 4 cents.

5. If 1 orange is worth 5 peaches, how many peaches are 6 oranges worth? Why?

6. How many cents will 6 citrons cost at 9 cents each? Why?

## REVIEW OF THE PRECEDING

- |                             |            |
|-----------------------------|------------|
| 1. How many are 2 times 4?  | 3 times 2? |
| 10. How many are 5 times 6? | 3 times 8? |
| 19. How many are 7 times 2? | 9 times 2? |
| 26. How many are 6 times 9? | 5 times 9? |

## WRITTEN REVIEW PROBLEMS

4. When eggs are 9 cents a dozen, how much must I pay for 8 dozen?

7. What will 7 combs cost, at 7 cents each? at 9 cents each?

10. Frank is 4 times as old as Mary: Mary is 3 years old: how old is Frank?

14. In a school-room there are 9 desks, and 4 boys at each: how many boys are there in the room?

#### PROMISCUOUS QUESTIONS

1. I had 14 cents and bought 2 oranges, at 6 cents each: how much had I left?

5. James bought 2 pups, at 3 dollars a-piece, and a dog for 4 dollars: how much did he pay for all?

6. John bought a sled for 10 cents: he sold it to Harry for 2 cents more than it cost: how many cents did he sell it for?

7. I bought a coat for 13 dollars, and sold it for 4 dollars less than cost: how much did I get for it?

9. James had 17 marbles: he lost 9 of them, and afterwards found 7 more: how many did he then have?

10. George owed me 19 cents: he gave me 2 oranges worth 5 cents each, and the remainder in money: how much money did I get?

The increase in the complexity of problem situation and solution is again evident in the preceding problems.

The mechanical analysis of the solution is also evident in the various types of problems which the author explains.

The Division topic will be illustrated as follows:

(1) reproduction of page 51, Lesson I, the developments of the division table; (2) a division table, page 59; (3) written problems as drill on the tables, pages 52 to 60, inclusive; (4) abstract and written problems as review on the preceding work done in Division, pages 61 and 62.



## DIVISION TABLE OF NINE

1 in 9, 9 times	9 in 9, once
2 in 18, 9 times	9 in 18, 2 times
3 in 27, 9 times	9 in 27, 3 times
4 in 36, 9 times	9 in 36, 4 times
5 in 45, 9 times	9 in 45, 5 times
6 in 54, 9 times	9 in 54, 6 times
7 in 63, 9 times	9 in 63, 7 times
8 in 72, 9 times	9 in 72, 8 times
9 in 81, 9 times	9 in 81, 9 times
10 in 90, 9 times	9 in 90, 10 times

1. How many apples, at 2 cents each, can you buy for 4 cents? Ans. 2. Why?

Because you can buy as many apples for 4 cents, as 2 cents, the price of 1 apple, is contained times in 4 cents: 2 in 4, 2 times: and, you can buy 2 apples.

2. If you have 6 balls, how many groups, of 3 balls each, can you make out of them?

Three in 6 how many times? Ans. 2 times. Why? Because 2 times 3 are 6.

9. How many rings, at 6 dimes each, can you buy for 54 dimes? Why?

## REVIEW OF THE PRECEDING PROBLEMS IN DIVISION

To Teachers.--The question, how many times? is to be put by the instructor, after each combination.

2 in 4	4 in 12	4 in 32	5 in 35	9 in 54
2 in 8	3 in 18	5 in 30	6 in 42	7 in 63
3 in 6	2 in 20	6 in 18	4 in 40	8 in 64
3 in 12	7 in 14	4 in 36	7 in 56	9 in 90

2. John gave 42 marbles for 6 cents; how many did he give for 1 cent?

8. If 3 lemons are worth 1 orange, how many oranges can you get for 24 lemons?

9. If 6 sheets of paper make 1 copy-book, how many books can you make out of 18 sheets?

The remaining portions of the book, pages 63 to 80, inclusive, deal with a review of the entire book. Examples of each type of review follow:

#### ABSTRACT PROBLEMS IN ADDITION

7. How many are 16 and 1? 2 and 16? 16 and 3?  
4 and 16? 16 and 5? 6 and 16? 16 and 7? 8 and 16?  
16 and 9? 10 and 16?

#### WRITTEN PROBLEMS IN ADDITION

13. Cora spent 47 cents for books, and 4 cents for pens: how much did she spend?
16. George bought a sled for 27 cents, and paid 5 cents to have it repaired: how much was it then worth?

#### WRITTEN PROBLEMS IN SUBTRACTION

7. Sarah had 31 needles, and lost 2: how many had she left?
13. Harry Lee owed me 53 cents for cakes: he paid 6 cents: how many cents did he then owe me?

#### ABSTRACT PROBLEMS IN ADDITION AND SUBTRACTION

21. Four and 4 and 6, less 7, are how many?  
24. How many are 26 and 7 and 9, less 7?  
29. How many are 44 and 4 and 8, less 7?

#### PROMISCUOUS QUESTIONS

1. Frank had 19 cents, and spent 5: how many cents had he left?
7. Frank paid 10 cents for 3 oranges, and 18 cents for 5 lemons: how many cents did he pay for all?
11. Mary had 50 cents: she gave 25 cents for a reader, 10 cents for a slate, and 5 cents for a sponge: how many cents did she pay for all and how many had she left?

## ABSTRACT PROBLEMS IN MULTIPLICATION AND SUBTRACTION

8. How many are 3 times 7, less 4?
20. How many are 4 times 9, less 7?
21. How many are 7 times 4, less 3?
28. How many are 10 times 3, less 8?
29. How many are 8 times 9, less 4?

## ABSTRACT PROBLEMS IN DIVISION

1. Thirty-five are how many times 5? How many times 7?
9. Fifty-four are how many times 6? How many times 9?
15. Seventy-two are how many times 8? How many times 9?
19. One hundred are how many times 10?

## PROMISCUOUS QUESTIONS

1. Charles and Henry have each 10 marbles: Charles gave 6 of his to Henry: how many did each then have?
2. William Jones owed me 20 cents: he gave me 3 peaches, worth 4 cents each, and an orange, worth 5 cents; how much was then due?
4. When 3 lemons sold for 15 cents, John gave 1 lemon, and 5 cents in money, for a book: how much did the book cost?
6. I bought 3 dozen eggs at 6 cents a dozen, and sold them, at 8 cents a dozen: how much did I make?
8. John gave 30 cents in money, and 3 peaches, worth 5 cents, for a sled: how much did the sled cost?

## WRITTEN PROBLEMS IN MULTIPLICATION AND DIVISION

1. If 2 peaches cost 6 cents, how much will 1 peach cost? If 1 peach cost 3 cents, how much will 3 peaches cost?
14. What will 63 marbles cost, if 14 marbles cost 2 cents?

9. How many slates, at 3 dimes each will pay for 2 geographies, at 6 dimes each?

A paragraph at the bottom of the last page of the text emphasizes the complete mastery of the text before taking up the study of the Ray's Arithmetic, Second Book.

Summary. The author says that arithmetic is a valuable study for young children because it is exciting and it develops, exercises, and disciplines the mind for the learning of other subject matter. His plan is to present each new topic by the use of practical questions, follow this by concrete questions, and conclude the topic by use of a review. The author uses small circles to illustrate the process of solution of problems when he introduces a new number combination. He emphasizes a set form of analysis or explanation for any question as follows: The pupil is to (1) read the question, (2) answer the question, and (3) give reasons for the answer. The author particularly emphasizes thorough mastery of each lesson before progressing to the next lesson.

In conclusion it may be said that the author's aim of arithmetic, namely that of mental discipline and faculty psychology, agrees with the educational aim of the period. The mental level of the learner is ignored and the content material becomes too difficult both in problem situation and



method of solution. This results in mere formalized drill. The author's plan and aim seem to harmonize with that of Colburn and Smith.

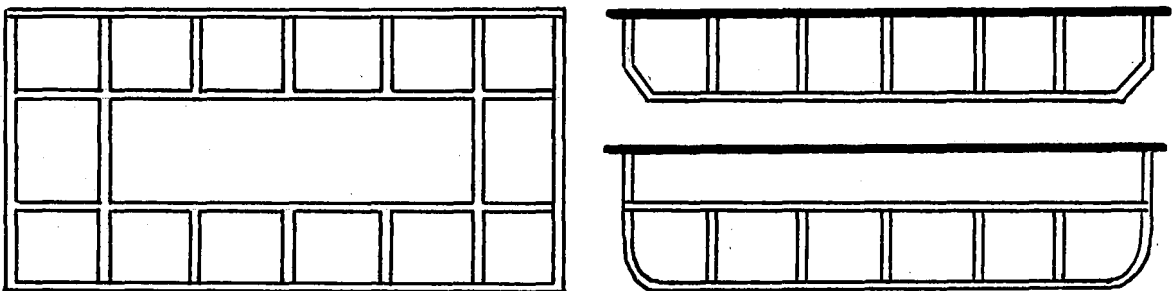
#### IV. FRENCH'S FIRST LESSONS IN NUMBERS

The first page of this little book is very interesting. Within an oval shaped outline, very intricately designed, is the picture of a teacher and three children. Two children are seated at tilted, individual tables or desks. One child is working with numbered blocks, and the other is counting the apples on his desk. The third child is standing in front of the teacher, who holds one apple in her left hand and two in her right. She is evidently attempting to teach him some number concept by using the concrete objects as an aid to learning. Around this oval picture and fitting the curve of the oval are the designed words of the title of the book. The words, "French's First Lessons," are at the top of the oval and "in Numbers" are at the bottom of the oval. The words, "New York," and, "Harper and Brothers," are in horizontal arrangement at the bottom of the page.<sup>4</sup>

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<sup>4</sup> J. H. French, First Lessons in Numbers (New York: Harper and Brothers, 1866 and 1871).

The second page includes a picture of a schoolroom, with counters and counting boards. Children and the teacher are standing around these counting boards. Below this picture diagrams of counting boards are shown. The reproduction of the diagrams for the counting board and the description follow:



Counters and Counting-Boards

Every primary school-room should be supplied with a variety of kinds of objects, and an abundance of each kind, to be used by the pupils in learning arithmetical combinations. For this purpose, walnuts, horse-chestnuts or buckeyes, pebbles, sea-shells, large flat beans, blocks of wood, etc., may be used.

A counting-board adds very much to the convenience of both pupils and teacher. This may be of any convenient length, about one foot in width if placed against the wall, and from two to three feet wide if standing away from it, and about two feet high. To prevent the counters from falling off, the edge should be raised a half inch or more by a thin strip of board, and the top should be divided into sections of about one foot in length, and from eight to twelve inches wide, by strips of lath or board. (See picture and diagram above.) When a counting board can not be had, a table or flat desk may be used in stead. (p. 2)

The third page contains the full title and information as follows:

French's Mathematical Series. First Lessons in Numbers, In the Natural Order: First, Visible Objects; Second, Concrete Numbers; Third, Abstract Numbers. By John H. French. Ll. D. New York: Harper and Brothers, 1871. (p. 3)

This title page gives an idea of the method used by the author in presenting his subject matter.

The fourth page is entitled, Publishers' Notice, and the names of the five series of arithmetics, First Lessons in Numbers, Elementary Arithmetic, Mental Arithmetic, Common School Arithmetic and Acedemic Arithmetic, the latter named text is cited as being preparation at this particular date. (p. 4)

The publishers say the following regarding this series:

The Publishers present this Series of Text-Books to American Teachers, fully believing that they contain many new and valuable features that will especially commend them to the practical wants of the age.

The plan for the Series, and for each book embraced in it, was fully matured before any one of the Series was completed; and as it is based upon true philosophical principles, there is a harmony, a fitness, and a real progressiveness in the books, that is not found in any other series of Arithmetics published. (p. 4)

Information concerning the publishing of this book is as follows:

Entered, according to Act of Congress, in the year 1866, by Harper and Brothers, In the Clerk's Office of the District Court of the United States for the Southern District of New York. (p. 5)

The preface includes two pages, 5 and 6, regarding

the aim of the author and the plan of the book. The exact words of portions of the preface follow:

This little book is intended to give to young children clear ideas of the elementary combinations of numbers, and some practical knowledge of their applications to the business affairs of life. As its general plan is unlike other works designed for the same grade of learners, it is important that teachers and parents should make themselves familiar with its peculiar characteristics, before using it in classes or families.

General Divisions.--Sections.--The book is divided into fifteen sections, the first one of which is devoted to lessons in counting; the next eight to examples and combinations in Addition, Subtraction, Multiplication, and Division; the next three to the fractional parts of numbers--halves, thirds and fourths; the thirteenth to miscellaneous problems, embracing all the classes of combinations in the preceding sections; the fourteenth section to the tables of denominations of money, weights and measures in common use; and the fifteenth to combinations embracing the tables of Addition, Subtraction, Multiplication, Division, Factors, and Aliquot or Fractional Parts. . . .

Tables of Money, Weights, and Measures.--These tables pages 97-99, contains only the tables and denominations in common use. The stereotyped schoolmaster and school-book-maker arrangement has been discarded, and the denominations are here presented as they are used in business. . . .

Natural Order of Mental Development.--All the combinations embraced in the tables, pages 100, 107, have been used in the previous sections of the book at least three times, and are presented, successively, in the natural order: first, Visible Objects; second, Concrete Numbers; third, Abstract Numbers. Every new combination is introduced either in connection with the picture of an object, or with the name of some object familiar to the pupil, and which the teacher, in many cases, may be able to place before him. The second time the combinations are used, they are associated with the names of familiar objects not in sight; and the third time they are made with abstract numbers. Thus the law of the natural order of mental development, viz., first,

Perception (Visible Objects); second, Conception (Concrete Numbers); third, Abstraction (Abstract Numbers), is strictly observed.

Illustrations and Examples.--The cuts are not mere counters, picked up at random; but are pictures which will cultivate the taste of the child, and impart useful knowledge, beside assisting him in his first steps in numbers; and the examples contain much valuable information upon the various occupations, trades, and branches of business, that can not fail to enlist the interest of children in the study of the book.

Manual for Teachers.--All forms of answer and solution, remarks, notes, etc., have been omitted from the body of the work, and are presented in the last thirteen pages, in the form of hints and suggestions, as a Reference Manual for teachers. They are not intended as arbitrary directions and rules; but are to be adopted, adapted, or rejected according to circumstances. (pp. 5-6)

The Manual of Methods and Suggestions, which the author mentioned in his preface on page 6, is found at the close of the book, pages 108 to 120, inclusive. In it the author makes interesting statements regarding a primary teacher's success, and the use of hints, methods and suggestions for the use of the book. Quotations from portions of the manual follow:

Oral Instruction.--Very much of your success as a primary teacher will depend upon the kind of oral instruction you impart, and the manner in which you conduct your classes.

If you make judicious use of the methods, hints, and suggestions contained in this Manual; if you enter into the work of oral instruction with earnestness and zeal, using proper discretion and judgment, you will find that your pupils will soon partake of your spirit, and your efforts will be rewarded with the most satisfactory results.

In attempting to be energetic, you must not forget to be persevering also. Do not pass too rapidly over a subject or lesson. Endeavor to fix every point in the mind of the pupil so thoroughly that he can dwell upon the point until you are sure the pupil understands it. And here you are cautioned not to confound memory with understanding. The learning of the Tables and Combinations is an act of memory; the solving of problems or examples intelligently, necessarily involves understanding, or reason and judgment. It is not enough that pupils repeat your words or the language of the book; you must extemporize questions, as tests of his understanding of the lesson or subject. The suggestions on teaching children to count (page 110) will indicate to you a course that you may pursue with any section or exercise in which the pupil encounters obstacles.

Incidental Instruction.--You will add much interest to this study by familiar conversations with the pupils about the pictures in the lessons, and various objects represented in them. Many of the pictures represent mechanical, manufacturing, and other business operations and industrial pursuits, about which children are interested. Encourage them to visit factories, shops, and other places of business of the kinds represented in the pictures, or suggested by them or by the examples, for the purpose of obtaining information. When either pictures or examples contain objects or terms with which a child is unacquainted, explain to him, consulting a dictionary whenever you do not understand the object to be explained, or the term to be defined.

Use of Books in Class.--The mere memorizing of the language of a problem or example, is no part of the true object for which Mental Arithmetic should be studied. The attempt to memorize and reproduce problems or concrete examples, verbatim, occupies and confines the mind, and thus prevents its free exercise in forming the combinations and discovering the reason for them. Therefore, generally, let the pupil use his book during recitation, unless the lesson is on abstract combinations.

Forms of Answer.--Abundant experience has fully established the fact that young children are not generally capable of understandingly making a rigid application of the principles of logical analysis, in the solution of arithmetical problems.

In most cases children who have had no previous instruction or training in number, will give the result of the problem first, and the because afterward. So generally this is the case, that it may be regarded as the natural order of development of mind in its first steps in concrete numbers. Hence, while several forms of answer or solution are given to one or more concrete examples in each of the different classes of combinations in this book, the first answer given, in any case, conforms to this view.

You should require only brief answers from young children, and you should not insist upon, or exact from them, formulated analyses logically stated. But you should always require pupils to give answers that are correct in language, and to form complete sentences, introducing the numbers contained in the question. For example: Sarah has 3 roses, and Eliza has 5. How many roses have the two girls?

Answers. (1) The two girls have 8 roses; because 3 roses and 5 roses are 8 roses; Or,

(2) The two girls have 3 roses and 5 roses, which are 8 roses.

Any questions involving but a single combination of abstract numbers, admits of only a brief answer. For example:

How many are 7 and 4? Ans. 7 and 4 are 11. (pp. 108, 109)

Section I, page 8, includes Exercises in Counting.

The manual gives the following directions regarding it:

Section I, page 8.--Exercise the class upon the objects represented in the pictures, and then with other objects or counters, until all of them can count to 10. Then name familiar objects not in sight, as men, animals, birds, fruit, flowers, houses, trees, etc., and require the class to count them or tell the number. This may be done by asking familiar questions relating to the objects, and involving definite numbers not exceeding ten. Thus: How many apples must I have to give to each girl in this class? . . . You may vary the model exercises, according to circumstances. (p. 110)

The first two pages of Section I have the pictures of hands, about which questions are asked for the purpose of having the children count. The following questions are typical:

1. If I hold up my right hand as you see in the picture, how many fingers do I hold up?
2. How many thumbs on your right hand?
6. How many fingers on your right hand?
7. Count the thumb with the fingers of your right hand. How many are there in all? (pp. 7 and 8)

Page 9 has a very detailed farm picture including a large house, birds in the tree, wagons, horses, cows, sheep, goats, ducks, dogs, pigs, fence, and people. Some of the questions about this picture are:

13. How many trees do you see in this picture?
14. Count the cows in the road.
15. Near the cows are some sheep. Count them, and tell me how many are there.
16. How many posts of the fence can be seen?

Section II deals with Addition and Subtraction of Numbers. The manual says that before leaving Article A of this section that the pupils should be thoroughly "exercised" upon the addition and subtraction table I, page 100.

On page 12 is a picture of two little girls playing in a room in which there are kittens, blocks, chairs, balls, a table, and books. The following problems are typical of



those regarding the picture:

1. How many girls are in this picture?
2. One girl and one girl are how many?
3. Ella had one letter block in her hand, but she has just put it upon the chair. How many blocks has she now in her hand?
9. In the room are three chairs, and one of them is an arm-chair. How many chairs are without arms? (pp. 11 and 12)

There are other pictures on the remaining pages of Article A of Section 1. The questions involve drill upon the addition and subtraction tables of 1.

Article B of Section 1, pages 14 and 15, contains pictures and problems regarding the addition and subtraction of the tables of three. One of the most interesting pictures is that of four children standing on a bridge watching three boys sail their boats below. Eleven questions are based on this picture. The following questions illustrate the type of problem.

3. On the bridge are two boys and three girls. How many children are on the bridge?
8. Four boats are sailing on the stream, and the boys have two in their hands. Four boats and two boats are how many boats? (pp. 14 and 15)

Article C, Section I, regards the addition and subtraction tables of 3. Various pictures are used as concrete counters for the problems. They include such things as animals, houses, birds, trees, and books.

Article D is a review of the addition and subtraction tables. The concrete objects are not used, but the names of objects (not in sight). The following questions, pages 18 and 19, are typical of the drill:

- D. How many are
1. One pen and five pens?
  10. Two boots and ten boots?

- How many will remain if you take
18. Two hats from two hats?
  26. Three shoes from twelve shoes?
  31. Three flies from eleven flies? (pp. 18 and 19)

Article E of Section I, pages 19 and 22, includes the written problems on addition tables of 1, 2, and 3, all based on a scene of an inn with stage coaches in front of it.

Section III with its Articles A, B, C, D, and E, pages 22 to 32, inclusive, is treated the same as that of Section II, except that the drill is upon the tables of 4, 5, and 6.

Section IV includes the drill on the tables of 7, 8, 9, and 10. This section is treated in a manner similar to the preceding ones, except that in Article B, page 33, the words, figures, and letters in Roman type, Italic script, and script type are given.

Section V consists of miscellaneous exercises in addition and subtraction. The problems contain both the figures and the words, but no pictures as concrete aids

are given. Article B of the section expresses the numbers from ten to fifty, inclusive, in words and figures. Article C gives the following directions regarding the counting, reading, and writing of numbers. Some examples, from pages 38 to 42, inclusive, follow:

1. Count twenty-one and back again. In the same manner count twenty-two; twenty-three; twenty-four; twenty-five; twenty-six; twenty-seven; twenty-eight; twenty-nine; thirty.

2. Read the numbers in the twelve columns below.

(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
21	8	47	36	25	12	1	40	29	5	4	44

Write the following numbers in figures:

(16)	(17)	(18)	(19)
seventeen	thirty-four	thirty-nine	four (pp. 38-42)

Section VI introduces multiplication with 2 or 3 as one factor or term.

The picture of a circus parade is the concrete basis for the first eight problems in multiplication, Article A, pages 42 and 43. The following problem is typical of the ones used in this section.

7. The band wagon is drawn by four pairs of horses. How many horses are 2 horses and 2 horses and 2 horses and 2 horses? How many horses are 4 times 2 horses? (pp. 42 and 43)

Three other pictures are used in Article A as the basis for problems of the multiplication combinations of the tables of 2 and 3.

Articles B and C of this section, pages 46 to 50, inclusive, consist of the division problems based on the tables

of 2 and 3. Pictures serving as concrete counters are included. A new feature is noted in Articles B and C. Following a question about certain things in a picture there is often another question which introduces the drill phase. This is written in smaller type. Problem 8 of Article C, in ordinary type, and problem 9, in the smaller type, illustrate this point:

8. How many soldiers in 3 squads of 7 soldiers each?  
(The picture illustrates this grouping)

9. How many are 3 times 7 flags? 7 times 3 tents?  
(pp. 46-50)

Section VII consists of the multiplication and division problems based on the tables 4 and 5, Section VIII, of those on 6 and 7, and Section XIX, those on 8 and 9, pages 55 to 71, inclusive. The new combinations continue to be furnished with the pictures as concrete counters. The review problems do not have the pictures.

Section X introduces the fraction, one-half. At the beginning of Article A the definition appears and the explanation follows. This explanation is based on the picture of a man dividing pineapples between two men. The definition and explanation, page 72, are as follows:

When any number of things is divided into two equal parts each part is ONE HALF of the number of things.

If I divide 2 pineapples between two persons, each person will receive one half of the two pineapples, which is one pineapple. If I divide 4 pineapples equally

between two persons, each person will receive one half of 4 pineapples, which is 2 pineapples. One half of 6 pineapples is 3 pineapples, and one half of 8 pineapples is 4 pineapples. (p. 72)

Article B is introduced by the following definitions and explanations and picture of apples, page 74:

When anything is divided into 2 equal parts, each of the parts is  $1/2$  of the thing.

Two halves of any thing make the whole of the thing.

If I cut an apple into two halves, each part will be  $1/2$  of the apple. (p. 74)

Problems involving the division of 3 apples, 5 apples and other numbers of objects follow in this article. The following problems, pages 74 and 75, illustrate this point.

9. If 2 barrels of sweet potatoes cost 7 dollars, how many dollars will one barrel cost?

15. If a train of cars runs 19 miles in one hour, how many miles will it run in  $1/2$  hour?

16. How much is  $1/2$  of 3 pounds of cheese? (pp. 74-75)

Articles C, D, E, and F have similar questions except the pictures do not accompany them.

Article G is purely drill. The following problems, page 80, illustrate this fact:

1.  $1/2$  of 2 is how many?
2. How many are  $1/2$  of 5?
26.  $2\frac{1}{2}$  are  $\frac{1}{2}$  of how many?
39.  $7\frac{1}{2}$  are  $\frac{1}{2}$  of how many?
40.  $\frac{1}{2}$  and  $\frac{1}{2}$  are how many? (p. 80)

Section XI deals with the fraction, thirds. Section XII deals with the fraction, fourths. Both these sections,

pages 81 to 90, inclusive, are treated in the same manner as Section X.

Section XIII includes the miscellaneous problems regarding all the preceding topics of the book. No pictures as aids to the solution are given in this review. The problems range in complexity as illustrated by the following, from page 91 to 96, inclusive:

1. A lady paid 9 dollars for a breastpin, and 4 dollars for a ring. How many dollars did she pay out?

2. Louise having 13 cents, paid 5 cents for a spool of thread. How many cents had she left?

3. How many bottles of ink, at 6 cents a bottle, can be bought for 42 cents?

26. A printer worked 6 weeks for 9 dollars a week, and 1 week for 10 dollars. How much did he earn in 7 weeks?

28. How many 10-dollar bills will be required to pay for 5 yards of broadcloth, at 4 dollars a yard?

49. The railroad fare from Boston to Buffalo is 11 dollars, and children between 5 and 12 years old are carried for half-fare. How much will 1 half-fare ticket cost?

81. Lemuel bought  $\frac{2}{3}$  of a dozen buttons for his coat, at 24 cents a dozen. How much did they cost him? How many buttons did he buy?

93. It takes a stage-coach  $\frac{2}{3}$  of an hour to run the distance between two villages, running at the rate of 54 miles in 9 hours. How many miles apart are the two villages?

96. If a blacksmith, by working 10 hours a day, can make 60 horseshoes, how many shoes can he make in  $1\frac{1}{2}$  hours?

97. 4 times 2 are  $\frac{1}{3}$  of what number?

110.  $\frac{3}{4}$  of 32 are how many times 6?

111. 7 is  $\frac{1}{3}$  of how many times 2? (91 to 96, inclusive)

Section XIV is entitled Tables of the Denomination of Money, Weight, and Measures, in Common Use. Following the table of United States Money are the definitions of a coin, and the denomination of the American coins, page 97. They are as follows:

A coin is a piece of metal, stamped by authority of the Government, to give it fixed value.

The American Coins are of Copper, Nickel, Silver and Gold.

Copper and Nickel coins.--Cent, 2-cent piece, 3-cent piece.

Silver Coins.--3-cent piece, 5-cent piece, or half-dime, 10-cent piece or dime, 25-cent piece or quarter-dollar, 50-cent piece or half-dollar.

Gold Coins.--Dollar,  $2\frac{1}{2}$ -dollar piece or quarter-eagle, 3-dollar piece, 5 dollar piece or half-eagle, 10-dollar piece or eagle, 20-dollar piece or double-eagle, 50-dollar piece.

Remark.--United States Money is also called Federal Money. (p. 97)

Section XV, includes drill on addition, subtraction, multiplication and division and aliquot or fractional parts of numbers by way of abstract review.

The Manual of Methods and Suggestions concludes the book. (pp. 108 to 120, inclusive)

Summary. The author clearly states that his book is

based on the theory of the natural order of mental development, namely (1) sense perception, (2) conception, or the association of old and new ideas, (3) the abstraction, or the formulation of new ideas. Therefore each new combination is presented either in connection with counters, pictures, or the names of objects which are visible to the child. The author calls these "visible objects." The second time the combination is presented only the names of familiar objects (not visible) are used. The author calls these "concrete numbers." The third time the combination is presented only numbers and number names are used. The author calls these "abstract numbers."

French's text differs from the Colburn, Smith, and Ray texts as follows: (1) the pictures of his book represent mechanical activities, manufacturing and other business operations in which children are interested; (2) the author suggests that the children should be encouraged to visit factories and shops and various places of business; (3) the author thinks only brief answers and not the analysis logically stated should be required of young children; and (4) the author feels that mere memorization of problems should be discouraged and not be confused with the learning of the tables and combinations, which is an act of memory.

French includes fractions and the tables of United



States Money, while Ray includes only the addition, subtraction, multiplication, and division of whole numbers. Although French furnishes numerous pictures as "visible aids" for problem solution, the increase in the complexity of problem situation and solution renders the material too difficult for children of the first and second grade level.

In conclusion it may be said that French emphasizes the natural order of teaching arithmetic. He carried out this plan reasonably well by providing visible aids in the text and by suggesting and explaining the use of counters. He emphasizes learning through sense perception. He does not stress, as did Colburn and Smith, learning by mere mental drill.

#### V. WHITE'S GRADED SCHOOL SERIES, PRIMARY ARITHMETIC

This book is four and one-fourth inches wide by six and three-eighth inches long. Its cover is of grayish green cardboard. At the top of the front cover are the words, "White's Graded Series, Primary Arithmetic."<sup>5</sup> Below this and extending to within approximately an inch of the lower edge of the book is the picture of a schoolmistress

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<sup>5</sup> E. E. White, A Primary Arithmetic Uniting Oral and Written Exercises in a Natural System of Instruction (Cincinnati: Van Antwerp, Bragg and Company, 1868), "Front cover."

standing at the table where one boy is seated and two other children, a boy and girl, are standing. The little girl is holding a section of blackboard which leans at an angle against the wall. The teacher is looking at the little boy who is seated, and is pointing to columns of figures on the blackboard. All three children are looking at the blackboard.

Below this picture is the company name of the publishers, "Van Antwerp, Bragg and Co., Cincinnati and New York." (Front cover)

The back cover advertises the two book series of Harvey's Language Course by the same publishers as of this particular arithmetic textbook. The recommendation covers the entire back of the book. (Back cover)

The title page is very interesting. On it appears the full title of the book, A Primary Arithmetic Uniting Oral and Written Exercises in a Natural System of Instruction by E. E. White, M. A. The title of the book gives an idea regarding the content of the book and the method of instruction. This book bears the name "Van Antwerp, Bragg and Co., Cincinnati and New York." (Title page)

The next page gives information regarding the series of arithmetics. The titles of these are: Primary Arithmetic, Intermediate Arithmetic, and Complete Arithmetic.

The plan of Primary Arithmetic follows the title of the series. This seems to serve as a table of contents.

The "Plan" follows:

Lessons I to XI develop the idea of numbers from one to ten and combine groups of objects.

Lessons XI to XX teach the Addition and Subtraction of numbers--results not exceeding ten.

Lessons XX to XXIX teach the Addition and Subtraction of numbers--results not exceeding fifty.

Lessons XXX to XLII teach the Addition of numbers to amounts not exceeding one hundred.

Lessons XLIII to LII teach the Subtraction of numbers--minuend in Oral Exercises not exceeding one hundred.

Lessons LII to LXXIII teach the Multiplication of numbers--product in Written Exercises not exceeding one thousand.

Lessons LXXIII to XC teach the Division of numbers--dividend in Written Exercises not exceeding one thousand.

Lessons XC to XCII, inclusive, contain a General Review.

The copyright is below and is as follows:

Entered according to the Act of Congress, in the year 1868, by Wilson Hinkle and Company. In the Clerk's Office of the District Court of the United States, for the Southern District of Ohio.

The author emphasizes the "characteristics" of his book as: (1) the "natural order" or "three step" method of teaching the concept of number value; (2) the inductive method of instruction; (3) the inclusion of oral and written problems as the characteristic feature; (4) the great

variety of exercises to make the pupil accurate and rapid in calculation involving small numbers; (5) the progressive character of the exercises; (6) the converse operations of the four fundamental processes; (7) the converse forms of the tables and their "non memoriter character;" and (8) the superior typography and number, utility, and beauty of the illustrations.

The author appears to have attempted to conform to the educational theory of training the mind to be accurate and rapid and to the inductive method of instruction. In order that the reader may have a clearer picture of the author's method of presentation and of the value he attaches to his arithmetic in carrying out the aim, the entire Preface, p. 3, follows:

The true method of imparting to a child a clear comprehension of the value of numbers, the foundation of arithmetical knowledge, consists of three steps, viz.: 1. The perception of numbers represented by objects in sight. 2. The conception of numbers applied to objects not in sight. 3. The conception of numbers not applied to objects. A knowledge of the elementary combinations of numbers is best communicated in the same manner.

A faithful observance of this natural order constitutes one of the characteristic features of this first book in arithmetic. Abstract numbers and operations are reached, in practice as well as in theory, as the final step. The plan every-where observed is, first, Physical Objects (in sight or represented by pictures); secondly, Concrete Numbers; and thirdly, Abstract Numbers. In this and other evident features, the book is a practical embodiment of the simplest and most vital principles of the inductive method of instruction.

But the distinguishing feature of the book, as well as of the Series of which it forms a part, is the complete UNION of Mental (Oral) and Written Arithmetic. This is secured, not by scattering a few miscellaneous slate exercises through the work, but by making every oral exercise preparatory to a written one, and by uniting both as the essential complements of each other. Slate and blackboard exercises are introduced at the very beginning of the course, and are continued, increasing in number and difficulty, to the end. Each lesson gives the pupil something to do, as well as something to study.

Two other noticeable features of the book are the great variety of exercises--the object being to make the pupil accurate and rapid in combining small numbers--and their preeminently progressive character. Attention may also be called to the presentation of converse operations, as Addition and Subtraction, Multiplication and Division, in connection with each other, as well as separately; to the converse forms of the tables and their non memoritor character; and, also, to the superior typography, and the number, beauty, and utility of the illustrations.

It is hoped that these and other features will commend the work to all intelligent and progressive teachers. (Preface, p. 2)

The page following the Preface is entitled Suggestions to Teachers. Regarding the amount of material to be covered, the rate of progress, and the first step in teaching a new combination the author says:

The first thirty lessons of this book may be mastered in the earlier part of the primary course. To this end, the pupils should be advanced very slowly, and the exercises should be multiplied until great rapidity and accuracy are secured.

The first step in every new combination is to combine groups of objects, and, both in the pupil's study and the teacher's instruction, the pictorial illustrations should be supplemented by the use of visible objects, as counters, blocks, beans, etc. The teacher should also refer

to other familiar objects in sight, as chairs, desks, slates, etc. Attention should be called to the difference between the picture in the lessons, and the objects which they represent. (p. 4)

The author opposed having the pupils give reasons for the answers, but did approve of complete statements as may be seen in the following quotation:

Pupils should be required to give answers in complete sentences. Suppose the question to be, "How many tops are 5 tops and 4 tops?" The answer should be, "5 tops and 4 tops are 9 tops." To secure rapidity, drills may occasionally be introduced in which only results are given, as "9 tops." Nothing is gained by requiring pupils to give reasons for answers to simple examples, and even problems which admit of a formal analysis should be solved briefly and concisely. See page 51. (p. 4)

The reference the author cites is as follows:

To Teachers:--The pupil should not be required to give a formal logical analysis of these problems. All that is necessary is a correct statement of the answer and the operation. Take, for example, the second problem below. The solution may be given thus:

Mary found 12 eggs: 4 eggs and 8 eggs are 12 eggs:  
Mary found 12 eggs. (p. 51)

The second problem is:

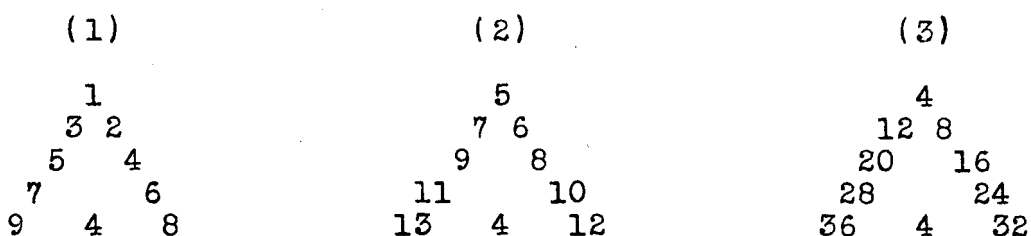
Mary found 4 eggs in one nest, and 8 eggs in another:  
how many eggs did she find? (p. 51)

The author continues to discuss the oral, written, and blackboard exercises in the following paragraphs of Suggestions to Teachers.

The written exercises are designed to go hand in hand with the oral, and should be taught with equal thoroughness. They are so easy and progressive that but little explanation will be found necessary. They should not only be copied and performed by the pupils on their slates,

but they should also be used as blackboard exercises. Such exercises are exceedingly valuable both as a means of awakening interest and of imparting skill in numerical calculations.

Blackboard exercises, affording a great variety of combinations, and requiring but little labor in copying, may be easily arranged. The following are given as illustrations:



By pointing successively to the figures in the oblique rows and to the figure between them, the first diagram will afford an excellent drill in adding or in multiplying, the "4", or any other figure in its place, being the number added or multiplied, as the case may be. The second diagram will afford a good exercise in subtraction, and the third in division. (p. 4)

Lessons three to ten, inclusive, deal with the concept of number value, from the single unit to the grouping of units from one to ten, inclusive. The directions given or the questions asked refer to (1) illustrations on the particular page of the lesson; (2) concrete "visible" articles, as counters, blocks, beans, or other objects in sight; or (3) directions which require concrete response by the pupil such as, holding up some object or objects or making the figures on the slate. These points are illustrated by the approximate reproductions of pages 5 and 7, which follow.

# PRIMARY ARITHMETIC.

## LESSON \*

Touch your head. How many hands have you? How many chins? Hold up one hand. Hold up two hands.

How many fingers do I hold up? Hold up two fingers. Hold up one thumb. Hold up two thumbs.



How many eyes have you? How many cheeks? How many tongues? How many lips? How many feet?



How many nuts do you see in this picture? How many leaves? How many stems has each nut?

Bring me one block. Bring me two blocks. Take away one block. How many blocks are left?

Hand me two books. Take one of them. How many books have I left? Make two marks on your slate.

Make the figure one on your slate, thus: 1

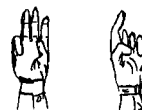
Make the figure two on your slate, thus: 2

Make the figure 2 two times.



### LESSON III.

How many fingers do I hold up on my right hand? How many on my left hand? How many on both hands? Hold up four fingers.



Here is a picture of a fine ox. How many horns has he? How many ears? How many legs? How many feet?









Bring me four blocks. Take one of them. How many blocks have I now? Make three marks on your slate. Make four marks.



How many groups of pears do you see? How many pears in the first group? How many in the second group? How many in the third group? How many pears in the second group more than in the first?



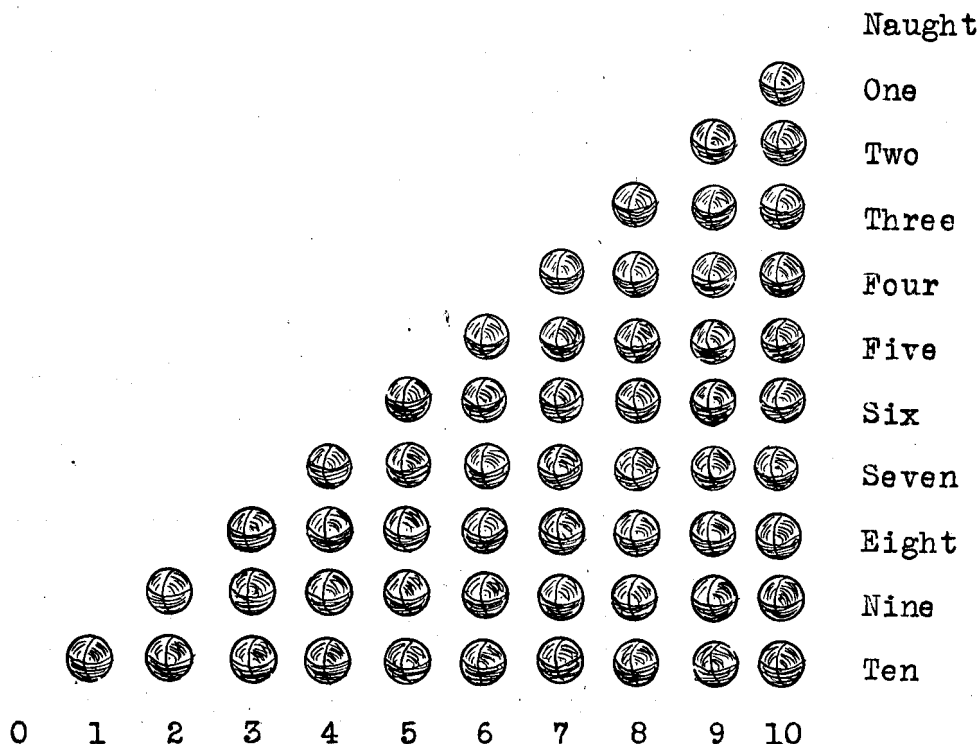
How many flowers are in each of these groups? How many more flowers in the second group than in the first?

How many blocks are    and  ?  
  and   are how many blocks?

How many balls are  and    ?

## LESSON X.

Count the balls in each row from left to right and from right to left. Count also the rows up and down.



How many balls are in the lowest row? How many in the third row from the top? How many in the fifth row? How many in the seventh? How many horizontal rows are there? How many vertical rows?

To Teachers.--Make this table on the black-board, using circles, squares, triangles or other simple figures, and drill the pupils in rapid counting, thus (pointing to the figures): One; one, two; two, one; one, two, three; three, two, one, etc.--each row being counted in two directions. Count first the horizontal rows and then the vertical. The drill should be continued until great rapidity and accuracy are secured.

Suggestions to teachers by the author appear occasionally at the beginning or at the close of a lesson in the series of lessons from one to ten, inclusive. Lesson two has the following suggestion at the beginning: "To Teachers.--Drill the pupils until they can tell the number of objects in each group, and combine the groups, without counting." (p. 6)

The preceding quotation clearly shows that the author emphasizes quick recognition of groups of things instead of the slow process of mere rote counting.

At the close of lesson three are the following suggestions relative to further drill on the same type of exercises as given in that lesson: "To Teachers.--Multiply these exercises until the pupils can add the groups instantly, without counting." The author's meaning of multiply is that of repeating or increasing the number of problems, drills, or exercises. (p. 8)

Again this shows the emphasis upon quick recognition of objects, not dependence upon rote counting.

Lessons eleven to nineteen, inclusive, deal with the addition and subtraction of numbers, the result of which does not exceed ten. These lessons include: (1) both oral and written exercises; (2) both the addition and subtraction tables; and (3) review lessons. Preceding lesson three are

the author's suggestions regarding the purposes of the lessons and the method of instruction. The suggestions follow:

The object of this and the next eight lessons is to teach the addition and subtraction of numbers not exceeding ten. The first step is to add and subtract groups of visible objects, or objects represented by pictures; the second, to add and subtract groups of objects not in sight (concrete numbers); and third, to add and subtract the corresponding abstract numbers. The exercises in each step should be multiplied until the results are given by the pupils instantly, without counting. The tables may be recited thus: 0 and 1 are 1; 1 and 1 are 2; 2 and 1 are 3, etc., and 1 from 1 leaves 0; 1 from 2 leaves 1, etc. (p. 16)

The addition and subtraction tables are introduced in lesson eleven following the pictures and the problems about them. The tables are succeeded by drill questions. The tables are arranged in parallel vertical rows, the addition being on the left and the subtraction on the right. The following is a reproduction of the tables 1 on page 17 of the textbook:

<i>How many are</i>		<i>Take</i>
0 and 1		1 from 1
1 and 1		1 from 2
2 and 1		1 from 3
3 and 1		1 from 4
4 and 1		1 from 5
5 and 1		1 from 6
6 and 1		1 from 7
7 and 1		1 from 8
8 and 1		1 from 9
9 and 1		1 from 10

The questions for drill on the preceding tables are:

How many are 2 and 1? 4 and 1? 6 and 1? 8 and 1?  
9 and 1? 3 and 1? 5 and 1? 7 and 1?

One from 3 leaves how many? 1 from 5? 1 from 6?  
1 from 7? 1 from 9? 1 from 10? 1 from 8? 1 from 4?

The drill in subtraction is the converse of addition and in the same consecutive order of the addition, a feature cited by the author in his preface, p. 4.

Written exercises consisting of addition and subtraction are introduced on page 20. Following these are the suggestions to the teachers. The written exercises and suggestions are as follows:

#### WRITTEN EXERCISES

	(1)	(2)	(3)	(4)	(5)	(6)		(1)	(2)	(3)	(4)	(5)	(6)
	2	2	2	2	2	2		1	2	2	1	1	2
Add	<u>2</u>	<u>4</u>	<u>6</u>	<u>8</u>	<u>7</u>	<u>5</u>		1	2	2	2	2	2
	(1)	(2)	(3)	(4)	(5)	(6)		1	2	2	1	2	1
							Add	<u>1</u>	<u>2</u>	<u>1</u>	<u>4</u>	<u>3</u>	<u>2</u>
From	4	6	8	10	9	7							
Take	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>							

To the teacher.--These exercises should be copied by the pupils on their slates, added, and the results properly written below. They, or similar exercises, should also be written on the blackboard, and the class drilled on them until the results are given instantly. In adding the columns at the right, the results only should be named; as (Ex. 3) 1, 3, 5, 7, 9. (p. 20)

The author uses fewer pictures as the lessons develop. These serve as examples on which he bases other problems and questions the answers of which do not exceed the original sum of illustrations; for example, on page 31, lesson 17, in the

textbook are groups of crowns, the first group containing one and the second group eight. These pictures serve as concrete counters for the succeeding questions which are as follows:

How many crowns are one crown and 8 crowns? 2 crowns and 8 crowns?

How many girls are one girl and 8 girls?

How many are 1 and 8? 2 and 8?

There are ten boys playing together: if eight of them go home, how many boys will be left? Eight boys from 10 leaves how many? 8 from 9?

How many chairs are 1 chair and 7 chairs? 1 chair and 8 chairs? 2 chairs and 7 chairs? 2 chairs and 8 chairs?

How many crowns will remain if you take 7 crowns from 10 crowns? 8 crowns from 10 crowns? 7 crowns from 9 crowns? 8 crowns from 8 crowns? (p. 31)

Lessons eighteen and nineteen, which are review questions of addition and subtraction, contain no pictures. The author carries out the second and third steps of his plan as mentioned in the preface,--the conception of numbers applied to objects not in sight and the conception of numbers not applied to objects. The questions of these lessons adhere to the author's statement regarding his method of the natural order of his book. He says, "Abstract numbers and operations are reached in practice as well as in theory, as the final step." (Preface, p. 3) The following questions illustrate the author's point:

There are 9 men in a stage coach: if 4 get out of the coach, how many men will be left?

How many are 2 and 2? 2 and 2 and 2? 2 and 2 and 2 and 2? 2 and 2 and 2 and 2 and 2 and 2 and 2? 2 and 1? 2 and 3 and 5? (p. 33)

Lessons eighteen to twenty-eight, inclusive, treat the addition and subtraction processes, the results of which do not exceed fifty.

Teacher aids precede the questions in lesson twenty, page 35, and relate to lesson twenty and twenty-one. These suggestions follow:

To Teachers.--The object of this and the next lesson is to develop the idea of each number from eleven to twenty, inclusive to teach its name, and the mode of representing it by figures. Make ten marks on the blackboard, and make beneath these, successively, one mark, two marks, three marks, etc. Add the lower group to the upper, and give the appropriate name to the result, as to four and ten, fourteen. Blocks, beans, etc., may also be used in a similar manner.

Before answering the three questions below the illustrations, the pupils should, in each case, add and subtract the groups of objects represented thus: "Ten trees and one tree are eleven trees; one tree from eleven trees leave ten trees." (p. 35)

The following reproduction of the illustrations and questions on page 35, is typical of the method of teaching the numbers from ten to twenty. (See the next page.)



1. How many boys are 10 boys and 1 boy?  
 How many are 10 and 1? Write eleven, thus:  
 One from 11 leaves how many?



2. How many stars are ten stars and 2 stars?  
 How many are 10 and 2? Write twelve, thus:  
 Two from 12 leaves how many?



3. How many balls are 10 balls and 3 balls?  
 How many are 10 and 3? Write thirteen, thus:  
 Three from thirteen leaves how many?



4. How many are 10 birds and 4 birds? 10 boys and 4 boys?

A reproduction of page 38 shows the first twenty numbers expressed by words, figures, and letters. It is as follows:

Five	Five	5	V	5
Sixteen	Sixteen	16	XVI	16
Seventeen	Seventeen	17	XVII	17
Eighteen	Eighteen	18	XVIII	18
Nineteen	Nineteen	19	XIX	19
Twenty	Twenty	20	XX	20



<i>By Words</i>		<i>By Figures</i>		<i>By Letters</i>	
ROMAN	SCRIPT	ROMAN	SCRIPT	ROMAN	ITALIC
One	<i>One.</i>	1	1	I	<i>I</i>
Two	<i>Two.</i>	2	2	II	<i>II</i>
Three	<i>Three.</i>	3	3	III	<i>III</i>
Four	<i>Four.</i>	4	4	IV	<i>IV</i>
Five	<i>Five.</i>	5	5	V	<i>V</i>
Six	<i>Six.</i>	6	6	VI	<i>VI</i>
Seven	<i>Seven.</i>	7	7	VII	<i>VII</i>
Eight	<i>Eight.</i>	8	8	VIII	<i>VIII</i>
Nine	<i>Nine.</i>	9	9	IX	<i>IX</i>
Ten	<i>Ten.</i>	10	10	X	<i>X</i>
Eleven	<i>Eleven.</i>	11	11	XI	<i>XI</i>
Twelve	<i>Twelve.</i>	12	12	XII	<i>XII</i>
Thirteen	<i>Thirteen.</i>	13	13	XIII	<i>XIII</i>
Fourteen	<i>Fourteen.</i>	14	14	XIV	<i>XIV</i>
Fifteen	<i>Fifteen.</i>	15	15	XV	<i>XV</i>
Sixteen	<i>Sixteen.</i>	16	16	XVI	<i>XVI</i>
Seventeen	<i>Seventeen.</i>	17	17	XVII	<i>XVII</i>
Eighteen	<i>Eighteen.</i>	18	18	XVIII	<i>XVIII</i>
Nineteen	<i>Nineteen.</i>	19	19	XIX	<i>XIX</i>
Twenty	<i>Twenty.</i>	20	20	XX	<i>XX</i>

Lesson twenty-two introduces a very detailed picture of a farm. Many of the objects are small and indistinct. Oral exercises relate to the picture, but many of the answers are number values more or less than the number of objects in the picture, e. g., There are seven cows in the picture. The first question which asked for the number of cows grazing, was followed by the one asking: "How many cows are 5 cows and 2 cows? 7 cows and 2 cows? 9 cows and 2 cows?" (p. 39)

The directions to copy numbers and written exercises occur more frequently as the number of lessons increases. This seems to carry out the idea that the pupil should have something "to do as well as something to study."--the idea expressed by the author in his preface. (Preface, p. 3)

Lesson twenty-nine, page 54, includes a diagram showing the consecutive progression of numbers to 100, by the arrangement of a square using ten objects wide and ten long. Suggestions for the use of this diagram precede it. The reproduction of the suggestions and diagram, page 54, of the text follow: (See the next page.)



Lessons thirty to forty-one, inclusive, deal with the addition of numbers in which the results do not exceed 100.

The suggestions preceding the exercises of lesson thirty state the object of the eleven succeeding lessons and the method of finding the sum of units. The suggestions are:

To Teachers.--The object of this and the next eleven lessons is to teach the addition of any number less than 10 to any number less than 100. The exercises are so arranged as to lead the pupil to add first the units. In finding the sum of 19 and 2, 29 and 2, or 39 and 2, for example, the 9 and 2 are first added. This gives the unit figure, and by adding 1 to the left hand figure the sum is obtained. (p. 55)

Definitions are introduced for the first time. Addition, sum, and amount are defined at the end of lesson forty-one. This seems to prove the author's idea of teaching by the inductive method, which he mentioned in the preface, page 3.

Lesson forty-two includes approximately two pages of abstract numbers arranged in columns for copying, reading, and adding.

Lessons forty-three to fifty-one, inclusive, deal with subtraction processes, the results of which do not exceed 100. The increased number of questions and written problems dealing with abstract numbers is very evident. The definitions of subtraction, difference, and remainder appear at the conclusion of lesson fifty, after much practice has been given on the processes of subtraction. This shows that the author

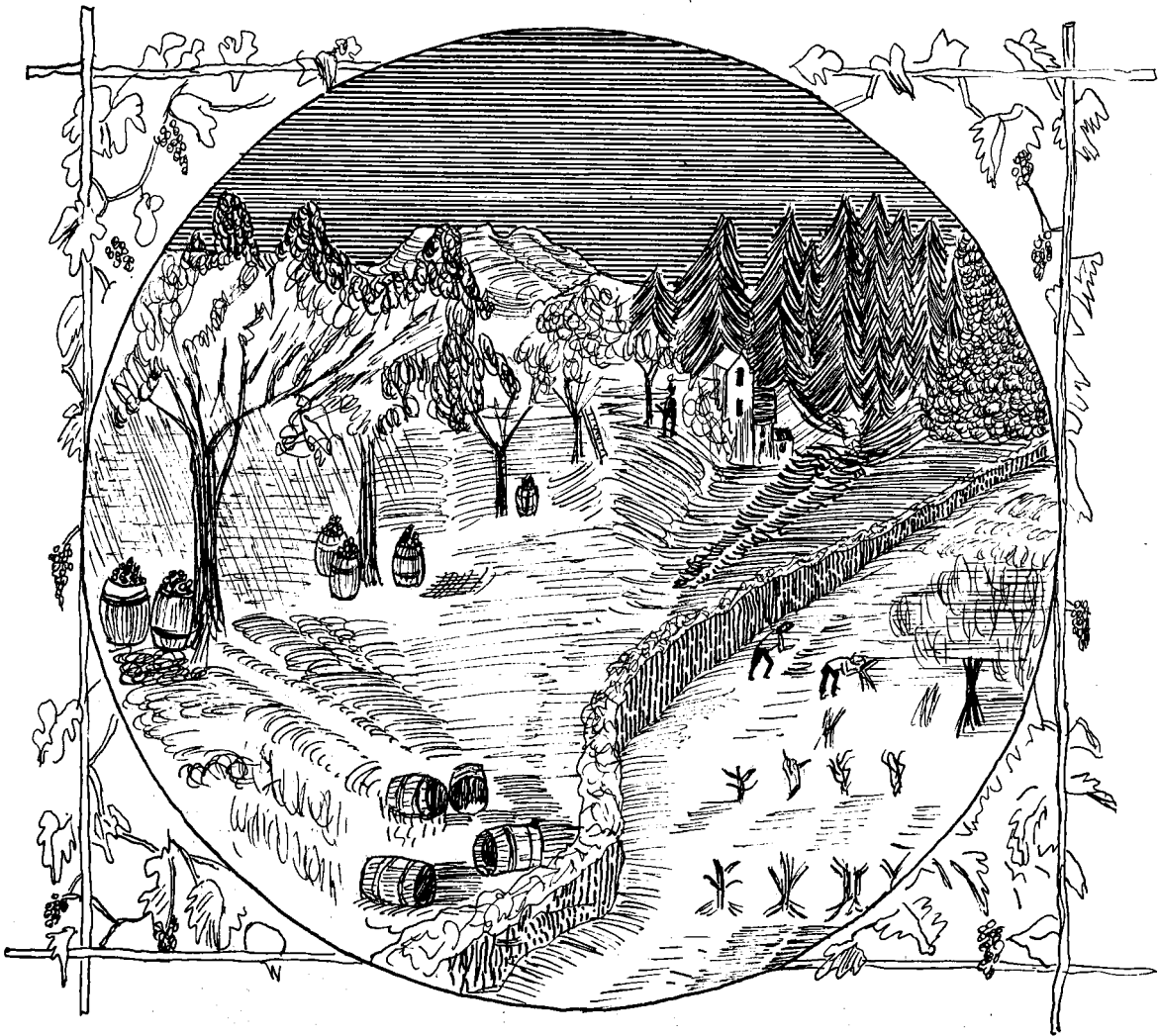
still holds to the principles of the inductive method of teaching.

Lesson fifty-one bears the title Promiscuous Oral Exercises which is the oral review of addition and subtraction. As stated in the above paragraph, there is an increased amount of abstract questions and exercises introduced. But the author still seems to remember that he stated that the first step in every new combination should be made by use of the "concrete or visible objects," (Preface, p. 4) and has used the picture of an urn on the page. However, it seems to be of no value in answering the only question of the lesson, which is: "If you add 9 urns and 6 urns, what will the sum be?" (p. 84) Other questions in lesson fifty-one introduce the words sum, amount, difference, and remainder. (p. 85)

Lessons fifty-two to fifty-eight, inclusive, teach the multiplication of numbers, whose products in the written problems do not exceed 1000.

A small, detailed farm picture of an orchard and field introduces development of multiplication. The oral exercises of the first lesson relate to the picture of the orchard and develop the multiplication table of 1. The oral exercises of the second lesson relate to the field and develop the multiplication tables of 2. A reproduction of the picture, the

oral exercise, suggestions, and tables of the two lessons on pages 86, 87, and 88 of the text follow:





—•••••—

## LESSON LIII.

### ORAL EXERCISES

1. How many boys are in the field? How many are twice 2 boys?

2. How many barrels are filled with apples? How many are 3 times 2 barrels?

How many are twice 2? 3 times 2?

3. How many are 4 times 2 barrels? 5 times 2 barrels? 6 times 2 barrels?

4. How many are 4 times 2? 5 times 2? 6 times 2?

5. How many are 7 times 2 apples? 8 times 2 apples?

6. How many are 7 times 2? 8 times 2?

7. How many are 9 times 2 trees? 10 times 2 trees?

8. How many are 9 times 2 men? 9 times 2 deer? 10 times 2 men? 10 times 2 deer?

How many are 9 times 2? 10 times 2?

9. How many are 3 times 2 tops? 4 times 2 tops? 6 times 2 tops? 8 times 2 tops? 10 times 2 tops? 9 times 2 tops?

To Teachers.--Show the pupil, that 3 times 2 is the sum of three 2's; that 4 times 2 is the sum of four 2's, etc.; and when the table is studied, let the pupil add the corresponding number of 2's at the right, if he does not know the product. The subsequent tables may be studied in the same manner.



—•••••—

*How many are*

1 times 2?	.	.	.	.	.	.	.	.	.	.	2
2 times 2?	.	.	.	.	.	.	.	.	.	.	2
3 times 2?	.	.	.	.	.	.	.	.	2	2	2
4 times 2?	.	.	.	.	.	.	.	2	2	2	2
5 times 2?	.	.	.	.	.	2	2	2	2	2	2
6 times 2?	.	.	.	.	2	2	2	2	2	2	2
7 times 2?	.	.	.	2	2	2	2	2	2	2	2
8 times 2?	.	.	2	2	2	2	2	2	2	2	2
9 times 2?	.	2	2	2	2	2	2	2	2	2	2
10 times 2?	.	2	2	2	2	2	2	2	2	2	2

10. How many times are 2 times 2? 4 times 2? 3 times 2? 5 times 2? 7 times 2? 6 times 2? 9 times 2? 10 times 2? 8 times 2?

11. How many are 5 times 1? 5 times 2? 7 times 2? 9 times 1? 9 times 2? 6 times 1? 6 times 2?

12. How many are 4 times 1? 4 times 2? 8 times 1? 8 times 2? 7 times 1? 7 times 2?

13. How many times 2 are 4? 6? 8? 10? 12? 14? 16? 18? 20?

The solution of a problem is mentioned for the first time in lesson fifty-four. The oral problem and solution are "A cart has two wheels: how many wheels have 4 carts? Solution.--4 times 2 wheels are 8 wheels: 4 carts have eight wheels." (p. 89) A picture of a man driving a cart is on this page near the picture. It seems that the author's idea is that this will assist the child in a concrete way in answering the question.

Following the development of the multiplication tables of 1 and 2, are written exercises which are tables to copy. The use of the multiplication sign and reverse order of the

multiplication process is to be explained by the teacher. The exercises are arranged in six columns, the first column of which serves as a model for the copying of the other five columns. (p. 90) On subsequent pages addition and subtraction tables appear in conjunction with the multiplication table exercises to be copied and completed by the pupils. The author suggests to the teachers the method of presenting the name and use of the addition, subtraction, and multiplication signs. (pp. 93, 97, 100, 104, 107, and 114)

The development of the table of 5 by the use of concrete illustrations is shown by the following illustrations and questions from page 98 in the text.

6. Edward has 5 marbles, and Albert 7 times as many as Edward: how many marbles has Albert?



7. Seven times 5 marbles are how many marbles? 8 times 5 marbles? How many are 7 times 5? 8 times 5? 5 times 8?



8. Mary kept the account in a spelling match between two classes. The first class misspelled 9 times 5 words; the second 10 times 5 words; how many words did each class misspell?

How many are 9 times 5? 10 times 5? 5 times 10?

The definition of multiplication and product and the explanation of the term, product, appears in the concluding oral review lesson.

The topic of Division deals with numbers not exceeding 1000. A very interesting picture of a winter scene appears at the beginning of the first chapter of division. Oral exercises relating to the picture are as follows:

1. Here is a gay winter scene. How many sleighs are in sight? How many times 1 sleigh are 2 sleighs?

2. How many boys are putting on their skates? How many times 1 boy are 4 boys?

3. How many boys are coasting? How many times 1 boy are seven boys?

4. How many times is 1 sled contained in 4 sleds? 1 sled in 6 sleds? 1 sled in 7 sleds?

5. How many times is 1 skate contained in 8 skates?

6. How many times is 1 contained in 2? 1 in 4? 1 in 6? 1 in 8? 1 in 7? 1 in 9? 1 in 10? (pp. 120-121)

The picture is of such small size that the skates can scarcely be seen in the picture, but the spirit of the author's theory of presenting concrete materials as the first step, is in evidence.

The division tables are at the end of the lessons in which that particular table has been developed by use of the illustrations and oral problems?

The division table of 1 on page 121 will serve as an example for the remaining division tables. It is as follows: (See the next page.)

the number of rows to be divided. 132, 144, 156, 168, and 180.

*How many times*

1 are 1?  
 1 are 2?  
 1 are 3?  
 1 are 4?  
 1 are 5?  
 1 are 6?  
 1 are 7?  
 1 are 8?  
 1 are 9?  
 1 are 10?

*How many times*

1 in 1?  
 1 in 2?  
 1 in 3?  
 1 in 4?  
 1 in 5?  
 1 in 6?  
 1 in 7?  
 1 in 8?  
 1 in 9?  
 1 in 10?

The division table of 1 is succeeded by the suggestions to teachers regarding the tables and other phases of division. The suggestions follow:

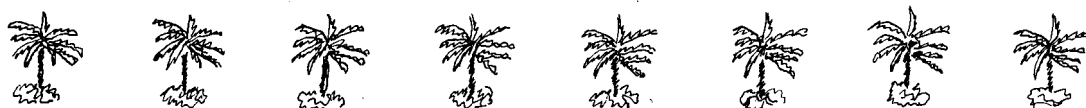
To Teachers.--These two tables should be recited together, thus: Once 1 is 1: 1 in 1 once. Two times 1 are 2: 1 in 2 two times. Three times 1 are 3: 1 in 3 three times, etc. The subsequent tables should be recited in the same manner.

Division here is treated as the converse of multiplication, but it may also be derived from subtraction. 4 is contained in 12 as many times as four can be taken from 12.

At this point pupils may be taught the division of a material unit into halves, thirds, fourths, etc. They may also be taught to add and subtract halves, thirds, fourths, etc., and to find the fractional part of small numbers. (p. 121)

Written exercises using the two forms of division problems are introduced after the division table of 2 and continue throughout the topic with a gradual increase in the size of numbers to be divided. (pp. 123, 125, 128, 130, 133, 135, 137, 139, and 140)

The last picture of the book appears on page 138, six pages previous to the conclusion of the book. This shows that the author has attempted to carry out his theory of presenting concrete examples as the first step in every new combination. The reproduction of this illustration and the question follows.



"How many palm trees will bear 72 leaves, if each tree bear 9 leaves?" (p. 138)

The definitions of division and quotient appear at the end of the oral review in the concluding lesson of the topic. (p. 140) This method is consistent with that regarding the definitions in the preceding topics of addition, subtraction, and multiplication.

The topic immediately following division is the General Review, which concludes the book. The review is given in three chapters and includes both the oral and written exercises. No pictures to be used as concrete illustrations are given in these lessons. (pp. 141-144) This seems to again emphasize the author's theory that the abstract numbers and operations are reached in practice in the third or final step in "natural order" of teaching. (Preface, p. 3)

Examples of progression from easy problems to difficult are as follows:

(p. 139)

Touch your head. How many hands have you? How many chins? Hold up one hand. Hold up two hands. (p. 5)

Here are two fine rabbits. (Picture accompanies the problem). How many ears have both of them? How many legs has each rabbit? How many have both? (p. 11)

One bird and one bird are how many birds? (Picture accompanies questions.)

Two birds and one bird are how many birds? How many are 1 and 1? 2 and 1? (p. 16)

Hand me two books. Take one of them. How many books have I left? Make two marks on your slate. (p. 5)

One from 3 leaves how many? 1 from 5? 1 from 6? 1 from 7? 1 from 9? 1 from 10? 1 from 8? 1 from 4? (p. 19)

	(1)	(2)	(3)	(4)	(5)	(6)		(1)	(2)	(3)	(4)	(5)	(6)
Add	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>		<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>1</u>	<u>2</u>
	<u>2</u>	<u>4</u>	<u>6</u>	<u>8</u>	<u>7</u>	<u>5</u>		<u>1</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>
								<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>
	(1)	(2)	(3)	(4)	(5)	(6)	Add	<u>1</u>	<u>2</u>	<u>1</u>	<u>4</u>	<u>3</u>	<u>2</u>
From	4	6	8	10	9	7							
Take	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	(p. 20)						

"How many boys are twice 1 boy? 3 times 1 boy? (p. 86)

The following are a few problems from one of the written exercises of the multiplication and division topics.

(p. 89)

Multiply	2	2	2	2	2	2	2	2
By	<u>3</u>	<u>5</u>	<u>4</u>	<u>6</u>	<u>9</u>	<u>8</u>	<u>7</u>	<u>2</u>
	<u>1</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>1</u>
	<u>6</u>	<u>9</u>	<u>7</u>	<u>8</u>	<u>4</u>	<u>6</u>	<u>7</u>	<u>5</u>

"How many times 1 sleigh are 2 sleighs? How many times 1 boy are 4 boys? How many times is 1 contained in 2?"

(p. 120)

Some problems in division follow:

2)8      2)6      2)10      2)14      2)16      2)12      2)18  
 2)20      2)22      2)24      2)28      2)42      2)82      2)62

Copy and complete:

2 ÷ 2	2 ÷ 2
4 ÷ 2	6 ÷ 2
6 ÷ 2	10 ÷ 2
8 ÷ 2	14 ÷ 2
10 ÷ 2	18 ÷ 2
12 ÷ 2	8 ÷ 2
14 ÷ 2	4 ÷ 2
16 ÷ 2	12 ÷ 2
18 ÷ 2	16 ÷ 2
20 ÷ 2	20 ÷ 2

A few problems from the last three lessons of the General Review, pages 141 to 144, inclusive, represent some of the most difficult of the written exercise. The problems follow: (p. 141)

2)676      3)588      4)872      5)725  
 6)474      7)476      8)512      9)648

Add 327, 303, 482, 206, 409, and 292.

From 736 subtract 345.

Multiply 165 by 3, 4, and 6. (p. 142)

Divide 492 by 3, 4, and 6.

What is the sum of 486 and 237?

What is the difference between 486 and 237?

What is the product of 348 multiplied by 6?

What is the quotient of 348 divided by 6? (p. 143)

A drover bought 564 sheep, and sold 288 of them: how many had he left?

A farmer bought 26 cows at 18 dollars a head: how much did they cost?

There are 7 days in a week: how many weeks are there in 427 days?

There are 267 boys and 289 girls in school: how many pupils are in school? (p. 144)

Summary. The author states in his preface that the "natural order" method of teaching the concept of number is used in the writing of the text. This is also called "the three step method." The first step is the presentation of concrete objects. The author attempts to conform to this by the use of pictures and illustrations he includes. However, many times the pictures are of such size that the objects are not easily seen and near the end of the book, often only one object is included in the picture when more than one object is mentioned in the problems. The second step is the conception of numbers regarding objects not in sight. In this step the concrete object is not presented but reference is made to a concrete object, such as "Mary found 4 eggs in one nest, and 8 eggs in another: how many eggs did she find?" (p. 51) The third step is the conception of abstract numbers without the use of objects, such as "How many are 2 and 1? 4 and 1? 6 and 1? 8 and 1? 9 and 1? 3 and 1? 5 and 1? 7 and 1?" (p. 19) The author refers to this three step method as being one of the "practical embodiments of the simplest and most vital principles of the inductive method of instruction." ("Preface," p. 3) Another example of the



inductive method is that every definition is given at the end of the topics. The tables are to be used for drill rather than for memorization. The author points out in the preface that (1) the distinguishing feature is the complete "union" of mental and oral arithmetic, (2) that the great variety of exercises is for the purpose of making the pupil "accurate" and "rapid" in combining small numbers, and (3) that both the converse and single operation of the four fundamental combinations as well as the tables are presented.

#### VI. NEW PRIMARY ARITHMETIC

This arithmetic is by John H. Walsh,<sup>6</sup> associate superintendent of schools, New York. The publishers are D. C. Heath and Company, Boston. The copyrights are 1895 and 1903. The introduction, page iii, states that: (1) The New Primary Arithmetic is for use in the second, third, and fourth elementary grades; (2) that chapter one is for the second grade; (3) that, in the distribution of subject matter, care has been taken to combine the best features of the spiral and topical arrangement; (4) that effort has been made to adapt the work at every stage to the "growing powers" of the pupil; (5) that, there is a large amount of drill material provided

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<sup>6</sup> John H. Walsh, New Primary Arithmetic (Boston: D. C. Heath and Co., 1895 and 1903).

under each subdivision before a new topic is taken up; (6) that graded reviews are continued throughout; and (7) that special attention is given to the grading and character of the problems. They deal with numbers smaller than those used in the corresponding abstract work; the conditions are limited to those within the experience and understanding of the average child; and the solution of the early problems involve but a single operation.

This book contains a table of contents, page v. The main topic of Chapter 1 is Addition and Subtraction, and the subtopics, Addition, Notation and Numeration to 999, Miscellaneous, Subtraction. These subtopics show the spiral arrangement which the author cites in his introduction.

Oral problems introduce the topic of Addition, page 1. Problems 1, 2, 3, and 5 of this group are illustrated by dots, pears, and apples arranged in the proper group as an aid to the solution. Problems 1 and 5 are as follows:

1. A girl pays four cents for a slate and one cent for a pencil. How much do both cost?

5. A boy lost three apples and had three apples left. How many had he at first?

The first problem is illustrated by the arrangement of four large dots in one group and one dot in the other group with the word "and" between the two groups.

The fifth problem has the illustration of three apples

in each of the two groups with the word and between.

The next topic is Notation and Numeration, which introduces the writing of the first nine numbers. Oral exercises, using these figures in addition combinations whose sums range from 1 to 9, inclusive, are next in order. An example of these on page 2 follows:

2	3	5	4	3	4	2	3	2
<u>4</u>	<u>2</u>	<u>1</u>	<u>5</u>	<u>3</u>	<u>2</u>	<u>5</u>	<u>6</u>	<u>3</u>

The author suggests in a note at the close of these oral exercises, (1) that brief drills should be given regularly on the preceding combinations, as well as those that follow.

Next in order are these exercises:

(1) The presentation of the figures for name of the numbers, page 3

(2) The writing of figures, pages 3 and 4

(3) The reading of lists of numbers, page 4

(4) The explanation of unit's and ten's figures in numbers, page 4

(5) The written exercises in the column addition of two numbers, pages 4 and 5, e. g.,

48	72	29	15	63
<u>51</u>	<u>23</u>	<u>40</u>	<u>62</u>	<u>15</u>

(6) Written problems, pages 5 and 6, e. g.: "11. There are 14 houses on one side of a street and 13 on the other side. How many houses are there on both sides?"

(7) Oral exercises, page 6, involving column addition of three and four numbers, e. g.,

$$\begin{array}{r}
 2 \\
 2 \\
 2 \\
 \underline{2}
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 2 \\
 3 \\
 \underline{2}
 \end{array}
 \quad
 \begin{array}{r}
 2 \\
 0 \\
 4 \\
 \underline{2}
 \end{array}
 \quad
 \begin{array}{r}
 4 \\
 2 \\
 0 \\
 \underline{3}
 \end{array}
 \quad
 \begin{array}{r}
 5 \\
 1 \\
 2 \\
 \underline{1}
 \end{array}
 \quad
 \begin{array}{r}
 3 \\
 1 \\
 1 \\
 \underline{3}
 \end{array}$$

The author says in a note prefacing these oral exercises that in these and subsequent examples, pupils should use as few words as possible. For example he should say in the second problem cited here, "five, seven, nine."

(8) Oral problems, page 7, with three or more number situations, such as: "10. William is 5 years old, James is 2 years older than William, Sarah is 2 years older than Sarah." This is too complicated a situation for the ordinary child.

(9) Original problems, page 8, with the following directions:

Make problems containing the following numbers:

$$\begin{array}{r}
 2 \\
 4 \\
 \underline{2}
 \end{array}
 \quad
 \begin{array}{r}
 3 \\
 2 \\
 \underline{1}
 \end{array}
 \quad
 \begin{array}{r}
 4 \\
 2 \\
 \underline{3}
 \end{array}
 \quad
 \begin{array}{r}
 5 \\
 1 \\
 \underline{2}
 \end{array}
 \quad
 \begin{array}{r}
 2 \\
 2 \\
 \underline{2}
 \end{array}$$

Thus: a duck has 2 feet, a cat has 4 feet, a hen has 2 feet. How many feet have they?

(10) Written exercises in column addition, page 8, such as the following:

$$\begin{array}{r}
 40 \\
 4 \\
 5 \\
 \underline{20}
 \end{array}
 \quad
 \begin{array}{r}
 64 \\
 2 \\
 3 \\
 \underline{10}
 \end{array}
 \quad
 \begin{array}{r}
 25 \\
 10 \\
 2 \\
 \underline{1}
 \end{array}
 \quad
 \begin{array}{r}
 50 \\
 25 \\
 10 \\
 \underline{2}
 \end{array}
 \quad
 \begin{array}{r}
 50 \\
 30 \\
 5 \\
 \underline{8}
 \end{array}
 \quad
 \begin{array}{r}
 72 \\
 4 \\
 2 \\
 \underline{1}
 \end{array}$$

(11) Written problems, pages 9 and 10, e. g.: "10. A boy paid 20 cents for firecrackers, 10 cents for torpedoes, 5 cents for pinwheels and 4 cents for sky-rockets. How much did he spend?"

The topic of Subtraction is presented in the following order:

(1) Oral problems, page 10, e. g.: "2. Mary wishes to buy a 5-cent doll. She has 3 cents already. How many more cents does she need? 5. What number must we add to 4 to make 7?"

(2) Oral exercises, page 10, e. g., "8 and what are 9?"

(3) Give the missing numbers, page 11, e. g.,

$\frac{2}{8}$	$\frac{0}{7}$	$\frac{0}{2}$	$\frac{?}{4}$	$\frac{?}{8}$	$\frac{?}{2}$	$\frac{?}{3}$
---------------	---------------	---------------	---------------	---------------	---------------	---------------

(4) Oral problems, page 11, such as: "5. A boy had 6 tops. He has 3 now. How many did he lose?"

(5) Sight exercises, page 11, with the direction, "Subtract," and the explanation of the process as follows:

Subtract:

$\frac{9}{8}$  We see that 8 and 1 are 9

$\frac{8}{1}$  The answer is 1.

(6) Original problems, and the directions and explanation, page 12, as follows: "Make problems in subtraction

containing the following numbers: Thus: there were 9 birds on a tree; 3 flew away; how many were left?"

$$\begin{array}{r} 9 \\ \underline{3} \end{array} \quad \begin{array}{r} 7 \\ \underline{4} \end{array} \quad \begin{array}{r} 8 \\ \underline{5} \end{array} \quad \begin{array}{r} 6 \\ \underline{2} \end{array} \quad \begin{array}{r} 4 \\ \underline{1} \end{array}$$

(7) Written exercises, with the explanation to write the larger number above the smaller and how to state the steps in the solution, pages 12 and 13, e. g.,

A boy has 25 cents. He pays 15 cents for a ball. How much will he have then?

$\begin{array}{r} 25c \\ \underline{15c} \end{array}$  Write the larger number above the smaller, the units figures in a line. Begin at the units column, and say 5 from 5 leaves 0 (write 0); 1 from 2 leaves 1 (write 1). The answer is 10 cents."

Subtract:

$$\begin{array}{r} 64 \\ \underline{52} \end{array} \quad \begin{array}{r} 87 \\ \underline{76} \end{array} \quad \begin{array}{r} 55 \\ \underline{15} \end{array} \quad \begin{array}{r} 28 \\ \underline{6} \end{array} \quad \begin{array}{r} 57 \\ \underline{16} \end{array} \quad \begin{array}{r} 46 \\ \underline{5} \end{array}$$

From 93 horses take 23 horses.

The next topic is entitled Miscellaneous and includes the following:

(1) Written problems involving addition and subtraction, pages 13 and 14, e. g.,

3. There are 32 boys in a class and 20 girls. How many more boys than girls are in the class?

4. There are 32 boys in a class and 20 girls. How many pupils are there in the class?

(2) The subtopic under Miscellaneous is Addition.

Sight exercises in Addition, both the column addition and the horizontal addition with the explanation of the plus

sign, pages 14 and 15, e. g., "The sign of addition is + , and is read plus.  $2 + 3 = 5$  is read, 2 plus 3 equals 5; or, 2 and 3 are 5.

(3) Oral problems, both addition and subtraction regarding concrete situations, and involving numbers small enough to be solved without use of pencil, page 15.

(4) An example of addition in which the sum of the units column is ten or more, and the left hand figure is added to the tens in the second column, page 15.

(5) The definition of sum is given thus on page 15: "The answer in addition is called the sum."

(6) Written exercises in addition as follows:

Find sum:

27	7	15	17	27	57
26	5	8	19	7	14
<u>25</u>	<u>14</u>	<u>42</u>	<u>3</u>	<u>21</u>	<u>2</u>

$15 + 3 + 12 + 36$

The Subtraction topic under the Miscellaneous topic includes

(1) The introduction of the minus sign and meaning of difference or remainder, pages 16 and 17.

(2) A group of problems using both the addition and subtraction signs, pages 17 and 18, e. g.,

Find answers:

35	64	27	36	49
<u>-20</u>	<u>-14</u>	<u>-21</u>	<u>+15</u>	<u>+25</u>

(3) Sight exercises

Give missing numbers:

$$2 + ? = 9 \quad 9 + 3 = ? \quad ? - 5 = 4$$

(4) Original problems involving both addition and subtraction similar to those found in the preceding original problems in addition.

(5) The introduction of the dollar sign and its meaning.

(6) Drills in addition by two and three, page 19, e. g.,

Add by twos:

0, 2, 4, 6, 8, etc., to 40;  
1, 3, 5, 7, 9, etc., to 39.

Add by threes:

0, 3, 6, 9, 12, etc., to 39;  
1, 4, 7, 10, 13, etc., to 40;  
2, 5, 8, 11, 14, etc., to 38.

Notation and Numeration, pages 20 and 21, is the next sub-topic review under the Miscellaneous topic. This deals with reading and writing of figures from 10 to 999. One exercise introduces a different method of direction for the writing of numbers, page 21, e. g., "4. Write the number that is one more than three hundred fifty. 5. Write the number that is one less than four hundred twenty."

The explanation of the units', tens', and hundreds' figures is found on page 22, as follows: "In the number



382, 2 is called the units' figure, 8 is called the tens' figure, 3 is called the hundreds' figure."

A Miscellaneous topic includes pages 22 to 26, inclusive, which is similar to the one on pages 13 to 19, inclusive, except the drill in addition by fours, fives, and sixes comes at the beginning of the topic, and the problems in addition and subtraction deal with larger numbers.

The last topic introduces subtraction by adding units and tens to the upper number in columns where the lower number in the unit and tens column is larger, page 27. This topic includes written exercises, oral exercises, and written problems in both addition and subtraction, and drills in counting by the adding of sevens, eights, and nines, pages 27 to 34, inclusive. There is a noticeable increase in the difficulty of the column addition exercises. Many of them have as many as six numbers ranging from the single unit to the hundreds.

Summary. The author of this arithmetic has for his aim the adapting of the material to fit stages of the growing mental powers of the child. He emphasizes (1) the importance of frequent graded reviews throughout the book, and (2) problems with simple situations and numbers small enough to be within the experience and understanding of the child. This shows a trend away from the formalized drill

and puzzle problems of the textbooks from the 1821 to 1892 period, in harmony with the educational movement beginning about 1892, which emphasizes concepts as the immediate end of education.

## VII. SCHOOL ARITHMETICS, PRIMARY BOOK

The Primary Book of the Wentworth-Smith Series is by George Wentworth and David Eugene Smith, 1919, and published by Ginn and Company, Chicago.<sup>7</sup>

The authors state the following in the Preface, pages iii and iv, relative to the purpose of their book and the materials included.

This work is prepared to meet a demand for a book which shall present within a reasonable compass but with sufficient thoroughness the great essentials of arithmetic required in the lower grades of our schools. There are various extreme positions from which the preparation of such a book may be approached, but, as is always the case, there is also a safe mean which is certain to produce satisfactory results. One extreme makes the arithmetic operation the sole purpose, neglecting to a large extent those applications that appeal to children. Another extreme attempts to make the application the sole purpose, neglecting the steady advance in power to compute that gives to pupils the consciousness of definite progress. A third extreme seeks to make all arithmetic a matter of play without serious purpose, and still another extreme attempts to frame a textbook that is merely a collection of methods for teachers, a book that is weak in drill and weak in serious applications. No one of these extremes has ever

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<sup>7</sup> George Wentworth and David Eugene Smith, School Arithmetics, Primary Book (Chicago: Ginn and Company, 1919).

given or, for psychological reasons, can ever give satisfactory results.

The safe mean proceeds on the supposition that the pupil should be led to his arithmetic through paths which are interesting; that he should see that he is studying a subject that is usable in school, in his play, in his home, and in all other phases of his daily life; and that, so far as possible, the applications should be real to the pupil, particularly in those grades in which his tastes are being formed and in which his outlook on life is very limited. It is possible to accomplish all this by arranging the work by arithmetic topics, showing the pupil the reason for studying each topic and the uses to which it can be applied.

In accordance with these principles this book stands for good, well-arranged mathematics, and not for scrappy presentation. . . . The book also appeals to the pupil's human interests by relating the subject to his personal needs and to the life in which he finds himself. It seeks to balance reasonably these two features, refraining on the one hand from devoting all the space to abstract drill, and on the other hand from failing, through the sacrifice of its space to methods of teaching, to give the amount of drill that is necessary. It recognizes that the children who study its pages have already been in school from one to two years, that they not only possess a fair knowledge of number but the kindergarten stage is already passing out of their lives.

The teacher is to use Chapters I and II in an abstract way for the first two grades and as a rapid review for grade three previous to Chapters II and III which are to be covered in this grade.

The table of contents includes the following general topics in Chapters I and II.

#### Chapter I

Counting to Twelve

Addition

Subtraction

Counting to 100

Addition  
 Subtraction  
 Measures and Fractions  
 Minimum Essentials  
 Little Examinations

## Chapter II

Numbers to 1000  
 Addition  
 Subtraction  
 Multiplication and Division Tables  
 Measures and Fractions  
 Minimum Essentials  
 Little Examinations

The topic Counting to Twelve in Chapter I, pages 1 to 4, inclusive, is presented as follows:

(1) Review of Counting, which consists of counting and reading of numbers, both Arabic and Roman, page 1

(2) Two pages of oral work, pages 2 and 3. The first page has a picture of four children on a donkey and a little girl standing nearby. Beneath the picture is the title, Taking a Ride. A list of questions to be answered by the children and a note to the teacher follow the title. Examples of these questions are:

1. Play you are taking a ride. How many children do you see in the picture?
2. How many children are riding the donkey?
6. If the little girl without a hat should get off, how many children would be left on the donkey?
7. Make up a problem about taking a ride.

The note to the teacher says:

These problems show what is called the dramatization

of arithmetic; that is, acting out a real situation. Such problems are more real when planned and suggested by the teacher than when they are given by the textbook. Oral work is so specified throughout the book.

Page 3 of the oral work has the topic, Playing Store.

Some of the questions are as follows:

1. Let us play store. How many are there to play?
3. How much shall we charge for apples?
8. John sells some blocks for building playhouses. How many are (picture of three blocks) and (picture of two blocks)?

The author states the following at the conclusion of this lesson:

Among the common rimes that can be dramatized at this time are the following: Bo-peep, Going to St. Ives, Old Mother Hubbard, Ten Little Indians, Three Little Kittens, and The Old Woman who lived in a Shoe. Among the common stories are Jack and the Beanstalk, The Three Bears, and Jack the Giant Killer. Among the dramatized occupations are running a grocery store.

(3) Page 4 is the Writing of Numbers, e. g.,

1. Write in figures the numbers from 1 to 10.
2. Write in figures the numbers from 1 to 12, and then from 12 back to 1.
5. Write in figures the number of letters in your teacher's last name.
9. Write in figures the number of doors in this room and then write the number of windows.

The note to the teacher regarding this topic says:

The pupils should close their eyes and hear the teacher tap the desk writing in figures the number of taps they hear. They should also close their eyes, touch a number of objects supplied or indicated by the teacher, such as fingers, pencils, or cubes, and then write the numbers.

The next topic is Addition, pages 5 to 8, inclusive.

The lessons are entitled:

(1) Playing School. This requires the answer to problems in the form of questions, in the horizontal combination, using the plus and equal signs, and in vertical column. The authors suggest that formal definitions be avoided. They also suggest the use of combination cards for drill.

(2) Skipping the Rope. This lesson is preceded by a picture of children skipping the rope. Directions for counting the skips are beneath the picture.

(3) The Story on the Lawn. This is a picture of seven children sitting on the lawn enjoying a story, which one little girl is reading. The questions below relate to number situations in the picture.

(4) Playing Number Games. Page 8 includes drill by means of a circle, a square, and a triangle with numbers placed at points on the outline and one in the center which is to be added to the numbers on the circle, square, and triangle.

The topic, Subtraction, is presented in a similar manner, pages 9 to 12, inclusive. The use of pictures and illustrations and number of games is continued to make the work concrete.

The topic, Counting to 100, follows Subtraction. This

includes:

(1) The reading and writing of numbers from 10 to 100, inclusive

(2) The introduction of six of the common coins of United States money by means of the picture of each. The directions beneath this picture, on page 14, are:

Read and learn:

5 cents = 1 nickel  
10 cents = 1 dime  
100 cents = 1 dollar

The Addition topic is again introduced, pages 17 to 26, inclusive, and includes not only the oral work, but the problems to be copied and answered, and pages of addition combinations for drill until they become familiar and easy to say. Pictures of squirrels on limbs of trees on which number combinations are placed, and ladders with numbers on the rungs, and posts with combinations on them are to be used in the learning of combinations. The exercise or drill is called games. The example of the game using the picture of the squirrel, page 22, is to be played thus: The child is told to be the squirrel and tell the combinations as quickly as he can without missing. If he is successful in giving the right answer to the combinations he is said to jump from branch to branch, but if he fails to give the right answer, he is said to have fallen off.

Subtraction is again introduced on pages 27 to 30,

involving larger numbers both in the oral and written problems.

The next topic is Measures and Fractions, pages 31 to 44, inclusive. Measuring is introduced by a picture of a boy measuring a plant and directions to carry out. Examples follow:

Draw on the blackboard lines these lengths:

5. 1 ft. 1 in.  
10. 3 ft. 6 in.

Write the number of feet in these lengths:

11. 2 yd. 1 ft.

Write the number of inches in these lengths:

15. 2 ft. 4 in.

In the lesson entitled Measuring Exercise some of the directions are:

1. Measure the width of the room in yards, omitting parts of a yard. Measure this distance in feet.
7. Draw a line you think is 1 ft. long. Measure it.
9. Is your desk more than 1 ft. long or is it less than 1 ft. long. Measure it.

The next subtopics under Measures and Fractions are as follows:

- (1) One Half an Object, which is explained by use of picture of single objects which have been divided
- (2) One Half of a Group, which has a number of objects and the directions calling for the grouping into halves
- (3) One Fourth of an Object, which is illustrated by



two pictures: (1) a square divided into four parts with the fraction one-fourth written in each part, and (2) the half of an apple, two-fourths, and the whole apple. The following questions are asked about the apples:

3. One of these apples has been cut into parts. Which part is  $\frac{1}{2}$  of the apple? Which part is  $\frac{1}{4}$  of the apple?

4. Which is larger,  $\frac{1}{2}$  of an apple or  $\frac{1}{4}$  of the same apple?

A page of exercises on one-fourth follows:

(4) Review of Fractions, including numbers 1 to 14, oral review

(5) The Dozen. The work is oral and includes numbers from 1 to 7. The terms 1 dozen and half dozen are emphasized.

(6) Pint and Quart. There is a picture on page 41 of a person pouring liquid from a pint measure into a quart measure. Questions relating to the picture are:

Which is the pint measure in the picture? Which is the quart measure? Which is the larger measure?

How many times must you pour the full pint into the quart measure to fill it?

The direction to read and learn: 2 pints = 1 quart is given in this lesson. The author suggests that the actual measures, should if possible, be used in the class, to make clear the meaning of the measures pint and quart.

(7) Weight, page 42, is illustrated in a picture.

The procedure of teaching the concept weight is similar to that of the measures pint and quart. The author suggests that it is desirable that the children should weigh various objects, using the pound, half pound, and quarter pound.

(8) Playing Store involves numbers from 1 to 4 in oral exercises. A picture of children playing store precedes the questions, page 43.

(9) Problems without Numbers. Questions relating to a picture of a little girl shopping in a store are asked for the purpose of making the child conscious of the process involved in answering the problems. The following questions are typical:

1. If Nora has some eggs in her basket and buys some more eggs, how do you find the number of eggs she then has?

2. If Nora knows the price of a dozen eggs and also of a half-dozen eggs, how does she find the price of a dozen and a half?

4. If Nora buys a pineapple and some oranges, how does she find how much she must pay for them?

5. If Nora find that the price of two pineapples are not the same, how does she find how much cheaper one pineapple is than the other?

The next topic, Minimum Essentials, page 45, consists of written exercises which include at least one question of each type preceding topics. The authors state in a note that the pupil cannot safely progress farther unless he can answer these questions and that such pages should be reviewed

frequently.

The questions are as follows:

1. Write in figures: seventy-eight, eighty and ninety.
2. Write in words: 0, 30, 22, 84, 60 and 98.
3. How many cents are there in 3 dimes?
4. I am thinking of two numbers whose sum is 17. One of the numbers is 9. What is the other number?
5. I am thinking of two numbers whose difference is 17. One number is 19. What is the other number?

Copy the following and add:

6. 15	7. 25	8. 45	9. 26	10. 56
<u>6</u>	<u>14</u>	<u>14</u>	<u>32</u>	<u>32</u>

Copy the following and subtract:

12. 59	13. 68	14. 57	15. 78	16. 59	17. 67
<u>20</u>	<u>32</u>	<u>27</u>	<u>70</u>	<u>39</u>	<u>15</u>

Copy the following and add:

18. 3	19. 6	20. 4	21. 2	22. 5	23. 8
4	7	0	3	1	2
<u>9</u>	<u>8</u>	<u>7</u>	<u>7</u>	<u>9</u>	<u>3</u>

24. Draw 14 short lines on paper and find, by counting, how many lines there are in  $\frac{1}{2}$  of 14 lines.
25. How many eggs are there in  $2\frac{1}{4}$  doz. eggs?
26. How many pints are there in  $1\frac{1}{2}$  qt.? in  $2\frac{1}{2}$  qt.?

The topic, Little Examinations, page 46, concludes Chapter I. The problems are abstract and include drill in addition, subtraction, and fractional parts of numbers. The purpose of these is review drill work on previous work and a basis for the next work. The author suggests that a time record be kept for each child, and that the child should try to improve his record.

Chapter II, pages 47 to 96, inclusive, begins with Notation and Numeration as did Chapter I. The numbers include numbers to 1000 under the subtopics Tens and Hundreds, Reading and Writing Numbers, both Arabic and Roman.

The second topic deals with Addition, introducing the terms addends and sum. Much drill work is given. Emphasis upon how long it takes to copy and add, the uses of addition, and the adding of three figure numbers is evident.

The third topic relates to Subtraction, pages 57 to 62, inclusive. The plan of presentation is the same as that for Addition just described. The meaning and use of the terms minuend, subtrahend, and difference is given at this time, and three figure numbers are introduced.

The fourth topic Multiplication and Division, includes the following subtopics and method of presentation: (pp. 63 to 84, inclusive)

(1) Need for Multiplication--This is introduced by the following questions:

1. Edward's father sent him to buy 3 two-cent postage stamps for local letters. How much must he pay for 3 stamps?

2. How much is  $2 + 2$ ? How much is  $2 + 2 + 2$ ? How much is  $2 + 2 + 2 + 2$ ?

3. If Edward's father buys 4 two-cent postage stamps, how much money must he pay?

4. If Edward's father has 5 local letters to mail, how much do the stamps cost for all these letters? Which

is better, to add  $2 + 2 + 2 + 2 + 2$ , or to know without adding how many five 2's are?

5. Edward did not know how many six 2's are, and his father told him to count by 2's like this: 2, 4, 6, and so on. Then Edward found how many six 2's are. How many are they?

The footnote at the conclusion of the lessons says:

When a new subject is begun, it is advisable to introduce real, concrete problems relating to the interests of the pupils, so as to the purpose of the work.

(2) Learning about Two, page 64. All this work is oral. The development of the multiplication table of 2 is shown by an illustration of the figure 2 arranged in five columns ranging from one 2 in the first column to five two's in the fifth column. The direction accompanying this is:

Add the columns of 2's, from one 2 to five 2's. This arrangement is as follows:

```

                2
               2 2
              2 2 2
             2 2 2 2
            2 2 2 2 2
  
```

The next directions follow:

2. On the blackboard and on paper build more columns of 2's, until you have twelve 2's in the last column. How many columns of 2's are there?

Other directions relate to the reading of the columns and the reading and learning of the tables of 2's. The multiplication sign is introduced.

(3) Multiplying by Two, page 65, deals with numbers 1 to 20, oral.

(4) Dividing by Two, page 66, is introduced by use

of the following arrangement of numbers in columns as in the Multiplication of Two:

$$\begin{array}{cccccc} & & & & & 2 \\ & & & & 2 & 2 \\ & & & 2 & 2 & 2 \\ & & 2 & 2 & 2 & 2 \\ & 2 & 2 & 2 & 2 & 2 \\ \hline \frac{2}{2} & \frac{2}{4} & \frac{2}{6} & \frac{2}{8} & \frac{2}{10} & \end{array}$$

The work is all oral. The following questions are typical of the questions asked:

3. How many 2's do you see in 8?
4. 8 contains 2 how many times?
5. Instead of saying that 10 contains 2 five times, we may say that 10 divided by 2 is 5, and we may write this  $10 \div 2 = 5$ ,  $\frac{1}{2}$  of 10 = 5, or as in the first of the following  $2 \overline{)10}$   
 $\underline{5}$

$$\text{Divisor} = 2 \overline{)10} = \text{Dividend} \\ \underline{5} = \text{Quotient}$$

The remaining subtopics under Multiplication and Division deal with the tables of 3, 4, 5, 6, 7, 8, and 9, in the same manner, pages 67 to 84, inclusive.

The next topic, Measure and Fractions, is treated in this chapter in the same manner as it was in Chapter I. The fractional parts involve thirds, fourths, fifths, and sixths.

The concluding topics of the chapter are Minimum Essentials and Little Examinations which are reviews of the previous work in multiplication and division.

Summary. The educational aim of education in the first part of the twentieth century, that of preparation for

living, with emphasis upon adjustment to the environment, and the practical phases of living, forms the basis for this textbook. The authors' aim, deduced from the study of the preface, may be stated as follows: (1) make arithmetic interesting; (2) lead the pupils to see that arithmetic is usable in school, in home, in play, and in all phases of activities; and (3) make the application of these problems as real to the child as is possible.

The amount of material, both concrete and abstract, involves much drill, which, according to the authors, is for the purpose of enabling the pupils to solve with speed and accuracy number situations they may meet in their environment.

In conclusion it may be said that while the authors' aim agrees with that of the educational aim of the period, the amount of material is too extensive for children of the first and second grade levels. Children of these grades have little practical use for numbers beyond one hundred even in finding page numbers, and for the simple processes of addition and subtraction.

#### VIII. A CHILD'S BOOK OF NUMBER FOR FIRST AND SECOND GRADES

John C. Stone<sup>8</sup> is author of this book. The copyright

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<sup>8</sup> John C. Stone, A Child's Book of Number for First and Second Grades (Chicago: Benj. H. Sandborn and Co., 1924).

date is 1924 and the publishing company is Benj. H. Sandborn and Co., Chicago. This book contains 137 pages. It has no preface but the plan and aims are included in the Suggestions to Teachers, pages 133 to 137, inclusive. The author says that at the end of the second grade the results attained should be classified under three heads: knowledge, skill, and attitude. Out of these three anticipated results grows the aim which may be stated as follows: (1) To give the children automatic control of all primary facts of the four fundamental processes, (2) to be able to use these facts in very simple situations of every day activities, and (3) to lead children to enjoy numbers, to desire to know more about them, to want to work with them, and to feel that they are necessary to them in many ways.

The author states that his book has three important phases: (1) The initial presentation of number concepts; (2) the drill; and (3) the use in simple everyday applications. The author discusses each of these phases as follows:

Presentation. The importance of the first presentation is two fold: through a concrete presentation of the fact the child knows what it means and how to find it for himself if he forgets it before getting an automatic control of it; and by the presentation the child learns how and when to use the fact in the simple problems that arise in his everyday affairs.

Thus the child finds the addition facts through objects of some kind. He is then shown "how figures say them"; or, when working upon the blackboard, the teacher shows him "how chalk says them."



Drill. The drill to give an automatic control of all the primary combinations should be both sight and oral drill. That is, the child should be able to recall a fact from seeing the figures or from hearing the numbers spoken, or from merely "thinking" them, as when he sees them written out in words or has occasion to "think" them in order to answer some question that arises in his own little activities.

Drills made up at random and lacking a definite purpose or system are a waste of time.

Among the principles governing drill are:

1. The work should be thoroughly motivated.
2. Drill without attention is useless.
3. Drill should include all facts of a series.
4. The most time should be given to the most difficult facts of the series.
5. There must be repetition.
6. The time space between drills should be very short at first and gradually lengthened as the facts become more permanent. . . .

Games. Games are used to motivate drill work,--that is, to secure and hold attention. The elements of a good game are: It must be interesting; it must be so arranged that even with the control of the combinations in the hands of the children all facts of a series will come up; it must not eliminate the slow pupil or the one who has made a mistake; and as nearly as possible, it must require all the children to make all combinations that occur in the game.

Not all drill can be gotten from games. There is too great a danger that certain combinations will not receive proper attention. Carefully planned drills like those in the text are the only means of assuring proper attention to all facts of a series. Hence all the drills in the text should be used over and over until the child has a perfect control of them.

The games, stories, and rhymes of the text are given for several purposes. They furnish interesting and valuable reading lessons and they suggest uses the child may make of number and thus motivate the study of number. But it will be observed that, as given in this text, they are expected to stimulate the child to want to do some of the things suggested by them. And thus they

suggest real "projects" in the true sense of this much-used word.

The little rhymes, then, are not to furnish drill, or even the application of number facts, but to encourage the child to make up rhymes using number and thus get pleasure from his work. . . .

The games are described in simple language, and it is hoped that by reading them many children will want to play them. . . .

Definitions and technical terms should not be given in these grades. All language should be the natural language of childhood. "Less," "minus," "plus," "times" and such terms are meaningless to children.

It is not unusual to hear children parrot-like say, "7 plus 3 equals 10," "8 less 3 equals 5," "3 times 5 equals 15," and so on, with no notion of what the expressions mean.

The child's only notions of the two inverse processes --subtraction and division--should be those given in this book, subtraction being only another form of an addition fact, and division only another form of a multiplication fact. Avoid technical language.

In teaching the common measures, have the measures present and use them in meaning things in which the children are interested.

There is no table of contents but the topics included are counting, addition, subtraction, multiplication, division, drills, rhymes, and reviews on each, and measures of time and lengths. The author suggests that multiplication and division facts may be reserved for the third grade work.

Counting is introduced by the use of pictures and rhymes involving numbers. The first rhyme is as follows:

One, two, three, four, five,  
 We're marching to Saint Ives;  
 Six, seven, eight, nine, ten,  
 We're marching home again.

This rhyme is illustrated by two pictures, one above and one below the rhyme. The picture above the rhyme shows five children marching up a hill to the castle, Saint Ives. The picture below shows the children marching down the hill from Saint Ives.

The next rhyme, which is also accompanied by a picture, is Ten Little Indians.

Page 2 has the picture of Little Boy Blue and the sheep in the meadow, and the cows in the corn. Below the picture is the rhyme and sentences with blanks to be supplied with the correct number word. Below these are the number names and figures from one to ten.

The sentences with the blanks are as follows:

"There are \_\_\_\_\_ sheep in the meadow

There are \_\_\_\_\_ cows in the corn."

Pages 3 and 4 are about adding one. There is a picture on page 3 of Bo Peep's sheep, and the following sentences and directions.

Little Bo Peep lost her sheep. She left them alone and one by one they all came home.

One sheep came home and then one more came. That made \_\_\_\_\_ sheep.

Two sheep came home and then one more came. That made \_\_\_\_\_ sheep.



An Outdoor Game has the picture of children playing a game, with the following information about the game:

These children have made up a number game. They form a circle as in "Drop-the-handkerchief." One child runs around the circle saying:

Hink-i-ty pink-i-ty, pank-i-ty, poo,  
 I had \_\_\_\_\_ pennies and now I have two.  
 Hink-i-ty, pink-i-ty, pank-i-ty, poo  
 Who knows how many I lost? Do you?

As she says "Do you?" she taps some one. The one tapped answers, and tries to catch her before she can get around the circle.

If she is caught, she has to go around again. If not, the other child goes around and taps some one in the same way.

Any number from two to ten may be used to fill the blank in the rhyme.

Succeeding pages are treated similarly to those just described except that the numbers added increase to 10, and through the money values of dimes and cents, the meaning of teens is introduced.

The subtraction process is taught by the additive method. The definition of the process, page 61, is as follows:

"When you tell how much to add to make a larger number, you are subtracting." Games, drills, and number stories are similar to those used in the addition topic.

Pages 72 to 79, inclusive, deal with the following concepts:

(1) Measures of time such as year, months, weeks, days, hours, and telling time by the clock, recognizing both Arabic and Roman numbers

(2) Measures of lengths, such as, yard, foot, inches

(3) Measure of quantity such as, pint, quart, pound, and ounce

(4) Parts of wholes, such as halves and fourths

Pages 80 to 104, inclusive, consist of the review of addition and subtraction, using larger numbers in the process, and reading larger numbers.

Multiplication, pages 105 to 107, inclusive, is introduced by the method of adding two equal numbers. This is followed by the exercise using the multiplication sign. This exercise explains that the short way to write: "Two threes are 6 is  $2 \times 3 = 6$ .

The Division topic, pages 108 to 111, inclusive, consists of concrete problems introducing the concept of the division of things into halves, and the reading of problems using the abstract form such as:  $2 \overline{)4}$   $2 \overline{)8}$ , etc.

The remaining pages of the text, pages 112 to 132, inclusive, have exercises which relate to the following topics: Dozen, Counting by Fives, Buying Five Cent Articles, A Ring Toss Game, Adding Four Numbers, A Number Race (drill), Adding Four Numbers, Adding Three Equal Numbers, Dividing by Three,

Exercises for Practice (addition), A Number Story (two pages, multiplication and division), A Review of Addition Facts (two pages), A Review of Subtraction (two pages), and Review of Multiplication and Division (one page).

Summary. Stone emphasizes knowledge, skills, and attitudes as the desired outcomes at the end of the second grade. His aim as seen by the foregoing statement appears to be threefold: (1) to help the pupils gain automatic control of all primary number facts of the four fundamental processes; (2) to provide for the practice of the facts in everyday life; and (3) to lead pupils to want to know more about these facts and to want to use them, and to feel that they are necessary to them in many ways in everyday life. The educational aim of the period beginning about 1919 and continuing to about 1933 emphasizes the preparation for the practical side of living, and the development of skill to enable the individual to meet the various situations. The author seems to have attempted to conform to this aim. But he seems to have failed to realize that the quantity of drill, the vocabulary, and the amount of material are not suited to the level of the first and second grades.

## IX. FIRST DAYS WITH NUMBERS

Clifford Brewster Upton,<sup>9</sup> professor of Mathematics, Teachers College, Columbia University, is the author of this arithmetic textbook. The copyright is 1933 and the publishing company is the American Book Company.

The preface of this book shows that there is an increasing sensitiveness to the fact that number is not a thing apart from everyday living and that as a subject matter it should be closely related to the experiences of children. The author states the following in the preface, pages 3 and 4.

Every child in the beginning grades has need for certain knowledge of number in connection with his everyday activities. He uses number in counting school supplies, in finding the pages in a book, in buying his lunch, in making change, in keeping scores in his games, in telling time, in reading the calendar, in measuring with a ruler, and in many other ways.

It is experiences such as these that form the basis of instruction in primary number today. According to our best city and state courses of study, this instruction is expected to give the child, by the end of the second grade the ability to count to 1000, a reasonable mastery of the 100 combinations in subtraction, some facility in applying this work to everyday problems, and an acquaintance with measures. It is the purpose of this book to provide instruction covering these topics and to do this in such a way that it is intimately related to the experiences of children.

This book may be introduced in the second half of the

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<sup>9</sup> Clifford Brewster Upton, First Days with Numbers (Chicago: American Book Company, 1933).



first grade or at the beginning of the second grade.

The following features are noteworthy:

1. The problems and number stories are all connected with the play, games, and other activities of children.

2. The illustrations sympathetically portray child life and will strongly appeal to the pupils.

3. This book is written for children. The vocabulary, which contains 495 words, is based on a critical analysis of vocabularies of a large group of primary readers. With the exception of a few words peculiar to arithmetic, the vocabulary contains only those words which the child is frequently meeting in his other reading. The vocabulary also harmonizes with the better-known word lists for primary reading, containing a very high percentage of the easier words in those lists.

4. The aim throughout has been to fix the best number habits from the beginning. Nothing is taught that is later to be undone.

5. The number facts are presented in small groups, each group being developed and drilled upon before another group is taken up. Frequent reviews are provided.

6. Special attention has been given to the treatment of subtraction, in order to familiarize the pupil with the more important ideas and number comparisons that are related to this operation.

7. Provision for the individual differences of pupils has been made through a series of diagnostic tests, with keyed references to remedial exercises, and also optional extra drill on each of the groups of number facts. (pp. 3-4)

Two workbooks by the same author, one for the first grade and one for the second grade, furnish supplementary materials and abstract exercises. These may or may not be used with the textbook according to the course of study outlined for the particular schools in which this author's

text is being used.

The table of contents of the First Days with Numbers includes the following topics:

Counting to 10  
Comparing Things  
Writing Numbers  
Numbers from 10 to 20  
Nine Easy Addition Facts  
Telling Time  
Nine New Addition Facts  
Buying Lunches  
Review of Addition  
Nine New Addition Facts  
At the Beach  
Review of Addition  
Diagnostic Tests  
Cents, Nickel, Dime  
Eighteen New Addition Facts  
Review of Addition  
Playing Tenpins  
Diagnostic Tests  
Zero in Addition  
Flash Cards  
Review Test in Addition  
Playing Hopscotch  
Column Addition  
Buying Dolls  
Adding 2-Figure Numbers  
Counting by 10's  
Buying Easter Presents  
Counting by 100  
Counting by 5's  
Days of the Week  
The Calendar  
Additive Subtraction  
Making Change  
Nine Easy Subtraction Facts  
Tom and His Rabbits  
Eighteen New Subtraction Facts  
Diagnostic Tests  
Pint and Quart  
Zeros in Subtraction  
Telling Time. Roman Numbers  
Review Test in Subtraction  
A Circus for the Dolls

Subtracting 2-Figure Numbers  
Counting by 2's  
Counting above 100  
Review of Addition  
At School  
The School Bank  
Ten New Addition Facts  
Addition and Subtraction  
The Pet Store  
Eight New Addition Facts  
Diagnostic Tests  
Nine New Addition Facts  
Review  
Nine New Addition Facts  
At the Penny Store  
Diagnostic Tests  
The Children Sell Pictures  
Additive Subtraction  
Comparing Ages  
Review of Subtraction  
Comparing Ages  
Review of Subtraction  
The Pigs Go to the Store  
Ten New Subtraction Facts  
The School Fair  
Addition and Subtraction  
Eight New Subtraction Facts  
Diagnostic Tests  
The Children's New Books  
Nine New Subtraction Facts  
One Half  
One Fourth  
Nine New Subtraction Facts  
Ring My Nose  
Comparing Numbers  
Diagnostic Tests  
Fun in Summer  
Quarter, Half Dollar, Dollar  
Dollars and Cents  
Review Test in Subtraction  
Buying by the Pound  
Inch and Foot  
A Number Game  
Column Addition  
Buying by the Dozen  
Adding Larger Numbers  
Subtracting Larger Numbers  
Review Test  
Drill Sheets (Remedial Exercises)

The 100 Addition Facts  
The 100 Subtraction Facts (pp. 5-8)

Counting by 1's to 10 is the first topic. This is introduced by a picture of a woman and three children holding balloons. Questions relating to the picture require counting to give the answers. Examples of these are: "1. How many balloons has the little boy? 5. Count all the balloons." (p. 9)

The next two pages have the illustration of the poem, "Ten Little Indians," the poem and directions for the counting involved. Questions relating to the picture are as follows:

1. How many Indians are there in all?
2. How many Indians are dancing?
3. How many Indians are playing drums? (p. 10)

The next direction on the same page is:

4. Read these numbers:

one	two	three	four	5
1	2	3	4	five
six	seven	eight	nine	ten
6	7	8	9	10

The author suggests, in a note below the poem, the method of using the poem. The note follows:

"Have the pupils count out ten pupils to be Little Indians. As the first stanza is sung, let the pupils creep up, one by one, to the front of the room." (p. 11)

The next method for counting is that of giving

directions for doing or finding out certain number information. The name of this exercise is Can You Do These Things?

A few of the directions follow:

1. Count your pencils.
6. Clap your hands three times.
9. Put five books on the table.
12. Tell the names of seven girls.
13. Tell how old you are.
15. Play that you are jumping rope. Jump and count to eight. (p. 12)

Counting by 2's is found on pages 91 to 93, inclusive; Counting by 5's, on pages 61, 92, 93; Counting by 10's, on pages 57, 59, 92, and 93; and Counting by 100's, on page 93. Some of the devices used for teaching the children to count are as follows:

The numbers from ten to twenty are:

ten eleven twelve thirteen, etc., to twenty.  
 10 11 12 13

1. Say the numbers from 5 to 20.
2. The number that comes after 11 is \_\_\_\_\_.
3. The number that comes before 15 is \_\_\_\_\_. (p. 17)

Underneath a picture of a little girl who is putting candy in boxes, is the following:

Ann is putting candy in boxes. She puts 10 pieces of candy in each box. How many pieces does she need for 10 boxes? To find out, count by tens like this:

ten twenty thirty forty fifty sixty  
 10 20 30 40 50 60

seventy eighty ninety one hundred  
 70 80 90 100

1. Count to 100. Count by 10's to 100.
2. A dime is      ¢. A nickel is      ¢.
3. Jane has 9 dimes. To tell how many cents she had, count like this:

10¢, 20¢, 30¢, --¢, --¢, 60¢, --¢, --¢, --¢.

4. Mary Ann has 2 dimes and 3 cents. How many cents has she?

The 2 dimes make 20¢. Mary Ann has  $20¢ + 3¢$  or 23¢.  
 $20 + 3 = 23$ .

5. Peggy has 3 dimes and 4 cents. Peggy has  $30¢ + 4¢$ , or 34¢.

6. Bobby has 4 dimes and 2 cents. How many cents does that make?

7. How many dimes and cents will it take to make 43¢, 51¢, 64¢, 32¢, 68¢? (p. 59)

1. Count by 1's to 100.
2. What number comes after 62?
3. What number comes after 17?
4. What number comes between 53 and 55? Between 46 and 48?
6. Tell the numbers that are left out: 52, 53, --, --, 56, --, --, 59, 60, --, 62, --, 64, --, --, 67.
7. In a book, what pages come between page 29 and page 33?
8. Tell what you see in the picture on page 19 of this book. (p. 60)

1. Ed has 5 nickels. To tell how many cents he has, count by 5's like this: 5¢, 10¢, 15¢, --¢, --¢. This shows that Ed has --¢.

2. Peggy's mother gave her 10 nickels. Count by 5's to see how many cents there are in 10 nickels. (p. 61)

Below a picture of children marching by two are the following directions:

1. Count the children in the picture by 2's like these: 2, 4, 6, 8, 10, 12.

2. Count the children's shoes by 2's.
3. Tell the numbers that are left out: 2, 4, 6, --, --, 12, --, 16, 18, --, 22, --, 26, --, --, 32, 34, 36, --, --, 42, --, --, 48, 50, 52, --, --, --, --, --, --, 66, 68, --, --, --, --, --, 80, 82, --, 86, --, --, 92, --, 96.

4. Count by 2's to 100. (p. 91)

1. Count by 1's from 100 to 150.
2. Count by 10's from 100 to 200 like this: 100, 110, 120, ---, ---,
3. What number comes before 120?
4. What number comes between 149 and 151? (p. 92)

1. Count by 100's from 100 to 1000 like this: 100, 200, 300, ---, ---,
2. Count by 10's from 200 to 400 like this: 200, 210, 220, ---, ---,
5. Count by 1's from 500 to 600.
6. What number comes after 337?
8. What number comes before 700? (p. 93)

Comparison of numbers dealing with the relationships, longer, older, more or less, larger, smaller, taller, cent, nickel, dime, are found on pages 13, 14, 15, 18, 38, 64, 80, 107, 123, 124, 131, 132, 134. A few examples follow:

Pictures of two apples, two glasses partially filled with milk, and two plates of oranges. The questions beneath them call for the comparison to be answered "yes" or "no."

The questions are:

- Is the first apple larger than the second?
- Does the first glass have more milk than the second one?
- Does the first plate have more oranges than the second one? (p. 13)

The following questions relate to the picture of a

boy and girl taking three dogs for a walk:

1. Who is older, Tom or Alice?
2. Who is taller, Tom or Alice?
3. Who has the larger hat, Tom or Alice?
4. Who has more dogs, Tom or Alice?
5. Who has the largest dog?
6. Who has the smallest dog?
7. Whose dog has the longest legs?
8. Whose dog can run the fastest? (p. 14)

On page 18 the author introduces the relationship more or less. e. g.,

1. Which is more, 9 tops or 6 tops?
3. Alice has 4 dolls. Mary has 7 dolls. Which one has less dolls?
4. Billy has 5 apples. Tom has 4 apples. Which has more apples?
5. Which number is more: 3 or 4? 6 or 7? 8 or 2? 5 or 2? 3 or 5? 8 or 10?
6. Which number is less: 2 or 3? 6 or 5? 4 or 2? 1 or 2? 8 or 4? 9 or 10? (p. 15)

On page 14 the term larger refers to sizes of objects or persons, while in the following examples larger refers to amount or number. e. g.,

1. Bobby has 16 pennies. Alice has 15 pennies. Which one has more pennies?
5. Which number is larger: 17 or 15? 11 or 13? 20 or 19? 14 or 18? 15 or 14? 12 or 14? (p. 18)

Addition is introduced through the use of a picture of two horses and two boys on each horse. The title under the picture is The Boys and Their Horses. The questions and information relative to the number facts in the picture are as follows:

1. How many horses do you see? 1 horse and 1 horse are 2 horses. 1 and 1 are 2.



2. How many boys do you see? 2 boys and 2 boys are 4 boys. 2 and 2 are 4.

3. 1 and 1 are --.

2 and 2 are --.

Write the numbers like this:  $\frac{1}{2}$   $\frac{2}{4}$  (p. 19)

Other pages dealing with addition are 20 to 23, 26 to 62, 94, 97 to 99, 101 to 106, 108 to 112, 143, 145, 146, 149, 150, 153, 154, and 157.

The meaning of adding is given in the following manner:

1. Jane had 2 small dolls. Her mother gave her 1 large doll. How many dolls has Jane now?

2 dolls and 1 doll are 3 dolls.

2 and 1 are --.  $\frac{2}{3}$   $\frac{1}{3}$

1 and 2 are --.  $\frac{1}{3}$   $\frac{2}{3}$

When you think, "2 and 1 are 3," you are adding. You call 3 the sum. (p. 20)

The method of adding three numbers is introduced by the picture of three children playing the Hopscotch game. The following information is beneath the picture.

The children are playing Hopscotch. Each child hops three times.

1. Bobby hops into 1, 6 and 2. To find his score, write the numbers under each other. Then add up like this: 2 and 6 are 8, and 1 are 9.  $\frac{1}{9}$

To make sure that 9 is right add down like this: 1 and 6 are 7, and 2 are 9.  $\frac{6}{9}$

This shows that Bobby's score is 9. (p. 52)  $\frac{2}{9}$

On the following two pages are abstract problems in which three numbers are to be added. (pp. 54 and 55)

The method of adding larger numbers is explained by use of a picture of a toy shop in which one little girl is buying dolls from another little girl, who is the clerk. The

following statements are below the picture:

1. Ann bought a rag doll for 25¢ and a baby doll for 42¢. Both dolls cost --¢.

$$\begin{array}{r} \text{Think } 2 + 5 = 7 \\ \text{Think } 4 + 2 = 6 \\ \hline 25 \\ 42 \\ \hline 67 \end{array}$$

The sum is 67. Both cost 67 ¢.

2. Each girl bought two dolls. Tell how much each spent for them.

Betty	Mary	Alice	Peggy	Jane	
<u>33¢</u>	<u>23¢</u>	<u>25¢</u>	<u>30¢</u>	<u>20¢</u>	
<u>64¢</u>	<u>15¢</u>	<u>21¢</u>	<u>45¢</u>	<u>32¢</u>	(p. 55)

On page 56 is a list of problems which are called Adding Larger Numbers.

In later exercises the author suggests the covering of the answers of lists of addition facts and saying the answers quickly after having learned them. (p. 108)

Subtraction facts are introduced by use of statements and questions about a picture of toys with the price marks. The title of the exercise is How Much More Do I Need? Two problems illustrate this point. They are as follows:

4. I have 7¢. How much more do I need to buy a kite that costs 9¢?

5. 6 and -- are 10. 3 and -- are 7. (p. 64)

Pages 65 and 66 require the same process but the activity is called Find the Missing Number. Pages 67 and 68 require the same thinking but the name of the activity is called Making Change. The activity on page 69 is entitled How many are left?

Examples of these pages just cited, immediately follow:

6 and -- are 9. 3 and -- are 8.

10. Tom wants 6 apples. He has 2 apples. He needs -- more apples. 2 and -- are 6. 2 and -- are 6. (p. 65)

1. Tell the missing numbers:

3	4	7	5	3	4	2	9	1	
*	*	*	*	*	*	*	*	*	
$\overline{7}$	$\overline{8}$	$\overline{10}$	$\overline{6}$	$\overline{9}$	$\overline{5}$	$\overline{5}$	$\overline{10}$	$\overline{10}$	(p. 66)

1. Sally bought a pencil for 3¢. How much change did she get from 5¢?

She got 2¢ because 3 and 2 are 5.

2. Ed bought a ball for 4¢. He got --¢ in change from 5¢. Think: 4 and 1 are 5.

4. Peggy had a dime. She bought some paper for 4¢. She got --¢ change.

5. How much change do you get from a dime if what you buy costs 3¢? 6¢? 4¢? 8¢? 7¢? 2¢? 9¢? (p. 67)

1. Ed had 2 books. He gave 1 to Mary. How many books had he left?

1 from 2 is 1. 1 from 2 is --.

5. Read these. You read the first one like this: "3 from 4 is 1."

2				
$\frac{1}{1}$				
4	2	4	3	3
$\underline{3}$	$\underline{1}$	$\underline{2}$	$\underline{1}$	$\underline{2}$

(p. 69)

The term subtracting and subtract is explained on the next page of the text. The explanation follows:

1. Jack has 5¢. He spent 1¢. How many cents did Jack have left?

1 from 5 is 4.

When you take 1 from 5 and find how many are left, you are subtracting. You say that you subtract 1 from 5. (p. 70)

5
$\frac{1}{4}$

Subsequent pages regarding subtraction use the terms, subtract, tell the numbers left out, less, and how many are left?

Other pages dealing with subtraction facts are pages 71 to 81, 83, 84, 86 to 90, 95, 106, 107, 114 to 117, 119 to 128, 131 to 135, 139, 147, 151, 152, 155, 156, and 158.

Measure of Time is introduced on pages 24 and 25.

Pictures of clock faces indicate various hours. The information about the telling of time and the direction to tell what each clock says, accompany the picture. (pp. 24-25).

The topics, The Days of the Week and the Calendar, as measures, appear on pages 62 and 63. A few of the directions relative to the days of the week are as follows:

1. Name the days of the week.
2. There are -- days in a week.
3. Name the days we go to school.
4. We go to school -- days a week.
5. The days we do not go to school are --- and ---.
6. The school week begins on ---.
7. Today is ---.
8. Tomorrow will be ---.
9. Yesterday was ---.
10. The day before Tuesday is ---.
11. The day after Tuesday is ---.
14. The day that comes between Friday and Sunday is ---. (p. 62)

Clocks, with Roman numbers for the purpose of practice in the telling of time, appear on page 85. A picture of the December calendar page is used to teach the facts about the number uses of the calendar. The directions follow:

1. This is the calendar for December. Read all the numbers on it.
2. How many days are there in December?
3. Read all the numbers that come on Sundays; on Wednesdays.
4. What day comes on December 25?

5. Find December 25 on your calendar.
6. What month is it now? What day of the month is it?
7. What day of the week is this?
8. When does your birthday come? (p. 63)

Measure facts relative to liquids are those of pint and quart. The author gives information about these measures by use of a picture in which a mother is pouring milk into two glasses from a pint bottle. Another pint bottle and two quart bottles are on the table. The information follows:

1 quart makes 2 pints.  
1 pint fills 2 glasses.

1. There are --- quart bottles and --- pint bottle of milk on the table.
2. Which is more, a quart or a pint?
3. A quart is the same as --- pints.
4. Mother put half of 1 pint into Tom's glass and the other half into Mary's glass. 1 pint fills --- glasses.
5. Tom and Mary each drink 1 quart of milk a day. 1 quart fills --- glasses. (p. 82)

The measure, one-half, is taught by use of a picture of an orange cut into two equal parts, an apple cut into two unequal parts, a cake divided into two equal parts and a stick of candy divided into two unequal parts. The reproduction of this page and the lesson follow: (p. 129)

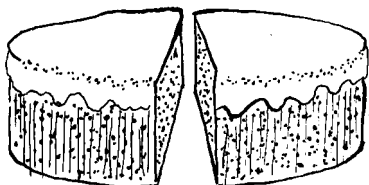


## One Half

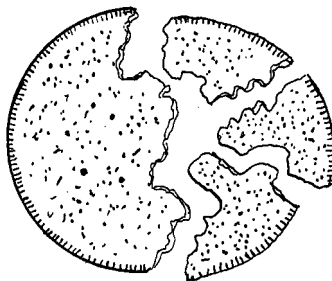
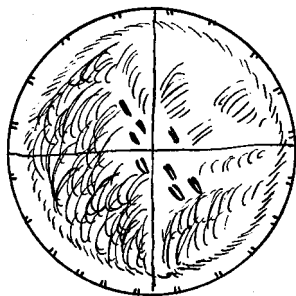
1. How many pieces of orange do you see? Are the two pieces of orange the same size? Each piece is called one half of the orange. You write one half like this:

2. The 2 pieces of apple are not half pieces because they are not the same size.

3. Look at the pictures below. Are the pieces of cake half pieces? Are the pieces of candy half pieces?



The following reproduction and questions on page 130 show how the measure, one-fourth, is presented:



## One Fourth

1. How many pieces are there in the pie? Are the pieces of the pie all the same size? Each piece is one fourth of the pie. You write one fourth like this:

2. How many pieces of cookie do you see? These pieces are not fourths because they are not the same size.

3. Cut an apple into fourths.
4. Cut a piece of paper into fourths. Are the 4 pieces the same size?
5. Cut a piece of string into fourths. (p.130)

The measure or money value relates to the dime, quarter, and half dollar. The picture of each coin is on the page. The information and questions below the picture is as follows:

1 quarter = 25 cents  
 1 half dollar = 50 cents  
 1 dollar = 100 cents

1. How many nickels make 1 quarter? (The value of the nickel has previously been given on page 61 of the text.)
2. How many quarters make 50 cents?
3. How many quarters make 1 dollar?
4. How many half dollars make 1 dollar?
5. How many dimes make 1 dollar?
6. One dime and --- nickels make 25¢.
7. Four dimes --- nickels make 50¢. (p. 137)

The measure pound is taught by use of a picture which shows Jean and her mother shopping. The topic is called Buying by the Pound. The lesson relating to the picture is as follows:

1. Jean's mother is buying a chicken. It weighs 3 pounds.
2. You buy butter by the pound. Tell some other things that you buy by the pound. When you want less than a pound of butter, sometimes you buy a half pound.
3. Which of these things below can you buy by the pound?
 

meat	sugar	eggs
milk	bread	oranges
4. How many pounds do you weigh? Ask your teacher to weigh you. (p. 140)

The next measure introduced is that of inch and foot. The directions are given for the use of the rule. The meaning of foot is explained and the relationship of 1 foot to twelve inches is expressed. The statement follows: "A ruler that is 12 inches long is called a foot ruler. 1 foot = 12 inches. (p. 141)

The quantity measures, dozen and half dozen, are represented by use of a picture of a bakery in which Alice is buying. The title of the lesson is Buying by the Dozen. The lesson is as follows:

12 things = 1 dozen

6 things = 1 half dozen

1. Alice bought a dozen cookies. The man gave her --- cookies.
2. She also bought a half dozen rolls. The man gave her --- rolls.
3. You buy eggs by the dozen. Tell other things that you buy by the dozen.
4. 1 dozen eggs is --- eggs.
5. 1 half dozen eggs is --- eggs. (p. 144)

Summary. The author believes that every child in the beginning grades should have certain number knowledges in connection with his everyday activities. There is evidence that the educational theories of the period have influenced the author's thinking relative to the type of arithmetic material for young children. From the study of the type of material in the text and of the preface, the author's aim seems to be to provide the materials that are



intimately related to children's experiences. To this end the author states that he has given particular attention to the following points in writing the book: (1) Problems and number stories connected with plays, games and activities of children; (2) illustrations that appeal to children and are true to child life; (3) a vocabulary based on the critical analysis of a large group of primary readers; (4) provision for suitable type and amount of material together with suggestions for the presentation and practice on it to insure the establishment of the best number habits from the beginning; and (5) provisions for the individual differences with keyed references to remedial exercises.

In conclusion it may be said that the author is sensitive to the fact that the understanding of number relation comes from the provision of material intimately connected with children's experiences. He has attempted to supply this in his text. However, the numbers included are larger than children of the first and second grades have occasion to use.

#### X. CHILD LIFE-NUMBER BOOK

The authors of this book are Charles E. Garner and Livingstone McCartney.<sup>10</sup> The copyright is 1938 and the

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<sup>10</sup> Charles E. Garner and Livingstone McCartney, Child-Life Number Book (Lyons and Carnahan, 1938).

publishing company is Lyons and Carnahan, Chicago.

An examination of this modern day text shows that authors are becoming conscious that: (1) a satisfactory textbook must be based upon not only educational theories of psychologists, administrators, and professors of elementary and secondary fields of education in colleges and universities, but upon research findings, and suggestions and criticisms of classroom teachers and supervisors; (2) that an arithmetic textbook must have a carefully graded vocabulary; (3) that each new subject or concept must be introduced gradually and informally; (4) that it is the whole child, in his daily experiences of living, that is important, and not the complete mastery of number facts or processes; (5) that provision must be made for individual differences; (6) that number concepts are not confined to the subject of arithmetic in an arithmetic textbook, but that they are coordinate in daily living; and (7) that interesting activities should be carefully organized and guided in such a way that the children may be able to develop thinking ability and clarify their understandings of number facts and processes through the discovery of their own social uses of number in these activities and others of daily living.

No mention is made by the authors of the fact that

the material of this text has been satisfactorily tested in actual classrooms, a practice quite common now, but they gratefully acknowledge the helpful suggestions and criticisms of many teachers and supervisors. The authors do not intend that this book shall be used in the formal study of arithmetic in the second grade but as a preparation, which they call a "getting ready for arithmetic."

In the preface the authors give the following:

Research studies show that children have considerable number knowledge when they first enter school, and they enjoy thinking in terms of number. Their number knowledge is fragmentary, however; and they need some means of organizing it before they take up a more formal study of arithmetic. Many teachers have expressed a desire for material that can be used for that purpose.

The Child-Life Number Book conforms to modern standards and modern requirements. The authors have developed number concepts by easy reading material, mostly in story form, clothed in the child's language and based upon the child's experiences.

Special consideration has been given to reading difficulties. The vocabulary of this book conforms to vocabulary requirements in the Buckingham-Dolch Word List and very closely to all standard reading word lists for the children in this grade. The language of this new subject is gradually introduced to the pupil by use of simple story material in which, informally, the child is made familiar with new words and terms within his experience and comprehension. (Preface, p. iii)

On page 230 there are further comments upon the Word List. The comments follow:

The language in the Child-Life Number Book is so controlled as to make the reading material approximately easier than the pupil's reading book. This is accomplished

by (1) checking the vocabulary against the Buckingham-Dolch List and comparing with other authoritative Vocabulary studies; (2) making all sentences free from involved structure and using the child's own form of expression. The general rule followed in making the material easy to read limits the number of new words per page to three and the number of words in any sentence to thirteen. . . .

Of the complete Word List for the Child-Life Number Book, only 15 words are placed higher than the second grade in the Buckingham-Dolch List. Most of these 15 words are arithmetical terms, such as column, eighth, examples, yardstick, zero, and their occurrence in this book is a part of the preparation for the use of an arithmetic text in the third grade. Excluding all proper names, only 4 words are used which are not in the Dolch List. They are two-figure, week-days, lunch-room, pinwheels, all of which are combinations of familiar short words. The Buckingham-Dolch List also omits names of persons, days, months, and holidays, which must be necessarily included in an arithmetic. (p. 230)

Unlike many authors previous to 1938, the authors of this book have limited the content to the 100 addition and the 100 subtraction facts in the simplest method. They provide practice to help the pupil in acquiring skill, and give informal tests to aid the teacher in watching individual differences in achievement, but they do not require complete mastery of number facts or processes as have some of the authors in the texts included in the present study.

Other outstanding features of the Child-Life Number Book are: (1) the beautiful, four-color pictures which lend life and interest to the content material of the text; (2) the development of number concepts and number language

previous to the study of processes; (3) the teaching of the meaning of words used in quantitative thinking; and (4) the emphasis on meaning of number relation.

The book is divided into four parts. The first part includes stories which develop the number concept from one to twenty and number language such as more, less, first, second, over, under, small, small enough, smallest, tall, tall enough, not enough, almost enough, etc. The number concept from 20 to one hundred, numbers in their order, numbers on the clock, half hours on the clock, half dozen, figures, and money are included in the stories of the second part. Number language, such as how many, which is more, which is less, and some, is either introduced for the first time or else recalled and further developed. The process of addition and subtraction is introduced.

The stories of part three introduce the concept of signs for addition and subtraction, the concept of pint, quart, ruler, the missing number, and the two-figure sum. They further develop number language such as how many and more or less. The process of addition and subtraction is further developed.

The stories of the fourth part further develop the concept of measures such as seasons, money, yard, foot, and inch. The processes and judgment are emphasized in this

part. The topics are When to Subtract, Adding Three Numbers, Subtraction Problems, Subtracting without the Sign, Writing Columns to Add, A Quick Way to Add, Adding Long Columns, Adding by Endings, 100 Addition Facts, 100 Subtraction Facts.

On pages 236 to 239, inclusive, the authors have included an analysis of the arithmetical development of number facts and processes. There are a few page references (1) to indicate the manner of early introduction and early treatment of number facts and (2) to show the method of teaching and use of some of the special terms in arithmetic to express relation and quantitative comparison. The authors have indicated the topics that are to be taught and those that are for incidental use. The teaching precedes the examples. Some of the topics indicated for teaching will follow in the order as given in the analysis:

1. Number concepts for teaching.--The story, Going to the Park, develops the concept of numbers from 1 to 7, inclusive. Pictures on each page aid in the teaching of these concepts. The first page shows Jane asking her father to take three of them to the park. The story on the page is as follows:

The children wanted to go to the park. Jane asked her father to take them.

"How many girls want to go to the park?" asked Father.  
"There are three of us. Betty, Sue and I," said Jane.  
Father took the girls in his car. (p. 1)

The story continues on the following pages and relates what the children see on the way to the park. They see a dog that could do one trick. This introduces the meaning of one. The little girl counts, "One, two, three," for the dog to do his trick, thus introducing the concept one, two, three and the idea of counting. The children see four boys, and five puppies. The direction on this page is, "Count the puppies." At the playground the children see seven slides and eight swings. Thus the concepts of numbers from one to eight are introduced in this story. (pp. 2-6) Other stories develop in a similar manner the number concepts to one hundred.

2. Counting.--The social uses of counting may be illustrated by the story on page 10. There is a picture of children playing hide and seek. The boy is counting from one to ten. The other children are hiding. This story shows the social uses for counting in playing games. The last three sentences in the story are: "What are the children playing? Why did the boy count? Show how you count to ten." (p. 8)

The incidental use is shown by the following:

The children are playing parade; they have gone around three blocks. Judy invites them to her house and serves ice cream and cake. The last two sentences say: "There are ten

children. Can you find them?" (p. 17)

The counting by 2's, and 10's is to be taught. In the story, The Girls Help Jane, Betty tells Jane how to count "10 pennies by 2's," after having arranged them in groups of 2's. Then Jane counts as follows: "Two, four, six, eight, ten." Three directions at the end of the story call for experience with counting. They are: "Put some books on the table by 2's. Count the books by 2's. Can you count by 2's to 10?" (p. 44)

Pages 66, 67, 68 teach by story the new way to count to 100. Father shows the children how to put ten pennies in each pile and count them by saying ten, twenty, etc., to one hundred.

3. The ordinals first, second, etc., are introduced in the story of Milk at Lunch Time. The children are shown at the lunch table. The story is as follows:

Some of the children drink milk at lunch time.  
The first one at the left is Don.  
Find Don.  
The second one is Betty.  
The third one is Jane.  
The fourth one is Tom, and the fifth is Judy.  
Find Betty, Tom, Jane and Judy. (p. 18)

4. Money concepts are introduced on many of the pages in various situations. e. g., Counting the money in the box which contains nickels by saying 5¢, 10¢, etc. (p. 217)

Making change in buying and selling is taught by the



story of Billy's experience in the bakery in which he gives a one dollar bill for his purchase amounting to 10 cents, and the baker gives Billy the change in dimes and counts as follows: "20, 30, 40, 50, 60, 70, 80, 90, one dollar." (p. 70)

5. Number order.--The first example is taught by use of the story Going on a Trip. The train man calls the train, on which Sue is going, for gate 9. The gates seen in the picture are numbered 9, 10, 11, and 12. The last three questions and sentences relating to the picture are: "Can you find 9 over one gate? Some of the people are going to gate 10 for their train. What gate is between 10 and 12?" (p. 48)

Another example which calls for concrete experience with this text is the lesson entitled Page Numbers, page 73. It is as follows:

Jane looked at this page.  
She said, "This is page 73.  
The page before 73 is 72.  
The page after 73 is 74."

Betty wants to turn from page 73 to page 34.  
Should she look nearer the front or the end of the book?

The numbers come the same as when you count.  
Which of these pages come nearest the front of the book? 75, 38, 45, 16, 19

Which way is each of these pages from page 73? 69, 80, 17, 90, 58 (p. 73)

The above teaching activities are especially valuable when children begin to use books.

6. The meaning of dozen is introduced by the story about Sue, who counts oranges which she and Grandmother have brought home. Mother tells her she has a dozen. Questions which follow show the use of dozen in other purchases and the comparison of numbers.

What other things could Sue and Grandmother get by the dozen?

Which is more, a dozen or 15?

Could you buy more with 12 cents or with 8 cents?

How many eggs are 1 dozen eggs? (p. 51)

7. The meaning of the term half of a thing is taught by means of a story in which a little girl assists her mother in preparing breakfast and learns to cut the oranges into halves. The concluding statement of the lesson is: "The two halves make one orange." (p. 88)

The term half dozen is taught in a similar manner to that of the dozen. (p. 89)

The meaning of the term half hour is taught by the use of a story and pictures of clocks showing the half hours, pages 94 and 95. The story tells about Don looking at the clock and seeing the position of the long hand. Then the time is stated. On page 95, there is an explanation of the position of the long and short hands when the time is half past the hour.

8. Missing numbers.--On page 65 is the picture of a blackboard on which are numbers from 1 to 20 with circles

indicating numbers left out. The children are to say the missing numbers. (p. 65)

9. Addition is treated in the following subtopics: Concepts (what add and sum mean), introducing addition facts, the form in addition examples, adding three numbers, adding in a quick way, adding in long columns, adding by endings, and the 100 addition facts.

The concept, what add means, is developed on page 100 by the story, Play after School, in which Tom and Billy count the white and black kittens and Billy says: "You have 2 white kittens and 3 black kittens. How many kittens do you have?"

Tom answers, "I have 5 kittens. You can add them. 2 and 3 are 5. 3 and 2 are 5." (p. 100)

The addition facts, 2 and 3 are 5, and 3 and 2 are 5, are also introduced in the story just cited.

The form in addition examples is developed by use of a story of children making pinwheels. The picture accompanies the story. The story is as follows:

The children made pinwheels to play with. They made 5 blue ones and 4 red ones. How many pinwheels did they make? 5 and 4 are how many?

5                      Dorothy added the numbers. She said,  
 $\begin{array}{r} 5 \\ +4 \\ \hline \end{array}$       "5 and 4 are 9. We made 9 pinwheels."  
   5  
 We call  $\begin{array}{r} 5 \\ +4 \\ \hline 9 \end{array}$  an addition example.

The term, sum, is explained in a story about Bobby who adds his score in a snowball game. The statements, "He added 1 and 3 to find the sum. The sum is 4," introduce the word in a meaningful way.

The addition of three numbers in a column is introduced by a story about Jane's dolls. Then the form and explanation of the addition is given as follows:

	Write the numbers one under another.
3	Add down.
1	3 and 1 are 4.
2	4 and 2 are 6.
<u>6</u>	The sum is 6.
	Begin with 2 and add up.
	Is your sum the same?

How many dolls does Jane have? In a column the numbers are under one another.

Write these numbers in columns. 2-5-4      3-2-4 (p. 208)

The meaning of the word column is thus explained at the same time that the addition is explained.

Adding long columns is introduced on page 223 in a similar manner as that of addition of three numbers.

Adding by endings is introduced by a picture and story relating to school. The teacher gives the children a new way to add. After the children answer what 2 and 3 are, the teacher tells them to take any number ending in 2 and add 3 to it.

The 100 addition facts appear in abstract form on pages 226 and 277 and are for drill. The directions preceding

them are: "Read these examples. Then cover the sums. Read again and say each sum." (pp. 226-227)

10. Subtraction has the following sub-topics:

Concept (what subtraction means), introducing subtraction facts, the form in subtraction examples, and the 100 subtraction facts. These are treated in a similar manner. (pp. 107, 108, 109, 140, 228, and 229)

11. Knowing why we subtract is developed by the lesson When to Subtract, in which questions are introduced and answered, and other questions which are to be answered in terms of how many are left, how many more, how many less. At the conclusion of the lesson are the following statements:

Subtract to find how many are left.

Subtract to find how many more.

Subtract to find how many less. (pp. 190-191)

12. Making number stories.--There are four pictures on page 119. The pictures are numbered. There is a story about the first picture. This is followed by this direction: "Make a number story for each of the other pictures. Make the stories so that you must add." (p. 119) More practice in this is found on other pages of the text. This includes the practice in both addition and subtraction.

13. Testing.--All the testing is informal. It includes number concepts, relationships of size, direction, and abstract number drill.

On page 21 is a picture and the exercises which test the number concept and the ability to count. The story is about Betty who is helping her mother. Blanks are to be filled with the right number when the sentences are read. e. g., "Betty put \_\_\_\_ books on the little table. Betty put out \_\_\_\_ blue cups. She put \_\_\_\_ flowers on the table." These directions follow the sentences: "Name four things in your room. Take six books in your hands. Put five books on the table." (p. 21)

An informal test relating to the meaning of position, and direction appears on page 29, and is entitled, You Should Know These. The test follows:

Where is the top of this page?  
 Where is the bottom?  
 Where is the middle of this page?

Read the first line on this page.  
 Do you read from left to right?  
 Where is the left end of the line?  
 Where is the right end of it?

Can you show where these are?

The top of the window  
 The bottom of the window  
 The left side of the window  
 The right side of the window  
 The middle of the window  
 The top of a box  
 The bottom of a box  
 The top of the table (p. 29)

Other informal tests regarding relationship of measures, position, etc., are similar to those explained in the previous paragraphs.

14. Reading and writing of numbers and arithmetical signs  $\phi$ , --, =, and - are developed through stories and pictures in a similar manner to that of other topics of the text.

15. Measures.--Pint, quart, inch, foot, and yard are introduced by stories and concluded with directions for activities in which the children are asked to do the actual measuring of a pint and a quart, of inches and finding the marks on the yardstick. (pp. 153, 168, 169, 218)

The meaning and use of terms which express relationship are introduced either in stories or informal tests. These include such terms as: more or less, many, more, most, large, larger, largest, small, smaller, smallest, little, big, left, right, top, middle, bottom, long, longer, longest, short, shorter, shortest, tall, taller, tallest, high, higher, highest, low, lower, lowest, far, farther, farthest, near, nearer, nearest, old, older, oldest, fast and faster, enough, not enough, almost enough.

16. Time.--The topics regarding time, on the clock, and on the calendar, the days of the months and seasons are likewise introduced and developed by story and practice in the using of the resultant learning. (pp. 49, 50, 82, 83, 127, 146, 185)

Summary. The authors of the Child-Life Number Books

base them (1) on research findings relative to children's number knowledges previous to entering school, and (2) on the theory that their number knowledge is fragmentary and that they need some means of increasing their knowledge and organizing it before beginning the more formal study of arithmetic above the second grade.

The authors state that many teachers have expressed a desire for material that can be used for this purpose. Therefore, the authors feel that this book is suited to such use.

The outstanding characteristics of the book are: (1) controlled vocabulary, checked by the Buckingham-Dolch Word List; (2) the four-color pictures, which add life and interest; (3) the development of number concepts and number language previous to the study of number processes; (4) the teaching of the meaning of words used in quantitative thinking; (5) the presentation of the 100 addition facts and the 100 subtraction facts, objectively, in situations that are life-like and within the children's experience; (6) emphasis on the meaning of number relation; (7) the provision of activities in which the children may discover number facts and processes and better understand them; (8) the introduction of many of the social situations in which children have need for number; and (9) the attempt to coordinate the number



knowledge within school with actual number contacts in the children's experiences, such as playing, planning, shopping, constructing, telephoning, dialing the radio, and assisting with duties in the home.

#### XI. THE WONDERFUL WONDERS OF ONE-TWO-THREE

The author of this book is David Eugene Smith.<sup>11</sup> Its copyright is 1937. The publishing company is McFarlane, Warde, McFarlane, Inc. This book is not an arithmetic textbook, but it is an excellent source of basic supplementary information to use in a present-day schoolroom. It may be used in correlation with the social science activities and interests. Its information will lead (1) to an understanding of the origin of numbers, of the social use of numbers, of the extent or universality of number usage, of the similarity of number, and (2) to an appreciation that the present system of numbers has its basis in needs of the peoples of the oldest civilization; that all the modern buildings, trains, inventions, science, etc., would not have been possible without the knowledge of numbers; that there is a common bond of relationship between all peoples instead of

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<sup>11</sup> David Eugene Smith, The Wonderful Wonders of One-Two-Three (New York: McFarlane, Warde, McFarlane, Inc., 1937).

a difference or queerness; and that the need for numbers is the same for all people at all times, namely, to show a relationship, value, or measure. Therefore, the analysis of this book is included in this study.

The vocabulary is too difficult for first grade children and even for second grade children possessing average reading ability. But the teacher can lead the children to use the book, although they may be unable to read it in its entirety. A conference period in which the teacher and children talk together offers an opportunity for the teacher to read this book with the children. Only the portions of the book that appeal to both the interest and understanding of the children and the portions that help them answer their questions and wonderings, need be read and discussed.

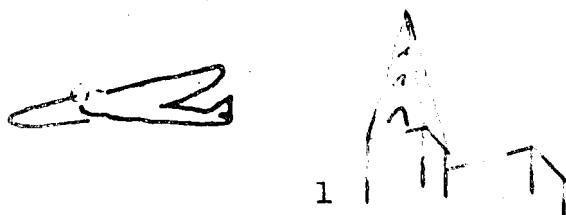
The illustrations assist in the understanding even though the pupils may be able to read only a small portion of the book without help. This is also an opportunity for the teacher to emphasize to the pupils that we do not always need to read an entire book, chapter, page, or paragraph in order to find an answer to some interest we have, but we often use only portions of the material. If this book is placed on the reading table or in some other place so that the pupils may have access to it, they will be able to make some use of the book in their own way.

The table of contents includes the following chapters:

- I. The Story of Numbers
- II. Playing Savage
- III. Where Our Written Numbers Came from
- IV. Fingers and Toes
- V. Roman Numerals and Place Value
- VI. What Is the Largest Number?
- VII. These Curious Numbers of Ours
- VIII. The Magic Square and Other Magic
- IX. Magic Squares and Circles
- X. Curious Sums and Curious Products
- XI. How Numbers Got Their Names
- XII. Nature Plays with Number

The use and value of the material in this book as a source of supplementary information in establishing a background for the understanding and appreciation of the social utility of number will be made clearer by a few reproductions and excerpts from the book.

The reproductions of pages 2 and 3 follow on the next two pages.

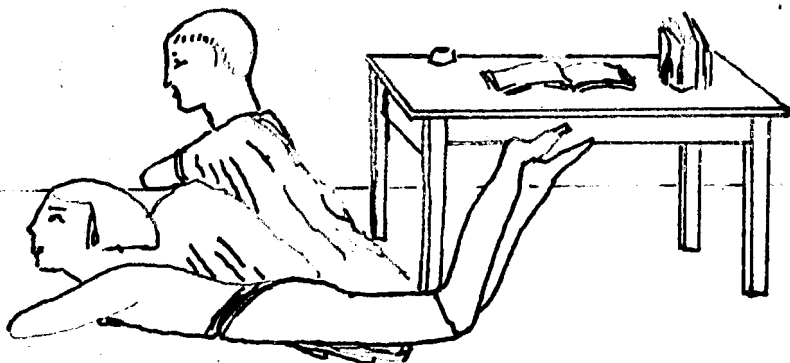
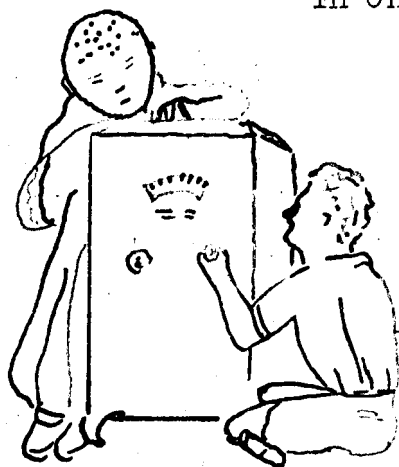



## THE STORY OF NUMBERS

You have all heard baby brothers and sisters learning to count: one, two, three. You all learned to count yourselves, and, when you learned to count you began by saying, one, two, three. But did you ever stop to think that there was a time when there was no one, two, three, or at least when numbers did not have names?

This book will tell you how numbers first came to be used and how they got their names. It will also tell you many strange things about numbers in many strange lands of long ago. It will also tell you how much alike the boys and girls in different lands find the problems of numbers here and now. You will see how John and Josie write their one-two-threes on the blackboard in their country, and how Gupta in India and Chen in China also write their one-two-threes.

2



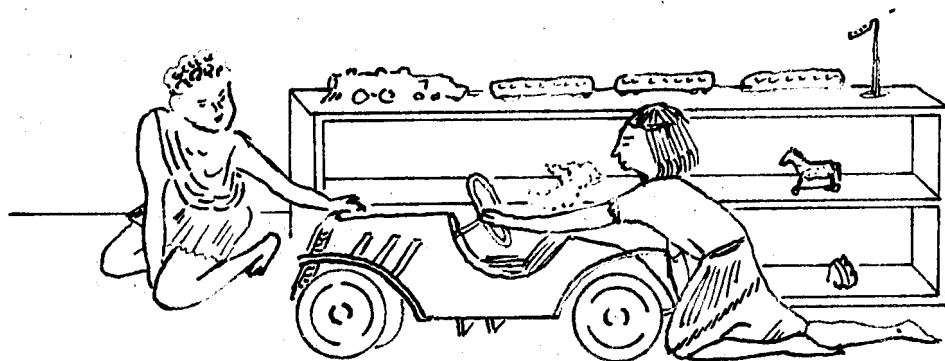


If people had not learned of the wonders of numbers, we would not have automobiles, or trains, or radios, or skyscrapers today.

Why do we say: "One, two, three and out?" Or, "One, two, three, go?" When we play hide-and-peek, why do we count by fives or tens? You will learn why we do these things. Every day you use numbers in some certain way, because for thousands of years people have been using some kind of numbers, since the time when the world was very young. This book will tell you how the cave men and their children came to use numbers. You will find out what you owe to India, Egypt, and China of five thousand years ago.

You will not only learn the wonderful wonders of numbers in this book, but you will learn how to have fun with them. You will learn how to solve number puzzles and how to multiply  $28 \times 15,873$  in your head as easily as you can add  $1 + 1$ .

3



It must be remembered that it is the teacher's part to explain sufficiently about the people of India, China, Egypt, and other countries, so that the children may better understand and appreciate the origin and use of numbers. The reference to the learning how to multiply " $28 \times 15,873$  in your head as easily as you can add  $1 + 1$ ," is not important to the understanding and appreciation of numbers and need not be emphasized. What is true of this reference is true of other references. Therefore, the value of this book as a source of information in the supplying of a general background for the understanding of number in its relation to various fields of knowledge and social activities of everyday living, depends upon the way in which the teacher uses the material of the book to enrich the understanding.

The following from the chapter entitled Playing Savage, if rightly used, will create an interest in how counting and writing of numbers originated. Portions from the chapter follow:

How did the world come to know numbers and how to write them? First, let us play that we are savages and see how we would go about the business of counting and writing numbers.

If you were a cage man's child, would you ever need to know what a million is, or a thousand, or even a hundred? Probably not. You would see that you had some fingers and toes; that you had more fingers than hands, and perhaps you would see that you had as many toes as

fingers. You would never see money or even hear of it. You would not know the number of hours in a day, or how to count twenty chickens. You would not have any use for the names of numbers nor would you need to know how to write them. You might make notches on a stick, or cut marks on a rock, or use pebbles to stand for three or four.

Our familiar numbers have many strange stories to tell us. Let us see what some of them are. Here are a few:

One is the beginning.

Two is next. . . .

It took the world many, many hundreds of years to think of the word "two" except in speaking of two things. For a long time words like "two," "three," or "four" without some other word like "men," "horses," or "houses" following them, were unknown, and to count "one, two, three, four" alone, was not thought of.

Many hundreds or even thousands of years must have passed from the time when early people cut notches on trees before numbers came to have signs that could be written as we write 1, 2, 3, 4, and so on. It will be interesting for you to learn how these characters came to have their present meaning. (pp. 5, 6, 7, 8)

The next chapter tells where our present numerals came from. Page 12 has an interesting illustration of the various numerals, accompanied by the explanation. The reproduction follows:

Our numerals

1      2      3      4      5      6      7      8      9

Later numerals used in India

१      २      ३      ४      ५      ६      ७      ८      ९

Boys and girls in Arabia use these figures.

۱      ۲      ۳      ۴      ۵      ۶      ۷      ۸      ۹

Oldest European numerals, Spain, 976

I      ۲      ۳      ۴      ۵      ۶      ۷      ۸      ۹

Chen uses these in China.

—      =      ≡      ≡      E      六      七      八      九

The Egyptians of 5000 years ago used these.

I      II      III      IIII      IIII      IIIII      IIIIII      IIIIII      IIIIII

These were used in Greece about 2000 years ago.

A      B      Γ      Δ      E      F      Z      H      Θ

These are Hebrew numerals.

א      ב      ג      ד      ה      ו      ז      ח      ט



The next chapter, Fingers and Toes, includes the interesting discussion of why we count by tens instead of nine, or seven, or eleven, or any other convenient number. The author gives an interesting line as answer to why we do this and then continues to discuss it more in detail. The line is as follows: "The answer is found in our fingers and toes." (p. 13)

The chapter, What Is the Largest Number, is as full of interest as the preceding one. A few paragraphs follow:

They are wonderful things these one, two and three, and they grow more wonderful as we count to the larger numbers.

Josie in America wonders what is the largest number in the world, and Chen in China and Gupta in India wonder, too. So do the children in many other lands. The wonderful thing is that there is no number that is the largest. If you write the largest number you can think of, there is still a larger one, because all you need to do is to add 1 to your largest number and you will have a larger one. It is the same with time. If you think of a thousand years as the longest time, just add another year and it will be longer.

Numbers are like time and space, they have no end. Who can say what is the greatest distance in space or the longest time?

We may know of the largest diamond that has ever been found in the world. We know the name of the world's largest ship and dirigible, and its largest building. But there is no largest number and there never will be. This is one of the wonders--but only one. (pp. 21-22)

The remaining chapters are equally as interesting

and valuable for use in supplying information and enrichment for a general background knowledge of number relationship in life.

Again it is well to emphasize the fact that this book is not an arithmetic textbook, but it is an excellent source of information to use in teaching children the origin of numbers and in leading them to an appreciation of the social usage of number in the present, everyday living of which he is a part. Although the vocabulary of the book is too difficult for first and second grade children, it has the following possible uses: (1) it may be placed where the children can have the opportunity to see the illustrations and read portions of it; (2) the teacher may read and talk about the parts of the book which answer questions which the children may ask regarding numbers.

## CHAPTER V

### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

#### I. SUMMARY OF ANALYSES

The arithmetic textbooks of Warren Colburn, Roswell C. Smith, and Joseph Ray represented the active period of formalized Pestalozzian education, from 1821 to 1857. The books were based on the theory: (1) that the mind could be developed and strengthened by mental drill to the degree that learning in all subject matter would be simplified; (2) that every new combination should be taught by the "natural method"-that of presenting first the visible objects, second the names of objects, and third the abstract numbers. To aid in the mental drill, reference was made to the use of counters (concrete objects) such as beans, pebbles, and even fingers, and to the use of illustrations and tables based on the Pestalozzian tables (plates). A set form of analysis of problems and reasons for the answer were required. The analysis and answer were to be memorized.

The books by J. H. French and E. E. White represented the static period of formalized education from 1857 to about 1892. These books revealed the same general plan as did those of Colburn, Smith, and Ray. They differed in the

following characteristics:

Each author included many pictures ("visible objects") to be used as the first step in presenting new number combinations. French emphasized the use of counting boards and tables as valuable aids to the teacher in teaching arithmetic and to the children in learning it. He included pictures of various occupations in order to interest children in the business affairs. French emphasized the fact that the success of the primary teacher depended upon the manner in which she used the textbook and upon the spirit and zeal manifested in her teaching of the subject.

The books of John H. Walsh, George Wentworth, and David Eugene Smith (co-authors), and John C. Stone represented the revival period of education from 1895 to 1933 in which there was a revolt against the formalized drill-type of education. The Herbartian theory of supplying content, the James theory of the impossibility of improving the memory and strengthening the mind by mere meaningless drill, coupled with the investigations regarding the practical uses of number in business, led authors to attempt to adapt arithmetic material and processes to the needs of the child in meeting future practical situations. The aim was to give sufficient practice in the processes which it seemed would be necessary in meeting future practical needs.

Therefore some of the topics, considered obsolete in business, were omitted. Practice and drill on abstract processes necessary in business computation were provided through artificially planned and motivated projects.

The books of Clifford Brewster Upton, Charles E. Garner and Livingstone McCartney (co-authors), and David Eugene Smith represent the modern social utility theory of education from 1933 to 1938, inclusive. The first two authors seem to have attempted to conform to the new psychological theory that it is a waste of time and useless drudgery to teach the child things he does not understand and for which he is unprepared; and that concepts and meaning are basic for living in the everchanging social culture. Therefore these books were based on: (1) research studies related to children's social usage of numbers, (2) a carefully checked and controlled vocabulary, and (3) children's interests and activities.

Smith provided the type of book that would furnish enrichment to the children's concepts and understanding of number and number relationships, and an appreciation of its use in all nations of all times.

## II. CONCLUSIONS

The summary suggests the following conclusions:

1. The authors of arithmetic textbooks have been sensitive to the changing aims of education and have attempted to provide textbooks in harmony with those aims.

2. There has been a tendency on the part of the recent authors to base their textbooks on: (1) research in the fields of education and of subject matter, (2) satisfactory results of materials previously tested in the classroom, (3) demands of teachers for suitable types of number material, (4) local and general social needs of society, and (5) criticisms and suggestions of classroom teachers, supervisors, and administrators in the field of education

3. Arithmetic textbook materials are artificial means for furnishing number experiences and do not meet varying local situations. Therefore it is impossible for authors to write textbooks which will completely satisfy every schoolroom need. The texts can be used only as guides in the teaching of arithmetic.

4. The unsatisfactory results of arithmetic teaching cannot justly be placed upon the authors. Teachers have failed to take cognizance of the realistic life problems in the evolving social culture, and they have failed to use the textbook as a supplementary source of information in the actual life situations or vicarious experiences. Teachers on the whole have not realized (1) that it is

important to know something about the pupil's concepts and understanding of number at every stage of instruction; and (2) that learning can be expressed only in terms of preestablished concepts. For example, the concept of the number five as a serial relationship, is quite different from the meaning of five as a result of addition or subtraction, as a page number, as a measure of time on the clock, or calendar, as a length on the ruler, as a number in every decade, and as a number included in every phase of his actual daily living.

### III. RECOMMENDATIONS

Since there is so little known about what concepts and understandings of number and number relationships children actually have, the following recommendations seem timely:

1. That teachers make a careful study of the concepts and meanings of number and number relationships established by individual children in formal and informal number situations; and that the teachers record and use this information as the basis for the type and amount of number experiences suitable to particular grade levels of learning.
2. That teachers be very alert to the vast amount

of everyday number situations in the environment; and that they utilize sources of first hand experiences from which children may form their concepts of number and learn its meaning in its various relationships.

3. That teachers use the textbooks and other materials pertaining to number as secondary and supplementary sources to life-like experiences and information to enrich the meaning of number in the ever changing social culture of which the children are a part.



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