AN EXPERIMENTAL COMPARISON OF TWO METHODS

OF TEACHING NINTH YEAR ALGEBRA

by

Grace M. McBeth

Contributions of the Graduate School
Indiana State Teachers College
Number 347

Submitted in Partial Fulfillment
of the Requirements for the
Master of Science Degree
in Education

1938

The thesis of _____ Grace M. McBeth _____,

Contribution of the Graduate School, Indiana State

Teachers College, Number _347, under the title ___

_____ An Experimental Comparison of Two Methods _____

_____ of Teaching Ninth Year Algebra _____

_____

is hereby approved as counting toward the completion

of the Master's degree in the amount of _8_ hour's

credit.

    Committee on thesis:

E. E. Ramsey

E L Abell

_O. E. Strong_____, Chairman

Date of Acceptance ___ July 29, 1938 _____

TABLE OF CONTENTS

INTRODUCTION

LIST OF TABLES

LIST OF GRAPHS

INTRODUCTION

High school mathematics, both in content and in method of teaching, has in the past few years undergone a storm of criticism. Among the contentions which have been advanced is that the large number of failures each year in high school algebra indicates either defective teaching methods or too rigid requirements in subject matter.

It has long been recognized that students learn at different rates and in different manners, and there has been a growing feeling that the traditional or daily recitation method of teaching does not provide ample opportunity for the differentiation of subject matter and instruction to suit the varying needs in any group of students.

In the light of this criticism it seemed a desirable project to ascertain by actual experimentation which of two outstanding methods of instruction in algebra would yield the better results if applied to a heterogeneous group in a small high school. It was felt that this thesis would have justified itself if it succeeded in accomplishing this purpose; and it was hoped also that it might give to teachers in the field a deeper interest in various methods of teaching and a little more inspiration to experiment, not by adhering to traditional material and method, but rather by teaching algebra as a mode of thought, as the basis of science and

engineering, as a means of understanding every day facts,'
and as a device for character building.

CHAPTER I

THE EXPERIMENT AND DEFINITIONS OF TERMS USED

Statement of the experiment. It was the purpose of
this experiment to determine whether the traditional recita-
tion plan or the modified unit plan of instruction would
yield greater returns when used with a heterogeneous group
in a small high school; the results to be measured (1) "on
the side of algebra, the ability to understand its language
and use it intelligently, the ability to analyze a problem,
to formulate it mathematically and to interpret the results,"[1]
(2) as a social subject, to make the world better by develop-
ing ideals of honesty, accuracy, industry, self-reliance,
persistency, and an enthusiasm for knowledge.

Importance of the study. Algebra is recognized as a
traditional subject in the secondary school. Probably be-
cause of this factor, few teachers of the subject have given
evidence of real interest in exploring some of the newer con-
cepts of educational methods. It is true that occasionally
one finds teachers experimenting with new methods, but for

---

[1]The Reorganization of Mathematics in Secondary Educa-
tion. A Report of the National Committee on Mathematical Re-
quirements. Under the auspices of The Mathematical Associa-
tion of America Inc. 1923, p. 11.

the most part algebra remains a traditional subject taught
in the traditional manner.

Effective teaching in any subject is dependent at
least in a measure upon the use of effective methods. It is
a common belief that the benefits which pupils derive from
any course depend in part on the ability and the personality
of the teacher, in part on the attitudes and abilities of
the pupils, but to a larger extent on the teaching methods.
In this connection Charles H. Judd says:

> The most promising subject in the curriculum can be
> turned into a formal and intellectually stagnant drill
> if it is presented by a teacher who has no breadth of
> outlook and no desire to teach pupils how to general-
> ize experiences.
>
> It is not far from the truth to assert that any sub-
> ject taught with a view to training pupils in methods
> of generalization is highly useful as a source of mental
> training, and that any subject which emphasizes particu-
> lar items of knowledge, and does not stimulate general-
> ization is educationally barren.[2]

With this broader view of teaching algebra in mind, and the
adoption of new methods, the present storm against the sub-
ject may be diverted.

While the present day secondary school has reduced the
required amount of mathematics to the minimum of one year in
many places, and many elective courses have been abandoned

---

[2]Charles H. Judd, _Psychology of Secondary Education_
(Boston: Ginn and Company, 1927), p. 422.

because of lack of interest in the subject on the part of'
pupils, we are still cognizant of the fact that all pupils
need some training in mathematical thinking. J. P. Everett,
writing on "The Fundamental Skills of Algebra" says:

> The pupil needs algebra for what it enables him to
> do but infinitely more does he need it for the orderly
> way in which in the midst of constant change, it enables
> him to think.[3]

A subject of this importance should not be open to criticism
or shunned as an elective subject if better methods of in-
struction will make it more effective and less distasteful to
the students.

Certain units of algebra are required for college
entrance, and because of that fact all pupils--particularly
in our smaller schools--are required to take the subject
despite the fact that the majority will not go to college.
Fear of failure has brought about a general dislike of the
subject, and this situation has brought us face to face with
the fact that some experimentation is necessary if this
attitude is to be changed. Writing on the subject, "The Gap
Between Promise and Fulfillment in Ninth Grade Algebra",
E. F. Lundquist said:

> A significant proportion of high school pupils are
> by reason of mental ability, previous training, and

---

[3]J. P. Everett, "The Fundamental Skills of Algebra",
Teachers College Record, Columbia University. 1929, p. 3.

present motivation, incapable of deriving enough value
from ninth grade algebra as it is being taught to justify
its being required of all pupils.[4]

"As it is being taught" seems to add emphasis to the idea of

many educators that our present methods leave much to be de-

sired. If there is to be improvement in our methods of in-

struction, more teachers in the field must be aroused to the

experimental program necessary. There must be enrichment of

teaching technique in order to create a revived interest, and

greater accomplishment on the part of the pupil. More effi-

cient schools should be the outcome of numerous studies of

this kind.

Definitions and discussion of terms. The terms used

in this thesis are "Recitation Method" and "Unit Method".

By the recitation method we refer to that mode of teaching

based on the Herbartian plan with which all teachers are

familiar, namely: preparation, presentation, comparison,

abstraction, generalization, and application. Morrison de-

scribes the ordinary recitation as follows:

> Common practice is to treat either the contents of a
> book or a syllabus of a course of study as the learning
> product to be achieved....the process, by the action of
> problems, or by other practices which tend to stimulate
> reflection, but in the end learning is simply a process

---

[4]E. F. Lundquist, "The Gap Between Promise and Ful-
fillment in Ninth Grade Algebra," School Review, December,
1934, pp. 762-771.

of covering a given body of narrative, descriptive, or exposition discourse, or of solving a given series of problems....The ground once covered is reviewed and the recitation form of testing the absorption of material is summarized in another test of the same sort and the same order, known as the examination.[5]

Thayer gives us a brief history of the evolution of methods of teaching. He warns us that although the individual system of teaching is older than the recitation method it must not be confused with our present method.

It was spoken of by Grimshaw in 1855 as "the old-fashioned and false method of teaching individuals instead of classes". He pictured it thus: "I notice in my visits to the schools many pupils sitting idle; sometimes part of the school is asleep, or what is worse, making a noise and disturbing the remainder who desire to be industrious".[6]

Thayer tells us that this system was replaced, a little more than a century ago, by a system of group teaching, which was originally devised by Joseph Lancaster. It was known as the monotorial system; and to it, Thayer tells us, we must give major credit for originating and developing the fundamental structure of the recitation. Of course, this structure has been greatly modified since that time; but the essential features are the same, and for such a procedure the activity and initiative are necessarily centered in the person of the teacher.

---

[5] H. C. Morrison, The Practice of Teaching in Secondary Schools (Chicago: University of Chicago Press, 1926), p. 37.

[6] V. T. Thayer, The Passing of the Recitation (Chicago: D. C. Heath and Co., 1928), pp. 1-12.

Now these ideas are in the process of change, and today a great deal is heard about the "child-centered school", and the teacher as "the guide". This has brought about the advancement of new methods of teaching to replace the traditional method. We must not lose the importance of the fact that

> No one acquainted with American education can seriously question the valuable contributions of the recitation method of instruction. Except for it....we should probably not have universal public education in this country. Consequently we should give all honor to the educational leaders who perfected it as a method of teaching and incorporated it as an integral part of their school organizations. They solved effectively the child educational problems of their day and laid the basis of future progress.[7]

When we have given due credit to this view, we still have to face the problems peculiar to our day. We must ask the question, "Is the recitation method best adapted for present needs?" What is valuable at one stage of development is frequently injurious in another. It is always worth while to raise the question of the suitability of an old tool for the realization of new purposes. It is for this reason that we wish to inquire whether the traditional "lock-step" method of teaching, with its daily assignments of like tasks for all pupils, is adequately designed to realize present day educational aims and ideals.

---

[7]*Ibid.*, p. 12.

There have been a number of definitions given of the unit method. Byrne J. Horton, writing in <u>Educational Method</u>, tells us that Henry C. Morrison, Professor of Education in the University of Chicago, who is responsible for the first treatise on the unit plan defines it as, "...a comprehensive and significant aspect of the environment of an organized science, of an art, or of conduct, which being learned, results in an adaption in personality".[8]

W. H. Burton[9] calls the unit of learning "an outcome". He further lists as the nature of units or outcomes: first, attitudes of understanding; second, attitudes of appreciation; third, special abilities. He states further that while many think the subject matter <u>is</u> the unit, it is really the assimilative material out of which the unit grows.

On the other hand, Reudiger[10] of George Washington University defines a unit as "any division of subject matter, large or small, that when mastered gives one an insight into and appreciation of, or mastery of some aspect of life".

---

[8]Byrne J. Horton, "A Brief Outline of the Unit Plan of Teaching," <u>Educational Method</u>, XII (December, 1932), pp. 75-77.

[9]W. H. Burton, "Unit Concept in Learning; an Attempt at a Simple Explanation," <u>Educational Outlook</u>, VII (May, 1933), pp. 206-213.

[10]Wm. C. Reudiger, "The Learning Unit," <u>School Review</u>, March, 1932, pp. 176-81.

An editorial[11] which reviewed the investigations of
Roy O. Billett, of Illinois State Normal University and a
member of the Staff of the National Survey of Secondary Edu-
cation, reported that of all the plans to provide for in-
dividual differences, the unit plan is the most widely dis-
tributed and the most widely used.

Billett himself tells of his investigations.[12] He says
that the unit plan is a prominent characteristic of at least
eleven plans in general use today, among which are the con-
tract plan, the Dalton plan, the laboratory plan, the project
method, the Morrison plan, and the Winnetka technique. He
found that in actual practice these methods "differ in name
only", irrespective of what the originators of these pro-
cedures may contend as to the differences represented. In
actual practice, the procedures are practically identical.

The purposes of education are, in general, the pur-
poses of the unit plan; namely, to attain "that stage at
which the student has realized the meaning and purpose of
study, has acquired the self-control which self-dependence
implies, and has further acquired the range of methods of

---

[11]Editorial, "The Unit Assignment in Actual Practice,"
School Review, XL (May, 1932), pp. 321-23.

[12]Roy O. Billett, "Directed Learning and the Unit
Assignment". Department of Secondary School Principals,
Bulletin 45, March, 1933, pp. 55-76.

thinking and of study which removes him from constant dependence on the teacher".[13]

The big objective in applying the unit plan to the teaching of algebra, according to E. Clarke Fontaine,[14] State Supervisor of High Schools in Maryland, is the understanding attitude leading the pupils to see _why_ rather than teaching them to do mechanical tricks in manipulation of algebraic symbols.

With these objectives in mind, it is necessary to give particular attention to the teacher's part in carrying out this plan. The instructor is dealing with learners in groups and these individuals differ widely in all the factors of learning experience. According to Grinstead:

> The teaching unit is merely a learning unit....in which the teacher assists the pupil in the attainment of his objectives. It is a cooperative enterprise in which the pupil....constitutes the center, the teacher functioning primarily as an expert adviser. The teacher's part is of no value whatever except as his help is effectively given. The goal, the learning activities, and the mastery are still the pupil's.[15]

[13]Morrison, op. cit., pp. 211-212.

[14]E. Clarke Fontaine, "The Unit Plan of Teaching," _Educational Administration and Supervision_,XVIII (January, 1932).

[15]Wren Jones Grinstead, "The Unit of Learning, Its Meaning and Principles," _Educational Outlook_, VII (November, 1932), pp. 9-20.

Finally we will define the unit plan as used in this experiment as that plan devised by the teacher in which:

1. The classroom becomes the work room
2. The pupil receives a study sheet containing:
    a. Topic references
    b. Page references
    c. Explanations and explanatory references
    d. References for practice outside the text
    e. Problems to be worked from the text
3. The pupil works out his unit at his own rate, individually or in a group of his own choosing
4. The pupil's mastery of the subject is tested by unit tests, and by final tests provided by the state.

The teacher's place in the plan is to:

1. Maintain favorable conditions for study in the room
2. Enlarge upon the assignment
3. Stimulate interest
4. Direct and supervise the work
5. Give special attention to the slow and backward
6. Administer tests at the request of the student.

In short the plan is one in which the pupil takes the initiative rather than the teacher, as is the usual procedure in the traditional recitation.

So far as educators have expressed themselves, their approval seems unanimous for the unit method. While many different reasons have been given, the most emphasis has been on these facts: that first, it will care for individual differences; second, it will place the emphasis on the pupil rather than on the teacher; third, it will increase interest on the part of the pupil by allowing him to advance according to his ability without being compared too critically with others of greater ability or being retarded by others of less ability.

CHAPTER II

REVIEW OF PREVIOUS STUDIES

Although there has been much discussion of various methods of teaching, comparatively little experimentation has been done in the field of algebra. A few of the more recent experiments, similar in nature to this one, will be reviewed briefly.

Barton,[1] studying the effect of group activity and individual effort in developing the ability to solve problems, experimented with two groups of eleven pupils each, for the short period of twenty-eight days. His conclusions were that, for pupils of normal or above normal intelligence, the group discussion was superior.

Smith[2] experimented with two groups, based on age, sex, and I. Q. The control group received instruction and assignments daily, while the experimental group received mimeographed assignments covering work for several days. After thirty-two weeks experimentation, he concluded that the con-

---

[1]W. A. Barton, "The Effect of Group Activity and Individual Effort in Developing Ability to Solve Problems in First Year Algebra," Educational Administration and Supervision, 12:512-519, November, 1926.

[2]O. O. Smith, "The Relative Efficiency of Two Methods of Instruction in Algebra," (unpublished Master's thesis, University of Kansas, 1927).

trol group achieved more, judging from test results. He '
attributed the difference to administration difficulties, and
to the teacher's lack of experience in individual instruction
technique. This conclusion seemed to indicate that the re-
sults of the experiment were not completely convincing to the
writer.

Mason[3] measured the difference in results between con-
tract assignments and recitation method in English, civics,
American history, seventh grade arithmetic, biology, and
first year algebra. He used two groups of eleven pupils each,
pairing on the basis of I. Q. and Hotz algebra scales. He
used the same scale at the end of the experiment as an end
test of mastery. He reached the conclusion that pupils
taught by the contract plan made greater achievement; the most
consistent gain being made by the middle fifty per cent of the
contract group.

Moore[4] made a similar comparison and decided that
neither method was significantly superior for the especially
bright or slow pupil though he made no report as to the rel-

[3]H. C. Mason, "A Comparison of Contract Assignment and
Recitation as Measured by Objective Tests," (unpublished Mas-
ter's thesis, University of Minnesota, 1930).

[4]R. U. Moore, "An Experimental Comparison Between the
Relative Effectiveness of the Individual Method in Mathematics,"
The High School, 6:86-89, February, 1929.

ative value as it concerned the middle group.

Lawton[5] reported the recitation plan better with less capable students, and the supervised study method better with able students.

Hunsiker[6] found negligible differences in favor of larger unit assignments in algebra.

R. E. Gadske,[7] on the other hand, using larger groups than any of the others, compared the unit system with the recitation system by testing with standardized tests and found a decided difference in favor of the individualized unit system. He used forty-six pupils equated on the basis of three standardized tests. The control group was taught by the lecture-demonstration method followed by tests and remedial instruction. The experimental group was taught by the unit method, individual assignments, and individual instruction. Achievement was measured by standardized tests. Mr. Gadske concluded:

---

[5]Robert C. Lawton, "Recitation Plan vs. Supervised Study," (unpublished Master's thesis, Boston University, 1936).

[6]C. W. Hunsiker, "A Comparison of Relative Effectiveness of a Plan of Supervised Study and Daily Recitation Method in Teaching Algebra and Geometry," (Master's thesis, University of Minnesota, 1933).

[7]R. E. Gadske, "A Comparison of Two Methods of Teaching First Year High School Algebra," School Science and Mathematics, XXXIII (June, 1933), pp. 635-640.

This conclusion is far from being a complete solution of all the relevant problems. It is simply one of many that must be made in order to reach a final conclusion .... Nevertheless, it seems apparent that some importance may be attached to the findings, particularly since the individual unit method has turned out so superior to the traditional group method.

Burton J. Stallard and Harl Douglass[8] made a rather extensive study of this nature in Wauwatosa, Wisconsin. They found a gain of 197 points as against 170 points in favor of the long unit groups. They discovered also, that the superiority of achievement of the exceptional group, from test to test, showed greater retention in those using the individual unit. In this experiment, the lower quartile of both groups had higher achievement according to ability than the upper quartile. Finally, they concluded in favor of the individual unit plan because it saved time for the teacher in explaining units; saved time for the gifted pupil; was adapted to any school without reorganization; and resulted in better relationship between teacher and pupil.

James V. Gainer[9] failed to prove statistically that

---

[8]Burton J. Stallard and Harl Douglass, "An Experimental Study of Two Plans of Supervised Study in First Year Algebra," Journal of Experimental Education, September, 1935.

[9]James V. Gainer, "Experimental Investigation into the Relative Difference in Two Plans of Teaching First Year Algebra," (unpublished Master's thesis, University of West Virginia, 1933).

there was any difference between the two methods, but agre'ed with Stallard, Gadske, and others that there was enough promise in the experiment to warrant a great deal more experimentation along this line.

Marguerite Linn[10] in her experiment took the broad view of the modern educator. In addition to the mastery of algebraic concepts, she studied the effect of the two methods on the character and attitudes of the pupils. As the measured test of achievement she used the results of standardized tests, and found in this little if any difference in the two methods. However, in her conclusions she stressed the fact that in the matter of attitudes and character building there was a decided difference in favor of the unit method.

This review of previous studies on the relative value of the unit plan and the recitation plan shows that there is much indecision. This very fact of indecision and disagreement found in reports of other experiments shows it to be an open field for further experimentation.

---

[10]Marguerite Linn, "An Experimental Comparison of Two Methods of Teaching Elementary Algebra," (unpublished Master's thesis, University of Southern California, 1934).

CHAPTER III

DESCRIPTION OF THE EXPERIMENT

This experiment was carried on with two groups of ninth year algebra students in the high school at Wingate, Indiana. Due to an administrative problem, these groups were maintained as the same heterogeneous groups which were promoted from the eighth grade. As a practical experiment this grouping was desirable because it represented the exact situation found in the small high school where the state requires all students graduating from a commissioned high school to have credit in one unit of mathematics. This requirement forces an unselected group into ninth grade algebra since the faculty in the small school is too limited to permit of an elective unit in mathematics. Heterogeneous grouping offers no disadvantage according to Luthur Purdom,[1] special educational investigator, who after investigating 442 high schools in Michigan, in algebra and English, reported that there was no significant advantage in homogeneous groups.

Data[2] accumulated at Palo Alto high school confirms

_____

[1]Luthur T. Purdom, Value of Homogeneous Grouping. Investigation for Warwick and York, Inc., Baltimore, 1929.

[2]Theron R. McCuen, "Predicting Success in Algebra," Journal of Educational Research, January, 1930, pp. 72-74.

what educators have contended for some time, that the I. Q.
as a determinant for grouping, is better than any other to
predict probable success in algebra. With this in mind, dur-
ing the first week of the term, Otis Self-Administering In-
termediate test, Form B was given. The results obtained
from this test were averaged with the results of Form A,
which had been given at the end of the eighth year. On the
basis of these averages, the eighteen pupils in each group
were ranged. In the control group, known hereafter as Group
A, the range was from 73 to 121. In the experimental group,
known hereafter as Group B, the range was from 70 to 120.
Following the method of Sorenson and Garrett, the former of
whom wrote:

> In most cases, the mean is the summary value generally
> accepted as the most accurate measure about which to cal-
> culate the measure of variability.[3]

the mean of each group was calculated.[4] In order to find the
reliability of the difference between the two groups in terms
of "chances that the obtained difference represents a true
difference greater than zero," the formulas of Garrett were
used. The mean I. Q. of Group A was 100.1 and of Group B,

---

[3]Herbert Sorenson, Statistics for Students of Psycholo-
gy and Education (New York: McGraw-Hill Book Co., 1936), p. 133.

[4]H. E. Garrett, Statistics in Psychology and Education
(New York: Longmans, Green and Co., 1926), pp. 127-35.

99.7. The reliability of the difference (.4) was found to be .02, which signified that the chances were 51 out of 100 that the true difference would be greater than zero. This is not considered a significant difference.[5] Table I and graph I (pp. 19-20) show the I. Q. of each member of the groups and the graphical picture of their comparative abilities. It is to be noticed that though, in general, Group A has a slightly higher mental ability, yet the line of Group B drops below the control group at the lower end of the scale, indicating that this is the group to which special attention should be given.

During the first five weeks of the term both classes were taught by the recitation method. At the beginning of the sixth week, Group B began instruction by the unit method, while Group A continued with the recitation method. This plan was carried on throughout the remainder of the school year.

At the beginning of the experiment, the Hotz Algebra Scale, Form B, was given to each member of both groups as a base from which to measure accomplishment. (Table II, p. 21)

At the beginning of the experiment, the plan was explained to both groups. Group B then received the instruction

---

[5]See Appendix.

TABLE I

INTELLIGENCE QUOTIENTS
OTIS SELF-ADMINISTERING TEST OF MENTAL ABILITY
INTERMEDIATE EXAMINATION--FORM A AND B

	GROUP A				GROUP B		
Number	Otis A	Otis B	Average I.Q.	Number	Otis A	Otis B	Average I.Q.
1.	116	127	121	1.	121	119	120
2.	123	111	117	2.	116	118	117
3.	120	112	116	3.	119	110	114
4.	109	118	113.5	4.	111	113	112
5.	100	118	109	5.	117	103	110
6.	109	103	106	6.	109	111	110
7.	97	107	102	7.	107	106	106
8.	103	97	100	8.	116	96	106
9.	105	93	99	9.	96	116	106
10.	100	96	98	10.	109	101	105
11.	106	88	97	11.	98	112	105
12.	100	88	94	12.	113	87	100
13.	96	92	94	13.	90	102	96
14.	90	98	94	14.	85	95	90
15.	88	94	91	15.	91	83	87
16.	108	72	90	16.	69	80	74
17.	91	83	87	17.	62	78	70
18.	76	70	73	18.	59	81	70

Median	98.5	105.5
Mean	100.1	99.7
S. D.	12.4	15.2
Diff. Mean	.4	
σM.	4.9	6.8
σD	8.48	
C. R.	.04	

C. R. .04 means that there are 51 chances in 100 that
the true difference is more than zero.

GRAPH I

AVERAGE INTELLIGENCE QUOTIENTS
OTIS SELF-ADMINISTERING TEST OF MENTAL ABILITY
INTERMEDIATE EXAMINATIONS A AND B

TABLE II

FIRST SCORES ON HOTZ ALGEBRA SCALE FORM B

	GROUP A					GROUP B			
Number	A.S.	M.D.	Eq.	F. Prob.	Number	A.S.	M.D.	Eq.	F. Prob.*
1.	12	16	11	7	1.	12	8	13	7
2.	13	7	13	8	2.	13	9	14	9
3.	10	5	12	7	3.	12	7	12	7
4.	12	6	12	5	4.	11	6	12	6
5.	11	4	8	5	5.	12	4	9	5
6.	10	5	9	4	6.	10	7	7	5
7.	8	4	7	3	7.	8	4	6	4
8.	9	5	6	4	8.	10	6	11	5
9.	12	5	11	6	9.	7	5	7	5
10.	10	6	5	4	10.	8	7	11	5
11.	11	6	8	5	11.	7	4	7	3
12.	7	4	5	3	12.	9	5	8	3
13.	7	5	4	2	13.	11	5	4	3
14.	6	4	3	1	14.	9	7	7	4
15.	10	5	4	3	15.	9	5	5	2
16.	5	4	2	1	16.	7	6	3	2
17.	5	4	2	2	17.	6	5	1	1
18.	4	3	0	0	18.	5	4	0	0

*A--addition; S--subtraction; M--multiplication; D--division; Eq.--equations; F--formulae; Prob.--problems.

Mean	9.2	4.88	6.75	3.88		9.2	5.8	7.6	4.2
Diff.						0	1.08	.85	.3
S. D.	2.74	1.14	3.8	3.1		2.2	1.43	3.94	2.2
σ M	.64	.27	.8	.73		.52	.34	.93	.52
σ D						.824	.43	1.23	.89
C.R.						0	2.6	.69	.33

C. R. 2.6 means that there are 99.5 chances in 100 that the true difference is more than zero.

C. R. .69 means that there are 75.5 chances in 100 that the true difference is more than zero.

C. R. .33 means that there are 63 chances in 100 that the true difference is more than zero.

sheets containing:

1. Topic under consideration
2. Page references to the text[6]
3. Explanation references to the text
4. Lists of problems to be solved
5. References to additional work on the same unit.

No definite number of problems was assigned, but it was explained that each one must master the unit and pass a satisfactory test[7] before being allowed to proceed to the next unit. The tests were to be given when each individual was satisfied in his own mind that he had mastered the unit. The passing grade was 75%, but if the individual was not satisfied that he had done the best he could, he was given an opportunity to do more work on the unit and take a second test over the same unit. When each unit was completed satisfactorily, a new unit was given the pupil.

Since no definite time limit was placed on any unit, an outline of the units to be covered by the end of the semester was posted on the bulletin board. This plan was used so that each individual could check on his own progress and thus develop his ability to plan and be responsible for his own progress. During the time set for the recitation, the pupils worked individually or in groups of their own choosing, al-

[6]Joseph A. Nyberg, First Course in Algebra, Revised Edition (New York: American Book Company, 1932).

[7]See Appendix for sample unit tests.

though it was noticed that comparatively little work was done in groups. No mention was made of work to be done outside of the class room, that being left to the judgment of the pupil: another device leading to the development of purposefulness, the development of which is unquestionably an element in character building. The class room became a work room in which there was always the hum of purposeful activity. The teacher assisted the pupils at their request, supervised the work, encouraged, and gave special attention to the slower and more timid pupils, who at first were somewhat reluctant to ask for help. At the request of the pupil the teacher administered a unit test.

Group A continued according to the traditional recitation method. With very little variation, the recitation program was as follows:

1. Definite assignment for the following day's work was made
2. Explanation of advance work was made by the teacher, using the explanations in the book and illustrating with supplementary problems from Betz and Hart [8, 9]
3. Correction of papers prepared on the previous assignment
4. Explanation of difficulties of the previous lesson

---

[8]William Betz, Algebra For Today, (New York: Ginn and Company, 1937).

[9]Walter W. Hart, Progressive First Algebra (New York: D. C. Heath Co., 1934).

5. Class drill on the previous lesson either by short
   tests or work on the board
6. Work on the advance assignment by the pupils and
   teacher, the class forming the rules of procedure
   to be used. These were put in simple form by the
   pupils so that everyone could understand them and
   use them in preparing the advance assignment.

The same units were used in this section as in Group B.

At the end of each unit the class was given the same test as

had been used in the other section. The results of these

tests were kept as a measure of progress by the teacher in

order to keep in close touch with what each pupil was doing.

The results of the unit tests were not used however in the

final measure of accomplishment because they had not been

administered in the same manner to both groups.

At the end of each semester, Manchester tests[10] were

used as a measure of accomplishment. These tests were used

because they are the tests used over a large part of the

state as a standard measure of ninth year algebra.

At the end of the year, both groups were given, as a

final test, the test which had been used by Indiana Univer-

sity in the 1937 State Sectional Algebra Contest.[11] As a

further test of accomplishment Hotz Scale, Form B, was again

given,[12] in order to measure accurately the gain made since

---

[10]See Appendix.

[11]See Appendix.

[12]See Appendix.

the beginning of the experiment.

A very definite attempt was made to hold the variables constant. The same teacher conducted both classes, both classes used the same text, and both recited in the morning; section A reciting the first period, and section B reciting the second period. The same time was devoted to the recitation. The same tests were given as a measure of final achievement, the testing material covering only the first thirteen units which had been covered by all members of both groups. All the tests were time limited to fifty minutes except the final test which was ninety minutes in length. All of the final tests were given in the same room and at the same time. Although the unit tests were given in such a way as to avoid as much as possible the probability of one member of the group who had taken the test early giving aid to another, the results of these tests have not been used in the final comparisons. In giving these tests, paper was supplied on which all the work was to be shown. All paper was handed in at the end of the test and no papers were returned until the end of the experiment.

CHAPTER IV

RESULTS OF THE EXPERIMENT

After performing an experiment such as this, the
logical question arises, "What are the results?" Of course
it is impossible to measure, by means of any test yet de-
vised, some of the most important values resulting from such
a study; but one thing which can be measured definitely, and
accurately, is achievement in the mastery of the fundamentals
of subject matter.

If the intelligence quotient is a true predictive
measure of probable success, these groups will be equally
successful in their mastery of the fundamental concepts of
algebra, unless the difference in the method of instruction
gives one group an advantage over the other. In order to
determine whether or not there is any difference, the re-
sults must be compared critically and according to scientific
method.

Success was measured at the end of the first semester
by the first semester Manchester Test,[1] and the results tab-
ulated in Table III.[2] The expected median of this test, as

---

[1]Manchester Tests in First Year Algebra, First Semester.
Manchester College, North Manchester, Indiana. December, 1938.

[2]Table III, p. 27.

TABLE III

FIRST SEMESTER SCORES
MANCHESTER TEST

Number	Group A	Group B
1.	88	98
2.	93	97
3.	83	93
4.	85	96
5.	73	80
6.	71	88
7.	47	68
8.	78	91
9.	85	83
10.	84	84
11.	86	87
12.	63	80
13.	74	76
14.	63	71
15.	63	68
16.	54	81
17.	51	32
18.	30	46
Median	73.5	83.5
Mean	70.6	78.8
Diff.		8.2
S. D.	16.5	16.7
σ M.	3.9	3.96
σ D.		5.5
C. R.		1.49

C. R. 1.49 means that there are 93.5 chances in 100 that the true difference is more than zero in favor of Group B.

indicated by the makers of the test, was 68. It is noticed that both groups exceeded this median; Group A by 5.5 points and Group B by 15.5 points, showing a ten point better median for the class doing work by the unit method. Comparing the mean or average of the two groups, the experimental group exceeded the control group by 8.2 points. When the reliability of this difference was computed,[3] it was found to be 1.49 which, according to Garrett's table,[4] means that there are 93.5 chances in 100 that the difference between the true measures is greater than zero. Unless the value of 3 is found, this ratio is not considered statistically as a significant difference.

Since this group is small, it is advantageous to study the results by means of graphical representation. Graph II (p. 29) shows that with but two exceptions the members of Group B equalled or exceeded the members of Group A. Particular attention is called to pupil 16 of Group B, who though having an I. Q. lower than pupils 13, 14, 15, 16, and 17, of Group A exceeded them in the semester test. Attention is also called to pupil 18 of Group B, who having an I. Q. of only 70

---

[3]H. E. Garrett, Statistics in Psychology and Education (New York: Longmans, Green and Co., 1926), pp. 118-134. See Appendix for formulas used in computation.

[4]Ibid., p. 134.

GRAPH II

FIRST SEMESTER SCORES
MANCHESTER TEST

exceeded the lowest member of Group A by several points. This graph would seem to indicate that, on the whole, the unit method gives better results and particularly with those of lower mentality.

At the end of the second semester, since Group B had covered more units than Group A, the Second Semester Manchester test was carefully checked to see that no problems were included which had not been included in the units completed by both groups. The results of this test were tabulated in Table IV.[5] The tabulations showed a median of 69.5 for Group A and 73 for Group B, as against an expected median of 56, which was designated by the makers of the test. Again both classes exceeded expectations by 13.5 points and 17 points respectively--a margin of gain of 3.5 in favor of the experimental group. At this time there was a difference of means of only 3.1 in favor of Group B. The reliability of this difference was .46 indicating 67 chances in 100 that the true difference was more than zero. Statistically this is not a significant difference.

The graphical representation[6] shows a very wide variation in grades in both classes, which probably accounts for

---

[5]Table IV, p. 31.

[6]Graph III, p. 32.

TABLE IV

SECOND SEMESTER SCORES
MANCHESTER TESTS

Number	Group A	Group B
1.	90	89
2.	95	96
3.	80	88
4.	81	94
5.	70	76
6.	76	56
7.	52	69
8.	69	89
9.	81	73
10.	69	89
11.	78	48
12.	60	58
13.	53	64
14.	51	76
15.	71	73
16.	39	55
17.	44	27
18.	23	19

	Group A	Group B
Median	69.5	73
Mean	65.7	68.8
Diff.		3.1
S. D.	18.3	21.4
σ M	4.3	5.04
σ D		6.6
C. R.		.46

C. R. .46 means that there are 67 chances in 100 that the true difference is more than zero in favor of Group B. This is not a significant difference.

Scores

Class Numbers

Black=Group A
Red= Group B
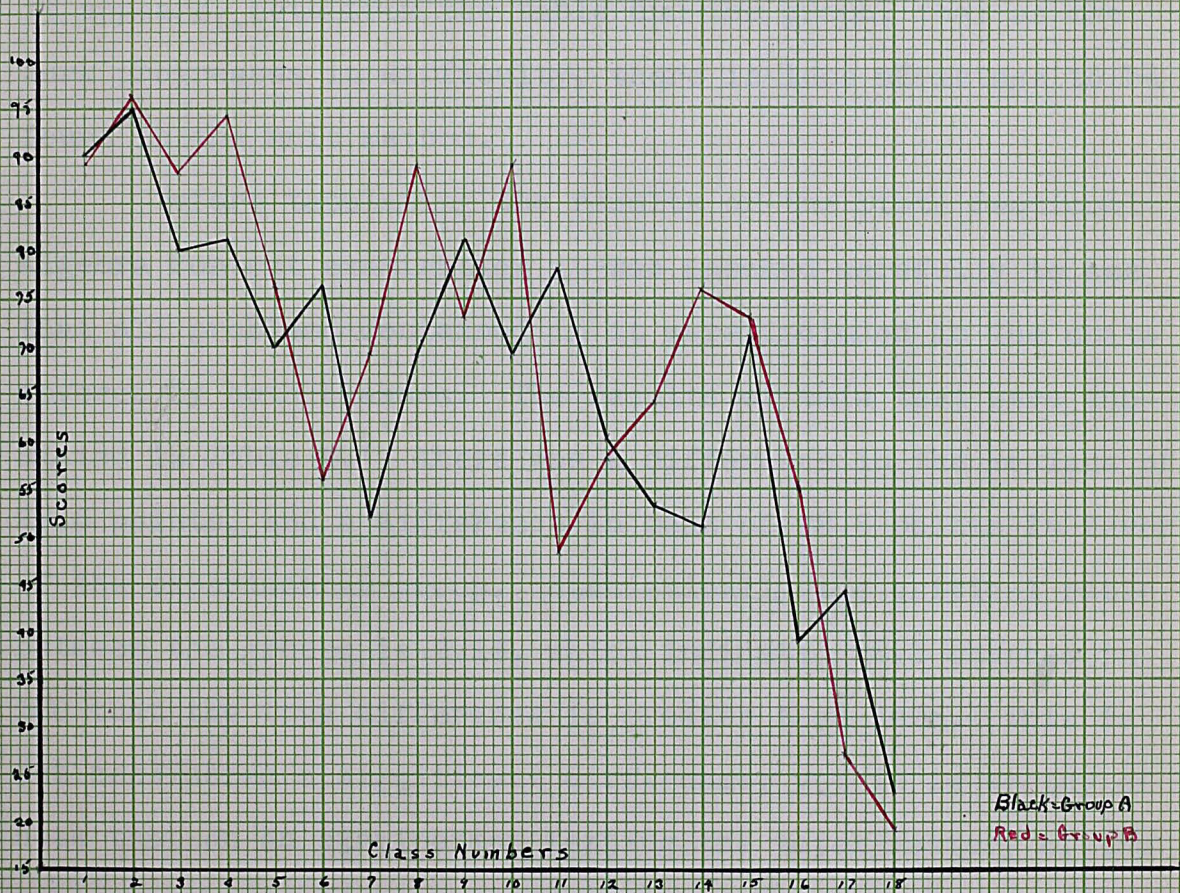
GRAPH III

SECOND SEMESTER SCORES
MANCHESTER TEST

the small statistical difference. Particular attention is
called to pupil 16 of Group B, who again exceeded pupils
13, 14, 16, and 17 of Group A. The graph of this semester's
grades shows less significant results than before. However,
it appears, both from the table and the graph, that as the
work progresses in difficulty, there is less advantage to be
found in the unit method.

Before tabulating the results of the State Sectional
Algebra Contest examination it was checked carefully to see
that all problems contained in it were included in the units
covered by both classes. The results of this test were tab-
ulated in Table V.[7] There was a difference in the medians of
29.5 points, and a difference in mean of 18.6 points in favor
of the experimental group. Statistically the critical ratio
between the groups is 2.14, which indicated 98.3 chances in
100 that the true difference is greater than zero. This is
not considered a significant difference; yet it is worthy of
note that on an examination testing the total mastery of the
subject, the statistical difference is very close to a sig-
nificant difference in favor of Group B.

The graphical representation[8] of these two groups of

---

[7]Table V, p. 34.

[8]Graph IV, p. 35.

TABLE V

STATE SECTIONAL ALGEBRA CONTEST
EXAMINATION SCORES

Number	Group A	Group B
1.	107	127
2.	125	137
3.	87	124
4.	120	113
5.	60	108
6.	94	101
7.	73	82
8.	79	105
9.	113	93
10.	56	95
11.	62	85
12.	46	88
13.	50	77
14.	47	72
15.	52	63
16.	42	60
17.	44	52
18.	35	42

Median	61	90.5
Mean	71.7	90.3
Diff.		18.6
S. D.	26.6	25.7
σ M.	6.2	6.1
σ D.		8.7
C. R.		2.14

C. R. 2.14 means that there are 98.3 chances in 100
that the true difference is more than zero in favor of Group
B.

GRAPH IV

STATE SECTIONAL ALGEBRA CONTEST
EXAMINATION SCORES

grades shows but two points where a member of Group B drops below the line of Group A. Study of the graph gives evidence that Group B, as a whole, has attained a higher degree of mastery in algebra than Group A; particularly is this fact evident in the case of the lower half of the class.

The results of the first Hotz test, given at the beginning of the experiment, are found in Table II (p. 21). Study of this table indicates that, at the beginning of the experiment, Group B was superior in multiplication and division, by a mean difference of 1.8; in simple equations and formulae, by .8; and in problem solving, by .3; while in addition and subtraction the groups were the same. In the same test, administered at the end of the experiment, there was a difference in favor of the experimental group in each division of the test. These differences,[9] however, were in no case great enough to prove, by statistical measurement, any significant difference in the groups.

Comparison of the results in the two Hotz scales[10] showed a total gain of 313 points for Group A, and 398 points for Group B--a difference of 85 points in favor of those instructed by the unit method. There was a difference of 4.7

---

[9]Table VI, p. 37.

[10]Table VII, p. 38.

TABLE VI

SECOND SCORES HOTZ TEST FORM B

GROUP A					GROUP B				
Number	A.S.	M.D.	Eq.	F. Prob.	Number	A.S.	M.D.	Eq.	F. Prob.*
1.	17	19	14	9	1.	22	22	17	9
2.	19	20	18	9	2.	25	23	21	11
3.	15	17	15	8	3.	21	21	16	9
4.	18	16	13	8	4.	20	19	16	7
5.	15	13	10	6	5.	19	15	15	5
6.	17	15	12	6	6.	19	14	16	7
7.	12	11	8	4	7.	9	9	10	5
8.	18	12	9	4	8.	18	16	13	6
9.	19	18	11	6	9.	15	11	14	5
10.	17	14	10	4	10.	19	17	15	7
11.	18	16	12	5	11.	17	12	9	3
12.	17	15	7	5	12.	16	16	11	6
13.	15	12	4	3	13.	13	14	9	4
14.	11	14	4	2	14.	20	15	10	5
15.	19	12	7	4	15.	17	13	7	4
16.	9	7	2	0	16.	17	14	7	5
17.	12	8	6	3	17.	9	9	2	1
18.	6	5	0	0	18.	10	8	0	0

*A--addition; S--subtraction; M--multiplication; D--division; Eq.--equations; F--formulae; Prob.--problems.

	A.S.	M.D.	Eq.	F.	Prob.		A.S.	M.D.	Eq.	F.	Prob.
Mean	15.2	13.6	9.05	4.77			17	14.8	16	5.5	
Diff.							1.8	1.2	6.9	.8	
S. D.	3.9	3.9	4.6	2.4			4.3	4.2	6.8	2.6	
σ M	.92	.92	2.05	1.07			1.01	.99	1.6	.61	
σ D							1.36	1.35	2.6	1.23	
C. R.							1.3	.88	1.8	.7	

C. R. 1.3 means that there are 91 chances in 100 that the true difference is more than zero in favor of Group B.

C. R. .88 means that there are 81 chances in 100 that the true difference is more than zero in favor of Group B.

C. R. 1.8 means that there are 96 chances in 100 that the true difference is more than zero in favor of Group B.

C. R. .7 means that there are 76 chances in 100 that the true difference is more than zero in favor of Group B.

TABLE VII

GAINS MADE ON HOTZ TEST

| | GROUP A | | | | GROUP B | | |
Number	Total First	Total Second	Gain	Number	Total First	Total Second	Gain
1.	46	59	13	1.	40	70	30
2.	41	66	25	2.	45	80	35
3.	34	55	21	3.	38	67	29
4.	35	55	20	4.	35	62	27
5.	28	44	16	5.	30	54	24
6.	28	50	22	6.	29	56	27
7.	22	35	13	7.	22	33	11
8.	24	43	19	8.	32	53	21
9.	34	53	19	9.	24	45	21
10.	25	45	20	10.	31	58	27
11.	30	51	21	11.	21	41	20
12.	19	44	25	12.	25	49	24
13.	18	34	16	13.	23	40	17
14.	14	31	17	14.	27	50	23
15.	22	42	20	15.	21	41	20
16.	12	18	6	16.	18	43	25
17.	13	29	16	17.	13	21	8
18.	7	11	4	18.	9	18	9

	GROUP A	GROUP B
Total gain	313	398
Mean	17.4	22.1
Diff.		4.7
S. D.	5.49	7.15
σ M	1.29	1.92
σ D		2.13
C. R.		2.04

C. R. 2.04 means that there are 97 chances in 100 that the true difference is more than zero in favor of Group B.

points in the means, and the reliability of this difference'
was found to be 2.04, signifying 97 chances in 100 that there
is a true difference greater than zero in favor of the unit
method. This scale, like the State Contest examination, is a
comprehensive test covering an entire years work. The sta-
tistical result on each of these tests was nearer a signif-
icant difference than in the case of either of the semester
tests. This would indicate that the unit plan gives better
results when retention and comprehension of the entire years
work are considered.

The graphical representation[11] shows but four points
where a member of Group A equalled or excelled the corres-
ponding member of Group B. In only one instance, that of
number 17, where the difference was 8 points, was there a
difference of more than two points.

For a more critical comparison, the experimenter
selected eleven pairs of pupils whose I. Q. rating differed
by no more than four points.[12] With this group we notice
that though the gains in the comprehensive tests did not
reach a statistical difference, the difference is much
greater than in either of the semester tests. The average of

[11]Graph V, p. 40.

[12]Table VIII, p. 41.
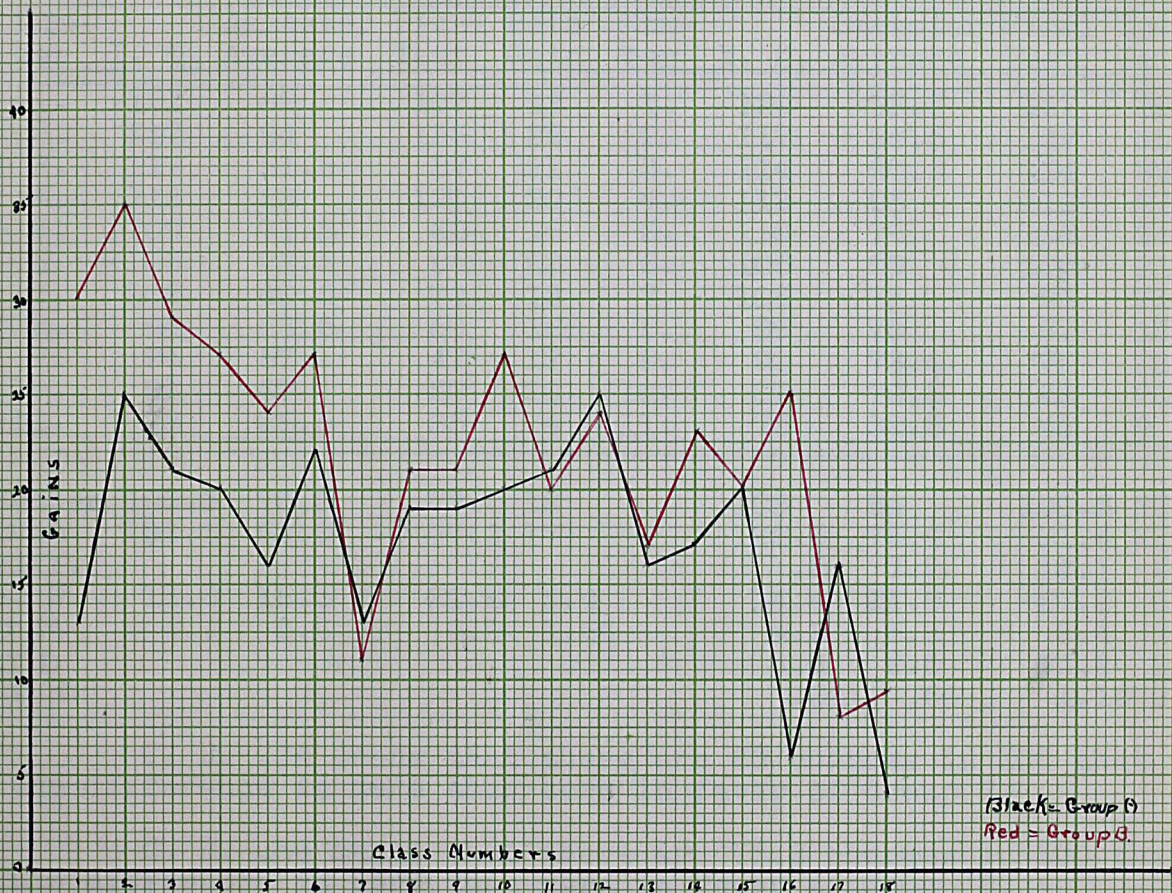
GRAPH V

COMPARATIVE GAINS HOTZ TEST

TABLE VIII

SEMESTER AVERAGES, STATE CONTEST EXAMINATION, AND HOTZ GAINS WITH 11 PUPILS PAIRED WITH NO DIFFERENCE GREATER THAN 4

	GROUP A					GROUP B			
Number	I. Q.	Semester Average	Final Test	Hotz Gain	Number	I. Q.	Semester Average	Final Test	Hotz Gain
1.	121	89	107	13	1.	120	93.5	127	30
2.	117	94	125	25	2.	117	96.5	137	35
3.	117	81.5	87	21	3.	114	90.5	124	29
4.	113	83	120	20	4.	112	95	113	27
5.	109	71.5	60	16	5.	110	78	108	24
6.	106	73.5	94	22	6.	106	90	105	21
7.	102	49.5	73	13	7.	106	78	93	21
8.	94	63.5	50	16	8.	96	70	77	17
9.	94	57	47	17	9.	90	73.5	72	23
10.	91	67	52	20	10.	87	70.5	63	20
11.	73	26.5	35	4	11.	70	32.5	42	9
Median	105.5	70.3	73.5	18		108	84	100.5	22.5
Mean	103.5	68.2	77.3	17		102.5	75.4	91.4	23.3
Diff.						.2	7.2	14.1	6.3
S. D.	13.6	18.5	29.8	5.4		14.4	21.6	27.9	6.7
σ M	4.1	5.6	8.9	1.64		4.3	6.5	8.4	2.02
σ D						5.9	8.5	12.2	2.6
C. R.						.34	.64	1.15	2.42

C. R. .34 means that there are 63 chances in 100 that the difference is more than zero in favor of Group B.

C. R. .64 means that there are 73.8 chances in 100 that the difference is more than zero in favor of Group B.

C. R. 1.15 means that there are 87 chances in 100 that the difference is more than zero in favor of Group B.

C. R. 2.42 means that there are 99.2 chances in 100 that the difference is more than zero in favor of Group B.
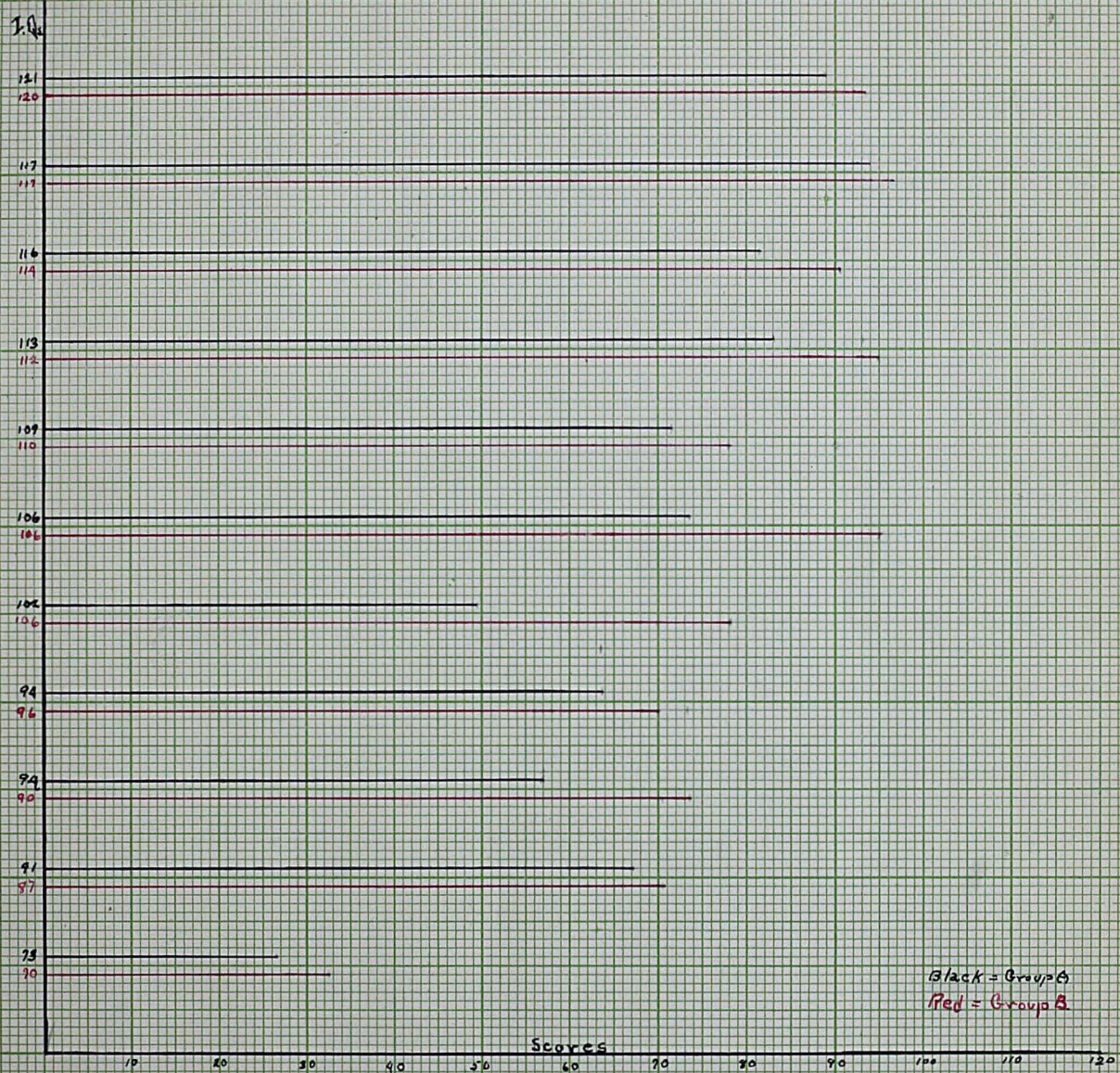
the two semester grades, the final grades, and the gains on the Hotz tests are represented graphically in pairs in Graphs VI, VII, and VIII[13] to show comparative accomplishment. In reading these graphs it is readily seen that in the semester averages and in the final averages, the members of Group B made the greater gain. In the case of the gains on the Hotz test, with this pairing, there was but one pair in which there was a small difference in favor of a Group A member.

Since the age of these two groups has not yet been taken into consideration, a glance at Table IX[14] and at Graphs IX and X[15] will show that, as a group, the control group is 7.1 months older than Group B, and mentally, 5 months older. The graphs show that at the lower end of the mental scale the members of Group B are younger than those of similar rank in Group A. Some school authorities consider age of sufficient significance to recommend that freshmen of low mentality wait until their second high school year to take algebra in the belief that the added year will in a measure offset the low mentality. This has been done in the Laboratory School, Indiana State Teachers College. The re-

---

[13]Graphs VI, VII, VIII, pp. 43, 44, 45.

[14]Table IX, p. 46.

[15]Graphs IX, X, pp. 47, 48.

I.Q.

121
120

117
117

116
114

113
112

109
110

106
106

102
106

94
96

94
90

91
87

75
70

Black = Group A
Red = Group B

Scores

10   20   30   40   50   60   70   80   90   100   110   120

GRAPH VI

SEMESTER AVERAGES ON MANCHESTER TESTS
ELEVEN PAIRS OF PUPILS: I. Q. WITHIN FOUR POINTS

IQs

121
120

117
117

116
114

113
112

109
110

106
106

102
100

98
96

94
90

91
87

73
70

Scores

15 20 45 60 75 90 105 120 135 150 165 150

Black = Group A
Red = Group B
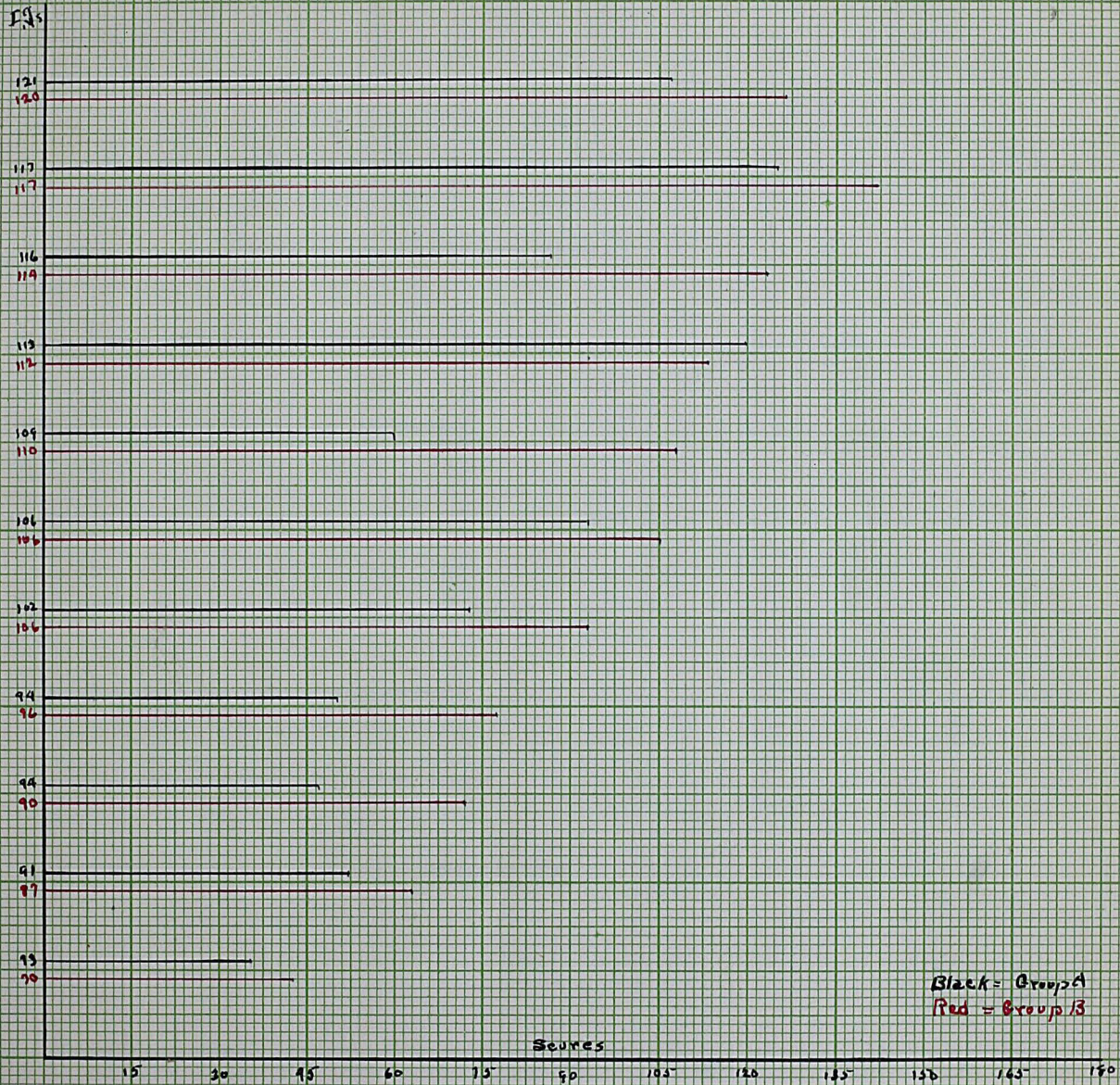
GRAPH VII

SCORES ON STATE CONTEST EXAMINATION
ELEVEN PAIRS PUPILS: I. Q. WITHIN FOUR POINTS

I.Qs.

121
120

117
117

116
114

113
112

109
110

106
106

102
106

99
96

99
80

91
87

73
70

Black = Group A
Red = Group B

Gains

5   10   15   20   25   30   35   40
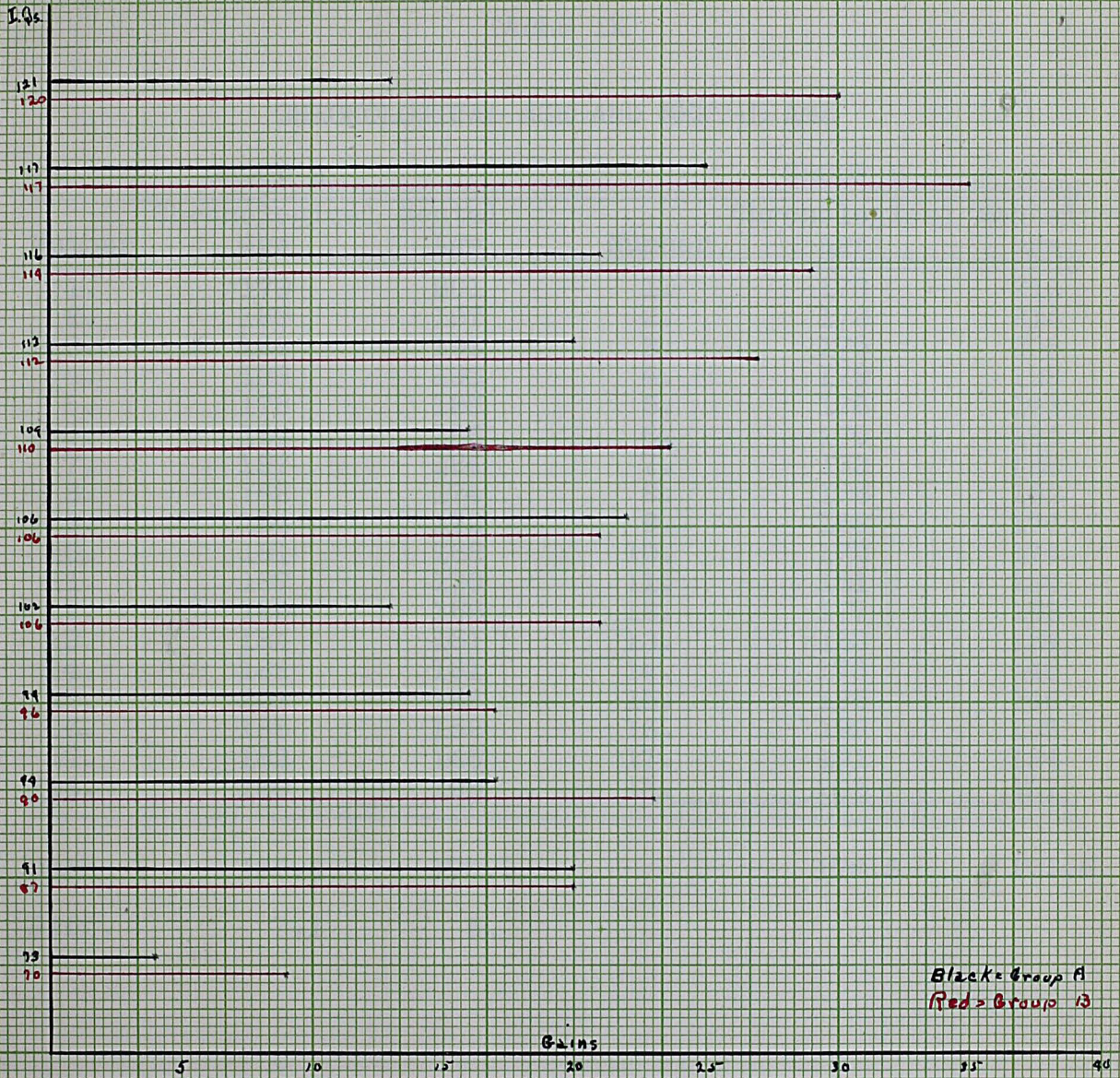
GRAPH VIII

GAINS ON HOTZ TEST
ELEVEN PAIRS PUPILS: I. Q. WITHIN FOUR POINTS

TABLE IX

CHRONOLOGICAL AGE AND MENTAL AGE

	GROUP A			GROUP B	
Number	Chronological Age	Mental Age	Number	Chronological Age	Mental Age
1.	179	201	1.	173	216
2.	188	197	2.	179	214
3.	188	198	3.	175	203
4.	183	192	4.	179	198
5.	178	188	5.	176	193
6.	184	186	6.	170	185
7.	197	184	7.	173	182
8.	182	174	8.	176	182
9.	184	172	9.	168	176
10.	181	168	10.	172	174
11.	187	170	11.	181	180
12.	189	166	12.	177	168
13.	184	161	13.	163	154
14.	182	161	14.	173	145
15.	201	162	15.	186	146
16.	185	152	16.	192	123
17.	182	146	17.	191	124
18.	189	108	18.	205	127
Mean	185.4	176.6		178.3	171.6
Diff.	7.1	5.			
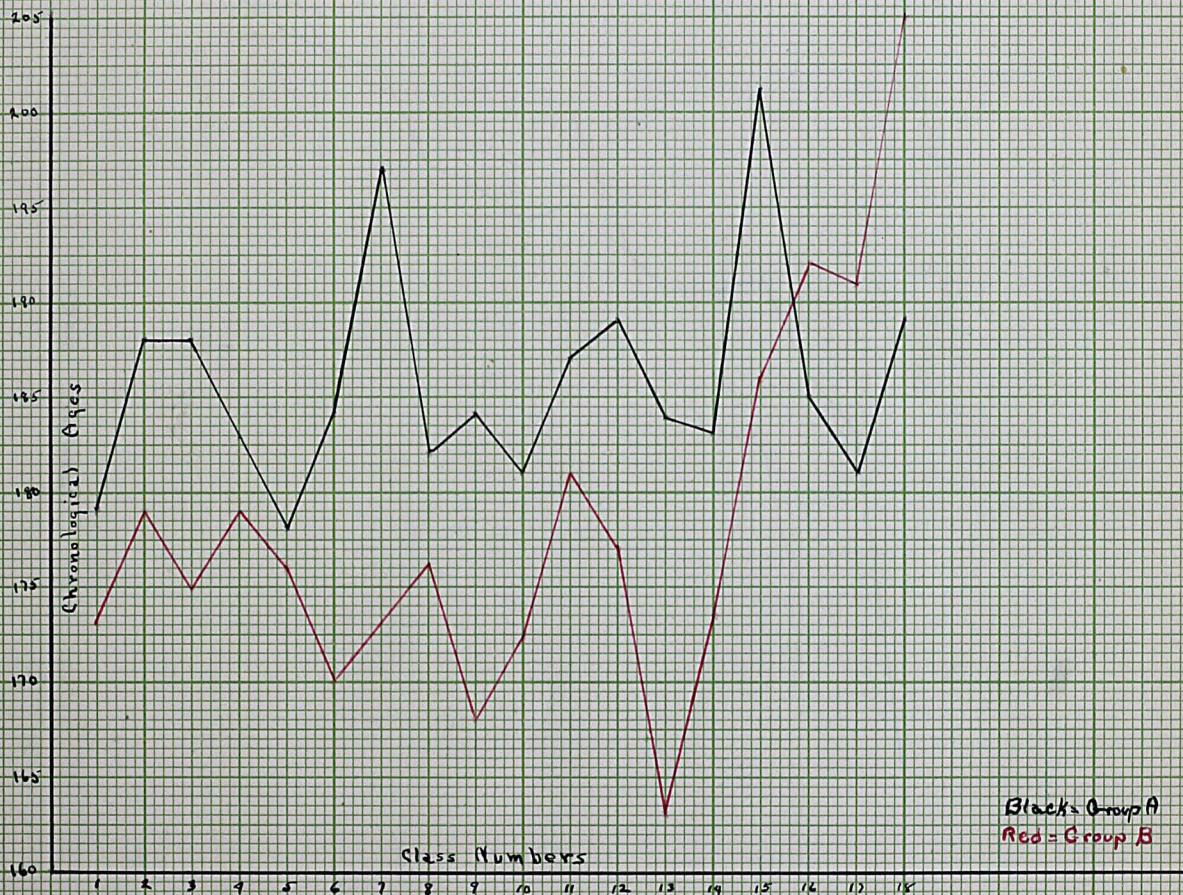
GRAPH IX

CHRONOLOGICAL AGES OF GROUPS A AND B
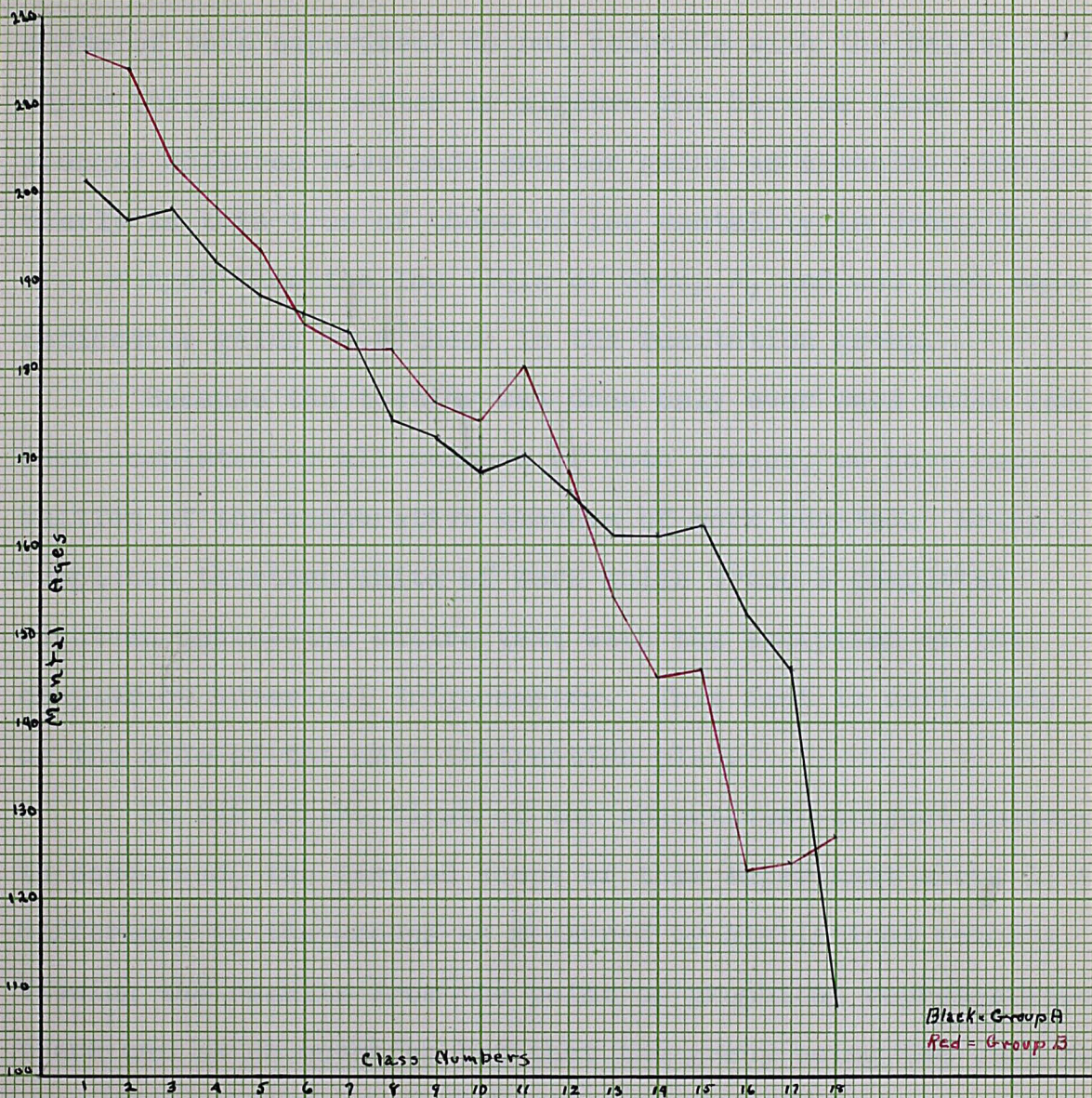
GRAPH X

MENTAL AGES OF GROUPS A AND B

sults of this experiment do not coincide with the belief that
older children do better in algebra for here we found the
younger children have made greater gains as a whole. Partic-
ularly is this true of those of low mentality in Group B, who
are younger than the low I.Q.s of Group A, and who have made
noticeable gains over those in the recitation group.

Because the testing was done only over work which had
been completed by all members of both groups, we have made no
mention of the comparative amounts of work covered. Group A
completed thirteen units of work while Group B completed
eighteen units--the entire text with the exception of the
chapter on Numerical Trigonometry. This fact is definitely
on the side of the unit instruction plan.

So far as the first of the objectives--subject mastery--
is concerned, the limited scope of this experiment shows little
difference in the groups as measured by statistical methods.
Viewing the results from the point of graphical representation,
there can be no question but that the group using the unit plan
of instruction made greater gains, and attained better mastery
of the subject.

No final conclusion should be drawn without considering
the second factor: the study as a social subject in which it
was the aim to test the attitude of the pupils toward the
acquisition of knowledge, and the development of ideals of

honesty, accuracy, industry, self-reliance, and persistency!
The measurement of attitude is hard except as the pupils
themselves express their opinions. In order to ascertain the
attitude of the pupils in an unbiased manner, the cooperation
of the English department was enlisted. The teacher in that
department asked the pupils in her classes to write a theme
in which they expressed their opinion of the two methods being
used in the algebra classes. She assured them that the con-
tent of their themes would not be divulged to anyone until
after the final grades had been published. In this way she
was unusually successful in getting some very original and
frank opinions.

The general concensus of opinion was in favor of the
unit plan. Every one in the experimental section except two
favored the plan. One of these said he did not like it be-
cause "there was too much to do". He further gave as a reason
for not liking the plan that he always lost his head on an ex-
amination and did not do as well as he thought he should. In
the class using the recitation plan there were seven who ex-
pressed a desire to try the unit plan because it sounded in-
teresting. This general favoring of the unit plan agrees with
the results of an investigation of Billett. He compared the
reactions of 263 pupils to the unit plan as contrasted with
classroom procedures, after six months experience with the

unit plan. He says:

> One must conclude that boys and girls of all levels
> of academic intelligence, accomplishment, and application,
> whether the subject matter be geometry or English, regard
> the unit plan as a distinct improvement over the tradi-
> tional recitation procedure.[13]

Some excerpts from the papers of the students seem to furnish the best proof that their attitude toward the subject and the plan has made the experiment of value as a social sub-ject. These excerpts, quoted from the pupils' papers, are of more value as evidence than the subjective opinion of the writer or of any teacher.

> I like the new plan because you learn to do things for
> yourself and be independent. If something happens like
> being sick or going to a party, so you don't get to work
> your problems, you don't get a zero. You work them the
> next day to make up.   (B)

> I like the units plan because it gives us a better
> chance to get a good grade because we have to get all the
> problems to pass the test. Sometimes when I don't have
> to, I only work a few of them. I like it, too, because
> there is not as much chance to cheat when we take the
> examinations alone. The members of the other class can
> cheat on the examination easier. Sometimes they don't
> get all the problems but wait until class and have the
> teacher explain them. In our class you have to get them
> all to learn how to do them on the examination.   (B)

> I like the new plan because the teacher is not always
> hounding me to get my problems. It looks as if she
> thought we had more sense when she leaves it up to us to
> get our lessons. If we don't get them we can't blame her
> for she is ready to help us when we need it.   (B)

---

[13]Roy O. Billett, "High School Pupils Opinions of the Unit Plan," _School Review_, XL (January, 1932), pp. 17-32.

I like the new plan because the teacher comes around
to see how we are getting along and helps us.   (B)

I like the units because I am afraid to recite and I
am so slow at the board that the rest always have to wait
for me and they call me "slow poke". I don't get as good
grades as the others but nobody knows it but the teacher
and me.   (B)

I don't know whether I would like the unit plan or not
but they all seem to like it because they work on algebra
a lot when they have their other lessons. Maybe if I had
that plan I would work more and not read so much. I wish
I had been in the other class.   (A)

I like our class better than I would the other class
because I know just what I have to do every day. If I
had to do it the way the others do I would never get my
lessons. Sometimes I don't anyway.   (A)

I like the recitations because I like to go to the
board and race the others to see who can get done first.
I don't think the other way would be any fun.   (A)

I wish I were in the other class. If I were I could
go just as fast as I want to. As it is, I get all my work
and have to sit around with nothing to do while she ex-
plains to some of those who do not understand. Some of
them haven't even tried. I think I could do as much as
Edwin if I had a chance. I think we should have been
asked which we wanted to do.   (A)

Regarding the third important factor, that of character

building, the conclusions must of necessity be purely subject-

ive. Of the eight other teachers who had observed these

pupils, either in class or in study hall, all but one were of

the opinion that the unit plan had helped the pupils to be-

come more independent in their thinking and in their actions.

They were unanimous in their opinion that the pupils had

learned how to discriminate between what they knew and what

they needed help to find out. All the teachers agreed that
the pupils, who were working under the unit plan, were more
industrious. The one who disapproved of the plan did so be-
cause he said that the pupils in Group B spent too much time
on algebra and neglected other subjects. When questioned
specifically about the results as observed in the less gifted
pupils, they were of the opinion that the less gifted were
more sure of themselves, and less timid about asking for help.

If the unit method will develop in the student inde-
pendence of thought and action, initiative, intellectual
honesty, and confidence in one's self, these are contributions
which are, without doubt, of value in educating citizens.
Other things being equal, if the unit plan does no more than
aid in character building, it has a very definite place in
the field of mathematics.

CHAPTER V

CONCLUSION

This experiment has certain shortcomings which cannot be overlooked. The small number of pupils, equated as groups rather than as individuals; and the wide range of variability due to heterogeneous grouping (although giving a life situation) made for unreliability in statistical measurement. The opinion of the writer on this point is at variance with that of M. R. Shibles, who after experimenting with twenty pupils, wrote in his conclusion:

> With these groups there was not a wide range of intelligence quotients and hence the difference between them is small. A heterogeneous group or one with a wide range would have been more valuable.[1]

In spite of these shortcomings, the findings in this experiment are so consistently in favor of the unit plan that the writer is inclined to draw tentative conclusions pending further experimentation along this same line.

The reliability of the difference in the first semester Manchester test was 1.49, indicating that there were 93.5 chances in 100 that the true difference in favor of Group B was more than zero; the second semester Manchester test scores showed a reliability of .46, indicating 67 chances in

[1]M. R. Shibles, "An Evaluation of Three Methods of Teaching Ninth Grade Algebra," (Master's thesis, Boston University, 1935).

100; and the state contest examination revealed reliability of 2.14 or 98 chances in 100 that the difference was more than zero in favor of Group B.

Comparison between the two Hotz tests showed that Group A made 313 points gain and Group B, 398 points. The reliability of this difference was 2.04, indicating 97 chances in 100 in favor of Group B.

In the smaller group of twenty-two students equated in pairs with no greater difference than four points in I. Q. rating, findings showed consistently better results in Group B. The reliability of the difference in average semester grades was .64, indicating 73.8 chances in 100 in favor of Group B; state contest grades showed 1.15 reliability, indicating 87 chances in 100; and gains on Hotz scale were 2.24, indicating 99.2 chances in 100 in favor of Group B. It is also to be noted that in every case but two the members of Group B made greater gains than those of Group A.

Chronologically and mentally, Group A exceeded Group B, but in spite of this advantage Group B exceeded Group A in every point of comparison.

Evaluating the groups on the bases of attitude toward the subject and development of good citizenship traits, the balance is decidedly in favor of the experimental group.

The unit plan of instruction is not an easy plan to

put in operation. The teacher's work is increased a great
deal in preparing the units, in keeping the records, and in
conducting the recitation hour. In spite of the increased
labor, and the small difference, as shown statistically, be-
tween the recitation and the unit methods, the writer is con-
vinced that the unit plan is best fitted to care for the in-
dividual differences in the ninth grade algebra classes. For
the teacher who is genuinely interested in teaching pupils
rather than subject matter, it holds immeasurable possibilities.
Pupil and teacher are brought closer together, and an opportu-
nity is created for better development of personality traits
along lines of good citizenship; and at the same time the pupil
is given a better mastery of the concepts of ninth grade algebra.

It cannot be said conclusively that the unit plan is the
best in all situations and under all circumstances; for, so long
as education is dealing with individual personalities, we will
never be able to use the same standard of measurement in all
situations. However, the writer is convinced that the unit plan
gave results enough better than those produced by the recitation
plan to commend it to the consideration of the teacher who is
trying to work out the method best qualified to truly educate
youth. It is perhaps not too much to hope that the unit plan
may serve as a means of giving a partial answer to the criti-
cism against algebra as a required subject; and therefore to
make the future of mathematics secure in our secondary schools.

BIBLIOGRAPHY

BIBLIOGRAPHY

A. BOOKS

Betz, William, Algebra For Today. Chicago: Ginn and Company, 1937. 555 pp.

Englehardt, Fred, and Leonard D. Haertter, First Course in Algebra. Chicago: John C. Winston Company, 1928. 450 pp.

Garrett, H. E., Statistics in Psychology and Education. New York: Longmans, Green and Company, 1936. 317 pp.

Hart, Walter W., Progressive First Algebra. Chicago: D. C. Heath and Company, 1934. 404 pp.

Judd, Charles H., Psychology of Secondary Education. Chicago: Ginn and Company, 1927. 545 pp.

Morrison, Henry D., Practice of Teaching in the Secondary School. Chicago: University of Chicago Press, 1926.

Nyberg, Joseph A., First Course in Algebra. Chicago: American Book Company, 1932. 333 pp.

Schorling, Raleigh, Mathematics in Life. New York: World Book Company, 1937. 437 pp.

Schorling-Clark-Smith, Modern School Algebra. New York: World Book Company, 1936. 462 pp.

Sorenson, Herbert, Statistics for Students of Psychology and Education. New York: McGraw-Hill Book Company, 1936. 344 pp.

Thayer, V. T., The Passing of the Recitation. Chicago: D. C. Heath and Company, 1928. 331 pp.

B. PERIODICAL ARTICLES

Barton, W. A., "The Effect of Group Activity and Individual Effort in Developing Ability to Solve Problems in First Year Algebra," Educational Administration and Supervision, 12:512-519, November, 1926.

Billett, Roy O., "Directed Learning and the Unit Assignment," Department of Secondary School Principals, Bulletin 45, March, 1933. pp. 55-76.

_____, "High School Pupils' Opinions of the Unit Plan," School Review, XL:17-22, November, 1932.

_____, "Plans Characterized by the Unit Assignment," School Review, 653-68, November, 1932.

Burton, W. H., "The Unit Concept in Learning: An Attempt at a Simple Explanation," Educational Outlook, VII:206-13, May, 1933.

Daggett, Clay J., and Florence A. Peterson, "A Survey of Popular Plans for Instruction," Educational Administration and Supervision, XVIII:21-30, January, 1932.

Editorial, "The Unit Assignment in Actual Practice," School Review, XL:321-23, May, 1932.

Everett, J. P., "The Fundamental Skills of Algebra," Teachers College Record, Columbia University, 1929. p. 3

Fontaine, E. Clarke, "The Unit Plan of Teaching," Educational Administration and Supervision, XVIII:21-30, January, 1932.

Gadske, R. E., "A Comparison of Two Methods of Teaching First Year High School Algebra," School Science and Mathematics, XXXIII:635-640, June, 1933.

Grinstead, Wren Jones, "The Unit of Learning, Its Meaning and Principles," Educational Outlook, 7:9-20, November, 1932.

Horton, Byrne J., "A Brief Outline of the Unit Plan of Teaching," Educational Method, 12:175-77, December, 1932.

Lundquist, E. F., "The Gap Between Promise and Fulfillment in Ninth Grade Algebra," School Review, 762-77, December, 1934.

McCuen, Theron R., "Predicting Success in Algebra," _Journal of Educational Research_, 72-74, January, 1930.

Moore, R. U., "An Experimental Comparison Between the Relative Effectiveness of the Individual Methods in Mathematics," _The High School_, 6:86-89, February, 1929.

Purdom, Luthur T., "Value of Homogeneous Grouping," _Investigation for Warwick and York_, Inc., Baltimore, 1929.

Reudiger, W. C., "The Learning Unit," _School Review_, 40:76-81, March, 1932.

Smith, Ross H., "The Unit Plan in Operation," _Journal of Education_, 112:548-49, December, 1930.

Stallard, Burton J., and Harl Douglass, "An Experimental Study of Two Plans of Supervised Study in First Year Algebra," _Journal of Experimental Education_, September, 1935.

E.   PUBLICATIONS OF LEARNED ORGANIZATIONS

Report of National Committee on Mathematical Requirements, under Auspices of _Mathematical Association of America_, Inc., 1923, p. 11.

G.   UNPUBLISHED MATERIALS

Gainer, James V., "Experimental Investigation into the Relative Difference in Two Plans of Teaching First Year Algebra." Unpublished Master's thesis, University of West Virginia, 1933.

Hunziker, C. W., "A Comparison of Relative Effectiveness of a Plan of Supervised Study and Daily Recitation Method in Teaching Algebra and Geometry." Unpublished Master's thesis, University of Minnesota, 1933.

Lawton, Robert C., "Recitation Plan vs. Supervised Study." Unpublished Master's thesis, Boston University, 1936.

Linn, Marguerite, "An Experimental Comparison of Two Methods of Teaching Elementary Algebra." Unpublished Master's thesis, University of Southern California, 1934.

Mason, H. C., "A Comparison of Contract Assignment and Recitation as Measured by Objective Tests." Unpublished Master's thesis, University of Minnesota, 1930.

Shibles, M. R., "An Evaluation of Three Methods of Teaching Ninth Grade Algebra." Unpublished Master's thesis, Boston University, 1935.

Smith, O. O., "The Relative Efficiency of Two Methods of Instruction in Algebra." Unpublished Master's thesis, University of Kansas, 1927.

APPENDIX

ABBREVIATIONS AND FORMULAS USED IN TEXT

S. D...........Standard deviation

σM...........Standard Error of the Mean

Diff...........Difference between Means

σD ....Standard Error of the Difference

C. R...........Critical Ratio

F...............Frequency

D...............Deviation

$$S. D. = \sqrt{\frac{FD^2}{n}}$$

$$\sigma M = \frac{SD}{\sqrt{n}}$$

$$\sigma D = \sqrt{\sigma M_1^2 + \sigma M_2^2}$$

$$C. R = Diff / \sigma D$$

C. R. must be 3 to be considered a significant difference.

Table--Garrett, p. 134.

H. E. Garrett, Statistics in Psychology and Education (New York: Longmans, Green and Company, 1926), pp. 118-148.

UNIT ONE

Review of Fundamental Operations pp. 47-54.

I. Addition of Polynomials
    1. See example p. 47
    2. Exercise 52
    3. Study carefully the method of checking p. 53
    4. Exercise 54
        a. Check each problem carefully
            (notice problem 7)

II. Subtraction of Polynomials
    1. Study example 55
        a. Define terms subtrahend, minuend, difference
        b. Make a rule for subtraction
    2. Exercise 56
        a. Read the problem carefully to notice which is
            the subtrahend

III. Multiplication
    1. What is the sign of the product when numbers with
        like signs are multiplied? Unlike signs?
    2. Exercises pp. 52, 53 through problem 21
    3. What is the purpose of the parentheses?
        a. What does $5(2a-3)$ mean?
    4. What relation do multiplication and division bear to
        each other? What then will be the rule for signs in
        division?

IV. Have you made a satisfactory grade on each exercise?

V.   Problems of this type are to be found in Wells, 1934
     edition, p. 93, 102
     Betz, p. 203, 205
     Nyberg, p. 69-72

VI. If you are satisfied you may ask for the unit test.

UNIT TWO

Linear Equations pp. 54-66.

I. Solve 3x=12
    1. What does the term 3x mean?
    2. If 3 times some number you do not know equals 12,
        what is the value of the unknown number? How did
        you get it? What fundamental process did you use?

II. -5x=20
    1. What is unusual about this equation?
    2. What rule must you keep in mind?
    3. What is your answer?

III. In this same manner solve problems 1-15, ex. 62.

IV. What is an equation?
     What do you call the parts of an equation?
     What different things do you think you could do to both
     members of an equation without changing its value?
     See ex. 63, 64, 65

V. Exercises 66, 67, 68, 69.

VI. Look up the meaning of the word transpose.
     Study articles 70, 71.

VII. What has been done in each step in the following?
      $12+8x=5x-3$
      $8x-5x=-12-3$
      $3x= -15$
      $x= -5$
      $12-40= -25-3$
      $-28= -28$

VIII. Solve and check Ex. 73, p. 59 and 60 through problem 42

IX. Write equations for problems 43-55

X. Parentheses
    $3(a+b)$ is read "three times the quantity $a+b$"
    1. Exercises 77-80

XI. Have you graded your exercises?
     Are you ready for your test?

UNIT THREE

Formula

I. What is a symbol?
   Write symbols for the following words.

1. length	11. volume	21. cost
2. width	12. area	22. multiply by
3. height	13. diagonal	23. Fahrenheit
4. altitude	14. distance	24. number
5. base	15. time	25. plus
6. perimeter	16. velocity	26. times
7. diameter	17. rate	27. minus
8. radius	18. interest	28. equals
9. pi	19. profit	29. percent
10. surface	20. loss	30. weight

II. Write in symbols
   1. Rate times time
   2. Gain plus loss
   3. Base times altitude
   4. Length plus width
   5. Two times pi times radius
   6. Circumference divided by pi
   7. Pi times diameter
   8. Distance divided by rate
   9. A number (n) minus three
   10. Principal times rate
   11. Volume divided by height
   12. Perimeter minus twice length
   13. Profit multiplied by two
   14. Area divided by length
   15. Amount minus principal
   16. 9/5 times centigrade (temperature) plus 32 degrees
   17. Fahrenheit is equal to zero
   18. Seven per cent of a number
   19. Rate times time equals distance
   20. Velocity times the time

III. Express the principle of each of the following formulas
   in a good English sentence:

1. $A = lw$
2. $A = S^2$
3. $V = Bh$
4. $I = prt$
5. $S = 6e^2$
6. $A = 1/2 bh$
7. $C = 2\pi r$
8. $A = \pi r^2$
9. $A = p + i$
10. $V = e^3$
11. $d = rt$
12. $S = 2\pi rh$

IV. What measurements do you make to find the area of a circle? The volume of a cylinder? Surface of a cube? Area of a triangle?

V. Exercises 81 and 82 in your book.

UNIT FOUR

Problem solving

You are now ready to put the symbols that you have been writing into problems. You will find that by the use of symbols will make the solving of problems much easier.

I.   How would you write in symbols one number increased by 3?

II. Write one number decreased by 7.

III. There are two numbers. One is three more than the other. Write expressions for both numbers.

    Let x = the smaller number

    then (x+3) = the larger number.

Now we will indicate the addition of these numbers as:

    x + (x+3)

Now we will try a problem--there are two numbers. One of the numbers is four more than the other. Their sum is ten. What are the numbers?

    Let x = the smaller

     (x+4) = the larger

     x + (x+4) = 10

     x + x + 4 = 10

     2x = 10-4

     2x = 6

     x = 3

IV. Study exercises 88, 89, 90 very carefully.

V.   Exercise 91.

The purpose of this exercise is to translate statements into equations.

Look back on page 20 for the form we used in simple equations at the first of the term.

Read each problem carefully, then with your book closed see if you can tell it in your own words. Ask yourself the following questions:

    What am I asked to find?

    What did the problem tell me?

    What shall I let x stand for? Why?

    Can I get expressions for the other unknown numbers?

    Is there a statement that will form an equation?

Solve problems and check all results.

Minimum work on this unit Grade D
    Exercises 91, 92, 93, 94.

Maximum work pp. 75-104.

Are you ready to try the test?

UNIT FIVE

<u>Equations</u> <u>Containing</u> <u>Fractions</u> <u>pp.</u> <u>105-124</u>.

I. Translate the statements into algebraic expressions.
   1. If the width of a field is expressed by the term x, how would you express 1/2 the width? 1/4 the width? 3/4 the width?
   2. If the length of a field is expressed by $(x+3)$ how would you express 1/2 the length? 3/4 the length?

II. Translate the problems in exercise 109.

III. Multiplication of fractional expressions. Ex. 110.

IV. Solution of equations containing fractions.
   1. Previous equations have contained no fractions. Notice principle IV, p. 108.
   2. What number could you use to multiply by in order to get rid of every denominator? Have you used the smallest number that would do?
   3. If your denominators were 3, 4, and 6, what number would you choose, 12 or 24? Why?

V. Procedure
   1. Select multiplier
   2. Multiply <u>every</u> term. (Why every term?)
   3. Solve by
      a. Transposing
      b. Collecting terms
      c. Finding value of the unknown
   4. <u>Check</u> (Substitute in the <u>original</u> equation) Why not use the equation in which you have cleared of fractions?

UNIT ONE TEST

I. Add the following

1. $6x - 7y + 3z$
    $-2x + 3y + z$
    $x - 2y - z$

2. $2a - 3b - 5c$
    $-5a + b - 3c$
    $3z - b + 9c$

3. $-2a^3 + a^2 + 5a - 6;$ $6a^3 - 2a + 3;$ $4a^3 + 8a^2$

4. $2A - 3B + 5C;$ $3A - 6C;$ $-5A - B + C$

5. $6a + b + 7c;$ $-3a - 6b + 4c;$ $5a + 4b - 6c$

Add the following polynomials and then check the results by letting $a=2$, $b=3$, $x=4$, $y=5$

6. $3x - 6y + 3a;$ $-5x + 2y - 5a;$ $7x + y - 6a$

7. $5a - 2b + 3y;$ $6a + 4b - 4y;$ $-3a - b + 2y$

Fill blanks with proper words

8. To _____ one number from another change the _____

of the _____ and then apply the rules for the _____

of numbers.

9. The quotient of two numbers with _____ signs is _____

10. The quotient of two numbers with _____ signs is _____

11. Plus times _____ equals <u>plus</u>
12. Minus times _____ equals plus
13. _____ times minus equals minus
14. Minus times plus equals _____
15. To add two or more numbers with unlike signs _____ the
    figures and _____ the answer the _____ of the _____
16. To add two or more numbers with like signs _____ the
    figures and prefix the sign of the _____ to the result
17. When you multiply expressions with exponents you _____
    the exponents
18. When you divide you _____ the exponents
19. (a) $2(-3x + 5y)$ (b) $y^2(y^4 - y^2)$
20. $-4(5x_3 + 3y)$ $x^3(x^2 - x)$
21. $-5x(x^3 - x)$ $5(-a + b)$
22. $-6x^2(3x - 2)$ $3(6x - 7y)$

UNIT TWO TEST

I. Solve the following
1. -5x = 75
2. 2x = -8
3. -5x = -15
4. 3x = 5
5. 5x = 5

II. Solve
1. 5x -8 = 32
2. 4x -19 = -3
3. 6x = 56 -2x
4. 7x = 50 + 2x
5. 6x -3 = 17 + 2x

III. Solve and check
1. 5x -2 + 3x = 14
2. -3y + 6 = 15 -5y
3. x -5 + 2x = 70
4. 5a -20 + a = 40
5. 6x + 2 = 30 -8x

IV. Write as equations the following:
1. The sum of x and 6 is 13
2. 35 equals x increased by 12
3. Three times a certain number equals 18 added to the number
4. Twice a certain number equals 11 subtracted from the number
5. If 5 is subtracted from 12 times a certain number, the result equals 15 more than 8 times the number.

V. Simplify:
1. 4(5x -3) -2
2. 6(-2x + 3) -5
3. 8(3 -4x) -7
4. 5(3a -2) + 4(3a + 1)
5. 2(5 -3x) -6(x + 4)

VI. Simplify:
1. (6x -2) -(-3x + 4)
2. 4 -5b -(2 -3b)
3. x -(x -3y) + y
4. 2x -y -(x + y)
5. 5x -(3x + 2) + 4x

VII. Solve: 6(x -3) = 5(x -1)

VIII. Solve: 2x = 9 -3(x -2)

IX. 1/2(8x -4) = 1/3(6x -9)

X. If r = 3, π = 3 1/7, h = 10, find the value of A = 2πr(r+h)

UNIT THREE TEST

1. Write a formula for the following rules:
   a. The perimeter of a rectangle is equal to twice the length plus twice the width
   b. The distance traveled is equal to the time, times the rate
   c. The area of a square equals the square of the side
   d. Pi is equal to the quotient of the circumference divided by the diameter
   e. The amount is equal to the principal plus the interest
   f. Area is equal to half the sum of the three bases times the height
   g. Volume is equal to pi times the square of the radius times the height.

2. Write formula for each of the following statements:
   a. The cost of N footballs at d dollars each
   b. The height of a building of 5 stories if each story is m feet
   c. An even number if n represents any odd number
   d. The number of inches in f feet and 4 inches
   e. The number of cents in r dollars
   f. The number of games won if a team played n games and lost 10
   g. The savings if a man earned D dollars and spent x dollars.

UNIT FOUR TEST

1. There are 46 pupils in a room, and the number of boys is 12 more than the number of girls. How many boys and how many girls are there in the room?

2. A board 31 feet long is divided into three pieces so that the second is 6 feet longer than the first, and the third is 3 times the length of the first. What is the length of each piece?

3. In an isosceles triangle, two of the angles are always equal. If the third angle of an isosceles triangle is 40 degrees more than one of the others, find the size of each angle.

4. The sum of three numbers is 30. The second is 12 more than the first, and the third is 4 times the first. Find the numbers.

5. A family spends twice as much for food as for rent, and $200 a year less for clothing than for rent. If these expenses total $3000, how much is spent for each item?

6. The capacity of one freight car is 4 tons more than that of another. A locomotive is pulling 10 of the smaller cars and 5 of the larger cars, making a total haul of 620 tons. What is the capacity of each kind of car?

7. One transport carried 400 more soldiers than another. In 5 trips of the larger and 3 trips of the smaller, 19,600 soldiers were carried. How many men did each ship carry?

8. A is 12 years older than B. Find the ages of each if 9 times A's age added to 4 times B's age equals 368 years.

9. John has 12 more nickels than dimes. How many nickels and how many dimes has he if he has $1.50?

10. A goes six miles an hour and B goes nine miles an hour. A starts two hours before B. How many hours has B been traveling when they are 102 miles apart?

UNIT FIVE TEST

Write expressions for the following:
1. 1/3 of x
2. 2/5 of the sum of x and 10
3. 2/3 of the sum of x and y
4. 2/5 of a certain number decreased by 10

5. A is x years old. B is 6 years younger. What is 1/5 of B's age?

6. Multiply $40\left(\dfrac{7-3x}{8}\right)$ 7. $20\left(\dfrac{x}{5} - 3/4\right)$

8. $15 - \dfrac{x+1}{3}$ 9. $-12 \cdot \dfrac{-5x}{4}$ 10. $16\left(\dfrac{5x-6}{2}\right)$

11. $\dfrac{x}{2} - 12 = \dfrac{x}{3}$ 12. $\dfrac{5x}{6} - 3 = x$

13. $\dfrac{x+2}{6} + \dfrac{x}{3} = \dfrac{x-4}{9}$

14. $\dfrac{x}{6} - \dfrac{x-13}{12} = \dfrac{2}{3}$

15. The sum of two numbers is 60. The sum of 3/4 of the smaller and 2/5 of the larger is 31. Find the two numbers.

16. The width of a rectangle is 3/5 of the length. Find the dimensions if the perimeter is 160 feet.

17. A can do a job in 12 days and B in 15 days. After A worked 5 days, B came to help and worked until the work was completed. How long did B work?

18. Solve $\dfrac{9x+1}{5} - \dfrac{7}{15} = x$

19. $\dfrac{6x+1}{3} - \dfrac{1-3x}{4} = 1$

20. State in words the meaning of $\dfrac{a+b}{2}$; $\dfrac{a}{b} - 2$; $ab + 2$;

   $a + bc$; $a(b+c)$.

MANCHESTER SEMESTER-END HIGH SCHOOL TESTS

FIRST YEAR ALGEBRA TEST

First Semester

NAME ....

Home Address...

School.. Teacher...

Directions: You should have several sharpened pencils. If you break your pencil points, raise your hand and the teacher will supply you with another.

First of all, fill in the above blanks. Write your name clearly and plainly. Do not turn the page over until the signal to begin is given. After you are told to "go," turn the page and read the instructions at the beginning of the first exercise. Do your best to follow the directions carefully. Ask no questions after the signal to begin is given. Continue doing your best throughout the entire test until after you reach the end of the test, unless the signal to stop is given before you have completed the test. **The time limit is 60 minutes.**

****

Directions for Scoring: The highest possible score is 100 for all of the Manchester Semester-end High School Tests. How much each item counts in scoring is indicated in the number of the item. Most of the items count one point; but some of the items in this test count more than one point. For example, if an item is numbered 70-73., it counts four points; if it is numbered 70, it counts only one point. Therefore, the way the item is numbered indicates the number of points each item should count in scoring this test.

Editorial Committee

J. G. Meyer, Dean, School of Education, Manchester College, North Manchester, Ind.
A. R. Eikenberry, Professor of Psychology, Manchester College, North Manchester, Ind.
O. Stuart Hamer, Professor of Education, Manchester College, North Manchester, Ind.
Isabelle V. Jones, Supervisor of Tests and Measurements, Gary, Indiana.

Algebra Test Committee

J. E. Dotterer, Professor of Mathematics and Methods, Manchester College.
A. Pryce Noe, Principal of Wheeler High School, Wheeler, Indiana.
L. J. Yoder, Teacher of High School Mathematics, North Manchester, Indiana.
Albert E. Harshbarger, High School Mathematics Teacher, Buck Creek, Indiana.

FIRST YEAR HIGH SCHOOL ALGEBRA

First Semester

Part I—Best Answer Exercise

Directions: Complete the following algebraic expressions by writing the number of the best answer in the parentheses on the right of each item.

Example: If $x = 2$, then the value of $2x$ is   (1) 8   (2) 4   (3) 6   (4) 9   (...2...)

1. If $b = 8$; $h = 4$, the value of $\frac{b}{h}$ is   (1) 1   (2) ½   (3) 2   (4) 32 (.........)

2. h^2 means   (1) h.h   (2) 2h   (3) ½h   (4) h   -   -   - (.........)

3. When you add $6x^2 + 4 + 3x^2 + 2$, you get   (1) $3x^2 + 2$ (2) $15x^2$ (3) $9x^2$ (4) $9x^2 + 6$ -   -   -   -   -   -   - (.........)

4. The equation for the following statement: "Three times a certain number equals 20" is (1) $3n = 20$ (2) $n = 20x3$ (3) $20n = 3$ (4) $3n + 20 = 0$ -   -   -   -   -   -   - (.........)

5. If one angle of a triangle is 40° and another 60° the third is (1) 10°   (2) 80°   (3) 90°   (4) 100°   -   -   - (.........)

6. If $x = -3$, $y = -2$, $a = -5$ then the value of $\frac{5xy}{a}$ is equal to   (1) 6   (2) —7.5   (3) —6   (4) —3   -   -   - (.........)

7. By adding $6x + 3a - 4x + 7a$ the result is   (1) 12 ax (2) 10x—10a   (3) 20a—x   (4) 2x+10a   -   -   - (.........)

8. By division $\frac{-15x}{5}$ simplifies to (1) 3x (2) —3x (3) —3 (4) —75x (.........)

9. The sum of 4x and 5y is   (1) 9xy   (2) 4x+y   (3) x+y (4) 4x+5y   -   -   -   -   -   - (.........)

10. The equation for "one angle of a triangle is 4 times as large as another angle; the third angle is 7 times the smaller number is (1) $4n + 7n = 180$ (2) $n + 4n + 7n = 180$ (3) $n + 4n + 7n = 90$ (4) $n + 4n = 7n$ -   -   -   -   - (.........)

11. The sum of 2a—3b+7c and 7a—9b—8c is   (1) 9a—12b—c (2) 5a—6b—15c   (3) —5a+6b—c   (4) 20abc   -   - (.........)

12. When you subtract $4a + 3b$ from $5a + 4b$ the difference is (1) 9a+7b   (2) —a—b   (3) a+b   (4) 16ab   -   -   - (.........)

—2—

13. By **multiplying** —4x by —3x⁴ the product is (1) 12x⁴ (2) —12x⁷
    (3) 12x⁵ (4) —12x   -   -   -   -   -   -   -   (..........)

14. By solving 2x = —8, x will equal (1) —4 (2) 4 (3) 16 (4) —¼ (..........)

15. By removing parentheses from 2x(3x—4) the product is
    (1) 6x²—4 (2) 3x—8x (3) 2x² (4) 6x²—8x   -   -   (..........)

16. The formula for finding the area of a circle is (1) $C = 2\pi r$
    (2) A = ½bh (3) $A = \pi r^2$ (4) A = bw   -   -   -   (..........)

17. If a⁵ is divided by a² the quotient is (1) a⁷ (2) a³ (3) $a^{\frac{5}{2}}$
    (4) a¹⁰   -   -   -   -   -   -   -   -   -   (..........)

18. The equation for finding interest is (1) A = P + Pr
    (2) i = Prt (3) $i = \frac{Pr}{t}$ (4) $i = \frac{Pt}{r}$ -   -   -   -   (..........)

19. If one ton of coal occupies 38 cu. ft., the formula for finding
    the number of tons T. of coal that a bin will hold is
    (1) $T = \frac{LWH}{38}$ (2) T = 38 LWH (3) $T = \frac{WH}{38L}$
    (4) $T = \frac{38}{LWH}$ -   -   -   -   -   -   -   -   (..........)

20. The equation for "A can do a certain piece of work in 12 days
    and B can do it in 8 days. If they work together how many
    days are needed?" is (1) $\frac{8}{x} + \frac{12}{x} = 1$ (2) $\frac{x}{8} = \frac{x}{12}$
    (3) $\frac{x}{8} - \frac{x}{12} = 1$ (4) $\frac{x}{8} + \frac{x}{12} = 1$ -   -   -   -   (..........)

21. In solving y—3x = 4, y equals (1) 4—3x (2) 3x+4 (3) $\frac{3x}{4}$
    (4) $\frac{4}{3x}$ -   -   -   -   -   -   -   -   -   -   (..........)

Part II—Completion

Directions: Below are statements with a word left out. You are to choose the word
that will best complete the thought. Place the number of that word in the parentheses
at the right.

Example: Algebra is a study in .... (1) English (2) history
    (3) mathematics (4) home economics   -   -   -   -   (.....3.....)

22. A rule expressed by literal numbers and mathematical symbols
    is called a .... (1) triangle (2) formula (3) square
    (4) fraction   -   -   -   -   -   -   -   -   (..........)

23. The 2 in 36h² is called the .... (1) coefficient (2) lit-
    eral number (3) decimal (4) exponent   -   -   -   (..........)

—3—

24. Such terms as **4x** and **3x** are called .... terms because in each term the same number x has been multiplied by some other number   (1) like   (2) unlike   (3) solution   (4) equation   (..........)

25. If one of the angles of a triangle is a right angle the triangle is called a .... triangle.   (1) protractor   (2) isosceles   (3) right   (4) equilateral   -   -   -   -   -   -   (..........)

26. The product of two numbers having like signs is ....
(1) negative   (2) unlike   (3) positive   (4) opposite   -   (..........)

27. The result of division is called the ....   (1) sum   (2) product   (3) difference   (4) quotient   -   -   -   (..........)

28. A single term, such as **5x**, is called a ....   (1) polynomial   (2) trinomial   (3) monomial   (4) coefficient   -   (..........)

29. If the sum of two angles is 180° they are   (1) complementary   (2) equilateral   (3) supplementary   (4) isosceles   -   -   (..........)

30. In graphs, the two numbers which locate a point are called .... of the point   (1) ordinates   (2) axis   (3) coördinates   (4) abscissa   -   -   -   -   -   -   (..........)

31. If one member of an equation is divided by any number the other member must be .... by the same number.
(1) added   (2) subtracted   (3) multiplied   (4) divided   -   (..........)

32. Transposing is the same as .... the opposite of any term to both members of an equation.   (1) adding   (2) subtracting   (3) multiplying   (4) dividing   -   -   -   (..........)

33. When numbers are compared by division the relation between the two numbers is called their ....   (1) proportion   (2) ratio   (3) sum   (4) difference   -   -   -   (..........)

34. In any .... the product of the extremes equals the product of the means.   (1) ratio   (2) proportion   (3) equation   (4) problem   -   -   -   -   -   -   -   (..........)

35. In graphs, the point where the axes cross is called the ....
(1) coordinate   (2) abscissa   (3) ordinate   (4) origin   -   (..........)

36. In solving two equations by the multiplication-addition method or substitution method the pair of numbers that satisfies both equations is called the .... of the two equations.
(1) solution   (2) graph   (3) multipliers   (4) method   -   (..........)

—4—

Part III—Matching Exercises

Directions: In the parentheses below, write the numbers of those left hand items which best match the right hand items. Not all left hand items will be needed.

Group 1

(1) $8x^6$

(2) $4x^6$

(3) $\frac{1}{4}$

(4) $8x^9$

(5) 2

37. $(\frac{1}{2})^2$ - - - - - (..........)

38. $(2x^3)^3$ - - - - - (..........)

39. $(2x^3)(2x^3)$ - - - - (..........)

Group 2

(1) $\frac{x-4}{x}$

(2) $4-x$

(3) $x-4$

(4) $7x^3-7xy$

(5) $7xy-7x^3$

40. $-1(4-x)$ - - - - (..........)

41. $-7x(x^2-y)$ - - - - (..........)

42. $-\frac{4-x}{x}$ - - - - - (..........)

Group 3

(1) $x = 7$

(2) $x = y$

(3) $x = -y$

(4) $x = 52$

(5) $x = 8$

43. $6x+5 = 47$ - - - - (..........)

44. $x-y = 0$ - - - - (..........)

45. $\frac{x}{2} = 4$ - - - - (..........)

Group 4

(1) x^9y^5

(2) -5

(3) x^3y^3

(4) $3a-5$

(5) 5

46. $\frac{x^5y^4}{x^2y}$ - - - - - (..........)

47. $\frac{-30a^3b^5}{-6a^3b^5}$ - - - - (..........)

48. $(6a^2-10a) \div 2a$ - - - (..........)

—5—

14. Solve for x: $-3x = 8 - (5 + 2x)$

Part IV—Graphs

Directions: On the squared space below draw on the set of axes the two graphs. (One point for each point located correctly, and two points for correct graph solution of the two equations represented by the graphs.)
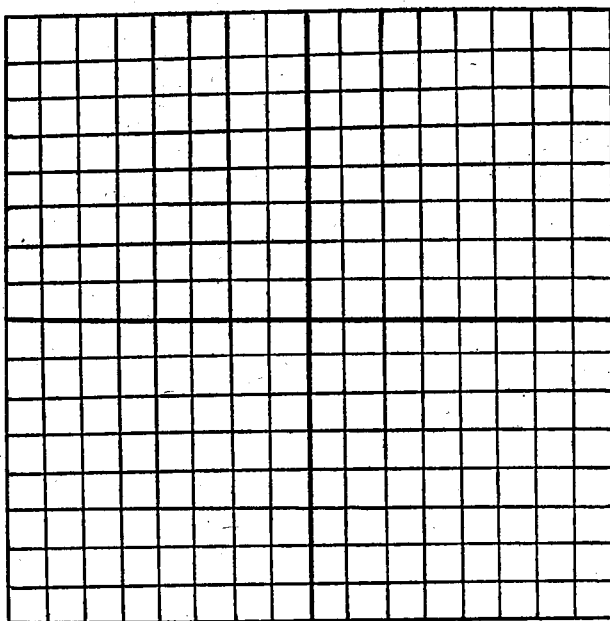
	x	y
49.	—3	4
50.	—1	3
51.	1	2
52.	3	1
53.	5	0

	x	y
54.	—3	—2
55.	—1	—1
56.	1	0
57.	3	1
58.	5	2

59-60. At what point do the graphs cross? (Place answers in parentheses.) - - - - - - - - - (.........)

Part V—Equations to Solve

Directions: Solve following equations and place answers in parentheses.

61-63. (3 points) $6x - 3 = 17 + 2x$ - - - $(x = \text{................})$

64-66. (3 points) $2 - (3x - 4) = 6x - 12$ - - - $(x = \text{................})$

67-70. (4 points) $\frac{x}{2} - 12 = \frac{x}{3}$ - - - - $(x = \text{................})$

71-74. (4 points) $\frac{2x - 9}{3} = \frac{x + 11}{5}$ - - - - $(x = \text{................})$

75-78. (4 points) $\frac{y + 4}{3} - \frac{y + 12}{6} = \frac{y - 2}{12}$ - - $(x = \text{................})$

Part VI—Story Problems

Directions: Solve the problems below in the space provided after each one. Each correct answer counts points as indicated at each problem.

79-81. (3 points) One number is twice another. Their sum is 30.
What is the smaller number? - - - - (..........)

82-84. (3 points) One angle of a triangle is 2 times as large as the
smallest, and the third angle equals the sum of the other two.
How many degrees are there in each of the three angles?
- - - First Angle (..........)
- - - Second Angle (..........)
- - - Third Angle (..........)

—7—

85-88. (4 points) An investment of $6000, part of it at 5% and the remainder at 8%, yields an income of $375. How much is the amount invested at each rate?

- - - - At 5% (.............)
- - - - At 8% (.............)

89-92. (4 points) A is 12 years older than B. Find the age of each if 9 times A's age added to 4 times B's age equals 368 years?

- - - - A's Age (.............)
- - - - B's Age (.............)

93-96. (4 points) John has 12 more nickels than dimes. How many nickels and how many dimes has he if he has $1.50?

- - - - Nickels (.............)
- - - - Dimes (.............)

97-100. (4 points) One number exceeds another by 39. Find the numbers if their sum is 67.

- - Smaller Number (.............)
- - Larger Number (.............)

Date of Test..   Total Score........................

MANCHESTER SEMESTER-END HIGH SCHOOL TESTS

FIRST YEAR HIGH SCHOOL ALGEBRA

Second Semester

NAME ....

Home Address...

School...   Teacher...

Directions: You should have several sharpened pencils. If you break your pencil points, raise your hand and the teacher will supply you with another pencil.

First of all fill in the above blanks. Here and throughout the test write clearly and plainly. Do not turn over the page until the signal has been given. Then turn the page, read the instructions carefully, and work thoughtfully through the exercises and questions until you have finished or until the signal to stop is given. The time limit is 60 minutes. Follow directions carefully, and do not ask any questions after you have started.

****

Directions for Scoring: The highest possible score is 100 for all of the Manchester Semester-end High School Tests. How much each item counts in scoring is indicated in the number of the item. Most of the items count one point, but some of the items in this test count more than one point. For example, if an item is numbered 70-73., it counts four points; if it is numbered 70, it counts only one point. Therefore, the way the item is numbered indicates the number of points each item should count in scoring this test.

Editorial Committee

J. G. Meyer, Dean, School of Education, Manchester College, North Manchester, Ind.
A. R. Eikenberry, Professor of Psychology, Manchester College, North Manchester, Ind.
O. Stuart Hamer, Professor of Education, Manchester College, North Manchester, Ind.
Isabelle V. Jones, Supervisor of Tests and Measurements, Gary, Indiana.

Algebra Test Committee

J. E. Dotterer, Professor of Mathematics and Methods, Manchester College.
A. Pryce Noe, Principal of Wheeler High School, Wheeler, Indiana.
L. J. Yoder, Teacher of High School Mathematics, North Manchester, Indiana.
Albert E. Harshbarger, High School Mathematics Teacher, Buck Creek, Indiana.
Helen Darley, High School Mathematics Teacher, Ossian, Indiana.

FIRST YEAR HIGH SCHOOL ALGEBRA

Second Semester

Part I—Best Answer Exercise

Directions: Complete the following algebraic expressions by writing the number of the best answer in the parentheses on the right of each item.

Example: The value of $a^2 \cdot a^3$ is (1) a^6 (2) $a^{3/2}$ (3) a^5 (4) $a^{2/3}$ - - (....3....)

1. The product of $-2xy \cdot -5x^2y$ is (1) $-10x^3y^2$ (2) $10x^3y^2$
    (3) $10x^2y$ (4) $-10x^2y$ - - - - - - (.........)

2. The product of $c^5 \cdot c^3 \cdot c^4$ is (1) c^{12} (2) c^{60} (3) $3c^{12}$ (4) $3c^{60}$ (.........)

3. $10^2 \cdot 10^3$ is equal to (1) 100^5 (2) 100^6 (3) 10^6 (4) 10^5 - (.........)

4. A simple way of writing $(x^3)^4$ is (1) x^{12} (2) x^7 (3) $4x^3$ (4) $3x^4$ (.........)

5. The indicated product of $a(a^2-3a+5)$ is (1) a^2-3a+5
    (2) a^3-3a+5 (3) a^3-3a^2+5a (4) $8a^2$ - - - (.........)

6. The product of $(x+2)(x-3)$ is (1) x^2+x-6 (2) x^2-5x+6
    (3) x^2-x-6 (4) x^2+5x+6 - - - - - (.........)

7. The product of $(2x-7)(x-1)$ is (1) $2x^2-9x+7$ (2) x^2-7x-7
    (3) $2x^2-5x-7$ (4) $2x^2-9x-1$ - - - - - (.........)

8. $(a+3)^2$ expanded, or written out in full is (1) a^2+9
    (2) a^2+6a+9 (3) a^2-6a-9 (4) a^2-9 - - (.........)

9. The factors of x^2-25 are (1) $(x-5)(x-5)$ (2) $(x+5)(x-5)$
    (3) $(x+9)(x+1)$ (4) $(x-3)^2$ - - - - - (.........)

10. The factors x^2+6x+9 are (1) $(x+3)^2$ (2) $(x-3)(x+3)$
    (3) $(x+9)(x+1)$ (4) $(x-3)^2$ - - - - - (.........)

11. x^4y^5 divided by x^2y is equal to (1) x^6y^6 (2) x^2y^4 (3) x^2y
    (4) xy^6 - - - - - - - - (.........)

12. The factors of x^2-x are (1) $x(x-1)$ (2) $(x-1)(x-1)$
    (3) $(x-1)(x+1)$ (4) $x^2(x-1)$ - - - - (.........)

13. The factors of $x^2+2x-80$ are (1) $(x-20)(x+y)$ (2) $x(2x-80)$
    (3) $(x+10)(x-8)$ (4) $(x-10)(x+8)$ - - - (.........)

14. $-7x^2y$ divided by $-7x^2y$ is equal to (1) 1 (2) 0 (3) xy (4) -1 (.........)

15. $\frac{15xy}{10x}$ can be written (1) $2xy$ (2) $150x^2y$ (3) $\frac{2y}{5}$ (4) $\frac{3y}{2}$ (.........)

—2—

16. $(2x+7)$ $(x+5y)$ is divided by $x+5y$ the result is (1) 0 (2) $x+5y$ (3) $2x+7$ (4) 1 - - - - - - - (...........)

17. $\frac{a^2-9}{a-3}$ simplified is (1) $a+3$, (2) $\frac{a+3}{a-3}$ (3) $a-3$ (4) $\frac{1}{2}$ (...........)

18. $\frac{2}{a}+\frac{3}{b}$ added together equals (1) $\frac{5}{a+b}$ (2) $\frac{2b+3a}{ab}$

(3) $\frac{6}{ab}$ (4) $\frac{2b+3a}{a+b}$ - - - - - - - (...........)

19. In the equation $a+2x = b$; x equals (1) $a(a+b)$ (2) $\frac{a-b}{2}$

(3) $\frac{ab}{2}$ (4) $\frac{b-a}{2}$ - - - - - - - (...........)

20. $3x+ax-bx$ added together equals (1) $3abx^3$ (2) $x(3+a-b)$ (3) $3(3ab)$ (4) 0 - - - - - - - (...........)

21. Solving $bx-a = b$ for x, x will equal (1) a (2) $\frac{b-a}{b}$

(3) $\frac{a+b}{b}$ (4) $-a$ - - - - - - - (...........)

Part II—Completion

Directions: Below are statements with a word left out. You are to choose the word that will best complete the thought. Place the number of that word in the parentheses at the right.

Example: A rule expressed by literal numbers and mathematical symbols is called a (1) triangle (2) formula (3) square (4) fraction - (......2......)

22. A factor which consists of just a simple term, like the term 2 a, is called (1) binomial (2) monomial (3) square (4) double (...........)

23. If the product of two or more factors is zero, then at least one of the factors must be (1) one (2) zero (3) quotient (4) sum (...........)

24. The kind of fraction we get when we change 5 3/8 to 43/8 is called (1) proper (2) mixed (3) improper (4) binomial (...........)

25. A fraction is not changed in value if both the numerator and ....?.... are multiplied by the same number (1) numerator (2) multiplier (3) denominator (4) fraction - - (...........)

26. To find the L. C. M. of several quantities, factor each one of the quantities and find an expression which is ....?.... by each of the quantities (1) divisible (2) multiplied (3) factored (4) subtracted - - - - - - - (...........)

27. A fraction whose numerator or denominator contains fractions is a ....?.... fraction (1) simple (2) complex (3) compound (4) convenient - - - - - - - (...........)

—3—

28. We may change the sign of both the numerator and denominator ....?.... changing the sign of the fraction (1) without (2) by (3) when (4) merely - - - - - - - (........)

29. If s is the sum of two numbers and d their difference we always know that the larger of the two numbers is always one-half of the ....?.... of s and d (1) difference (2) quotient (3) product (4) sum - - - - - - - - (........)

30. The sign $\sqrt{}$ is called a ....?.... sign (1) division (2) multiplication (3) radical (4) adding - - - - (........)

31. Radicals like $2\sqrt{3}$ and $5\sqrt{3}$ are called like radicals because the ....?.... is or are the same (1) root (2) radicands (3) coefficients (4) sum - - - - - - (........)

32. A right triangle is one in which one of the angles is ....?.... angle (1) a right (2) an obtuse (3) an acute (4) an equal (........)

33. The law for multiplication of terms having exponents is that you ....?.... the exponents (1) subtract (2) divide (3) multiply (4) add - - - - - - - (........)

34. For a fractional exponent the ....?.... indicates the root (1) denominator (2) numerator (3) exponent (4) coefficient (........)

35. In a distance problem the rate times the time always gives ....?.... (1) hours (2) distance (3) time (4) rate - - - (........)

36. $24 = 3 . 2 . 2 . 2$ The $3 . 2 . 2 . 2$ are called the ....?.... factors of 24 (1) prime (2) monomial (3) binomial (4) fractional (........)

Part III—Matching Exercises

Directions: In the parentheses below, write the numbers of those left-hand items which best match the right hand items. NOT ALL LEFT HAND ITEMS WILL BE NEEDED.

Group 1—Factoring

(1) $(x-6)(x+4)$
(2) $(x-12)(x+2)$
(3) $(x-4)(x+3)$
(4) $(x-4)(x-3)$
(5) $(x+3)(x+4)$

37. x^2-x-12 - - - (........)

38. $x^2-2x-24$ - - - (........)

39. $x^2-7x+12$ - - - (........)

Group 2—Factoring

(1) $(x-25)(x+1)$
(2) $(x-5)(x-5)$
(3) $(x+25)(x+1)$
(4) $(x-5)(x+5)$
(5) $(x+5)(x+5)$

40. x^2-25 - - - - (........)

41. $x^2-10x+25$ - - - (........)

42. $x^2-24x-25$ - - - (........)

-4-

Group 3—Square of Binomial

(1) $a^2+2ab+b^2$

(2) $9a^2-6ab+b^2$

(3) $4a^2+8ab+4b^2$

(4) $a^2-2ab+b^2$

(5) $9a^2-4a^2-b^2$

43. $(a+b)^2$ -   -   -   -   (..........)

44. $(2a+2b)^2$ -   -   -   (..........)

45. $(3a-b)^2$ -   -   -   (..........)

Group 4—Literal Equations to Solve

(1) $x = a-4$

(2) $x = 4-a$

(3) $x = 4a$

(4) $x = a/4$

(5) $x = 4/a$

46. $4x = a$ -   -   -   -   (..........)

47. $4+x = a$ -   -   -   (..........)

48. $4-x = a$ -   -   -   (..........)

Group 5—Addition of Like Radicals

(1) $\sqrt{16}$

(2) $10\sqrt{5}+\sqrt{6}$

(3) $2\sqrt{6}$

(4) $12\sqrt{5}$

(5) $\sqrt{5}+\sqrt{6}$

49. $3\sqrt{5}+7\sqrt{5}+2\sqrt{5}$ -   (..........)

50. $3\sqrt{5}+\sqrt{6}-2\sqrt{5}$ -   (..........)

51. $\sqrt{6}+\sqrt{5}+\sqrt{6}-\sqrt{5}$ -   (..........)

Group 6—Reduction of Radicals

(1) $a^4\sqrt{a}$

(2) $a^2\sqrt{a}$

(3) $2a^3\sqrt{3a}$

(4) $3a\sqrt{a}$

(5) $12a\sqrt{1}$

52. $\sqrt{a^5}$ -   -   -   -   (..........)

53. $\sqrt{9a^3}$ -   -   -   -   (..........)

54. $\sqrt{12a^7}$ -   -   -   (..........)

Group 7—Fractional Exponents

(1) $y\sqrt{y}$

(2) $y^3\sqrt{y}$

(3) $\sqrt[3]{y^2}$

(4) $\sqrt[2]{y^4}$

(5) $\sqrt[4]{y^3}$

55. $y^{\frac{3}{2}}$ -   -   -   -   (..........)

56. $y^{\frac{2}{3}}$ -   -   -   (..........)

57. $y^{\frac{3}{4}}$ -   -   -   (..........)

—5—

Part IV—Graphs

Directions: On blank space provided solve each problem and place answers in parentheses at the right locate each point and draw the graph for equation $y = x^2-2x-3$. One point for each point correctly found and one point for points correctly located.

	x	y
58-59.	5	
60-61.	4	
62-63.	3	
64-65.	2	
66-67.	1	
68-69.	0	
70-71.	—1	
72-73.	—2	
74-75.	—3	
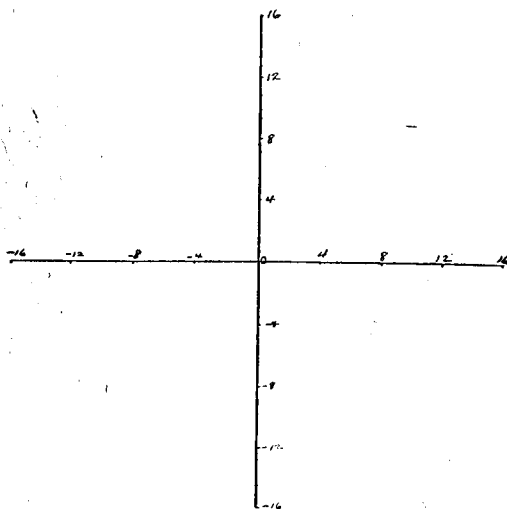
Part V—Equations to Solve and Written Problems

Group 1—Equations to Solve

Directions: On blank space provided solve each problem and place answers in parentheses at right.

75-78. (4 points) $\dfrac{4}{x+2} + \dfrac{7}{x+3} = \dfrac{x^2+44}{x^2+5x+6}$

x = (...............)

or (...............)

79-80. (2 points) Solve by factoring. Show work. $3x^2-5x-2=0$

x = (................)

or (................)

81-84. (4 points) Solve by completing the square. Show work.
$4x^2-8x+3=0$

x = (................)

or (................)

85-89. (5 points) Solve by use of the formula. Show work. $2x^2-3x+1=0$

x = (................)

or (................)

—7—

Group 2—Written Problems

90-94. (5 points)   If twice the square of a number is decreased by three times the number the result is 35.   Find the numbers.

Number = (..............)

or = (..............)

95-100. (5 points)   A can do a certain work in 15 days, B can do the same work in 12 days. A works for 3 days and asks B to help him finish the work. How many days will B have to work with A to complete the work?

B works (.............. days)

—8—

INDIANA UNIVERSITY EXTENSION DIVISION
State High School Mathematics Contest

Algebra Sectional Test, 1937   . Identification No. _____

INSTRUCTIONS TO STUDENTS:
This examination consists of five parts. The problems are grouped
according to difficulty and time required for solving. Below is a
table showing value of correct answers in each part.   Score:

Part I   Each problem counts 1 point (Total, 35) _____
Part II   Each problem counts 2 points (Total, 50) _____
Part III Each problem counts 3 points (Total, 30) _____
Part IV   Each problem counts 4 points (Total, 20) _____
Part V   Each problem counts 5 points (Total, 15) _____

Part I.   (Each problem, 1 point)   Total Score _____

1. Simplify $12a - (-4a)$.   1. _____

2. Subtract $9a^2b^2$ from $-8a^2b^2$.   2. _____

3. Solve for x: $\dfrac{5x}{6} - 3 = x$ 3. _____

4. Multiply $\left(\dfrac{a}{b} + 1\right)$ by $\left(\dfrac{a}{b} - 1\right)$.

4. _____

5. Simplify $(6x^2y^2 - 3xy) \div 3xy$.   5. _____

6. Find two factors of $8x - 12y$.   6. _____

7. Reduce to simplest form $\dfrac{\dfrac{1}{x}}{\dfrac{3}{x^3}}$

7. _____

8. Solve for x: $\dfrac{2x}{3} = 0.8$ 8. _____

9. What is the perimeter of a square field
with an area of $16a^2$ sq. rods?   9. _____

10. Express in dollars the sum of d dollars
and c cents.   10. _____

11. Find the value of $\left(\dfrac{8ab^4}{5c^3}\right)2$

11. _____

12. What distance does an automobile travel in
$(t + 2)$ hours at 40 miles per hour?   12. _____

13. If c parts of cement and s parts of sand
are mixed, what part of the mixture is cement?   13. _____

14. Solve for x: $-3x = 8 - (5 + 2x)$ 14. _____

15. Multiply: $(6x - 5y)^2$ 15. _____

16. Divide: $-28r^3s$ by $-7rs$ 16. _____

17. If $x = 3$, $y = -2$, $x^2y^2 =$

17. _____

18. If $a = 2$, $b = 3$, $c = -4$, find the value
of $2c^2 - ac + b^2$

18. _____

19. Perform the indicated operation:
$$\frac{14x^3}{15y^5} \div \frac{7y}{5x}$$

19. _____

20. Subtract $2x^3 - 3x^2 + 4x - 5$ from 0.

20. _____

21. If 10 is 4 more than $2a$, what is
the value of $3a - 5$?

21. _____

22. Write the following statement as an
equation: If 18 is subtracted from
7 times a certain number, the result
equals 5 times the number.

22. _____

23. Simplify $\left(\dfrac{-1}{2} a^3\right)^3$

23. _____

24. Find the value of L:

$L = a + (n-1)d$, when $a = 10$, $d = 5$, $n = 6$

24. _____

25. Write as an improper fraction:
$3x + 2y - \dfrac{5}{x}$

25. _____

In the following 5 problems answer true or false.

26. The square of the sum of a and b is $a^2 + b^2$

26. _____

27. $3a^2$ means $3a \cdot 3a$

27. _____

28. In the expression $-5xy$, the coefficient of
x is $-5y$

28. _____

29. The product of two numbers with like signs
will always be a number having this same sign.

29. _____

30. The ratio of one number to another is found
by comparing them by division.

30. _____

In the next 5, fill in blanks with suitable words.

31. The terms $2x$, $7x$, $-9x$ are called _____ terms.

31. _____

32. In the expression $4a^3$, the _____ of a is 3.

32. _____

33. An algebraic expression in which the parts
are not separated by the signs $+$ or $-$ is
called a _____.

33. _____

34. Doubling the length of the radius of a
circle multiplies the area by _____

34. _____

35. Two angles are called supplementary if
their sum is _____ $^\circ$.

35. _____

Score Pt. I _____

Part II.   (Each problem, 2 points)

36. Factor $2x^2 - 11x + 12$ 36._____

37. Simplify: $\dfrac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$ 37._____

38. Divide $6x^3 - 25x^2 - 8 + 26x$ by $3x - 2$ 38._____

39. Solve for x: $5(4x + 3) - (9x + 5) = 7x - 30$.   39._____

40. Reduce to lowest terms: $\dfrac{x + y}{x^2 + 2xy + y^2}$ 40._____

41. Find the least common denominator of $\dfrac{4x}{x^2 - y^2}$,
    $\dfrac{3x}{x-y}$, $\dfrac{5x}{x^2-2xy+y^2}$.   41._____

42. Simplify: $\dfrac{3ab^2 - 6a}{5b^3 - 10b}$ 42._____

43. Simplify: $\dfrac{x}{x - 4} - \dfrac{x}{x + 4}$ 43._____

44. Factor $9a^5 - 36a^3$ 44._____

45. A tower casts a shadow of 20 feet. A man 6 ft. tall casts a shadow of $1\frac{1}{2}$ ft. Write the proportion that would be used in solving the problem.   45._____

46. Divide $8x^3 + y^3$ by $2x + y$ 46._____

47. Multiply: $(3x^2 - 7 + 4x)(2x - 3)$.   47._____

48. Solve by factoring: $x^2 = 4x$ 48._____

49. Find prime factors: $a^2 + 2ab + b^2 - x^2$. 49._____

50. The sum of two numbers is 53, and 3 times the larger added to 4 times the smaller equals 173. Find the numbers.   50._____

51. Simplify: $\dfrac{x^2 - 13 + \frac{36}{x^2}}{x + 1 - \frac{6}{x}}$ 51._____

52. Solve for y: $(y - 3)^2 - (y - 2)^2 = y - 1$ 52._____

53. A and B together have \$50. If 9 times A's money is \$10 more than twice B's, how many dollars has each?   53._____

54. Simplify: $\dfrac{24x^2 - 26x - 5}{6x^2 - 9x} \cdot \dfrac{2x - 3}{6x + 1}$ 54._____

55. Perform indicated division and simplify:

$$\frac{2a^2 + 5a - 3}{2a^2 - 5a + 3} \div \frac{a^2 + 3a}{a - 1}$$

55. _____

56. What is the principal if the amount is $280 for 2 years at 6%?

56. _____

57. Solve for x and y: $\begin{cases} x + 3y = -7 \\ 4x - 5y = 23 \end{cases}$

57. _____

58. Combine into one fraction and simplify:

$$\frac{x - 5}{x^2 + 6x + 8} + \frac{x + 1}{3x + 12}$$

58. _____

59. Add: $2a - 3b + 7c$; $5b + 3c - 6a$; $-9b + 7a - 8c$

59. _____

60. Multiply and simplify:

$$18 \left(\frac{1}{3}a - \frac{1}{6}b + \frac{1}{2}c\right) - 12 \left(\frac{1}{4}a + \frac{1}{2}b - \frac{1}{6}c\right)$$

Score Pt. II _____

60. _____

Part III.   (Each problem, 3 points)

61. Solve for x and check:
(Solution 2 points, check 1 point)
$$2x - (4x - 9) + 5(x + 4) = 41$$

61. _____

Check:

62. Solve for x: $\dfrac{x^2}{2} - \dfrac{3x}{5} = \dfrac{19}{2}$

62. _____

63. Solve for V; $S = Vt - \frac{1}{2}gt^2$ when $S = 1328$, $g = 32$, $t = 4$.

63. _____

64. Find the prime factors of $a^8 - b^8$

64. _____

65. A can plow a field with his team in 10 days, and B can plow it in 2 days with a tractor. How many days will it take them to do it together?

65. _____

66. A sum of money amounting to $2.20 consists of nickels and quarters. There are in all 16 coins. How many of each coin are there? Write equation and solve. 66. Eq. _____

Nickels _____

Quarters _____

67. Find prime factors of $2x^4 - 26x^2 + 72$.   67._____

68. Missing a train by three hours, a man
    hires an airplane to catch it. The
    airplane can go 30 miles an hour fast-
    er than the train. If the man catches
    the train in 4 hours, what is the rate
    of the train in miles per hour?   68. _____

69. The capacities of two motor trucks are
    3 tons and 5 tons respectively. The
    larger truck made 6 more trips than the   69. Smaller_____
    smaller one. How many trips did each
    make if they hauled a total of 86 tons?   Larger_____

70. A man needs 20 hr. for a trip if he goes
    45 miles an hour. How fast must he go to   70. Prop._____
    make the trip in 18 hours? Write propor-
    tion and solve.
                        Score Pt. III   Ans._____

— —
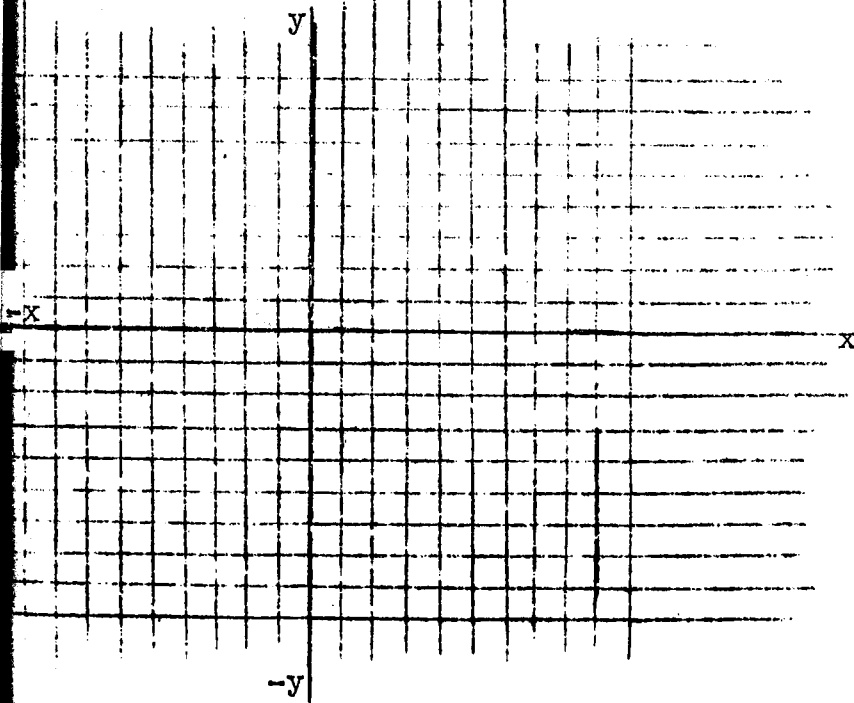
Part IV. (Each problem, 4 points)

71. Solve by graphing: 71. x = _____

    $2x - y = 4$ y = _____

    $2x + 3y = 12$

72. A grocer has coffee which he sells at 36¢ per pound, and other coffee which he sells at 20¢ per pound. How many pounds of the better coffee will there be in a mixture of 100 lbs. which he may sell at 26¢ a pound?

72. _____

73. Solve for x and y:

$$\frac{x}{4} + \frac{y}{6} = \frac{1}{2}$$

$$\frac{x}{3} - \frac{y}{2} = -1$$

73. x = _____

    y = _____

74. A man invests $6000, part at 4% and part at 7% and obtains an average of 6% on his money. How much is invested at each rate?

74. _____

75. A's age now is 2/3 of B's age, but in 9 yrs. A's age will be 3/4 of B's age. Find their present ages.

75. A _____

                                   Score Pt.IV_____   B _____

- -

Part V.   (Each problem, 5 points)

76. Solve:
    (See below)

76. a _____

    b _____

77. Simplify: $$\frac{\dfrac{a}{a-b} - \dfrac{a}{a+b}}{\dfrac{b}{a-b} + \dfrac{a}{a+b}}$$

77. _____

78. A crew can row 10 miles downstream in 50 minutes, and 12 miles upstream in $1\frac{1}{2}$ hrs. Find the rate in miles an hour of the current and of the crew in still water.

Current:

78. _____

Crew:

_____

76.(cont'd) $$\begin{cases} \dfrac{4a + b}{5} - \dfrac{6a - 3b}{3} = -3 \\[2mm] \dfrac{8a + 5b}{2} - \dfrac{10a - b}{7} = -4 \end{cases}$$

FIRST YEAR ALGEBRA SCALES

By HENRY G. HOTZ

School............. Date.............. City........... County...............

Write your name here..................................... Age...................

When did you begin to study algebra? Month............... Year..................

ADDITION AND SUBTRACTION

Carefully perform the operations as indicated.

1. $4r + 3r + 2r =$

2. $2x + 3x =$

3. $12b + 6b - 3b =$

4. $2c + \frac{1}{2} c =$

5. $7x - x + 6 - 4 =$

6. $3a - 4b + 5a - 2b =$

7. $5m + (- 4m) =$

8. $20x - (10x + 5x) =$

9. $(4r - 5t) + (s - 3r) =$

10. $8c - (- 6 + 3c) =$

11. $3a^2 - 3b - (2a^2 + 3b - 4) =$

12. $5x - [\, 4x - (3x - 1) \,] =$

13. $\dfrac{3c}{4} - \dfrac{3c}{8} =$

14. $\dfrac{3x - 2}{3} + \dfrac{x + 4}{6} =$

(Turn over. There are exercises on the other side)

15. $\dfrac{1}{a-x} - \dfrac{3x}{a^2 - x^2} =$

16. $\dfrac{r}{r+z} + \dfrac{r}{r-z} =$

17. $\dfrac{5a+1}{6a} - \dfrac{3a-2}{2a} =$

18. $3 - \dfrac{3-2x}{4} - 2x =$

19. $\dfrac{10x+3y}{2x^2y} - \dfrac{3x+5y}{xy^2} =$

20. $\dfrac{1}{a+1} - \dfrac{a}{a^2-a+1} - \dfrac{a-4}{a^3+1} =$

21. $\dfrac{2}{x^2-5x+6} - \dfrac{15}{x^2+2x-15} =$

22. $\dfrac{3-2x}{(x-1)^3} + \dfrac{x+1}{(x-1)^2} - \dfrac{1}{(x-1)} =$

23. $\sqrt{20} + \sqrt{45} + \sqrt{\tfrac{1}{5}} =$

24. $\dfrac{a}{a-2} - \dfrac{a-2}{a+2} + \dfrac{3}{4-a^2} =$

FIRST YEAR ALGEBRA SCALES

SERIES B

By HENRY G. HOTZ

School................................Date..............................City.......................County.......................

Write your name here..Age.......................

When did you begin to study algebra? Month..Year.......................

MULTIPLICATION AND DIVISION

Carefully perform the operations as indicated.   Reduce all answers to their *simplest* forms.

1. $3 \cdot 7y =$

2. $\dfrac{12n}{4} =$

3. $2a \cdot 4ab^2 =$

4. $6c^3 \div 2c^2 =$

5. $\frac{2}{3}$ of $9m =$

6. $\dfrac{-8a^2b}{4a^2} =$

7. $4x \cdot (-3xy^3) =$

8. $a^3 \cdot (-3a) \cdot (-2a) =$

9. $\dfrac{18m^2n - 27mn^2}{9mn} =$

10. $\dfrac{4x^4}{5} \div 2x^2 =$

11. $(2a^2 + 7a - 9)(5a - 1) =$

12. $\dfrac{n^4 + 7n^2 - 30}{n^2 - 3} =$

13. $\dfrac{7a}{15} \div \dfrac{7a^2}{20} =$

14. $\dfrac{-12x^2y^2 \cdot (x - 2)}{-3x^2y^2} =$

(Turn over.   There are exercises on the other side)

15. $\dfrac{m+n}{a} \cdot \dfrac{b}{m^2 - n^2} =$

16. $(-3xy^3)^4 =$

17. $\dfrac{c^4 - d^4}{(c-d)^2} \cdot \dfrac{c-d}{c^2 + d^2} =$

18. $3x^{\frac{1}{2}} \cdot 4x^{\frac{3}{2}} =$

19. $\dfrac{a^2 + \frac{3}{2}a - 1}{a + 2} =$

20. $\dfrac{p^2 + 4p - 45}{p^2 + 2p + 4} \cdot \dfrac{p^3 - 8}{p^2 - 81} \cdot \dfrac{1}{3pr - 15r} =$

21. $\dfrac{x^3 + 27}{x^2 + x - 12} \div \dfrac{3x + 9}{x + 4} =$

22. $64^{\frac{2}{3}} \times 27^{\frac{1}{3}} =$

23. $\dfrac{3\sqrt{6a}}{2a\sqrt{18}} \cdot \sqrt{12a} =$

FIRST YEAR ALGEBRA SCALES

BY HENRY G. HOTZ

School_____Date_____City_____County_____

Write your name here_____Age_____

When did you begin to study algebra? Month_____Year_____

EQUATION AND FORMULA

Solve the following equations and formulae:

1. $2x = 4.$

2. $7m = 3m + 12.$

3. $3x + 3 = 9.$

4. $5a + 5 = 61 - 3a.$

5. $7n - 12 - 3n + 4 = 0.$

6. $10 - 11z = 4 - 8z.$

7. $\frac{2}{3}z = 6.$

8. $c - 2(3 - 4c) = 12.$

9. $\frac{1}{2}x + \frac{1}{4}x = 3.$

10. $\frac{2x}{3} = \frac{5}{8}.$

11. The area of a triangle $= \frac{1}{2}bh$, in which
$$b = \text{length of the base}$$
$$\text{and } h = \text{height of the triangle.}$$
How many square feet are there in the area of a triangle whose base is 10 feet, and whose height is 8 feet?

12. $\frac{y}{3} = \frac{5}{2} - \frac{y}{4}.$

13. $\frac{1}{4}(x + 5) = 5.$

(Turn over.   There are exercises on the other side)

14. $3m + 7n = 34$
$7m + 8n = 46$

15. $\dfrac{4}{3 - x} = \dfrac{2}{1 + x}$.

16. The area of a circle $= \pi r^2$, in which
$r =$ radius of the circle
and $\pi = 3\frac{1}{7}$.

· Find the area in square feet of circle whose radius is 7 feet.

17. In the formula $RM = El$, find value of M.

18. $\dfrac{x + 3}{x - 2} = \dfrac{x + 5}{x - 4}$.

19. $p^2 - 5p = 50$.

20. $\dfrac{2}{x^2 + 4x + 3} = \dfrac{3}{x^2 + 3x + 2}$.

21. $\dfrac{1}{x} + \dfrac{2}{y} = 1$

$\dfrac{4}{x} - \dfrac{4}{y} = 1$

22. $F =$ temperature in Fahrenheit degrees
$C =$ temperature in Centigrade degrees
and $F = \dfrac{9C}{5} + 32°$.
Solve for C when $F = 70°$.

23. $\dfrac{6x - 2}{x + 3} - 3 = \dfrac{3x^2 + 13}{x^2 - 9}$.

24. $S = \frac{1}{2}gt^2$; solve for t.

25. $\sqrt{x^2 - 1} - x = -1$.

FIRST YEAR ALGEBRA SCALES

By HENRY G. HOTZ

School.................................. Date.............................. City.............................. County..............................

Write your name here.. Age..............................

When did you begin to study algebra? Month.............................. Year..............................

PROBLEMS

Do not work out the answer to the problem—merely *indicate* the answer or *state* the equation in each case.

1. If one coat cost x dollars, how much will 3 coats cost?

$$\text{Answer}...$$

2. A man is m years old; how old was he r years ago?

$$\text{Answer}...$$

3. A boy has a marbles and buys b more; how many has he then?

$$\text{Answer}...$$

4. A gold watch is worth ten times as much as a silver watch, and both together are worth $132. How much is each worth?

$$\text{Equation}...$$

5. The distance from Chicago to New York by rail is 980 miles. If a train runs v miles an hour, what is the time required for the run?

$$\text{Answer}...$$

6. The width of a basket ball court is 20 feet less than its length. The perimeter of the court (distance around) is 240 feet. Find the dimensions.

$$\text{Equation}...$$

7. The total number of circus tickets sold was 836. The number of tickets sold to adults was 136 less than twice the number of children's tickets. How many were sold of each?

$$\text{Equation}...$$

8. A rectangular box is d inches deep, w inches wide, and contains r cubic inches. What is its length?

Answer...

9. The area of a square is equal to that of a rectangle. The base of the rectangle is 12 feet longer and its altitude 4 feet shorter than the side of the square. Find the dimensions of both figures.

Equation...

10. A tower casts a shadow of 20 feet. A man, 5 feet 9 inches high, who is near at the same time, casts a shadow of 2 feet 6 inches. Find the height of the tower.

Proportion...

11. Five thousand dollars is invested in two banks, part in one at 3 per cent and the rest in the other at 4 per cent. The annual income from the two investments is $172. How much is each investment?

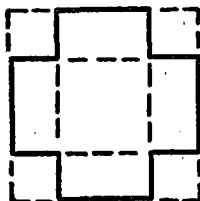Equation...

12. A train leaves a station and travels at the rate of 40 miles an hour. Two hours later a second train leaves the same station and travels in the same direction at the rate of 55 miles an hour. Where will the second train pass the first?

Equation...

13. A merchant has two kinds of tea, one kind costing 50 cents and the other 65 cents per pound. How many pounds of each must be mixed together to produce a mixture of 20 pounds that shall cost 60 cents per pound?

Equation...

14. An open box is made from a square piece of tin by cutting out a 5-inch square from each corner and turning up the sides. How large is the original square, if the box contains 180 cubic inches?

Equation...