Ten Questions Concerning the Large-Eddy Simulation of Turbulent Flows

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ABSTRACT

There have been many and substantial advances in LES since the pioneering works of Smagorinsky (1963), Lilly (1967), Deardorff (1974), Schumann (1975) and others. Advances have been made in: modelling the unresolved processes; accurate numerical methods on structured and unstructured grids; detailed comparison of LES calculations with DNS and experimental data in canonical flows; extensions to include additional phenomena, e.g., turbulent combustion; and in computational power, which has increased by about four orders of magnitude since the 1970's.

In spite of these advances, there remain fundamental questions about the conceptual foundations of LES, and about the methodologies and protocols used in its application. The purpose of this talk is to raise and to discuss some of these questions.

Before posing the questions to be addressed, it is necessary to introduce the terminology used to describe LES. This needs to be done with some care in order to include existing divergent views on LES, and to avoid pre-judging some of the questions raised. The fundamental quantity considered in LES is a three-dimensional unsteady velocity field which is intended to represent the larger-scale motions of the turbulent flow under consideration. We refer to this as the *resolved velocity field* and denote it by $\mathbf{W}(\mathbf{x}, t)$. In the "filtering approach" introduced by Leonard (1974), $\mathbf{W}(\mathbf{x}, t)$ is identified as the *filtered velocity field*, denoted by $\overline{\mathbf{U}}(\mathbf{x}, t)$, obtained by applying a low-pass spatial filter of characteristic width Δ to the underlying turbulent velocity field, $\mathbf{U}(\mathbf{x}, t)$. The effects of the sub-filter scales are modelled, and the resulting evolution equation for $\mathbf{W}(\mathbf{x}, t)$ is solved numerically on a mesh of spacing h. In contrast, in the "MILES approach" advocated by Boris et al. (1992), the Navier-Stokes equations are written for $\mathbf{W}(\mathbf{x}, t)$ and are solved on a mesh of spacing h which is insufficiently fine to resolve the smaller-scale motions, using a numerical method designed to respond appropriately in regions of inadequate spatial resolution. In order to accommodate all viewpoints we refer to $\mathbf{W}(\mathbf{x}, t)$ as the *resolved velocity field*, and to Δ as the *turbulence-resolution lengthscale*, which for MILES we define as $\Delta = h$. Turbulent motions that are not resolved are referred to as *residual motions*, and we use the term *residual stress* for the quantity often referred to as the sub-grid scale (SGS) stress, or the sub-filter scale stress.

The fundamental quantity in LES–namely the resolved velocity field $\mathbf{W}(\mathbf{x}, t)$—is an extremely complex object. It is a three-dimensional, time-dependent random field, which has a fundamental dependence on the artificial (i.e., non-physical) parameter Δ, and which (in some approaches) depends also on the mesh spacing h and on the numerical method used. It is not surprising, therefore, that LES raises non-trivial conceptual questions.

We address the following ten questions:

1. Is LES the right approach?

2. Can the resolution of all scales be made tractable?
3. Do we have sufficient computer power for LES?
4. Is LES a physical model, a numerical procedure, or a combination of both?
5. How can LES by made complete?
6. What is the relationship between **U** and **W**?
7. How do predicted flow statistics depend on Δ?
8. What is the goal of an LES calculation?
9. How are different LES models to be appraised?
10. Why is the dynamic procedure successful?

There is insufficient space here to discuss these questions fully. Prior to the Workshop, in November 2003, a paper addressing these questions will be available at `http://mae.cornell.edu/~pope/Reports`.

For flows in which rate-controlling processes occur below the resolved scales (e.g., near-wall flows and combustion), LES calculations have a first-order dependence on the modelling of these processes. Approaches that include a statistical resolution of all scales provide a more fundamental description of the rate-controlling processes; but it remains a challenge to devise such approaches that are computationally tractable and free of empiricism.

The relationship between the resolved LES velocity field $\mathbf{W}(\mathbf{x}, t)$ and the turbulent velocity field $\mathbf{U}(\mathbf{x}, t)$ can only be statistical. Corresponding to a turbulence statistic Q, the LES provides a model Q^m for Q of the form

$$Q^m = Q^w + Q^r, \tag{1}$$

where Q^w is the contribution from the resolved motions (which is obtained directly from $\mathbf{W}$) and Q^r is the modelled contribution from the residual motions. In LES, the turbulence resolution lengthscale $\Delta(\mathbf{x})$ is an artificial parameter of prime importance. As a rule, as Δ decreases, Q^w increases and Q^r decreases. Unless demonstrated otherwise, there is every reason to suppose that LES predictions Q^m depend (maybe strongly) on Δ. As a consequence, characterizing the dependence of predictions on Δ must be part of the overall LES methodology.

As currently practiced, LES is incomplete because the turbulence resolution lengthscale $\Delta(\mathbf{x})$ is specified subjectively in a flow-dependent manner. It can be made complete through *adaptive LES*. The variation of $\Delta(\mathbf{x}, t)$ is controlled (by grid adaption) so that a measure $M(\mathbf{x}, t)$ of turbulence resolution (e.g., the fraction of the kinetic energy in the resolved motions) is everywhere below a specified tolerance ϵ_M.

An alternative principle is advanced to justify the dynamic procedure, namely: the LES model coefficients should be chosen to minimize the difference between $Q^m(\overline{\Delta})$ and $Q^m(\widetilde{\overline{\Delta}})$ (where $\overline{\Delta}$ is the value of Δ used in the LES, and $\widetilde{\overline{\Delta}}$ is somewhat larger). It is shown that this principle applied to the Smagorinsky model results in essentially the same formula for the coefficient c_s as the standard dynamic model. Rather than depending on scale similarity, the procedure selects c_s to minimize the dependence of Q^m on Δ in regions where scale similarity does not apply.

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