MODELING OF THERMAL BEHAVIOR INSIDE A BUBBLE

Boonchai LERTNUWAT[*], Kazuyasu SUGIYAMA[**] and Yoichiro MATSUMOTO[***]

*, ***Dept. of Mechanical Engineering, The University of Tokyo, Tokyo, Japan

**Ship Performance Division, National Maritime Research Institute, Tokyo, Japan

Abstract

Cavitation occurs by the sudden expansion and the volumetric oscillation of bubble nuclei in the water due to the ambient pressure change. The size of bubble nuclei is O(10μm) and the thermal damping effects on the bubble motion is dominant in comparison with the acoustic and viscous damping effects. Because the thermal damping effect strongly depends on the thermal phenomena inside the bubble, it is important to simulate the detailed thermal behavior inside the bubble. The full DNS (Matsumoto and Takemura (1994) and Takemura and Mastumoto (1994)) is the most useful method to obtain the detailed structure. However, since it requires a long computational time to conduct the full DNS, a simple modeling for the internal thermal behavior is required. As we have known that there are many simple models for calculating the bubble oscillation such as the Rayleigh-Plesset equation etc. Nevertheless, most of previous works have assumed the thermal process of content inside the bubble to be adiabatic or isothermal and have neglected the thermal damping effect. In the present study, a simple model of the thermal behavior inside a spherical bubble is developed coupling with the Rayleigh-Plesset equation. The behavior of a spherical bubble in an acoustic field is numerically obtained by the full DNS, the present model and the conventional ones under adiabatic and isothermal assumptions.

1 Introduction

Cavitation is a serious problem in mechanical engineering field. To predict how the cavitating flows occur, there are 2 ways of simulation on cavitating flow problems. One is Cavity closure model and the other is two-phase flow model. It is well known that a cavitation bubble is formed from small bubble in water, so-called cavitation nuclei. Cavitation is understood as a process of rapid growth and collapse of a gas bubble with evaporation and condensation at the bubble surface according to the surrounding pressure, which decreases and increases rapidly. In the case that we want to follow the fact, two-phase flow simulation is a better choice. Besides if ones would like to know whole the flow field properties rather than only cavity attaching on solid surface imposed in liquid flow, two-phase flow simulation can give whole the flow field properties. Once Two-phase flow simulation is chosen to use, thermal behavior in oscillating bubble is to be considered. DNS gives theoretically the best result. However it needs to discretize the equation of bubble motion to solve for pressure and velocity distribution in it. This makes the computation code required more memory and needs more CPU time. Moreover, in simulating cavitating flows, there is not only one bubble but also a large number of bubbles so it takes much more CPU time and memory so it is expensive to due with. It is often the case that researchers assume that the thermal behavior inside a bubble is adiabatic or isothermal. This makes easier

programming and quicker simulation. But the results of those conventional methods (adiabatic and isothermal model) are not good enough in several cases. Therefore, making a simple model that gives good agreement with DNS but requires less CPU time and memory is a big problem to simulate cavitating flows. Yongliang (1995) proposed an empirical model of pressure difference to the change of cavitation bubble size (density). Matsumoto (1998) used switching model to predict the processes of an oscillating bubble. Prosperetti (1991) studied the polytropic model to calculate the proper polytropic index used to describe the thermal behavior in bubble. Anyway, Matsumoto (1999) concluded that this model is well defined only in the framework of a linear theory. By the way, there is, at least, a method that can avoid using bubble Dynamics equation (as well as the model of thermal behavior inside a bubble). Alajbegovic (1999) used a method to compute bubble number density directly without taking care of bubble Dynamics equation. But this method is not in the scope of our consideration. The aim of this work is to make a model of thermal behavior inside a oscillating bubble that is easy to use, requires less computational time and memory but gives results reliable (herein comparing to DNS results).

2 Simulation Methods

On solving the bubble motion, the following assumptions are employed:

(1) Gases inside the bubble and the surrounding liquid move maintaining spherical symmetry.

(2) Gases inside the bubble obey the perfect gas law.

(3) The vapor, mist generation and diffusion of non-condensable gas in liquid are neglected.

In the DNS, the full conservation equations for mass, momentum and energy in gas are solved numerically. The motion of the liquid phase is estimated by solving the first-order approximate equation for the bubble motion with respect to the liquid compressibility and the phase change at the bubble wall (Fujikawa (1980)). In the present model, the liquid phase is solved using the Rayleigh-Plesset equation and the constitutive equation of the pressure inside a bubble is proposed. The following relation at the bubble-water interface (Prosperetti (1988)) is used to consider thermal effect on the pressure inside a bubble.

$$\frac{d}{dt}\left(P_G R^{3\gamma}\right) = 3(\gamma-1)R^{3\gamma-1}K\left.\frac{\partial T}{\partial r}\right|_{r=R} \tag{1}$$

According to the order estimation written in a paper of Prosperetti (1988), we propose the temperature gradient model at the bubble-water interface expressed below.

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = \frac{\left(T_b - T_s\right)}{\left(Dt_0\right)^{1/2}} \tag{2}$$

Where T_b is average temperature of bubble. This temperature is calculated by assuming that gas inside bubble behaves itself as ideal gas.

$$\frac{P_G R^3}{T_b} = const \tag{3}$$

And T_s is temperature at bubble surface assumed to be equal to that of surrounding fluid, D is diffusivity of gas inside bubble and t_0 is a characteristic time of a bubble calculated by Eq.(4).

$$t_0 = \frac{2\pi}{\omega_0} \tag{4}$$

in which

$$\omega_0 = \left(\frac{3\gamma(P_\infty - P_V)}{\rho_L R_0^2} + \frac{2(3\gamma - 1)S}{\rho_L R_0^3} - \frac{4\upsilon_L^2}{R_0^4} \right)^{\frac{1}{2}} \tag{5}$$

P_∞	Pressure at infinity
P_V	Vapor pressure in bubble
S	Bubble surface tension
υ_L	Liquid viscosity
γ	Polytropic index
R_0	Initial bubble radius

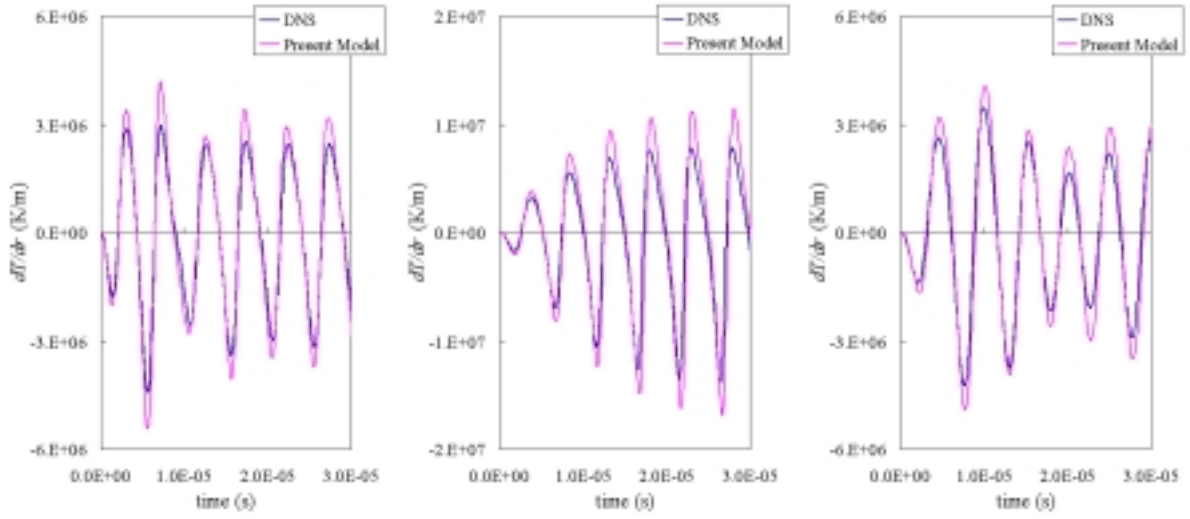
From Eqs. (1) to (5), the pressure of non-condensable gas inside the bubble is obtained. Then Rayleigh-Plesset equation (Eq.(6)) is used to compute the movement of bubble wall.

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_G + P_V - P_\infty}{\rho_L} - \frac{2S}{\rho_L R} - \frac{4\upsilon_L \dot{R}}{R} \tag{6}$$

3 Bubble-Oscillation Behavior

We conducted simulations for three cases of the initial bubble radii (R_0=1.0,1.4 and 2.0 x 10^{-5}m), whose driving frequency (f_d) is 200kHz. The initial pressure (p_0) is 100kPa and the pressure amplitude (δp) is 10kPa. Under the assumption of isothermal change (γ=1.0 constantly), the resonant bubble radius is 1.42 x 10^{-5} m. The time histories of temperature gradient at bubble-water interface obtained by the DNS and the present model are shown in Figure 1. In all cases of the initial bubble radii, the results of the present model agree well with ones of the DNS. The frequency of oscillation is the same, comparing between DNS and the present model results. But there is some difference on the amplitude of oscillation. The difference occurs when bubble changes process from rebounding to be collapsing (and collapsing to be rebounding). It is because when the process is changing, gas content near bubble wall is disturbed. This disturbance makes temperature distribution near bubble wall varied violently and the assumption of linear temperature gradient at bubble wall is temporally not valid. This is because Eq.(2) is not proper in this condition. However the difference is not big and occurs within only short period of oscillation so its effect is not so much to affect oscillation behavior.

The time histories of bubble radius obtained by the DNS, the conventional methods under the isothermal or adiabatic assumption and the present model are shown in Figure 2. In all cases of the initial bubble radii, the results of the present model agree well with those of the DNS compared with the conventional methods. This is because the non-linear characteristic of thermal behavior is sensitive to pressure variation. When bubble changes its size from equilibrium, the temperature inside it computed by Eq.(3) will changes. Then the temperature inside bubble defers from outside temperature results in the thermal behavior of oscillating bubble varies according to the changed size of bubble. Conventional methods do not take care of this non-linear characteristic. Therefore, bubble oscillates differently due to the lack of thermal damping effect in conventional models.



R = 1.0x10^{-5}m R = 1.4x10^{-5}m R = 2.0x10^{-5}m

Figure 1: Time histories of the temperature gradient at the bubble-water interface obtained by the DNS and the present model (f_d=200(kHz), p_0=100(kPa), δp=10(kPa))

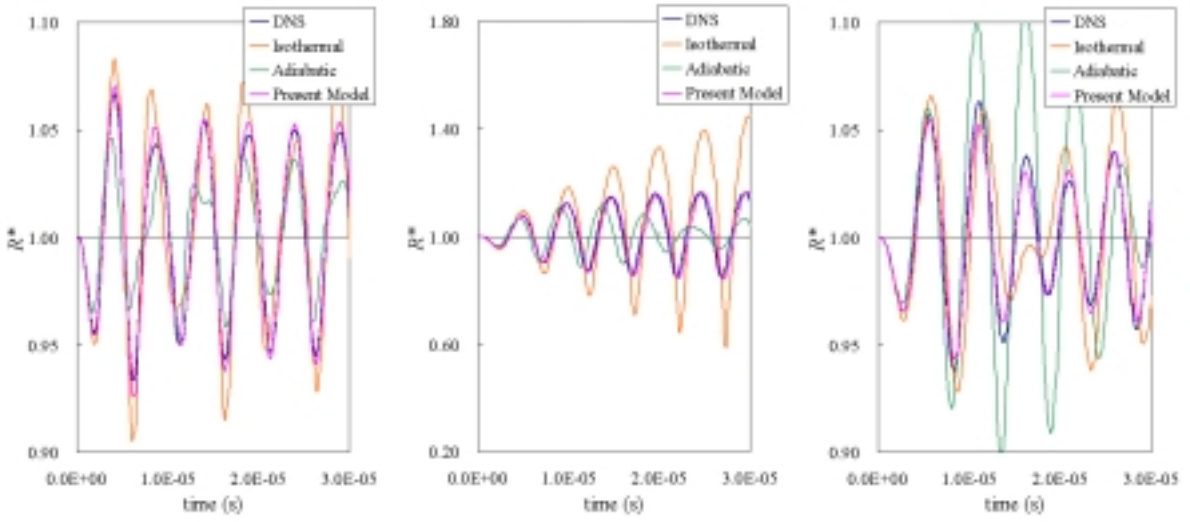


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Figure 2: Time histories of the bubble radius obtained by the DNS, conventional methods and the present model (f_d=200(kHz), p_0=100(kPa), δp=10(kPa))

Figure 3 and Figure 4 present the results of bubble oscillation under larger pressure amplitude (δp=20kPa). And Figure 5 and Figure 6 present the results of bubble oscillation under much larger pressure amplitude (δp=50kPa). From Rayleigh-Plesset equation (Eq.(6)), it shows that the higher $\ddot{R}, \dot{R}$ and R occur when the larger P_∞ occurs. And since larger δp makes larger P_∞ so R is larger. Considering Eq.(3), larger R gives larger T_b. This will make larger temperature gradient in Eq.(2). Finally it affects thermal behavior in Eq.(1) more than in the case of smaller δp. Therefore the error occurs in the case of large δp will be more than of small δp.

However the results of present model still agree well with DNS model and they are better when compared to two other conventional methods. This means that the model is robust for wide range of δp (but there is a limitation up to the condition of oscillation).

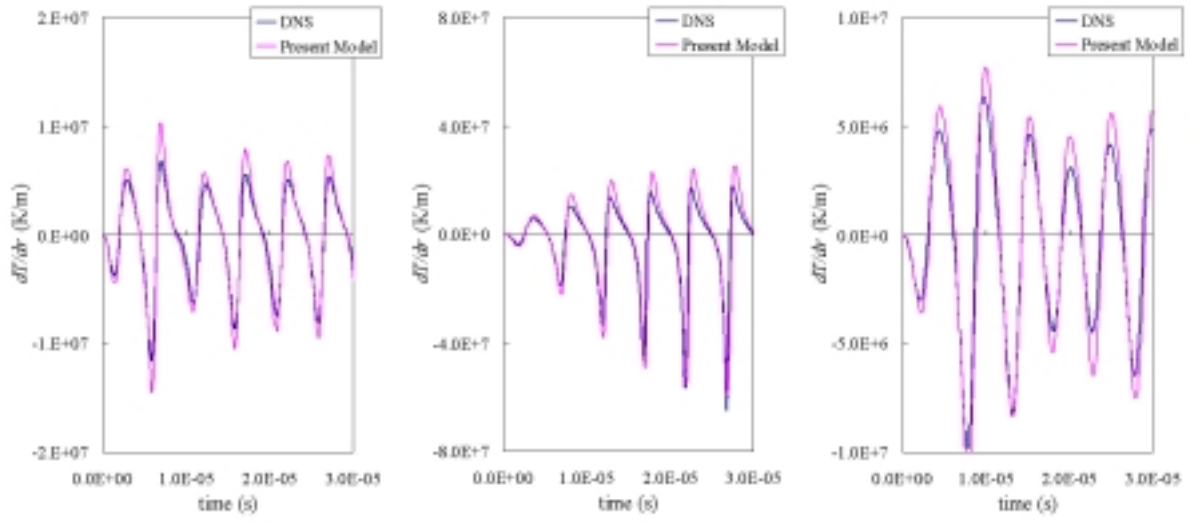

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Figure 3: Time histories of the temperature gradient at the bubble-water interface obtained by the DNS and the present model (f_d=200(kHz), p_0=100(kPa), δp=20(kPa))

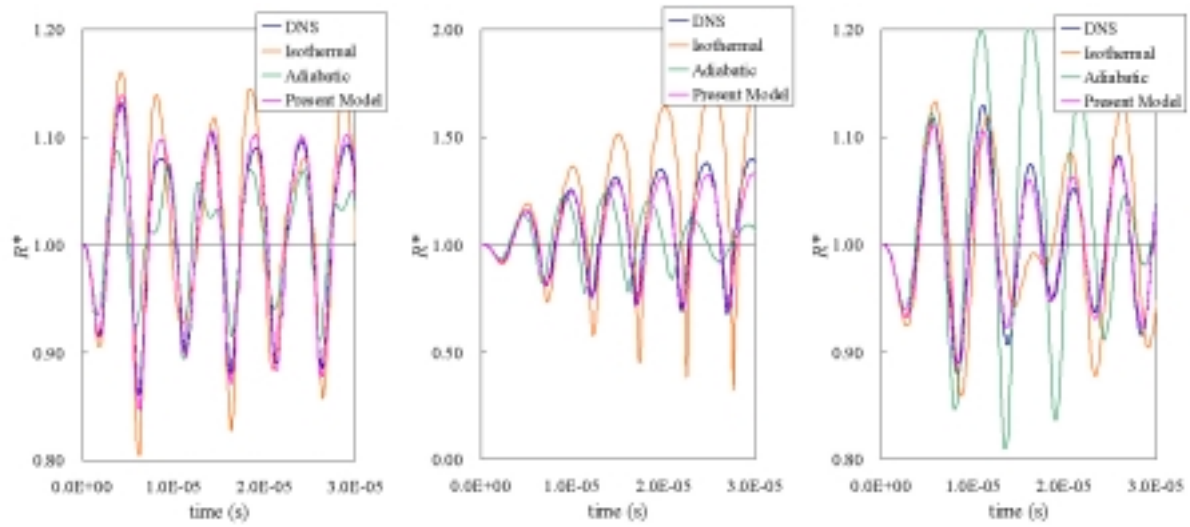

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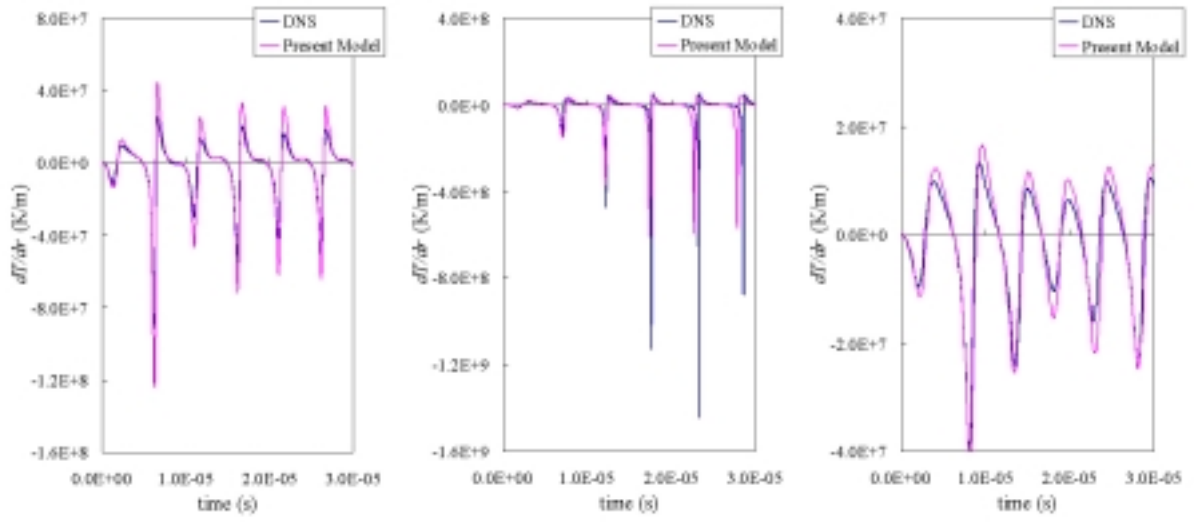
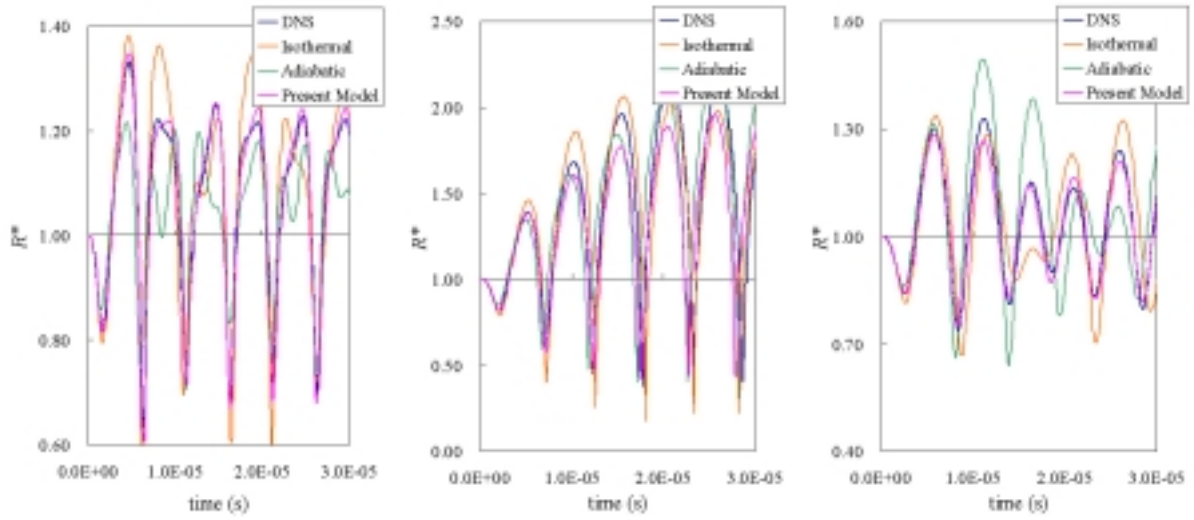
Figure 4: Time histories of the bubble radius obtained by the DNS, conventional methods and the present model (f_d=200(kHz), p_0=100(kPa), δp=20(kPa))



R = 1.0x10^{-5}m R = 1.4x10^{-5}m R = 2.0x10^{-5}m

Figure 5: Time histories of the temperature gradient at the bubble-water interface obtained by the DNS and the present model (f_d=200(kHz), p_0=100(kPa), δp=50(kPa))



R = 1.0x10^{-5}m R = 1.4x10^{-5}m R = 2.0x10^{-5}m

Figure 6: Time histories of the bubble radius obtained by the DNS, conventional methods and the present model (f_d=200(kHz), p_0=100(kPa), δp=50(kPa))

4 Resonance

By varying bubble radius, we can determine the resonant bubble radius given from the different models. Figure 7 shows comparison of the largest radius amplitude of different initial bubble radius under and the pressure amplitude (δp) of 10kPa. DNS and the present model give almost the same resonant bubble radius for all cases and liltle difference on radius amplitude size. It is obvious that the accuracy of result for the present

model is better. Furthermore the larger size of conventional methods may be the cause of diverging when computing bubbly flow problem. Hence the present model prevents computation not to diverge when it encounters the condition close to natural frequency because of thermal damping effect.

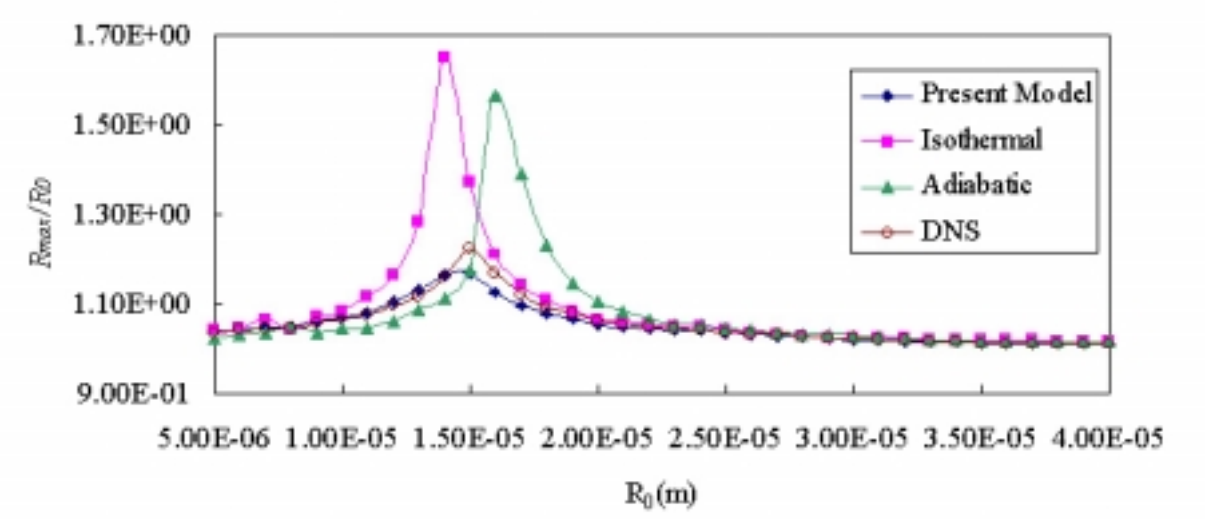


Figure 7: Maximum bubble radius driven by sinusoidal pressure field (f_d=200kHz, p_0=100kPa, δp=10kPa)

Figure 8 and 9 are the same graphs as of Figure 7 but with different pressure amplitude (δp), 20 and 50kPa successively. The results still show the same fashion as shown in Figure 7. All cases shows that the present model gives smaller resonant bubble size and smaller radius amplitude of oscillation. Anyway present model gives results agreeing quite well with of DNS whereas other conventional methods give much more difference. And in these both latter cases, we can see another harmonic at smaller bubble.

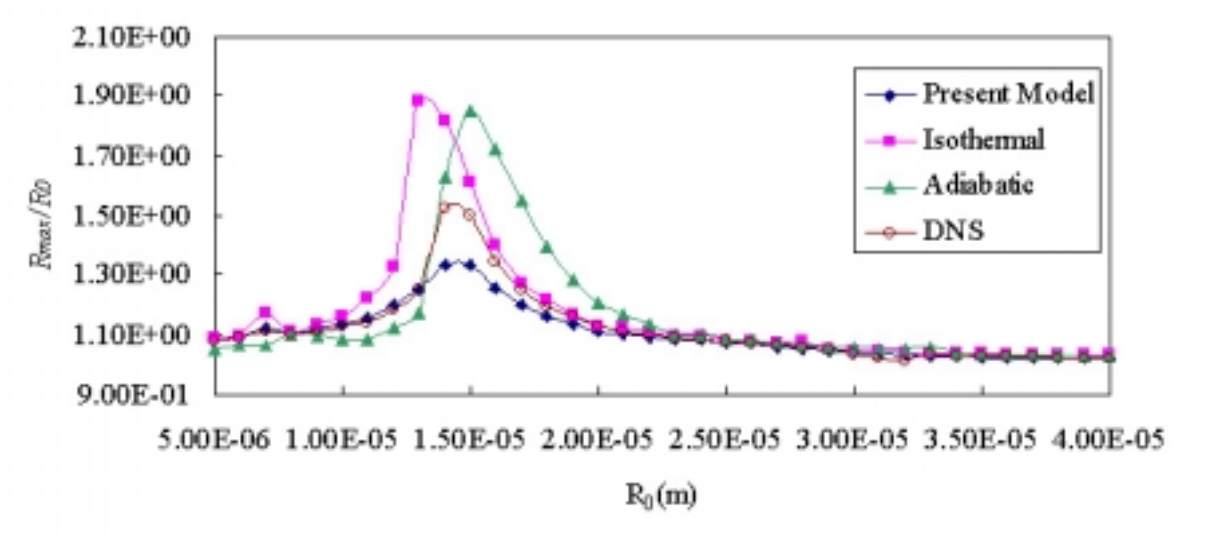


Figure 8: Maximum bubble radius driven by sinusoidal pressure field (f_d=200kHz, p_0=100kPa, δp=20kPa)

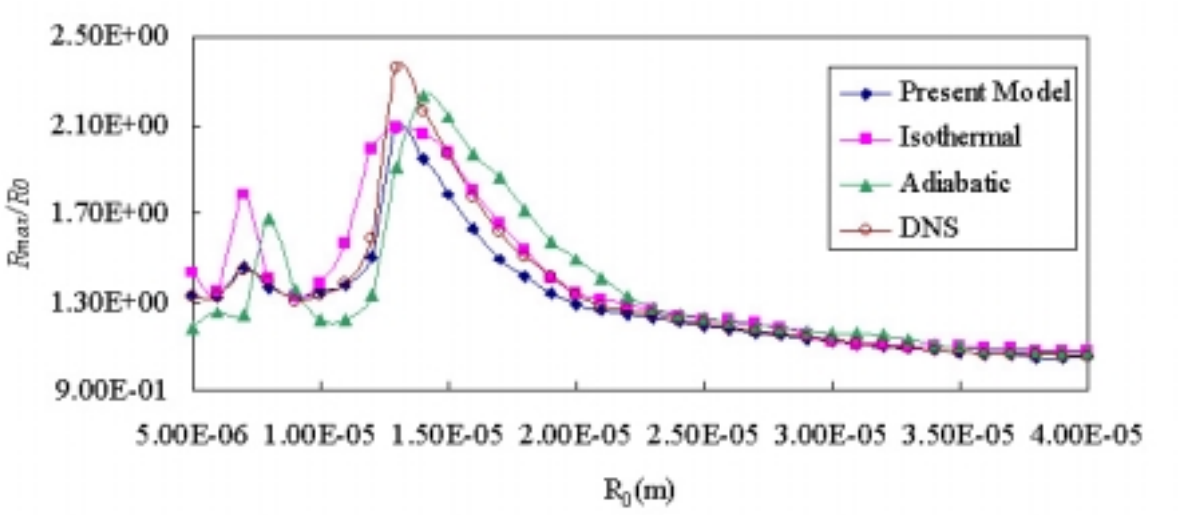


Figure 9: Maximum bubble radius driven by sinusoidal pressure field (f_d=200kHz, p_0=100kPa, δp=50kPa)

5 Conclusions

The results of the present model agree well with those obtained by DNS on the frequency of bubble oscillation. Time history of temperature gradient on the bubble wall and its radius show good agreement with DNS except the peak value. The discrepancy of the peak value comes from the simple modeling of the temperature gradient on the bubble wall in the present study. Although this modeling causes relatively larger error under the resonant condition, the error is not large in most cases. Therefore the present simple model given by Eq.(1) and (2) reasonably reproduces the thermal effect inside bubble. And since it has much less time consuming, it is useful to apply this simple model into numerical models of cavitating flows.

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