

# EXPERIMENTAL STUDY OF CAVITATION IN A KAPLAN MODEL TURBINE

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## Abstract

The cavitation processes present in a Kaplan model turbine was studied with the aim to identify mechanisms that promote erosive cavitation. The studies were carried out with high-speed filming, video filming and visual observations with stroboscopic light. A periodic pattern of the cavitating tip vortex was observed. The main feature of this pattern is that the cavitating vortex is bent towards the blade surface and transformed into cloud formations. These clouds were collapsing in a manner that suggests this to be an erosive process. The blade periphery, where the cloud collapses was observed, is known as an area likely to sustain damage. It was also found that these cloud formations appears in bands, with a periodicity which corresponds approximately to the spacing of the guide vanes. Bubble clouds and cavitating vortices was found to be shed from the sheet cavities which showed signs of re-entrant jets. The cavities at the blade root seem to be mainly of sheet or travelling bubble type, depending on running condition, some facts indicate the contribution of vortex motion as well.

## 1 Introduction

To accomplish the basic aim of making a compact turbine with a high setting, a certain amount of cavitation usually has to be allowed and the erosion risk must then be predicted. Cavitation may cause rapid erosion of the exposed parts, which in turn requires increased maintenance and, consequently, a reduced energy production. The damage caused by imploding cavities is usually associated with cloud cavitation. Attached sheet cavitation at the leading edge is often reported to shed bubble clouds or cavitating vortex structures. Cloud cavitation, in which the bubbles collapse under collective interaction and may generate shock waves of very high pressure amplitudes, is known as one of the most harmful processes (e.g. Brennen, Reisman & Wang, 1992; Brennen, 1994). One explanation of the generation of cloud cavitation is that a re-entrant jet, originating from the closure end of the attached cavity, moves upstream inside the cavity and strikes the cavity interface from the inside. If the jet has momentum enough, it may tear off a part of the cavity. This part is then transformed into a cloud of bubbles which is convected downstream where it collapses. The direction of the re-entrant jet is determined by the cavity closure line. It mirrors the direction of the main flow streamlines moving into the re-entrant jet, see de Lange, de Bruin & van Wijngaarden (1994). The velocity component normal to the sheet cavity closure line is reversed, whereas the tangential component is preserved. A convex cavity closure line may therefore cause a focusing of the re-entrant jet and a growth of its thickness. This accumulation of fluid may cause the re-entrant jet to interact with the cavity interface, see Schön (2000).

The re-entrant jet mechanism, which has been suggested, by Kubota (1988), de Lange *et al.* (1994) and Kawanami, Kato & Yamaguchi (1998), for example, to be responsible for shedding of bubble clouds, is most likely just one of the mechanisms that may cause cavitation shedding. To suppress the shedding of cloud cavitation, Kawanami *et al.* installed small bars at the wall, normal to the flow, just upstream of the sheet cavity closure. This was made to prevent the re-entrant jet from extending further upstream. Although the cloud cavitation shedding ceased, many small cavitating vortices were shed. Kawanami *et al.* (1997) show that the shed cavity structure and shedding frequency may depend on how far the re-entrant jet extends inside the sheet cavity. Their result shows that the structures shed changed from relatively small vortex cavities to larger ones closer to the bubble cloud type as the penetration length of the re-entrant jet increased. They also noted that the penetration length increased as the thickness of the sheet cavity increased.

## 2 Experimental methods

The aim of the experiments is to gain general knowledge of the different cavitation processes that appear in a Kaplan turbine. It is of particular interest to identify cavitation processes that may result in erosion.

The experiments were carried out on a Kaplan model turbine in a cavitation tunnel at the Hydraulic Machines Laboratory of GE Energy (Sweden) AB. The model runner had four blades and a diameter of 500 mm. A discharge ring with windows, offering visual access from four circumferential positions around the runner, was used to trace any flow inhomogeneities. The discharge ring was made of plexiglas and provided with a strengthening shell of steel with four openings allowing visual access to the runner. The tip clearance was somewhat larger than normally is used by GE Energy (Sweden) AB, because of the reduced mechanical strength of the discharge ring selected. It was also not possible to exclude deformation effects on the tip clearance under running conditions either.

Video filming, used primarily for general documentation, was also employed to trace flow inhomogeneities, because the video camera was small enough for filming through all four windows in the tight space between the discharge ring and the spiral case. The video recordings made in stroboscopic light, with one flash per turbine revolution, were enough for a general overview of the cavitation behaviour; for a more detailed study of the dynamics of individual cavities, high speed filming was done from two different circumferential positions. Although the high-speed camera was capable of reaching a frame rate of 10 000 frames per second, the maximum frame rate was somewhat reduced by the film length used.

It was hard to arrange powerful continuous lighting because of the narrow space around the model turbine. Flash bulbs were connected in series of three or four bulbs for sequential flashing. The bulbs were attached with tape to the observation windows. A slightly overlapping flashing sequence gives a moving and slightly pulsating light. The number of flash bulbs is limited by the dimensions of the observation window to which they are attached. This arrangement offered a compact and powerful lighting arrangement. The drawbacks are that the duration of the lighting is limited and that the exact distribution of light in time and space is hard to predict. With the maximum number of flash bulbs that could be attached to the observation window, the duration of the light covered almost one turbine revolution. Because of random fluctuations in the cavitation, a certain minimum number of films per running condition is needed for statistically reliable evaluation. Experience led to the conclusion that this procedure requires a lot of time and film. Unfortunately, the resources available for the present experiments extended to a total of only seven high-speed films in addition to the video filming.

Finally, a method that has been used successfully at SSPA Sweden AB for erosion assessment of ship model propellers was tried. The blades were covered with a soft coating, and thereafter the turbine was operated for three hours. Some minor damage, probably caused by the cavitating tip vortex, was found in the soft coating in the peripheral area of the blades. But, in general there was too little damage to support an estimate of the erosion potential of any of the cavitation processes. The method needs to be adapted for turbine applications before it is useful for that purpose. This matter will not be discussed further in this paper.

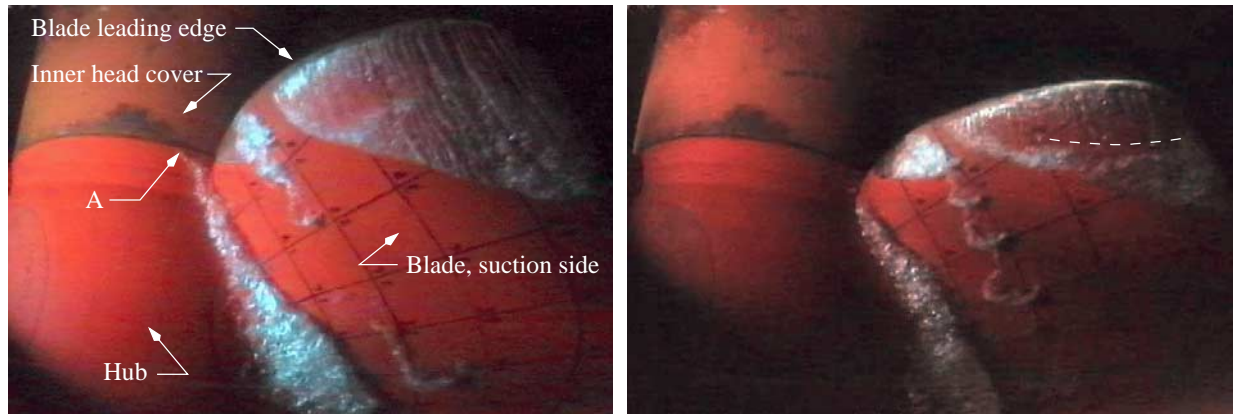
## 3 Result and discussion

This study concentrates on cavitation appearing at the blade inlet, at the blade root and at the blade tip, all on the suction side of the blades. Reasons for this limitation are that the cavitation types treated are often the dominant ones, and that erosion on the runners occurs mostly on the suction side. Cavitation, however, may appear in other areas than studied here.

### 3.1 Cavitation starting at the blade leading edge

Attached sheet cavities starting at the leading edge are usually the result of a mismatched angle of attack, causing a flow separation close to the leading edge. At this particular runner, two attached sheet cavities were usually found, one small and one large, at the leading edge on the suction side. Although the sheet cavities are near each other, they are more or less detached from each other depending on the running condition. The small sheet cavity is located closer to the hub, whereas the large sheet cavity is located further out. Both cavities have convex closure ends and are relatively stable with minor volume fluctuations, however the surface of the small sheet cavity is much more disturbed than that of the large one.

The distorted cavity interface in the closure region is linked with a disintegration of the sheet into small parts which are convected downstream. The disintegrated voids can be characterized as cavitating bubble clouds, vortex cavities of horse shoe shape (also called ring vortex, hairpin-shaped vortex and inverse U-shaped vortex) or long stream oriented cavitating vortices (see photographs in Figure 1).



(a) A small attached sheet cavity at the leading edge, closer to the hub, and a large sheet cavity further outward towards the blade periphery. The surface of the small sheet cavity is more disturbed than the surface of the large one. Cavitation originating from the join between the rotating hub and the stationary inner head cover is also shown at A. The flow is from above.

(b) Relatively large cavitating ring vortices broken off from the small attached sheet cavity fairly regularly. The cavitating vortices are approximately of same size as the sheet cavity from which they originate. The trace of a re-entrant jet front in the large sheet cavity has been marked with a white dashed curve.

Figure 1: The sheet cavities have convex closure ends and disturbed surfaces.

### 3.1.1 Re-entrant jets

While the experiments reported here were not ideal for detailed observations of re-entrant jets, some observations were nevertheless made from the video recordings. These indicated the existence of re-entrant jet fronts in the upstream, fairly smooth part of the large sheet; see Figure 1(b) where the jet front is marked with a white dashed line. Because of the distorted surface, it was not possible to find out if there is any re-entrant jet inside the small cavity. However, the distorted surface and the broken off structures can both be results of a re-entrant jet. Perhaps the visual indications on a re-entrant jet front, the lack of large disintegrated bubble clouds, the disturbed cavity surface, and the shedding of several small cavitating vortex structures all indicate a thin sheet cavity with a re-entrant jet that causes shedding by interaction with the cavity interface. This implication is made with the results of Kawanami *et al.* (1997) in mind, that a loss of momentum will decrease the penetration length of the re-entrant jet which will cause the shed structures to be smaller.

### 3.1.2 Bubble formations

Bubble formations or cloud cavitation that were observed to shed from the sheet cavities were relatively small and of different shape. The photograph in Figure 2(a) shows an example of a coherent bubble cloud, at A, that just has broken off from the closure end of the sheet cavity. Further downstream, at B, a foamy structure indicates an already collapsed bubble formation. Figure 2(b) shows a similar example, with the difference that the broken-off bubble structure is not as coherent as the one in Figure 2(a). In the next photograph, Figure 2(c), there are somewhat smaller cloud formations that are almost regularly broken off from the main sheet cavity; one of them, at D, resembles a ring-like cavitating vortex. Figure 2(d) also shows ring-like cavitating vortices, although they are thinner than the one in Figure 2(c).

All of the views in Figure 2 were made under the same running conditions, however Figure 2(c) is from another inspection window, than the three others.

### 3.1.3 Cavitating vortices

The third category of cavitation at the leading edge of the blade is cavitating ring vortices which are broken off from the closure end of the sheet cavities, after which they are convected downstream with the vortex



(a) Coherent cloud cavitation at A has just broken off. The foamy remainder of an already collapsed bubble cloud is shown at B.

(b) Small shed structures of cloud cavitation are indicated at C.

(c) Scattered bubble formations, one of them, at D, resemble a cavitating vortex.

(d) Relatively small cavitating ring vortices, E and F, broken off from at a point where the sheet cavity closure line is locally convex.

Figure 2: Examples of bubble formations shed from an attached sheet cavity.

ends against the blade surface. Since the vortices are convected downstream with the vortex ends touching the blade surface, the collapse probably occurs very close to the blade surface, which is a basic requirement for erosion.

The ring vortices are usually broken off at the convex closure region of a sheet cavity and are of approximately the same size as the convex part of the cavity. Cavitating ring vortices of dissimilar relative sizes were observed, Figures 1(b) and 2(d), show ring vortices of different relative dimensions. Ring vortices shed from the small sheet cavity are of approximately the same size as the whole cavity from which they are shed, whereas the ring vortices shed from the large sheet cavity are smaller: they are of the same size as the locally convex part of the cavity closure line from which they are broken off. For short time intervals the ring vortices are sometimes shed very regularly, as can be seen in Figure 1(b). Occasionally the formation process degenerates, and long stream oriented vortices are formed instead, see Figure 1(a).

Since the vortices seem to be generated only at regions where the sheet cavity closure line is convex, the shedding is perhaps caused by a focusing of a re-entrant jet as discussed in the *Introduction*. Another possible scenario is that a focusing of a re-entrant jet causes a shed void which generates vorticity at its collapse. Gopalan & Katz (1999) has shown that collapsing voids can cause substantial vortex production.

Although the authors are uncertain about the origin of the ring vortices observed here, they were found to rotate in the same direction as the inverse U-shaped ones reported by Avellan & Karimi (1987).

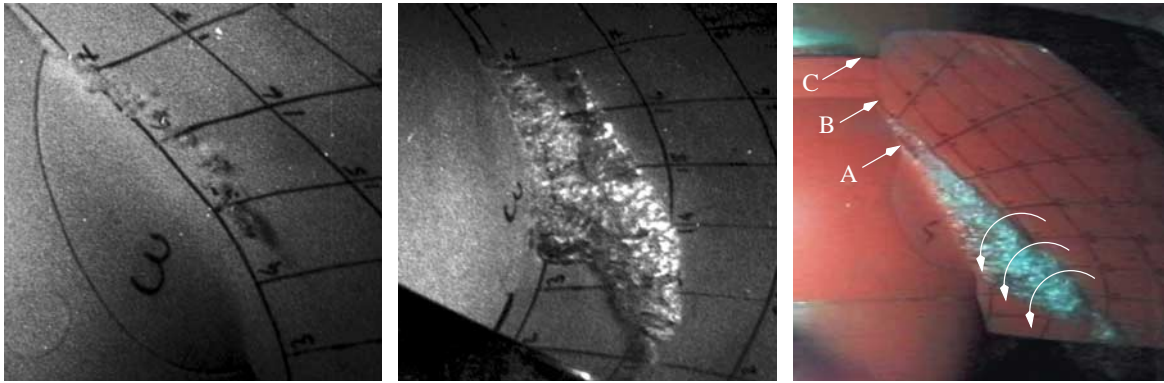
### 3.2 Cavitation at the blade root

Cavitation at the blade root appears in the experiments reported here mainly as two different types of cavitation (see Figure 3(a) and 3(b)): travelling bubbles and sheet cavitation that may have some superposed vorticity. The type of cavity that appears depends on the running condition. The travelling bubble cavitation shifts towards sheet cavitation as the bubble concentration is increased by decreasing the cavitation number.

The root cavities appears at two locations, one is located close to the blade flange on the blade surface (inner root cavity), while the other is located further out on the blade surface. The latter has a shape resembling a cavitating vortex but, as far as could be detected with the high speed films, it may have only a slight vorticity. The inner cavity, on the other hand, shows no signs of any rotation: it seems to be either of travelling bubble or of sheet type.

Some flow mechanisms influencing the cavitation at the blade root were identified by studying various running conditions. The flow mechanisms were found to originate from the following places:

- A The fillet between the blade and the blade flange at the upstream end of the flange (A in Figure 3(c));
- B The corner between the blade leading edge and the hub (B in Figure 3(c)); and
- C The join between the rotating hub and the stationary inner head cover (C in Figure 3(c)).



(a) Cavitation initiated close to the fillet between the blade and the flange.

(b) More severe cavitation started at the fillet between the blade and the flange.

(c) Cavitation at the blade root, with the inner cavity incepted at the fillet and the outer one at the inner corner of the blade leading edge. The direction of the possible weak rotation of the outer cavity observed is shown by the arrows.

Figure 3: Cavitation at the blade root. The flow is from above.

### 3.2.1 Flow disturbance from the upstream end of the blade flange fillet (Mechanism A)

The cavitation that originates from point A in Figure 3(c) and is located close to the blade flange fillet is here called the inner root cavity. Figure 3(a) shows an example of it at an incipient stage, appearing as travelling bubble cavitation.

A primary reason for cavitation in the region of the blade flange fillet is a decrease of pressure caused by the body shape at the fillet. There may also be a contribution caused by a mixing of two boundary layers, i.e. the blade boundary layer and the hub boundary layer. Perhaps the mixing process causes a flow separation and, consequently, a low pressure area where the inner root cavity is located. The blade boundary layer is initiated at the blade leading edge, whereas the hub boundary layer is initiated much further upstream.

Under conditions of moderate cavitation development, as in Figure 3(a), bubbles are initiated at the fillet, whereupon they are convected downstream while expanding. With a somewhat higher concentration of bubbles (lowered cavitation number or altered running condition) the growth process remains the same, but the bubbles will then merge together more with neighbouring bubbles before collapsing. The collapse is sometimes followed by minor rebounds.

Under conditions of more developed cavitation, as in Figure 3(b), the inner root cavity that is initiated at the fillet shows no broken-off structures at the cavity closure. On the other hand, the outer cavity in the figure shows some minor shedding of bubble clouds. When the amount of cavitation at the blade root is increased further by changing the running condition (as in Figure 3(c)), the shedding of bubble clouds from the outer cavity appears to increase, whereas the inner cavity still does not show any shed structures. It should be noted that similar root cavitation formations on propellers can exhibit significant shedding of cloud cavitation that results in erosion and noise. The origin of the inner root cavity remains at the fillet even under conditions of more severe cavitation development, as can be seen in Figures 3(b) and 3(c).

No rotation was detected in the inner root cavity. Accordingly, the possibility that a leakage flow vortex, which is generated by a leakage in the clearance between the blade and hub, is a major dominating mechanism can be eliminated. A leakage vortex may, however, be superposed on another dominating mechanism, e.g. a flow separation caused by boundary layer mixing, as already discussed. Perhaps this is the reason why the resulting cavitating flow was not clearly found to be affected by vorticity.

### 3.2.2 Flow disturbance from the inner corner of the blade leading edge (Mechanism B)

The influence of the flow disturbance from the corner, between the blade leading edge and the hub (B in Figure 3(c)), on the root cavitation is not obvious under all running conditions. In Figure 3(c), one can see that the outer cavity originates from the corner between the leading edge of the blade and the hub. When looking for vorticity, it appears that the outer root cavity is not strongly influenced by this either. Nevertheless, a very weak rotational motion, shown in the figure, can be traced with some difficulty. The direction of rotation, however, is opposite to that expected for a leakage flow.

If a leading edge vortex (Lakshminarayana, 1996) is present, it should be a very weak vortex, because in this application, the leading edge radius is small. However, such a leading edge vortex would have the same direction of rotation as observed. Moreover, Půlpitel, Skoták & Koutník (1996) has reported a similar vortex, starting at the vicinity of the inner corner of the blade leading edge and extending diagonally over the suction side of the blade. The rotational direction and angular speed of the vortex are not discussed in the paper, although Půlpitel *et al.* call their observed flow pattern a vortex.

The results from calculations made by Nilsson (1999) of the flow in a Kaplan turbine also show a vortex structure at the same location. The rotation direction of the vortex in his calculations is the same as that indicated in Figure 3(c). The calculation shows also that it is a very weak vortex which is in agreement with the observations made in our experiments (the rotation direction and the strength of the vortex were confirmed by private communication; they are not included in the report by Nilsson (1999)).

### 3.2.3 Flow disturbance from the join between the hub and inner head cover (Mechanism C)

A flow disturbance originating from the join between the rotating hub and the stationary inner head cover (shown at A in Figure 1(a)), was observed under conditions of severe cavitation development. It looks like a cavitating vortex which is superposed to the cavities originating from the inner corner of the blade leading edge. The high-speed films did not make it possible either to confirm or to deny any vorticity in the cavity originating from the join. That the flow structure starts upstream from the leading edge and also seems to cavitate is confusing. No reasonable explanation for this to be caused by some fault in our model turbine (e.g. air leakage) was found.

Perhaps the mechanism originating from the join is of a more general nature. The boundary layer flow at the join is complicated: the boundary layer formed over the stationary upper part passes over the join to a surface that is rotating with the runner angular velocity. Půlpitel, Skoták & Koutník (1996) has reported an axisymmetric ring vortex to appear in that area, just upstream of the blade leading edge in a Kaplan runner. The ring vortex was attached to the inner head cover and had, what they called, wave-like modulations of the core in the axial direction. Perhaps these axial modulations are the same mechanism as we observed as a vortex-like flow structure originating from the join.

## 3.3 Cavitation at the blade tip

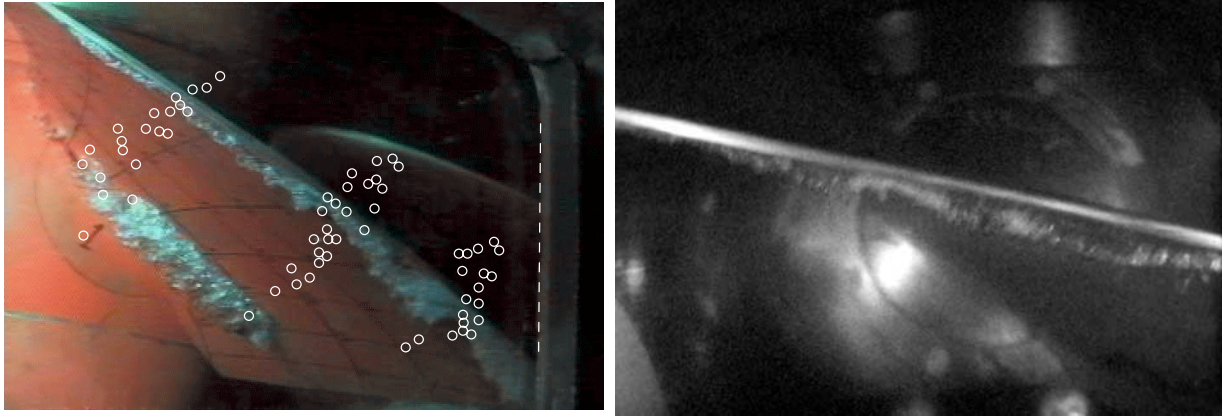
Important mechanisms that influence the cavitation development at the blade tip area are: tip clearance flow, scraping of the boundary layer on the discharge ring wall by the blade tips. It should be noted that a tip vortex, which is caused by tip clearance flow, and a scraping vortex, formed by boundary layer scraping, have opposite rotation directions (e.g. Lakshminarayana, 1996, page 339–340) and (Raabe, 1985, page 376).

The tip vortex, which is rolled up by the leakage flow, had a clearly visible rotation, in the experiments reported here, consistent with the theoretical direction for a leakage flow. The cavitating tip vortex revealed an interesting pattern. The vortex had a periodic waviness with a periodicity corresponding approximately to the spacing of the 24 guide vanes, a fact suggesting that it is caused by wakes or vortices which originate from the guide vanes. As can be seen in Figure 4, the cavitating tip vortex starts a little bit downstream from the blade leading edge, leaves the blade surface and makes a turn, after which it again approaches the blade surface further downstream in a place where erosion damage is commonly reported to appear in Kaplan turbines (e.g. Klein, 1974; Sotnikov & Harriman, 1986).

The closeness to the blade surface and the fine scale of the cavitation at the points where the tip vortex approaches the blade surface may indicate erosive cavitation. By studying the blades in different positions, it was also found that the waviness appears in bands at a small angle relative to the runner axis. The bands are shown in Figure 4(a) by white dots marking out the points where the tip vortex approaches the blade surface. The bands, which are fixed in a turbine casing frame of reference, have the same slant direction, relative to the runner axis, as the main flow. However, it was not possible to determine an absolute value of the angle of the bands.

A high-speed film, made by GE Energy (Sweden) AB on another occasion, with a different set-up of the same cavitation tunnel, was studied to find out if the pattern observed was unique for our set-up. Basically





(a) Points where the tip vortex makes contact with the blade surface are marked for several blade passages. The contact points appear in bands. Running condition no. 51.

(b) A frame from a high-speed film with dissimilar turbine set-up showing the same basic pattern of the cavitating tip vortex. This frame is from a high-speed film by GE Energy (Sweden) AB, with permission.

Figure 4: The pattern of the cavitating tip vortex is perhaps caused by vortices or wakes originating from the guide vanes.

the same pattern appeared also in the other set-up, although perhaps not as clearly as in our own. Figure 4(b) shows a frame from that high-speed film. The flow comes from the left in the photograph, in a blade frame of reference, and the lower side of the blade is the suction side. One can see how the cavitating tip vortex leaves the blade surface, makes a turn and then approaches the blade surface again. This procedure is repeated a couple of times over the blade length.

It is suggested that the observed pattern is caused by interaction with vortices or wakes, originating from the guide vanes. Wakes that originate from the guide vanes have been reported to appear in Kaplan turbines at the outer cylindrical sections just upstream of the runner, as well as downstream of the runner where they are sliced into pieces (Raabe, 1985, page 362-364). Two mechanisms that conceivably promote vortex generation at the guide vanes are: (1) passage vortices (also called channel vortices) caused by secondary flow, and (2) leakage vortices caused by leakage flow from the pressure side to the suction side of the guide vanes at the lower end of them.

### 3.4 Flow inhomogeneities

Video filming through four inspection windows was carried out sequentially under two different turbine running conditions to trace any flow inhomogeneities by the cavitation formation. Apart from the tip vortex pattern already discussed, one may find some variation in the amount of cavitation at the four different circumferential positions. This is an indication of some circumferential variation in the flow to the runner.

The root cavities are smaller at one of the windows for both running conditions. One may also note that there are intermittent variations of the cavitation at the blade root; the cavity at the blade flange fillet, which is mainly of sheet or travelling bubble type, appears intermittently. The other cavities do not seem to be exposed to such a clear volume change. The leading edge sheet cavities, for example, are approximately of the same size in all positions. Observed flow inhomogeneities do not seem to be of major importance for the generation of erosive cavitation except for the cavitating tip vortex.

## 4 Conclusions

To identify and study cavitation processes on the suction side of the blades, experimental studies of cavitation processes and their possible evolution towards eroding cavitation were carried out in a Kaplan model turbine. It is suggested that there are re-entrant jets in the attached sheet cavities at the blade leading edge observed here, and that they are of major importance to the shedding of cavitating vortices and cloud cavitation.

Although the cavities at the blade root seem to be mainly of sheet type or travelling bubble type, depending on running condition, several facts indicate the contribution of vortex motion as well. However it was not possible to draw any final conclusion as to the type of these cavities. Some flow disturbances that most likely influence the root cavitation were also found.

The cavitating tip vortex behaved strangely, possibly due to influence of wakes or vortices originating from the guide vanes. After its initial formation, the cavitating tip vortex leaves the blade surface, whereupon it turns back and approaches the blade surface again at a place where erosion damage is commonly reported to appear. Moreover, the tip vortex cavitation is characterized by fine-scaled cloud cavitation at the points where it approaches the blade surface, which indicates an erosion risk.

Although some minor inhomogeneity did modulate the root cavitation, no major flow inhomogeneities were observed to influence the cavitation processes specifically.

## Acknowledgments

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