LOCAL BUS DEREGULATION AND TIMETABLE INABILITY

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Abstract

In this paper we present a model of competition between operators on urban local bus routes in which passengers always board the first bus to arrive, and it is costly to revise timetables. The model predictds that timetables are unstable, the operator whose bus was boarded by fewest passengers is the most likely to change its arrival time, and to try to leapfrog its rival by arriving just before, and that there is a tendency for bus arrival times to be clustered together. The predictions are consistent with observed features of on-the-road competition on urban local bus routes. On express coach routes, where passengers are more likely to research departure times before travelling, and to arrive at the coach station in order to catch their preferred coach, instability does not arrise in the model, and has not been noted as a feature of competition in practice.

Keywords: Bus deregulation; timetable; instability; urban bus routes; competition; express coach routes.

1 Introduction

The 1930 Road Traffic Act created a bus and coach market in which all aspects of service were tightly regulated. In order to run a service, an opera-

*I owe thanks to John Sutton, who suggested I read the Select Committee report into the effects of local bus deregulation, to Peter White, with whom I discussed the bus market and who steered me away from some blind alleys, and Volker Nocke and Tommaso Valletti for many useful comments. I would also like to thank STICERD at the LSE for financial support and for providing a stimulating working environment..

tor had to meet prescribed standards of vehicle safety and driver competence and, more restrictively, acquire a Road Service Licence from the Traffic Commissioners. A licence would only be issued if the applicant could show that its service was in the public interest. In practice permission would often not be granted if existing licence holders, or British Rail[1], objected, creating a barrier to the entry of independent operators. Permission was also required for changes to fares or timetables for existing services, and again the onus was on the applicant to prove that such changes were in the public interest.

The 1985 Transport Act deregulated local bus markets[2]. Subsequent to the Act an operator needed only to register its timetable and satisfy basic safety requirements in order to run a local bus service. The main remaining restriction was that the Traffic Commissioners had to be notified of all new services, and changes to existing services, 42 days in advance.

In 1995 the Select Committee on Transport produced a report on the effects of bus deregulation [3]. Among the many issues raised by the Committee was the stability of bus timetables[3]. It is clear from the report that where there is competition between operators on the same route (on-the-road competition), timetables are frequently updated. Moreover the frequency of these updates is a considerable source of irritation to bus users. The rise in timetable instability was the first point raised by the National Federation of Bus Users (NFBU) in its evidence to the Committee and the Road Traffic Commissioners agreed that this was the problem which was of most concern to passengers. Indeed those Commissioners questioned by the Committee would have liked powers to restrict the frequency with which changes to the timetable could be made. In many places the cost of keeping passengers

[1]Or, prior to 1948, the various railway companies.

[2]Express coaches had previously been deregulated by the 1980 Transport Act. The 1985 Act also privatised the old incumbent express coach operator, National Express.

[3]There are many other issues raised in the report, which we do not discuss here. The Committee commented that deregulation had not generated the expected falls in fares, that frequency had risen, that the market remained surprisingly concentrated. The Committee spent a lot of time considering whether or not bus markets are contestable, the issue which dominated the theoretical debate while the 1985 legislation was being drawn up, and which is relevant to the policy question of whether the government should view consolidation in the industry with equanimity. Another feature of the deregulated market which had surprised observers is the relative failure of high quality minibus services to develop alongside full size bus services. Such differentiated markets are common in South East Asia. The Committee also considered the vexed question of whether or not there has been predation in local buses.

informed about timetables is borne by the local authority and those Councillors from the metropolitan areas, where on-the-road competition is most common, testified to the Committee that the frequent timetable changes were very expensive to them. Councillor McLellan from Strathclyde, where four large operators and numerous small ones competed, testified that whereas before deregulation timetables were produced at 3 or 6 month intervals, since deregulation 5 timetable changes had been notified to the Strathclyde Traffic Commissioners every working day. The NFBU observed that frequent changes to the timetable are associated with another feature: bus departure times tend to be bunched together. They noted that two operators running practically identical timetables did little to increase customer choice. In its final report the Committee made a similar point, stating that entrants into bus markets typically registered times just before those of the incumbents.

There is anecdotal evidence from previous periods of unregulated bus services of similar behaviour. Glaister, in his evidence to the Committee, cites the example of the horse bus Associations in 19th century London, which had to make great efforts to enforce service regularity on their members in order not to alienate their passengers. Another example is given by Chester [2][4] in a book written 6 years after the 1930 Act which first introduced some control into London's bus markets. There he described the ills of "unfettered competition", and argued that such competition means

> the running of vehicles to a regular timetable will become impossible.

In contrast, there is no evidence in the secondary literature that deregulation has lead to instability in the timetables of express coaches[5]. This paper puts forward the hypothesis that this difference arises from a basic difference in passenger behaviour. On the one hand the time at which passengers on urban local bus routes arrive at the stop is taken to be independent of the arrival times of buses, while on the other hand passengers on express coach routes are assumed to arrive just in time to board their most preferred coach. The way in which this leads to timetable instability was clearly

[4]Cited by Mackie & Preston [10]

[5]Papers on the effects of express coach deregulation include those by Jaffer, Thompson and Whitfield [9], [14], Barton and Everest [1], Robbins and White [12], [15], and references therein.

stated by Chester, who argued that 'unfettered competition' on local bus routes would undermine regular timetables because:

> if any operator fixed definite times, rival operators will seek to reach stopping places a few minutes earlier and take the traffic [2].

Instead, each operator will keep its rival guessing as to its arrival time, and will choose different times each day.

The assumption underlying timetable instability, that on that on urban local bus routes passengers arrive at a bus stop independently of the arrival time of buses, has independent support. Savage [13] cites work on passenger waiting times in Greater Manchester. It was found that when the intervals between buses are comparatively short there is a random element in the arrival patterns of potential passengers at stops, so that as frequency increases the average waiting time falls. In fact if the intervals are less than around 12 minutes, then arrivals become totally random. Savage in his own empirical work on competition on selected bus routes assumes that a bus arriving just before its rival will get all the market. This feature of local bus markets was cited by witnesses to the Select Committee as an explanation for various effects in the deregulated urban bus market. Witnesses[6] testified that passengers on high density bus routes take the first bus to arrive, regardless of price differentials, and that this undermines attempts by bus operators to win market share through cuts in fares. This explanation was also put forward by Mackie & Preston [10] as a reason why fares remained so high. Many go on to note that competition on local bus routes focuses on being first rather than cheapest, and as a consequence operators put many buses on a route.

Despite the obvious way in which timetables may be unstable when passengers behave in this way, early attempts to formally examine whether timetables would be stable under deregulation excluded such behaviour, and so, not surprisingly, predicted no instability. Foster & Golay [8] used a Hotelling framework in which passengers have an ideal departure time. One component of the cost of making a journey is an item which increases as the difference between the actual departure time and the ideal gets larger. To the passenger it is unimportant whether a bus arrives before or after its

[6] See the evidence of White and the TGWU

ideal time. It follows that the benefit to be gained by pre-empting the rival is offset by the loss incurred as a result of the increased separation from the preceding service. They identify instability with lack of pure strategy equilibrium, and since a pure strategy equilibrium *does* exist in this location model[7], they conclude that there will not be instability. Subsequent work on the choice of arrival time has used a similar framework[8][9].

In this paper we examine a location model in which times at a bus stop are located around a day. Operators choose a time at which their bus will arrive, and can change times between days. Passengers arrive at the stop uniformly throughout the day and cannot board a bus which leaves before they arrive. This last assumption is the important departure from Foster and Golay's framework, and instability arises in a natural way. Operators keep their rivals guessing as to their arrival time by playing mixed strategies, and since the realisation of the randomising variable differs from day to day, timetables are unstable.

That each bus operator will keep its rival guessing would, on its own, induce bus operators to choose all possible arrival times with equal probability. We also assume, however, that there is a cost to revising the timetable. This is in fact likely to be the case. Timetable changes must be registered, giving rise to at least some administrative costs. Other costs arise from the managerial time needed to decide on a change, and on the form of the new timetable. Moreover, since timetables do not come into effect until 42 days

[7] The authors assume sequential entry. Moreover in proving the existence of equilibrium the authors rely on the modified zero conjectural variation introduced in Novshek [11], so that they do not show that a pure strategy Nash Equilibrium necessarily exists. Their result can be seen as part of the debate about the conditions under which a pure strategy Nash Equilibrium exists in Hotelling location games, when firms choose both price and location. d'Aspremont et al [4] pointed out that when transport costs are linear there is not neccessarily a pure strategy price equilibium when locations are too close together, but there is an equilibrium when costs are quadratic.

[8] See the papers by Foster & Golay [8], Evans [7], Dodgson et al [5], [6].

[9] A slightly different perspective on timetable choice is provided by Glaister in his evidence to the Select Committee. He suggests that irregularity arises because there are revenue benefits from service regularity which are external to the individual operator but internal to the market as a whole. Presumably there is a market benefit because demand is higher for a regular service, and this demand benefits all operators, not just the one whose choice of timetable led to a more stable and regularly spaced service. This observation will only imply an underprovision of regularity, however, if there is a private gain to creating irregularity. Glaister leaves the source of this gain unexplained.

after initial registration, deciding on a change will involve planning and research into the rival's planned actions. The cost has a striking effect on the pattern of timetable changes: the bus operator whose bus, yesterday, arrived just before its rival's, and so had most passengers, is more likely not to revise its timetable today at all, while the other is more likely to change so that its buses arrive just before the time its rival's arrived yesterday. The tendency is for buses to leapfrog each other in order to arrive earlier and earlier. One result of this behaviour is that bus arrivals tend to be bunched together as each bus operator, if it revises its timetable at all, will choose a new time just before its rival's old one.

The model is highly stylised in order to draw out the effects on timetable stability of the twin assumptions that passengers board the first bus to arrive, and timetable revision is costly. In particular we do not endogenise passenger boarding behaviour, it is just taken as a primitive of the model. Also we will treat a day as circular in order to focus attention purely on the question of whether firms want their buses to arrive before or after those of their rivals, without the complications caused by end effects.

We first give a simple discrete example which shows instability, bunching and leapfrogging. The full continuous model draws out the underlying mechanisms more clearly.

As in the earlier work on timetable choice, instability in the model here will arise when a pure strategy equilibrium does not exist. However, the lack of such an equilibrium here has a different source than that discussed in early formulations of the Hotelling location game. In those cases the lack of equilibrium arose because of problems in optimal pricing when firms were located too close to each other, and the problem could be resolved through a suitable choice of cost function[10]. Here there is no pricing problem. Instability results directly from a lack of pure strategy equilibrium in the choice of location.

2 An Example

Two buses, A and B, compete to pick up passengers during each of an infinite number of days. Each day has 4 minutes arranged around a circle, so that

[10]See Footnote 7.

minute 3 is just before minute 0. Figure 1 illustrates a day.

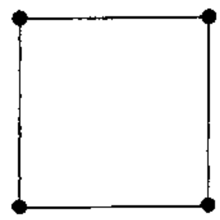


Figure 1: A day

In period t bus i, $i = A, B$, picks an arrival time a_i^t, $a_i^t \in \{0, \ldots, 3\}$. The state in period t, denoted k^t, is the arrival times of buses in the previous period, $k^t = a^{t-1} = (a_A^{t-1}, a_B^{t-1})$.

One unit of passengers arrive every minute and if a bus arrives in the same minute they board it, otherwise they board the first bus to arrive. Two buses arriving at the same time share the waiting passengers equally.

A bus gets a gross profit of 1 for every unit of passengers which board. Let $\pi_i(a_A, a_B)$ be the gross payoff of bus i when arrival times are a_A, a_B. The complete gross profit matrix is given in Table 1 which shows the pair π_A, π_B for each possible combination of arrival times.

		a_B							
		0		1		2		3	
	0	2	2	3	1	2	2	1	3
a_A	1	1	3	2	2	3	1	2	2
	2	2	2	1	3	2	2	3	1
	3	3	1	2	2	1	3	2	2

Table 1: Table of values of π_A, π_B

Buses also incur a cost of c if they revise their timetable. Let $C_i(a_i^t)$ be the cost bus i pays if it arrives at a_i^t in period t. Then:

$$C_i(a_i^t) = \begin{cases} 0 & \text{if } a_i^t = k_i^t \\ c & \text{otherwise} \end{cases}$$

For simplicity we assume that each bus i is myopic: it seeks only to maximise the expected current profit net of any revision cost. However, we will see that this assumption is not as restrictive as it appears: even if buses maximised the sum of discounted future net profits the equilibrium strategies would be the same as the ones we find here. The problem is essentially a one period one (though the past influences the present through the state) and we now drop the time superscript. A strategy for bus i, denoted s_i, specifies the probability that i chooses each arrival time, so that $s_i(m)$ is the probability that $a_i = m$. Strategies can be conditioned on the state, but on nothing else. In any equilibrium $s^* = (s_A^*, s_B^*)$ any arrival time chosen with positive probability must maximise expected profit net of cost, given the rival's strategy, i.e. if $s_i^*(m) > 0$ then m maximises $E\left[\pi(m, a_j) - C(m) \mid s_j^*\right]$.

It is easy to see that if the revision cost is less than 1 firms will never choose a pure strategy in equilibrium, no matter what the state. Suppose A arrived in minute 0 with certainty. If B arrived at 3 it would collect 3 units of passengers, and pay a maximum cost of c, giving a net profit of more than 2. If B arrived at any other time it could collect at most 2 units, and so it will certainly arrive at 3. But if B is arriving at 3, A does best to arrive just before at 2, and so on. When $c < 1$ the only equilibrium is in mixed strategies. Each bus randomises to keep its rival guessing as to exactly when it will arrive. Equilibrium strategies are shown in Tables 2 and 3 below. In

k_B	$s_A^*(0)$	$s_A^*(1)$	$s_A^*(2)$	$s_A^*(3)$
0	$\frac{1}{2}$	—	$\frac{1}{2} - \frac{c}{2}$	$\frac{c}{2}$
1	$\frac{1}{2} + \frac{c}{4}$	—	$\frac{1}{2} - \frac{c}{4}$	—
2	$\frac{1}{2}$	$\frac{c}{2}$	$\frac{1}{2} - \frac{c}{2}$	—
3	$\frac{1}{2} - \frac{c}{4}$	—	$\frac{1}{2} + \frac{c}{4}$	—

Table 2: A's equilibrium strategy

k_B	$s_B^*(0)$	$s_B^*(1)$	$s_B^*(2)$	$s_B^*(3)$
0	$\frac{1}{2}$	—	$\frac{1}{2} - \frac{c}{2}$	$\frac{c}{2}$
1	—	$\frac{1}{2} - \frac{c}{4}$	—	$\frac{1}{2} + \frac{c}{4}$
2	$\frac{1}{2} - \frac{c}{2}$	—	$\frac{1}{2}$	$\frac{c}{2}$
3	—	$\frac{1}{2} - \frac{c}{4}$	—	$\frac{1}{2} + \frac{c}{4}$

Table 3: B's equilibrium strategy

both tables we assume that A arrived in minute 0 yesterday, i.e. $k_A = 0$. The state is therefore summarised just by k_B. We can always ensure that $k_A = 0$ simply by relabelling the minutes at the start of the current period.

Before examining what these strategies imply for the pattern of timetable revisions, we first confirm that they do form an equilibrium. To do this we need just show that each firm only chooses an arrival time with positive probability if arriving at that time maximises its expected net profit given its rival's strategy. Table 4 sets out the expected net profit of A when B plays the strategy given in Table 3. Denote A's expected profit when it arrives at m and B plays s_B^* by $\Pi_A(m)$. It is clear by inspection that, given the state

k_B	$\Pi_A(0)$	$\Pi_A(1)$	$\Pi_A(2)$	$\Pi_A(3)$
0	$2-\frac{c}{2}$	$2-\frac{3c}{2}$	$2-\frac{c}{2}$	$2-\frac{c}{2}$
1	$2-\frac{c}{2}$	$2-c$	$2-\frac{c}{2}$	$2-c$
2	$2-\frac{c}{2}$	$2-\frac{c}{2}$	$2-\frac{c}{2}$	$2-\frac{3c}{2}$
3	$2-\frac{c}{2}$	$2-c$	$2-\frac{c}{2}$	$2-c$

Table 4: A's expected profit, given s_B^*

k_B, the strategy given in Table 2 only assigns positive probability to those arrival times which maximise A's expected profit. A similar table to Table 4 could readily be found for B and this would show that the strategy given in Table 3 likewise only assigns positive probability to those arrival times which maximise B's expected profit. This confirms that Tables 2 and 3 do specify an equilibrium in mixed strategies. Moreover this is the only equilibrium when $0 < c < 1$, though to check this requires an exhaustive search of other possibilities and the results of this search are not repeated here. One final general feature of the equilibrium is that a bus operator's expected net profits do not depend on the state: they are always $2 - c/2$. Since the current period can only affect the future through the state, this means that the current period has no effect on future net profits and firms would not change their behaviour if they were not myopic[11].

Turning to the implications of these equilibrium strategies for the pattern of timetable revision, the case that is of particular interest is when the buses arrived in two successive minutes yesterday, so either $k_B = 1$ if B arrived just after A, or $k_B = 3$ if it arrived just before. The strategies of the two buses are shown diagrammatically in Figure 2 for the case where $k_B = 1$. According to equilibrium strategies either A arrives at a particular time with positive

[11]Suppose buses maximise the discounted sum of net profits, the discount rate is δ and the revision cost $c\delta$, then there is a perfect eqilibrium in which strategies are identical to those found here.

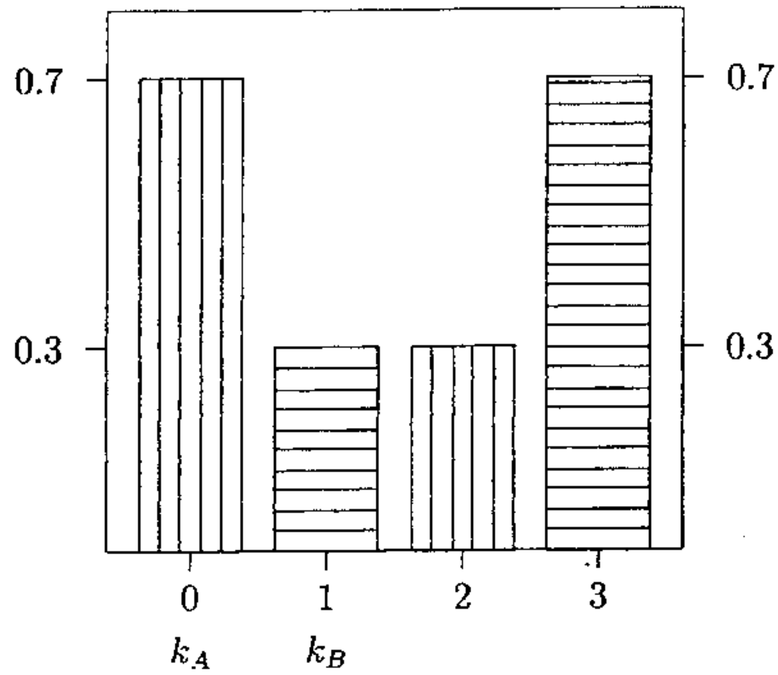


Figure 2: Strategies when $k_B = 1$ and $c = 4/5$

probability, or B does, but not both. In the Figure there is a bar at each minute whose height is proportional to the probability that a bus arrives at that minute: if it is bus A the bar has vertical stripes, if B horizontal. We assume that $c = 4/5$.

The reason why this case is the most important is that whatever the actual realisations of firms' random strategies, they will never arrive at the same time, and neither will they arrive evenly spaced: if buses arrived one after the other in the previous day, they are bound to arrive one after the other in the current day, and so in all future days. When $k_B = 0$ or 2, the only other possible cases, firms randomise over three possible arrival times, and so with positive probability arrive one after the other in the current day, and if not in the current day, then with positive probability in the next day, and so on. In the long run, buses will always arrive one after the other every day. This phenomenon resembles the bunching of bus arrivals that many commentators have noted is a feature of deregulated local bus markets.

In the long run not only do buses always arrive one after the other, but we see a tendency for buses to leapfrog each other backwards round the day. In Figure 2 bus A, which arrived just before B and collected most passengers yesterday, was most likely to arrive at the same time today, whereas B was most likely to revise its timetable in order to arrive just before A's arrival time yesterday. The continuous time model in the next section explores the mechanisms underlying this leapfrogging and bunching more fully.

3 The Model

Two buses, A and B, compete to pick up passengers during each of an infinite number of days. Each day has length 1, and is circular, with later times being further clockwise round the circle. In each period t each bus i, $i = A, B$, picks an arrival time a_i^t, $a_i^t \in (0, 1]$. At the start of period t all times are relabelled so that A's arrival time in period $t-1$ is at 0, which just has the effect that all times in t are measured in terms of minutes later than A's arrival time in the previous period. The state in period t, denoted k^t, is the (relabelled) arrival time of B in the previous period. We will assume henceforth that A arrived 'before' B in the sense that $k \leq 1/2$. By symmetry, this is without loss of generality.

Passengers arrive at a uniform rate throughout the day, with a total mass of 1 per day, and board the first bus to arrive after they do, unless both arrive at the same moment, in which case half board each bus.

A bus gets a gross profit in the day equal to the mass of passengers which boards and the mass boarding a bus is just the minutes after the previous bus that this bus arrives. Let $\pi_i(a_A^t, a_B^t)$ be the gross profit of bus i when arrival times are a_A^t, a_B^t. Then:

$$\pi_A(a_A^t, a_B^t) = \begin{cases} a_A^t - a_B^t & \text{if } a_A^t > a_B^t \\ 1/2 & \text{if } a_A^t = a_B^t \\ 1 + (a_A^t - a_B^t) & \text{if } a_A^t < a_B^t \end{cases}$$

and similarly for π_B.

If buses must pay when they update their timetable different arrival times will entail different costs. Denote the updating cost incurred by i should it arrive at x when the state is k by $C_i(x, k)$.

We assume that buses are myopic and seek only to maximise the expected current profit net of any revision cost. The problem is essentially a one period one (the past is summarised by the state), and we now drop the time superscripts. We restrict attention to Markov strategies which depend only on the state. A pure strategy for bus i is a function $s_i(k)$ which gives the arrival time chosen when the state is k. A mixed strategy is a distribution function $F_i(x,k)$ which gives the probability of arriving in the interval $[0,x]$. We consider Nash Equlibria where each bus chooses a strategy which maximises its expected net profit given the strategy chosen by its rival.

The first point is that for updating costs sufficiently low there is no equilibrium in pure strategies. Consider the extreme case where the updating cost is everywhere zero. In this case the state does not affect current payoffs and Markov strategies will not depend on it. The best reply function is not even defined here. If B chooses $s_B = a_B$, A will maximise the profit from boarding passengers by arriving as late as possible while still arriving before a_B, i.e. by setting its arrival time as the largest a_A such that $a_A < a_B$. When time is continuous there is no a_A which satisfies this. However, even without this technical problem there would be no equilibrium in pure strategies. Consider whether an ε-equilibrium (s_A^*, s_B^*) exists, where if j arrives at s_j^*,no arrival time gives i a payoff of ε more than $\pi_i(s^*)$, for an arbitrarily small ε. No such ε-equilibrium exists. To see why, simply note that in any ε-equilibrium A will arrive no more than ε minutes before B, and B will arrive no more than ε minutes before A. When ε is small, these conditions cannot both be met.

From now on we will consider only mixed strategies. Denote the average arrival time of bus i by $\bar{a}_i$. Let $\Pi_i(x, F_j)$ be the expected gross profit of bus i when it arrives at x and its rival's strategy is F_j. This will be given by:

$$\begin{aligned}\Pi_i(x,F_j) &= \lim_{\epsilon\to 0}\left[x-(\bar{a}_j|a_j<x)\right]F_j(x-\epsilon)\\ &\quad+\left[1+x-(\bar{a}_j|a_j>x)\right](1-F_j(x))\\ &\quad+\frac{1}{2}\left(F_j(x)-F_j(x-\epsilon)\right)\end{aligned}$$

Let $\lim_{\epsilon\to 0}(F_j(x)-F_j(x-\epsilon)) = \mathrm{Pr}_j(x)$ (this will be zero when there is no atom in the distribution at x). Then:

$$\begin{aligned}\Pi_i(x,F_j) &= \left[x-(\bar{a}_j|a_j<x)\right](F_j(x)-\mathrm{Pr}_j(x))\\ &\quad+\left[1+x-(\bar{a}_j|a_j>x)\right](1-F_j(x))\\ &\quad+\left[x-(\bar{a}_j|a_j=x)\right]\mathrm{Pr}_j(x)\\ &\quad+\frac{1}{2}\mathrm{Pr}_j(x)\end{aligned}$$

which rearranges to:

$$\Pi_i(x, F_j) = 1 - \bar{a}_j + x - F_j(x) + \frac{1}{2}\Pr{}_j(x) \tag{1}$$

Using these expressions we first find equilibrium when timetable revision is costless.

When there is no cost to revising the timetable, so that in the absence of other considerations all arrival times are equally attractive, there is an equilibrium in which both operators choose arrival times according to a uniform probability distribution:

$$F_i^*(x) = x,\ i = A, B$$

To confirm that these strategies do form an equilibrium, substitute into the expression for expected profit above to give:

$$\Pi_i^*(x, F_j^*) = 1/2 \quad \forall x, \quad i = A, B$$

Since, when its rival chooses arrival times according to a uniform distribution over all times, an operator earns the same expected profit no matter what time it chooses, it is indifferent over all possible strategies, including arriving according to a uniform distribution. Here we see a radical instability in the timetable. Buses choose any arrival time with equal probability, independently of their rival's or their own previous arrival time. This instability arises from the desire on the part of both buses to arrive just before their rival, when there will be many passengers waiting at the stop.

Once we assume that it is costly to adjust the timetable more structure on the probability distribution chosen by firms emerges.

For technical reasons we assume that the cost of choosing different arrival times changes continuously. In particular we assume that if the arrival time is the same as in the last period there is no updating cost, and that the cost rises linearly at a rate m with the absolute change in the arrival time until a maximum updating cost of c is reached, at which point the updating cost remains constant. When m is large this function will approximate the situation where a firm pays c for every arrival time except that at which it arrived in the previous period, for which it pays nothing, and henceforth we assume $m > 1$. The updating cost functions are:

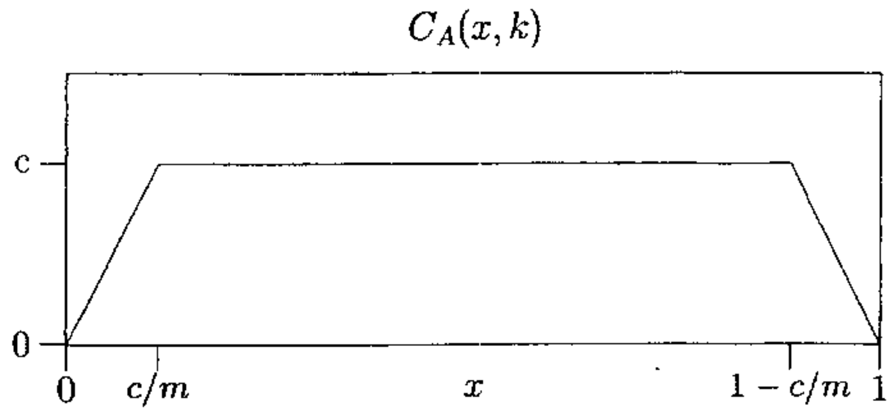
$$C_A(x,k) = \begin{cases} mx & \text{if } x \in [0, c/m] \\ c & \text{if } x \in [c/m, 1-c/m] \\ m(1-x) & \text{if } x \in [1-c/m, 1) \end{cases} \tag{2}$$

$$C_B(x,k) = \begin{cases} c & \text{if } x \in [0, k-c/m] \\ m(k-x) & \text{if } x \in [k-c/m, k] \\ m(x-k) & \text{if } x \in [k, k+c/m] \\ c & \text{if } x \in [k+c/m, 1) \end{cases} \tag{3}$$

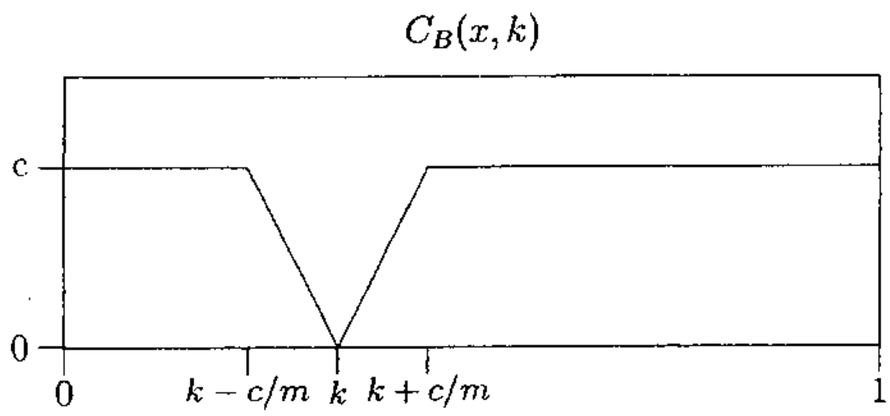
An example of these costs are illustrated in Figure 3 below. In writing and illustrating $C_B(x,k)$ we have assumed that the interval between bus arrivals in the previous period, k, was not too small, specifically that $k \geq c/m$. In the limit where $m \to \infty$ this will almost always be true, but in any case what is at issue is notation rather than results. Looking at the illustration of C_B in the second panel of Figure 3, note that the circular day has been mapped to a line in the diagrams by cutting it at the point where 1 and 0 meet up and placing 0 at one end and 1 at the other. If $k < c/m$ the diagram is essentially the same, but the two ends of the line will lie in a region where the updating cost is less than c. The exact expression for C_B would differ from the one given in Equation 3, though the function is, in essence, the same. We will ignore this notational complication in what follows. The reader should be able to construct the exact expressions relevant to the case $k < c/m$ from the results that follow.

Now that the cost of arrival times varies, there can no longer be a completely mixed strategy equilibrium in which A chooses to arrive according to a uniform probability distribution. If A did so, B would choose to arrive at the same time in one period as it did in the previous one, i.e. at k, since all times give the same expected profit from boarding passengers, and by arriving at k bus B avoids all updating costs.

There are two cases, depending on whether the buses were bunched together in the previous period or were evenly spaced, specifically on whether $k > c$ or not. We consider the simpler case first where buses were fairly evenly spaced and $k > c$ (this case is only possible when $c \geq 1/2$). We will first formally state and prove the result before describing its implications and providing some intuition as to why it is true. We have:



Updating cost function: bus A



Updating cost function: bus B

Figure 3: An example of the updating cost as a function of arrival time x for a given k

Result 1 *There is an equilibrium* (F_A^*, F_B^*) *in which for* $k \in (c, 1/2]$:

$$F_A^*(x,k) = \begin{cases} c & x \in [0,c] \\ x & x \in [c, k - c/m] \\ (1+m)x - mk + c & x \in [k - c/m, k] \\ k + c & x \in [k, k+c] \\ x & x \in [k+c, 1) \end{cases}$$

$$F_B^*(x,k) = \begin{cases} 0 & x \in [0,c] \\ x - c & x \in [c, k) \\ k & x \in [k, k+c] \\ x - c & x \in [k+c, 1 - c/m] \\ (1+m)x - m & x \in [1 - c/m, 1) \end{cases}$$

Proof. To show that these strategies form an equilibrium, we need to show that the net profit a bus operator expects to earn is the same, no matter what time in the support its bus arrives, and that this net profit is no less than that from arriving at any time not in the support. Substituting B's strategy into the expression for A's profit from boarding passengers gives:

$$\Pi_A(x, F_B^*) = 1 - \bar{a}_B + \begin{cases} x & x \in [0,c] \\ c & x \in [c,k) \\ c/2 & x = k \\ x - k & x \in (k, k+c] \\ c & x \in [k+c, 1 - c/m] \\ m(1-x) & x \in [1 - c/m, 1) \end{cases}$$

which gives an expected net profit for A as a function of its arrival time of:

$$\Pi_A(x, F_B^*) - C_A(x) = 1 - \bar{a}_B + \begin{cases} x(1-m) & x \in [0, c/m] \\ x - c & x \in [c/m, c] \\ 0 & x \in [c,k) \\ -c/2 & x = k \\ x - k - c & x \in (k, k+c] \\ 0 & x \in [k+c, 1) \end{cases}$$

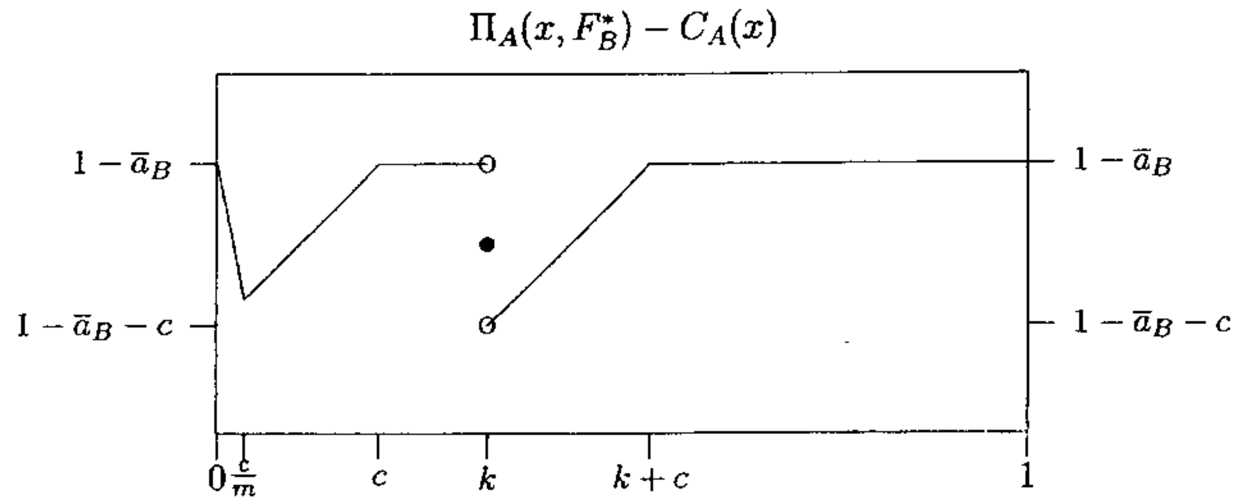
Similarly the expected net profit of B as a function of its arrival time is:

$$\Pi_B(x, F_A^*) - C_B(x) = 1 - \bar{a}_A + \begin{cases} x - 3c/2 & x = 0 \\ x - 2c & x \in (0, c] \\ -c & x \in [c, k] \\ (x-k)(1-m) - c & x \in [k, k + c/m] \\ x - 2c & x \in [k + c/m, k + c] \\ -c & x \in [k + c, 1) \end{cases}$$

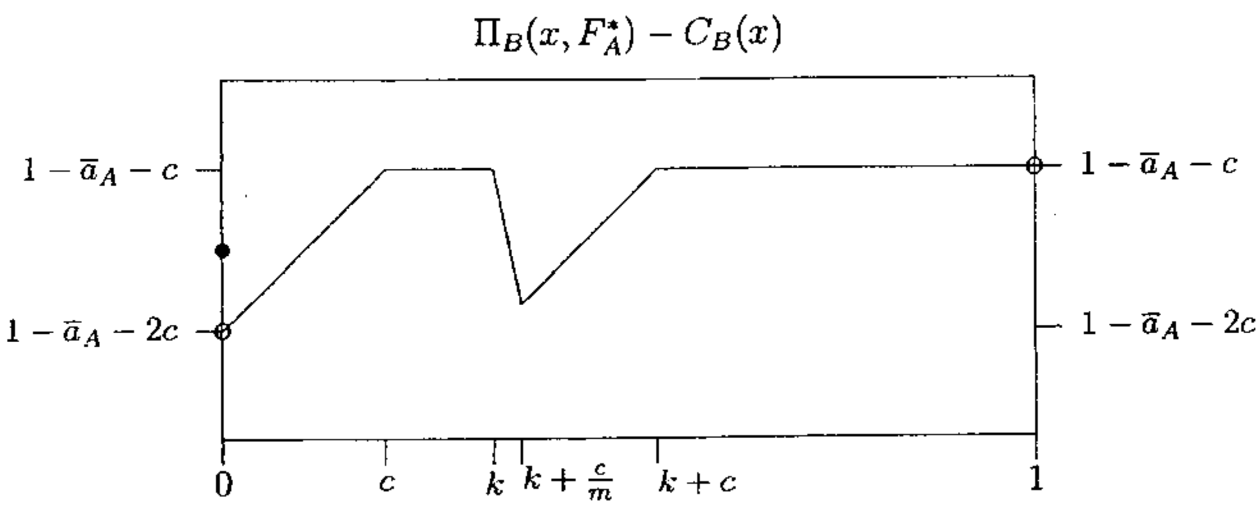
This net profit is shown in Figure 4 below. Inspection of the expressions and the Figures reveals that the expected net profit is $1 - \bar{a}_B$ if A arrives at any time in the support of $F_A^*(x)$, and is less than this should A arrive at any other time. This confirms that F_A^* is a best response to F_B^*. Similar reasoning confirms that F_B^* is a best response to F_A^* and so that these strategies are an equilibrium. ■

Figure 5 below shows the equilibrium strategies. Since marginal probabilities are simpler to interpret than the related distribution function, the figures give the marginal probability chosen by each firm in equilibrium, where this is defined. A filled square at the top of a line means that there is a probability mass at that point, and the probability in that mass is marked. The probability mass of c in the distribution means that each bus arrives at the same time as it did in the previous period with probability c. Thus, not surprisingly, the higher the updating cost, the higher the probability that a bus chooses not to incur it. If a bus does update its arrival time, it never chooses to arrive a little later than previously, but may arrive little earlier. Also we see that a bus never arrives a little later than its rival's previous arrival time, but may arrive a little earlier. In particular with probability $c + c/m$ it will arrive in an interval of width c/m immediately before its rival's arrival time in the previous period[12]. The implication of these strategies is that buses tend either not to update their arrival times, or if they do, to arrive just before their rival's previous arrival time. They never arrive later than either their own or their rival's previous arrival time. Since a lot of probability is concentrated close to the same two arrival times for each bus, this behaviour will cause a tendency for buses to choose close arrival times this period and so to bunched arrival times. Moreover we see some leapfrogging to earlier

[12] $c + c/m = (k - (k - c/m))(1 + m)$



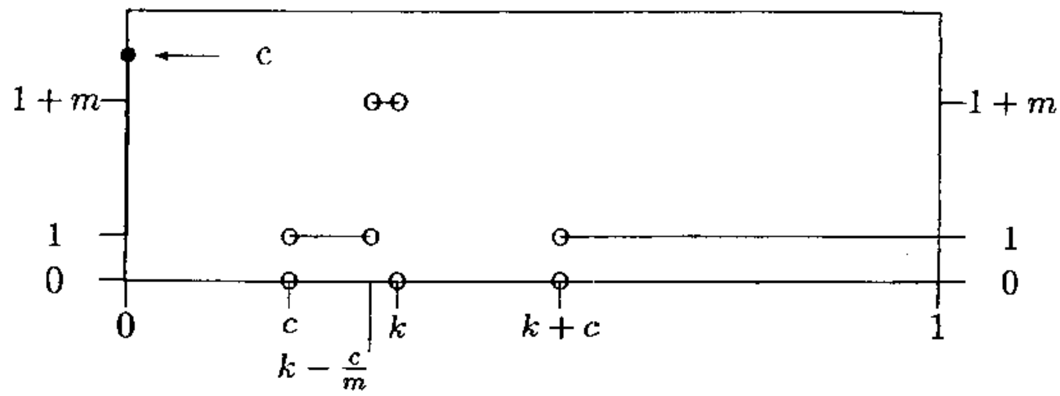
Expected net profit: bus A



Expected net profit: bus B

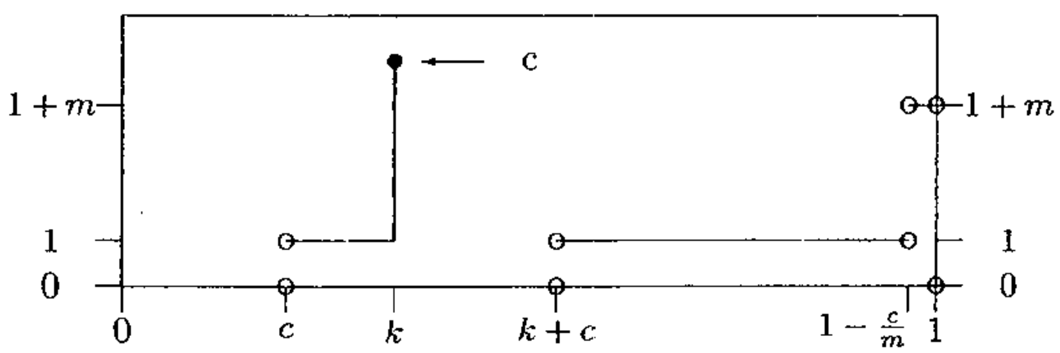
Figure 4: Expected net profit as a function of arrival time: $k < c$

$f_A(x)$



Marginal probability of arriving: bus A

$f_B(x)$



Marginal probability of arriving: bus B

Figure 5: Equilibrium marginal probabilities of arrival time for given k

and earlier times as bus operators avoid arrival times later than their rival's previous time, but sometimes choose an arrival time before.

As stated in the proof, to show that these strategies do form an equilibrium we need to show that the expected net profit of each bus is maximised by its arriving at any time in the support of its equilibrium distribution function. Suppose A arrives according to F_A^* and consider B's expected payoff. All other things equal B would arrive at k and avoid all updating costs. However, with a relatively high probability A arrives just before k which increases B's expected payoff if it arrives a little earlier still. A's distribution function is such that this inventive exactly offsets the disincentive from having to pay an updating cost. Also the atom in A's distribution at 0 makes the expected profit from arriving just before this higher so that these times also lie in B's support.

So far we have just considered the case where the buses were not too bunched together in the previous period. Now we turn to the case where their arrival times were separated by less than c in the previous period and so $k < c$. In this case:

Result 2 *There is an equilibrium* (F_A^*, F_B^*) *which, when* $k < c$, *has the form:*

$$F_A^*(x,k) = \begin{cases} k+c & x \in [0, k+c] \\ x & x \in [k+c, 1) \end{cases}$$

$$F_B^*(x,k) = \begin{cases} 0 & x \in [0,k) \\ k & x \in [k, k+c) \\ x-c & x \in [k+c, 1-c/m] \\ (1+m)x - m & x \in [1-c/m, 1) \end{cases}$$

Proof. To confirm that these form an equilibrium we can calculate the expected net profit from arriving at different times, assuming the rival's times are given by these distributions, the same way as above. This gives:

$$\Pi_A(x, F_B^*) - C_A(x) = 1 - \bar{a}_B + \begin{cases} x(1-m) & x \in [0, c/m] \\ x - c & x \in [c/m, k] \\ k/2 - c & x = k \\ x - k - c & x \in (k, k+c] \\ 0 & x \in [k+c, 1) \end{cases}$$

$$\Pi_B(x, F_A^*) - C_B(x) = 1 - \bar{a}_A + \begin{cases} -k/2 - 3c/2 & x = 0 \\ x - k - 2c & x \in (0, k - c/m] \\ (x-k)(1+m) - c & x \in [k - c/m, k] \\ (x-k)(1-m) - c & x \in [k, k + c/m] \\ x - k - 2c & x \in [k + c/m, k + c] \\ -c & x \in [k + c, 1) \end{cases}$$

These expected net profits are illustrated in Figure 6 below. Inspection of the Figure and the expressions confirms that each bus' expected net profit is maximised at any point on the support of its equilibrium distribution function. ∎

These equilibrium strategies are illustrated in Figure 7: When bus arrivals were close together in the previous period, the bus which arrived just before its rival and so had more passengers, i.e. bus A, is less likely to have its timetable updated this period than is bus B which had fewer passengers. This is shown by the fact that the atom at 0 in A's equilibrium strategy has mass $k + c$, whereas the atom in B's equilibrium strategy at k only has mass k. Moreover if bus A does have its timetable updated, it will arrive earlier than its own previous arrival time, but will avoid time either a little earlier or later than B's previous arrival time. Bus B on the other hand will, with relatively high probability, arrive in the interval c/m just before A's previous arrival time. The leap-frogging to earlier and earlier times first seen for the case when buses were fairly evenly spaced previously, $k > c$, is a much stronger feature of the equilibrium when buses were bunched together previously. The later bus is both more likely to have its timetable revised than its rival, and if it is revised at all, is relatively likely to arrive just before the previous arrival time of the early bus.

4 Express Coaches

The assumption that passengers arrive independently of the times at which buses arrive is fundamental to the finding that timetables are unstable. It is also unlikely to be the case for passengers travelling by express coach. It is much more plausible that such passengers know the timetable and travel on that coach which they most prefer. Consider a model identical to the one

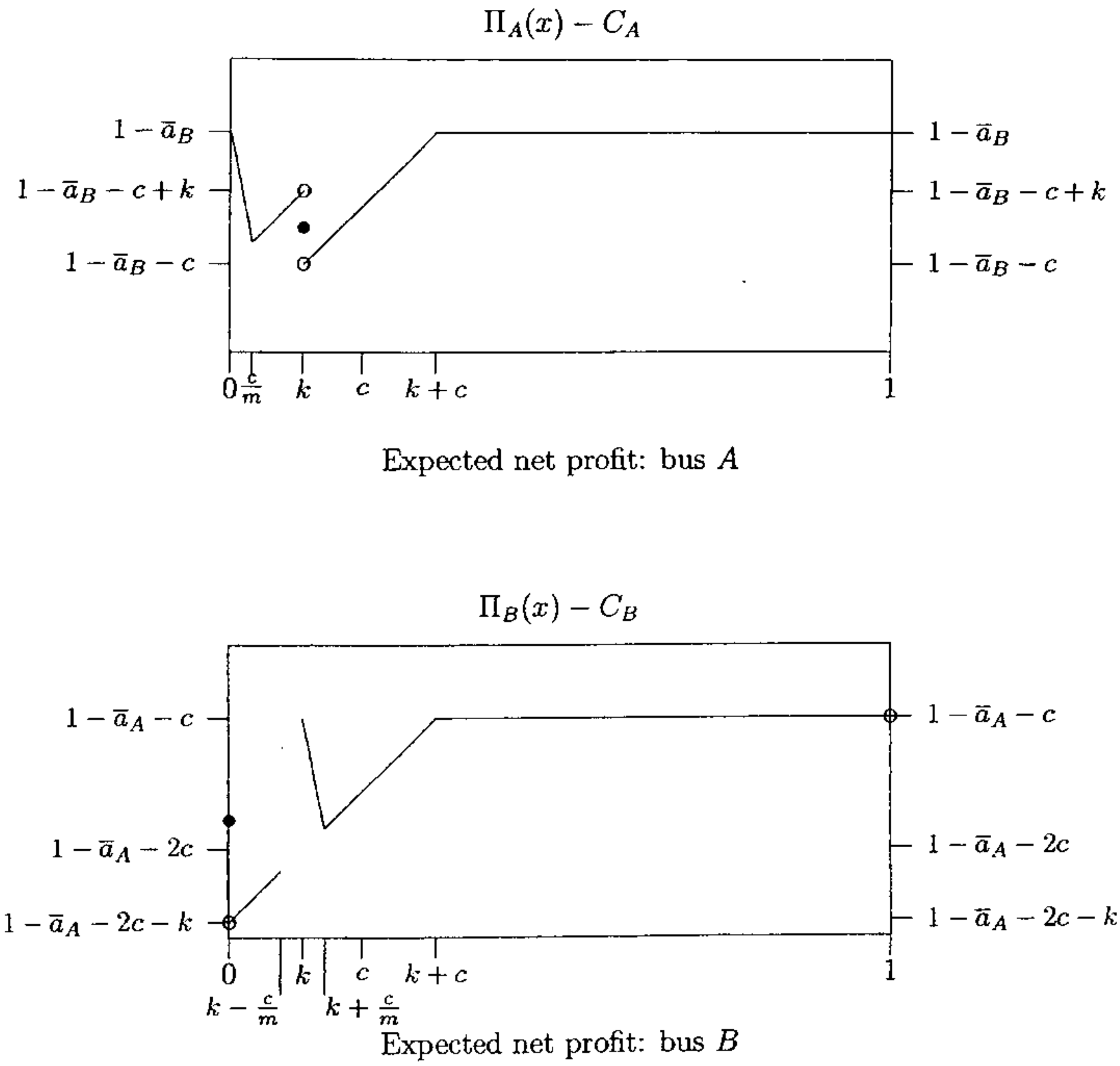
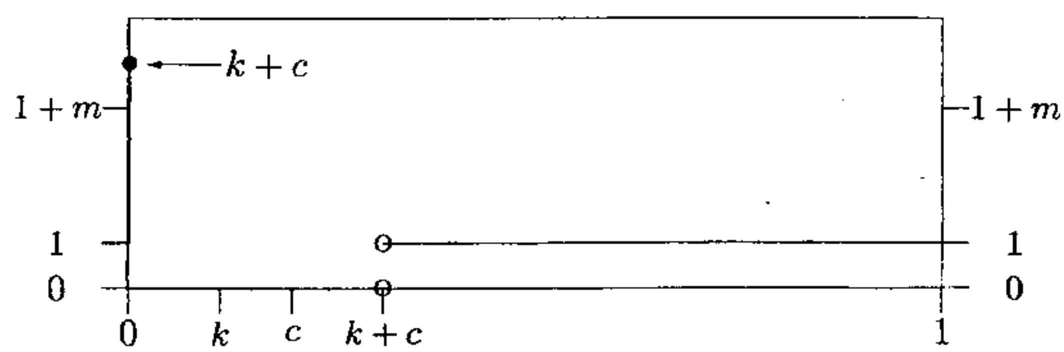


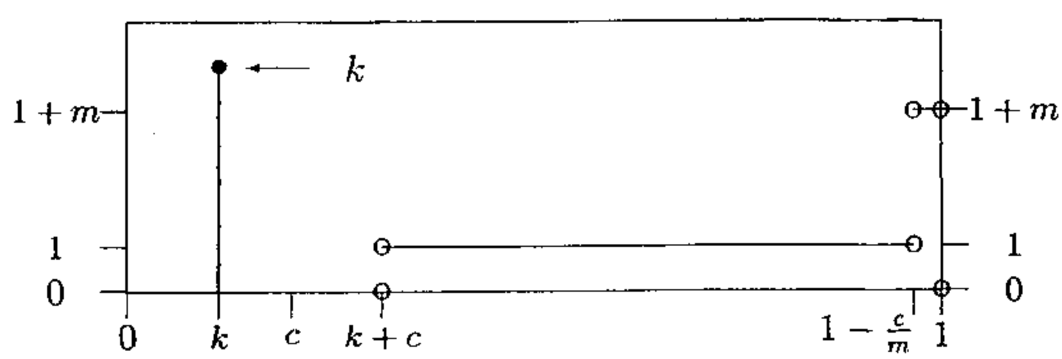
Figure 6: Equilibrium marginal probabilities of arriving: $k < c$

$f_A(x)$



Marginal probability of arriving: bus A

$f_B(x)$



Marginal probability of arriving: bus B

Figure 7: Equilibrium marginal probabilities of arriving: $k < c$

for urban local buses above except that the specification of demand differs. Assume that passengers are located evenly over the day and that the mass of passengers boarding a bus is equal to the mass which is closer to that bus than to its rival. For both buses this will be $1/2$ no matter what the arrival times chosen. So long as the updating cost is at a minimum when buses do not update their arrival times, there will be an equilibrium in which both buses arrive at the same time from day to day.

5 Conclusions

This paper has explored the implications of assuming that passengers on a bus routes arrive at the bus stop independently of the times at which buses arrive, and that it is costly to revise bus timetables. These implications are that when there is on-the-road competition timetables are unstable, the operator whose bus was boarded by fewest passengers is the most likely to change its arrival time, and to try to leapfrog its rival by arriving just before, and that there is a tendency for bus arrival times to be bunched together. The assumptions have independent support in the case of local urban bus routes, which are those where bunching and instability are noted features of on-the-road competition. On express coach routes, where passengers are more likely to research departure times before travelling, and to arrive at the coach station in order to catch their preferred coach, instability does not arise in the model, and has not been noted as a feature of competition in practice.

References

[1] A J Barton and J T Everest. Express coaches in the three years following the 1980 Transport Act. Technical Report 1127, Transport and Road Research Laboratory, 1984.

[2] D N Chester. *Public Control of Road Passenger Transport.* Manchester, 1936.

[3] Commons Select Committee on Transport. *Local Bus Deregulation.* HMSO, 1995.

[4] C D'Aspremont, J J Gabszewicz, and J F Thisse. On Hotelling's 'stability in competition'. *Econometrica*, 47:1145–1170, 1979.

[5] J Dodgson, Y Katsoulacos, and C Newton. An application of the economic modelling approach to the investigation of predation. *Journal of Transport Economics and Policy*, 27:153–170, 1993.

[6] J Dodgson, C Newton, and Y Katsoulacos. A modelling framework for the empirical analysis of predatory behaviour in the bus services industry. *Regional Science and Urban Economics*, 22:51–70, 1992.

[7] A Evans. A theoretical comparison of competition and other economic regimes for bus services. *Journal of Transport Economics and Policy*, 21(1):7–36, January 1987.

[8] C Foster and J Golay. Some curious old practices and their relevance to equilibrium in bus competition. *Journal of Transport Economics and Policy*, 20:191–216, 1986.

[9] S M Jaffer and D J Thompson. Deregulating express coaches: A reassessment. *Fiscal Studies*, 7(4):45–68, November 1986.

[10] P Mackie and J Preston. *The Local Bus Market: A Case Study of Regulatory Change*. Avebury, 1997.

[11] W Novshek. Equilibrium in simple spatial (or differentiated) product models. *Journal of Theoretical Economics*, 22, 1980.

[12] D Robbins and P White. The experience of express coach deregulation in Great Britain. *Transportation*, 13(4), 1986.

[13] I Savage. *The Deregulation of Bus Services*. Gower, Aldershot, 1985.

[14] D Thompson and A Whitfield. Express coaching: Privatisation, incumbent advantage and the competitive process. Mimeo, 1990.

[15] P R White. Express coach services in Britain since deregulation. *Transport Policy*, 1983. proceedings of seminar M, PTRC summer annual meeting.