Risk- & Regret-Averse Bidders in Sealed-Bid Auctions*

Takashi Hayashi† and Hisayuki Yoshimoto‡

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Abstract

Overbidding, bidding more than risk-neutral Bayesian Nash Equilibrium, is a widely observed phenomenon in virtually all experimental auctions. The scholars within the auction literature propose the risk-averse preference model to explain overbidding structurally. However, the risk-averse preference model predicts underbidding in such important classes of auctions as all-pay auctions. To solve this discrepancy, we construct a structural model of bidding behavior in sealed-bid auctions, one in which bidders may regret their decisions. Our model nests both risk-averse and regret-averse attitudes and aims to explain overbidding in a wider class of auctions. We first derive equilibrium first-order conditions, which are used for estimation and calibration analyses, and show monotonic increasing properties of equilibrium bidding functions. Second, we carry out structural estimation and calibration analyses based on experimental data from Kagel and Levin (1993) and Noussair and Silver (2006). With these structurally estimated parameters, we test the significance of bidders' risk-averse and regret-averse attitudes. The estimation results show that bidders exhibit weak risk-averse (close to risk-neutral) and strong regret-averse attitudes. Furthermore, regret-averse attitudes are significant when bidders anticipate losing. Calibration results demonstrate that our risk- & regret-averse model can explain overbidding across all of the above IPV auctions. Third, we simulate our model with the estimated parameters and obtain revenue rankings numerically. This allows us to confirm the revenue supremacy in all-pay auctions reported in experimental auction literature. We discuss extensions to asymmetric and Common-Value (CV) auctions in our online Appendix.

Key Words and Phrases:

Overbidding, Risk Aversion, Anticipated/Expected Regret, Auction Rule Dependent, Auction Designs

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†Department of Economics, University of Texas at Austin: th925@eco.utexas.edu

‡Job Market Candidate, Department of Economics, University of California, Los Angeles: hisayoshimoto@ucla.edu (corresponding author)

1 Introduction

In this paper, we propose a risk- & regret-averse model to explain widely-observed overbidding phenomenon across different auction rules. The proposed risk- & regret-averse model nests the canonically-accepted risk-neutral and risk-averse models, and it predicts a revenue ranking that is consistent with experimental observations.

A significant number of experimental auction researchers report that bids observed in sealed-bid auction experiments are higher than predictions from risk-neutral Bayesian Nash Equilibria (BNE). Such experimental phenomena is called *overbidding*, and observed in first-price auction experiments conducted by Cox, Robertson, and Smith (1982) [9] and Cox, Simth, and Walker (1988) [10]. Scholars have observed many other instances of overbidding. Kagel and Levin (1993) [27] noted observations of overbidding in first-, second-, and third-price auctions. Noussair and Silver (2006) [34] discussed observations of overbidding in all-pay auction experiments.

Based on these experimental observations, researchers have been proposing theoretical and behavioral models to explain overbidding. The literature offers three major explanations, and this research combines these three separate explanations into one structural model. The first explanation, that bidders have a risk-averse preference, remains the best-accepted explanation. Bidders prefer to raise probabilities of winning at the costs of higher payments. Higher payments are associated with higher bids which are more than risk-neutral BNE bids. The second explanation is that bidders have preference for winning, apart from their valuations of goods. Scholars refer to this preference for winning as the *Joy of Winning* (JOE). The third explanation is that bidders are driven by *anticipated ex-post regrets*. When a bidder loses, and if she is willing to pay the resulting price, the bidder perceives the resulting forgone surplus as a loser regret. On the other side, when a bidder wins, and if she realizes that she overprices, scholars see such overpayment as a winner regret.

1.1 Motivation: Discrepancies in Observed Bids and Risk-Averse Model Predictions

To clarify our motivation, herein we compare observed bids in experimental auctions and theoretical predictions from the risk-averse preference model. We demonstrate that the risk-averse model predicts both overbidding (bidding above risk-neutral BNE) and underbidding (bidding below risk-neutral BNE), depending on auction rules. Then, we illustrate discrepancies between observed bids in experiments and risk-averse model predictions.

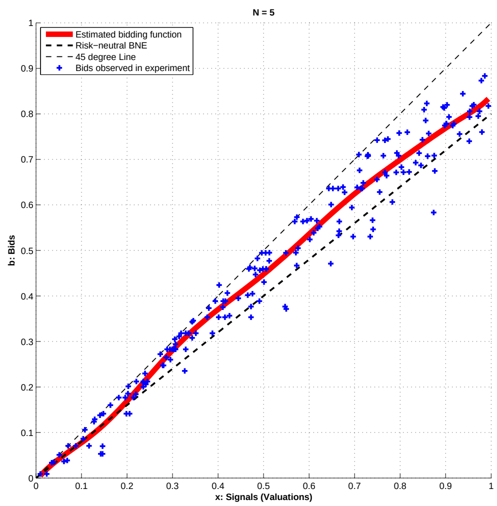
Figures 1 to 4 depict experiment results in symmetric independent private value first-price, all-pay, third-price, and second-price auctions (listing in the order of figures) from Kagel and Levin (1993) [27] and Noussair and Silver (2006) [34][1]. In their experiments, bidders' valuations of auctioned objects are exogenously given (controlled) by experiment administrators, but bidders choose bids by themselves. Valuations are from i.i.d. uniform distributions.

First-Price Auction Experiments: Figure 1 summarizes Kagel and Levin (1993) [27]'s first-price auction results and indicates that significant amounts of bids are above risk-neutral BNE in both five- and ten-bidder fist-price auction experiments. These experimental results perfectly agree with the theoretical prediction in Riley and Samuelson (1981) [37], in which they proved that risk-averse bidders bid more than risk-neutral BNE. Therefore, the risk-averse preference can predict bidders' behavior in experimental first-price auctions well[2].

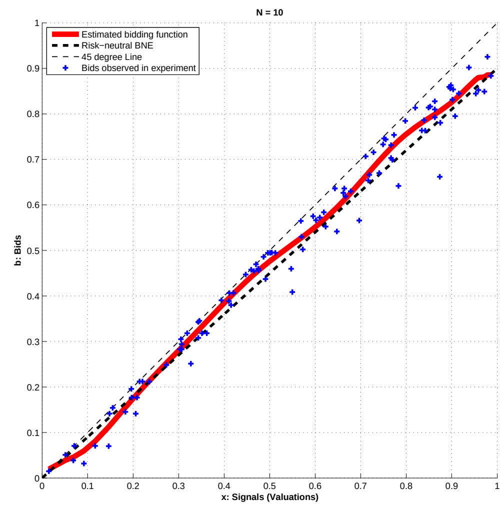
[1] We explain the details of their datasets in Section 6. We remove extreme outliers such as non-serious bids (close to zero bids). We also normalize their data to [0,1] scale for comparison purposes.

[2] Accordingly, first-price auction models with risk-averse preferences are widely applied in empirical auction research. See Guerre, Perrigne, and Vuong (2000, 2009) [20] [21] and subsequent empirical studies.

Figure 1: Kagel and Levin (1993): Frist-Price Auction Experiments

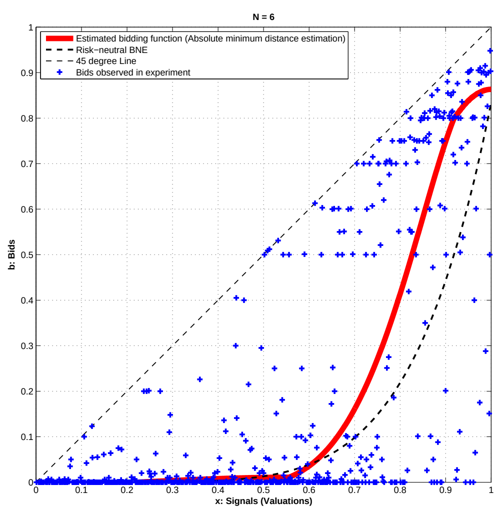


(a) Frist-Price: Five Bidder Experiment Result

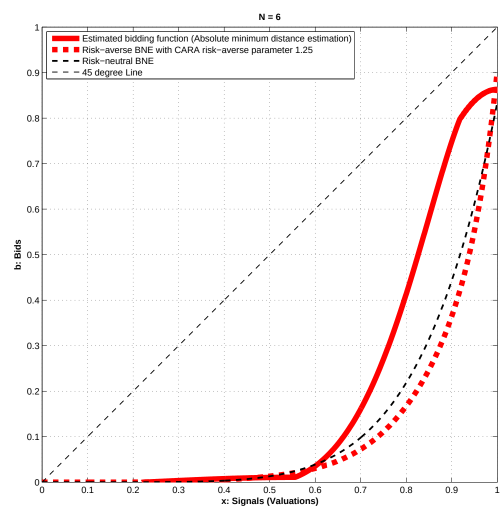


(b) Frist-Price: Ten Bidder Experiment Result

Figure 2: Noussair and Silver (2006) - (a) Six Bidder All-Pay Experiment and (b) Calibrated Risk-Averse BNE



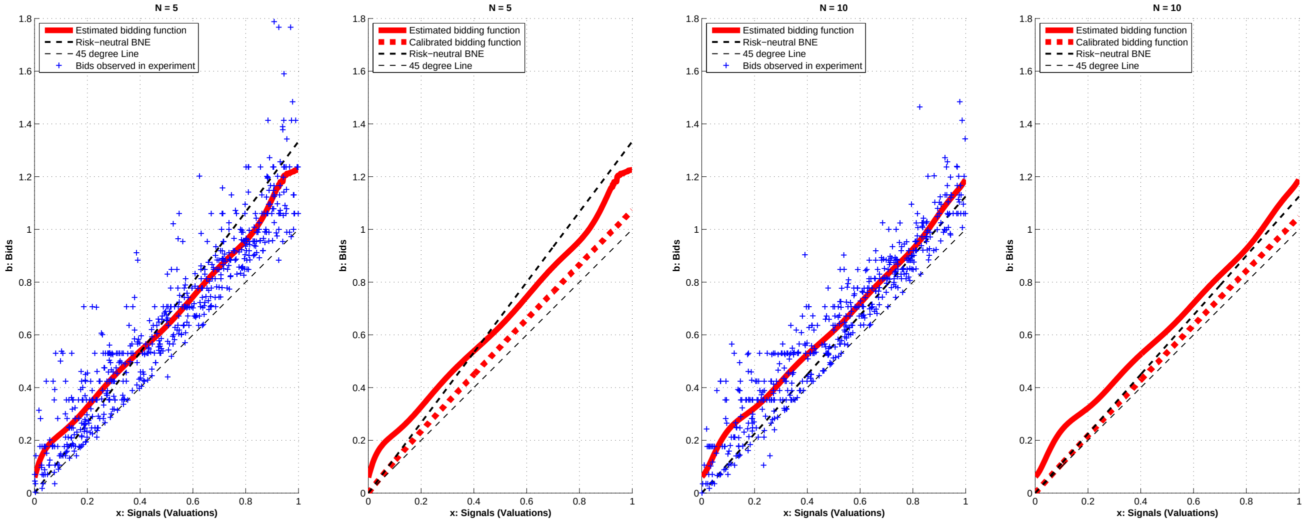
(a) Noussair and Silver (2006) - All-Pay: Six Bidder Experiment Result



(b) Theoretical Six Bider All-Pay Auction BNE with CARA risk-averse parameter 1.25

All-Pay Auction Experiments: Figure 2(a) depicts the results from all-pay auctions from Noussair and Silver (2006) [34]. They experiment with six-bidder all-pay auctions. In theoretical all-pay auctions, the risk-averse preference predicts two different bidding behaviors, depending on bidders' valuations, as illustrated in Figure 2(b). A risk-averse bidder with a low or middle valuation bids less than risk-neutral BNE to avoid payments caused by the all-pay payment rule when she loses. On the other hand, a risk-averse bidder with an extremely high valuation tries to secure her winning payoff by increasing her bid and increasing her winning probability. As a result, a risk-averse BNE bidding function single-crosses the risk-neutral one at very high valuation. Therefore, the risk-

Figure 3: Kagel and Levin (1993): Third-Price Auction Experiments Results - Comparisons to Risk-Averse BNEs



(a) Third-Price: Five-Bidder Experiment Result

(b) Theoretical Third-Price Five-Bidder BNE with CARA Risk-Averse Parameter 1.25

(c) Third-Price:Ten-Bidder Experiment Result

(d) Theoretical Third-Price Ten-Bidder BNE with CARA Risk-Averse Parameter 1.25

averse preference predicts underbidding in all-pay auctions, except bidders with extremely high valuations. Figure 2(b) plots the risk-averse BNE. However, Figure 2(a) shows that significant amounts of bids in experiments fall above risk-neutral BNE[3]. The estimated bidding function lies above the risk-neutral BNE. To explain this observed overbidding, one needs a risk-loving preference, which is not consistent with other empirical economics findings[4]. Since all-pay auctions have such important real-world applications as patent races and political elections, we believe that the risk-averse model's prediction inability in all-pay auctions poses a significant problem.

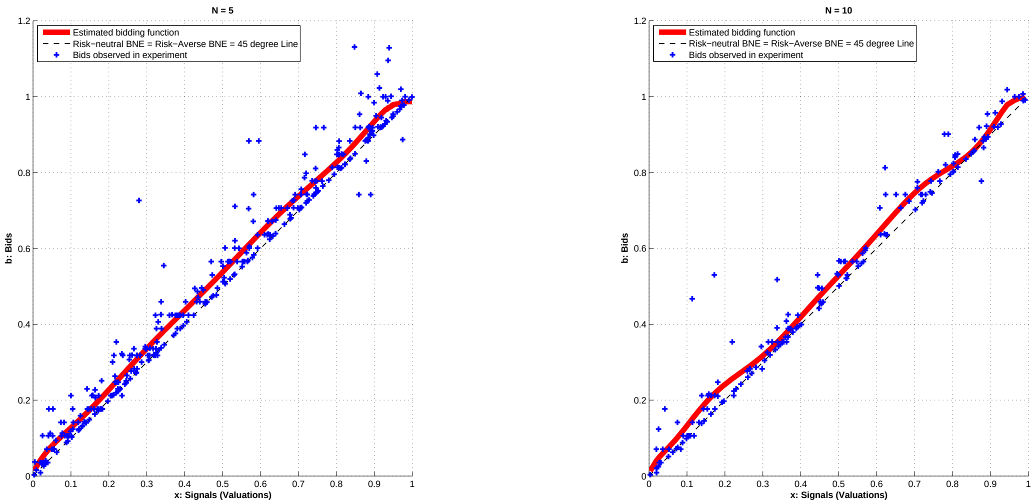
Third-Price Auction Experiments: Figures 3(a) and 3(c) illustrate third-price auction experiment results with five and ten bidders. Third-price auctions, unlike other auctions, are hypothetical auctions never seen in the real world. However, third-price auction experiments provide useful insights into bidders' behavior, especially into their risk-averse attitudes. The theoretical risk-averse model prediction is as follows. In third-price auctions, a winner pays the value of third highest bid, and bidders bid higher than their valuations. If a bidder wins, a third-highest bid can be more than her valuation, and her payment can exceed her valuation. Thus, payoffs in third-price auctions can be negative. As a consequence, risk-averse bidders try to avoid negative payoffs by decreasing their bids and lowering probabilities of winning. Therefore, the risk-averse preference predicts underbidding. Figures 3(b) and 3(d) plot the risk-averse BNE[5]. By comparing the estimated bidding functions and risk-averse BNE, we observe overbidding among lower valuation bidders in five-bidder experiments and among entire valuation bidders in ten-bidder experiments. If one sticks to the risk-averse framework and strategic interactions, and if one wants to explain these observed overbiddings in third-price auction experiments, one needs to accept the risk-loving preference. This

[3] The estimated bidding function (by absolute minimum distance estimation) has a "jump" around valuation 0.6 to 0.8.

[4] In particular, in the insurance literature, one rarely observes risk-loving behaviors.

[5] The analytic bidding function with the CARA preference is derived in the appendix of Kagel and Levin (1993) [27]. The CARA risk-averse parameter 1.25 is used to create Figures 3(b) and 3(d).

Figure 4: Kagel and Levin (1993): Second-Price Auction Experiments



(a) Second-Price: Five Bidder Experiment Result (b) Second-Price: Ten Bidder Experiment Result

generates two immediate problems. First, the risk-loving preference is not consistent with empirical literature in economics, because we never observe such empirical evidences of risk-loving attitudes as negative insurance premiums. Second, bidders in Kagel and Levin (1993) [27]'s experiments[6] reveal a risk-averse preference in the first-price auction experiment. Thus, we have preference inconsistency in bidders' risk-averse attitudes. As in the all-pay auction experiment, the risk-averse preference alone cannot explain overbidding in third-price auctions.

Second-Price Auction Experiments: Figure 4 depicts the second-price auction experiment results. In theory, bidding one's valuation is a weakly dominant strategy in second-price auctions regardless of the bidders' risk-attitudes. However, the experiment results in Figure 4 demonstrate that bidders tend to bid more than their valuations. Therefore, the risk-averse preference alone cannot explain the observed overbidding in the second-price auction experiments[7][8].

These comparisons between experimental results and theoretical predictions illustrate that the risk-averse model alone cannot explain widely-observed overbidding phenomena across different types of auction rules. The risk-averse preference predicts both overbidding and underbidding, depending on specific auction rules. This insufficiency of the risk-averse preference concerns us about reliabilities of auction designs. An auction design with a risk-averse preference model can provide opposite revenue predictions due to the model's underbidding natures under some auction rules. As a result, such auction designs are likely to be unreliable. To design better auctions in the real world, we need to have a model that consistently explains overbidding phenomenon across a variety of auction rules.

[6] They are University of Huston undergraduate and MBA students.

[7] Researchers have developed such concepts to explain observed overbidding in second-price auction experiments as non-strategic interactions and the existence of spiteful feelings. Among such proposed explanations, the most well-accepted explanations is the Joy of Winning, where bidders obtain payoffs that are separated from their valuations. See Cooper and Fang (2008) [8] for an example. We will return to the Joy of Winning in Section 4.

[8] We concentrate on the framework of strategic interactions in this research.

1.2 Contributions

Based on the above motivation, we propose *risk- & regret-averse* model in the rest of this paper. Our model nests canonically accepted risk-neutral and risk-averse preferences in the auction literature. This paper makes two main contributions. First, by numerical calibrations, we qualitatively demonstrate that the risk- & regret-averse model can explain overbidding across a variety of symmetric Independent Private Value (IPV) auction rules. Section 7 reviews the calibration figures. Second, by revenue comparisons, we quantitatively demonstrate that the risk- & regret-averse model predicts a revenue ranking among standard Independent Private Value (IPV) auctions as[9]

$$\text{all-pay} > \text{first-price} \approx \text{second-price} > \text{third-price}.$$

This ranking drastically contrasts with a prediction based on risk-averse preference[10], as our model demonstrates the revenue supremacy in all-pay auctions. Table 2 in Section 7 summarizes the results of revenue comparisons. This new revenue ranking is consistent with the experimental auction literature, especially results from all-pay auction experiments in Noussair and Silver (2006) [34] and Gneezy and Smorodinsky (2006)[22], who report surprisingly large revenues generated in all-pay auction experiments.

1.3 Literature

Sizable amounts of experimental auction research have analyzed overbidding, and these analyses propose a variety of explanations. Since we have already discussed Kagel and Levin (1993) [27] and Noussair and Silver (2006) [34] with their experimental data, and since we focus on regret-averse attitudes among auction bidders, we here concentrate on the three most-related studies with respect to regret-averse attitudes.

Engelbrecht-Wiggans (1989) [14] is the first theoretical study of bidding that incorporates regret[11][12]. Recent papers by Engelbrecht-Wiggans and Kotok (2007)[16] and Filiz-Ozbay and Ozbay (2007) [18] experimentally investigate the effect of feedback policies on bidding behaviors. These studies explain observed overbidding by anticipated ex-post regret. In particular, in Filiz-Ozbay and Ozbay (2007) [18], they experiment with first-price auctions under different post-auction information disclosure environments. Using the reduced-form regressions, they find significant differences in bidding behaviors caused by changes in post-auction information disclosure policies. Bids are significantly higher in the environment where winning bids are publicly announced at the end of each auction, compared to those in a no post-auction information disclosure environment. Based on the reduced form regression results, they assess and verify existences of anticipated regrets under different information disclosure environments. Although we do not discuss differences in information disclosure environments in this paper, we recognize our risk- & regret-averse model is the structural extension of Filiz-Ozbay and Ozbay (2007) in the sense that we statistically verify the significance of regret-averse attitudes in auctions.

[9] Revenue computations are based on the uniform valuation distribution and with estimated risk-averse and regret-averse parameters.

[10] With a uniform valuation distribution and with reasonable risk-averse attitudes, the risk-averse preference predicts a revenue ranking as

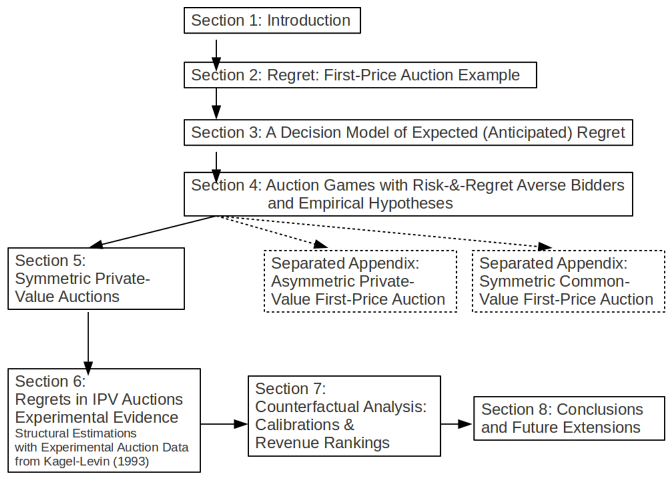
$$\text{first-price} > \text{second-price} > \text{third-price} > \text{all-pay}.$$

Programs that compute revenues with risk-averse preference are posted on Yoshimoto's website.

[11] In this research, we use terminologies (1) "Regret", (2) "Anticipated Regret", and (3) "Expected Regret" interchangeably. These are all ex-ante evaluations of ex-post regret. Formal definitions of regret are in Section 3.

[12] See also Engelbrecht-Wiggans and Kotok (2005)[15]

Figure 5: Roadmap of the Paper



We emphasize that we restrict our attention to strategic interactions among bidders. We acknowledge that one may explain the failure to choose a dominant strategy by arguments at a cognitive level, such as imperfect reasoning, or many other behavioral models. We believe such behavioral models are orthogonal to our research. Nevertheless, we view them as adequate explanations of overbidding phenomenon[13].

1.4 Organization of Paper

We organize this paper as follows. In Section 2, we illustrate bidders' regret-averse attitudes with the simple first-price auction example. In Section 3, we define the formal models of expected (anticipated) regret. In Section 4, we apply the risk- & regret-averse model to the auction environment. We also propose several testable empirical hypotheses. Section 5 discusses Independent Private Value (IPV) auctions with the risk- & regret-averse model. In Section 6, we structurally estimate model parameters and test empirical hypotheses. In Section 7, we compute calibrations and counterfactual revenue rankings. Section 8 concludes and mentions possible future extensions. A separate Appendix discusses extensions to asymmetric and common-value auctions.

[13]We are confident that many behavioral models, such as level-K and quantal-response models, remain compatible with our model.

2 Regrets: Simple First-Price Auction Example

Before formally introducing definitions, it is worthwhile to cultivate the intuitions behind regrets in auctions. In this section, we consider a simple first-price auction example in which perceptions of regrets naturally emerge.

We consider a fictional auction story. Mrs. Robinson is a big fan of baseball and she participates an auction where an auctioneer offers a vintage Joe DiMaggio[14] autographed bat. This auction is in the sealed-bid first-price format, and we hypothetically assume valuations among bidders are private, to simplify the explanation[15]. All such information such amounts of winning and losing bids are publicly announced after the auction. Mrs. Robinson evaluates the bat as worth $\$1,000$, and for some equilibrium reasons, that we will intensively investigate throughout this paper, she submits a bid of \$800. After such an auction, Mrs. Robinson ex-post experiences one of the following three mutually exclusive events.

Event 1: *Mrs. Robinson wins the auction but she overprices*

It turns out that Mrs. Robinson's submitted bid (\$800) is the highest and she obtains the bat. However, from the public announcement, she ex-post learns that the second-highest bid fell far below her bid, say \$600. Then, Mrs. Robinson ex-post realizes that she only needed to bid $\$(600+\varepsilon)$ to win the auction, where ε is a smallest monetary unit[16]. This means she ex-post realizes she overprices the bat to the amount of $\$800 - \$600 = \$200$. A regret in this event is defined as the difference between an ex-post best payoff (which *could* be achievable if Mrs. Robinson *knew* that the second highest bid *was* \$600 before an auction) and a realized payoff (which Mrs. Robinson actually obtains after the auction). If Mrs. Robinson has the risk-neutral preference, her regret is defined by

$$\underbrace{1000-600}_{\text{ex-post best payoff}} - \underbrace{(1000-800)}_{\text{realized winning payoff}} = 200, \tag{1}$$

which is nothing but the amount Mrs. Robinson had overpriced[17]. Next, if Mrs. Robinson has a risk-averse preference with a CARA $\left(u(z) = \frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha z)\right)$ von Neumann-Morgenstern (henceforth, vNM) payoff function[18] with a risk-averse parameter α, her regret is similarly defined by the difference between an ex-post best payoff and a realized payoff

$$\left[\underbrace{\frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha(1000-600))}_{\text{ex-post best payoff}} - \underbrace{\left(\frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha(1000-800))\right)}_{\text{realized winning payoff}}\right]. \tag{2}$$

Furthermore, if Mrs. Robinson has not only risk-averse but also regret-averse attitudes, meaning she amplifies a regret (which is the difference between an ex-post best and a realized payoff) with an index function with a regret-averse parameter γ, her regret is defined by

$$\left[\underbrace{\frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha(1000-600))}_{\text{ex-post best payoff}} - \underbrace{\left(\frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha(1000-800))\right)}_{\text{realized winning payoff}}\right]^{1+\gamma}. \tag{3}$$

[14] Joe DiMaggio (1914–1999), a Hall of Fame baseball player for the New York Yankees.

[15] We will relax the private value assumption in online Appendix of this paper.

[16] We follow the convention in auction literature. We assume that monetary units are continuous and signals (valuations) are drawn from atom-less distributions. Ties happen with probability measure zero, and we simply ignore ε in discussions.

[17] Some researchers call this event as "Money Left on a Table."

[18] We normalize a CARA payoff function so that $u(0) = 0$ and $u'(0) = 1$.

We call numerical amounts defined by (1), (2), and (3) as **winner regret** in order to capture a winner's perception over a forgone payoff caused by an overpayment that could be avoidable[19].

Event 2: *Mrs. Robinson loses the auction but she underprices the affordable bat*

It turns out that Mrs. Robinson's submitted bid ($800) is not the highest and she loses the auction. However, from the public announcement, she ex-post learns that the highest bid is below her valuation (which is $1,000), say the highest bid is $900. Then, Mrs. Robinson ex-post realizes that she underpriced her bid. In other words, if she *had submitted* a bid of $(900 + \varepsilon)$, she *could have won* an auction with a positive payoff. In the manner similar to Event 1, a regret is defined as the difference between an ex-post best payoff (which *could* be achievable if she *knew* the highest bid *was* $900 before an auction) and a realized losing payoff of zero. If Mrs. Robinson has the risk-neutral preference, her regret is defined by

$$\underbrace{1000 - 900}_{\text{ex-post best payoff}} - \underbrace{0}_{\text{realized losing payoff}} = 100. \tag{4}$$

Furthermore, as we saw in Event 1, if Mrs. Robinson has not only a risk-averse preference but also a regret-averse attitude, her regret is defined by

$$\left[\underbrace{\frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha(1000-900))}_{\text{ex-post best payoff}} - \underbrace{\left(\frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha \cdot 0)\right)}_{\text{realized losing payoff }=0}\right]^{1+\gamma}. \tag{5}$$

We call numerical amounts defined by (4), and (5) as **loser regret** to capture losers' perceptions over a forgone positive payoff that could be achievable.

Event 3: *Mrs. Robinson loses the auction and the bat is not affordable to her*

It turns out that Mrs. Robinson's submitted bid ($800) is not the highest and she loses the auction. Furthermore, by the public announcement, she ex-post learns that the highest bid is far above her valuation (which is $1,000), say the highest bid is $1,300. This means she ex-post realizes that the bat is unaffordable to her. Here, unaffordable means that even if she knew the highest bid was $1,300, she would have had no way to obtain a positive payoff. If Mrs. Robinson has the risk-neutral preference, her regret is defined by

$$\underbrace{0}_{\text{ex-post best payoff}} - \underbrace{0}_{\text{realized losing payoff}} = 0. \tag{6}$$

In this event, even if Mrs. Robinson has both risk-averse and regret-averse attitudes, her regret is still zero, because a regret is defined by the difference between an ex-post best payoff and a realized payoff.

Intuitively, the concept of regret numerically converts the perception of "if a person knew the true state" stories. Ex-post best payoffs provide reasonable reference points to describe potentially attainable payoffs that a bidder may consider when she chooses her bid in an action.

[19] Note that if $\gamma = 0$ (no regret amplification), (3) collapses to (2). In addition, if $\alpha \to 0$, (2) collapses to (1). These nesting properties provide testabilities in risk-averse and regret-averse attitudes. We will explore these testabilities in the empirical sections.

3 A Decision Model of Anticipated (Expected) Regret

In this section, we define the regret minimizing decision rule proposed by Hayashi [23] that we use throughout this paper. The regret minimization problems nest expected payoff maximization problems.

3.1 Definition

The formal definition of the regret minimizing decision rule is described as follows.

Definition 1 : Regret minimizing decision rule (Hayashi, 2008 [23])
An agent chooses an action b^* with the criterion

$$b^* = \underset{b \in A}{argmin} \left\{ \int_X \left[\underbrace{\underset{\tilde{b} \in A}{sup} \left\{ u(\tilde{b}, \mathbf{x}) \right\}}_{\text{ex-post best payoff}} - u(b, \mathbf{x}) \right]^{1+\gamma} dG(\mathbf{x}) \right\}, \tag{7}$$

where γ is a regret-averse parameter, $u(\cdot)$ is a vNM function, A is a set of available actions, X is the set of entire states that can be discrete, continuous, or both, and $G(\cdot)$ is the probability measure over a state space.

Notice that we define the integrating object as a homothetic[20] index function with a regret-averse parameter γ. A regret-averse parameter γ can be categorized and named by its sign as

$$\begin{cases} \gamma > 0 & : \quad \textbf{(i)} \quad \textbf{regret-averse} \\ \gamma = 0 & : \quad \textbf{(ii)} \quad \textbf{regret-neutral} \iff \textbf{expected payoff maximization} \\ \gamma < 0 & : \quad \textbf{(iii)} \quad \textbf{regret-loving} \end{cases}.$$

Three interpretations can explain this parameter. (i) If an agent is regret-averse ($\gamma > 0$), she tends to amplify the difference between an ex-post best payoff she *could* obtain (which is achievable if she *knew* a true state is $\mathbf{x}$) and a payoff she obtains by choosing action b. (ii) If an agent is regret-neutral ($\gamma = 0$), she solves an expected payoff maximization problem, as we explain greater detail shortly. (iii) If an agent is regret-loving ($\gamma < 0$), she tends to diminish her regrets[21].

Regret minimization problems nest expected payoff maximization problems[22]. If $\gamma = 0$, the portion of the ex-post best payoff in equation (7) becomes a constant, and it can be removed from a minimization problem. More precise, if $\gamma = 0$, we can write a regret minimization problem as

$$b^* = \underset{b \in A}{argmin} \left\{ \int \left[\underbrace{\underset{\tilde{b} \in A}{sup} \left\{ u(\tilde{b}, \mathbf{x}) \right\}}_{\text{constant}} - u(b, \mathbf{x}) \right]^1 dG(\mathbf{x}) \right\} = \underset{b \in A}{argmin} \left\{ - \int u(b, \mathbf{x}) dG(\mathbf{x}) \right\} = \underbrace{\underset{b \in A}{argmax} \left\{ \int u(b, \mathbf{x}) dG(\mathbf{x}) \right\}}_{\text{expected payoff maximization}}. \tag{8}$$

In the last equality, we change a minus of minimization problem to a maximization problem. The last expression is nothing but an expected payoff maximizing problem. This nesting property has significant empirical implications.

[20] Because of a homothetic index function, the regret minimization problem is robust to an affine payoff transformation. A constant part of an affine transformation is canceled out by the subtraction. A multiplication part of an affine transformation can be moved to the outside of minimization problem.

[21] We define a regret-loving attitude for purely theoretical purposes, and we view it is unrealistic to assume such attitudes.

[22] If states are discrete, it also nests Savage (1951) [38]'s minmax regret decision rule. As $\gamma \to \infty$, the regret decision rule gradually emphasizes a state that provides a maximum regret and eventually becomes Savage's minmax criterion in the limit.

4 Auction Games with Risk- & Regret-averse Bidders and Empirical Hypotheses

One can straightforwardly apply the concept of regret to the auction environment by specifying a state space to be a signal (valuation) space. In this section, we define the Bayesian Nash Equilibrium (henceforth BNE) equilibrium concept with regret minimizing criterion in symmetric Independent Private-Value (henceforth IPV) auctions[23]. We also introduce testable empirical (behavioral) hypotheses that we test in the empirical application sections.

4.1 General Auction Settings

One unit of the indivisible object is being sold via a specific rule of sealed-bid auctions, for example, first-price or all-pay[24]. We assume the existence of symmetric equilibria with no reservation price. There are n bidders participating in an auction. Each bidder $i = 1, \cdots, n$ receives a signal (valuation) x_i, where $x_i \in [0, \bar{x}]$, and $\bar{x}$ is an upper bound of signals which can be normalized to one. Let $F : [0, \bar{x}] \to [0, 1]$ be the continuously differentiable cumulative distribution function and $f : [0, \bar{x}] \to \mathbb{R}_+$ be its derivative. Signals are i.i.d. and valuations are private. We here focus on a symmetric equilibrium with a continuously differentiable and strictly increasing bidding function denoted by $\beta : [0, \bar{x}] \to \mathbb{R}_+$. We denote bidder i's signal as x_i, and also denote the vector of opponents' signals as $\mathbf{x}_{-i} = (x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_n)$. The payoff for bidder i (given her own choice of bid b_i with her realized signal x_i and given that her opponents employ a bidding strategy β with their realized signals $\mathbf{x}_{-i}$) is denoted by a vNM function $u(b_i, x_i, \mathbf{x}_{-i}|\beta)$. For the simplicity of mathematical notations, we drop subscripts of bidder i's signal and her bid by denoting $x = x_i$ and bid $b = b_i$.

4.2 Equilibrium Concepts

We now introduce the BNE concept in auctions with the regret minimizing criterion.

Definition 2 : Symmetric BNE with Regret Minimizing Criterion
A symmetric Bayesian Nash Equilibrium (BNE) strategy is a monotone increasing and continuously differentiable bidding function β that satisfies

$$\beta(x) = \underset{b \in A}{argmin} \left\{ E_{\mathbf{x}_{-i}} \left[\left[\underbrace{\underset{\tilde{b} \in A}{sup} \left\{ u(\tilde{b}, x, \mathbf{x}_{-i}|\beta) \right\}}_{\text{ex-post best payoff}} - u(b, x, \mathbf{x}_{-i}|\beta) \right]^{1+\gamma} \right] \right\} \tag{9}$$

for all $i \in (1, \ldots, n)$ and for any $x \in [0, \bar{x}]$, where A is a bid space that is common to all bidders and $u(\cdot, \cdot, \cdot|\cdot)$ is a vNM function.

Here, we apply the standard symmetric auction BNE interpretations. Given other bidders employ an equilibrium strategy β, bidder i minimizes her expected regret by optimally choosing her bid b, and her optimal choice b coincides

[23] We relax symmetry and private-value assumptions in online Appendix of this paper. Extensions to asymmetric and common-value auctions straightforward.

[24] Since Krishna (2002) is a standard reference in the auction literature, we try to follow his notations closely.

with $\beta(x)$. As we have seen in the previous section (equation (8)), if we assume regret-neutrality $\gamma = 0$, the equation (9) collapses to a standard auction expected payoff maximizing BNE. In this sense, our auction BNE with the regret minimizing criterion nests the standard auction literature. Next, we explore the intuitions behind this BNE.

4.3 Understanding Regret Minimizing BNE

In the standard auction theory, one derives a BNE bidding function by solving an expected payoff maximizing first-order condition. The beauty of theoretical auction literature is that composing parts of such a first order condition usually have clear economic interpretations. Researchers usually interpret these as marginal benefit and marginal cost of increasing bids. In this subsection, we first review marginal benefit and cost analysis in an expected payoff maximizing first-price private-value auction, which may appear trivial but holds great importance in fostering the intuitions behind regret. Then, we examine a regret minimizing first-order condition that one can also interpret as marginal benefit and cost.

4.3.1 Marginal Benefit and Marginal Cost in Expected Payoff Maximization

In a private-value first-price auction expected payoff maximization problem, the expected payoff (EP) for bidder i (given her signal x and bid b, and given other bidders employ a strategy β with their signals $\mathbf{x}_{-i}$) is written as

$$\mathrm{EP}(b, x|\beta) = u(x-b) \cdot E_{\mathbf{x}_{-i}}\left[\mathrm{ALLOC}(b, \mathbf{x}_{-i}|\beta)\right] \tag{10}$$

where $u(\cdot)$ is a vNM function and $\mathrm{ALLOC}(b, \mathbf{x}_{-i}|\beta)$ is a binary allocation function. The first order condition of an expected payoff maximization problem is

$$\underbrace{u(x-b) \cdot \frac{dE_{\mathbf{x}_{-i}}\left[\mathrm{ALLOC}(b, \mathbf{x}_{-i}|\beta)\right]}{db}}_{\text{marginal expected benefit of increasing bid }>0} \quad = \quad -\underbrace{\left[-u^{'}(x-b) \cdot E_{\mathbf{x}_{-i}}\left[\mathrm{ALLOC}(b, \mathbf{x}_{-i}|\beta)\right]\right]}_{\text{marginal expected cost of increasing bid }<0}. \tag{11}$$

We intentionally keep negative signs on the RHS for consistency in interpretation. A bidder i is choosing her optimal bid to equate her expected marginal benefit (LHS, an increase in winning probability) and marginal cost (RHS, an increase in payment) to maximize her expected payoffs.

4.3.2 Marginal Benefit and Marginal Cost Interpretations of Regret Minimization

Such expected marginal benefit/cost interpretation is carried over to regret minimization problems. Define a Regret function $\mathrm{R}(\cdot,\cdot|\cdot)$, a winner Regret function $\mathrm{WR}(\cdot,\cdot|\cdot)$, and a loser Regret function $\mathrm{LR}(\cdot,\cdot|\cdot)$ as

$$\begin{aligned} \mathrm{R}(b, x, \mathbf{x}_{-i}|\beta) \quad &\equiv \quad \left[\sup_{\tilde{b}\in A}\left\{u(\tilde{b}, x, \mathbf{x}_{-i}|\beta)\right\} - u(b, x, \mathbf{x}_{-i}|\beta)\right]^{1+\gamma} \\ \mathrm{WR}(b, x, \mathbf{x}_{-i}|\beta) \quad &\equiv \quad \mathbf{1}_w \cdot \mathrm{R}(b, x, \mathbf{x}_{-i}|\beta) \qquad \text{and} \qquad \mathrm{LR}(b, x, \mathbf{x}_{-i}|\beta) \equiv \mathbf{1}_l \cdot \mathrm{R}(b, x, \mathbf{x}_{-i}|\beta) \end{aligned}$$

where $\mathbf{1}_w$ and $\mathbf{1}_l$ are binary winning and losing indicators such that

$$\mathbf{1}_w \quad = \quad \begin{cases} 1 & \text{if bidder } i \text{ wins} \\ 0 & \text{else} \end{cases} \qquad \text{and} \qquad \mathbf{1}_l = \begin{cases} 1 & \text{if bidder } i \text{ loses} \\ 0 & \text{else} \end{cases} . \tag{12}$$

Since an object auctioned is indivisible, winning and losing are mutually exclusive events. Using this mutual exclusivity, we can decompose a regret function as

$$\mathrm{R}(b, x, \mathbf{x}_{-i}|\beta) \quad = \quad \mathrm{WR}(b, x, \mathbf{x}_{-i}|\beta) + \mathrm{LR}(b, x, \mathbf{x}_{-i}|\beta).$$

Then, one can rewrite the Definition 2 (equation (9)) as

$$\beta(x) \quad = \quad \underset{b \in A}{argmin} \left\{ \underbrace{E_{\mathbf{x}_{-i}} \left[\mathrm{R}(b, x, \mathbf{x}_{-i}|\beta) \right]}_{\text{expected regret}} \right\} = \underset{b \in A}{argmin} \left\{ \underbrace{E_{\mathbf{x}_{-i}} \left[\mathrm{WR}(b, x, \mathbf{x}_{-i}|\beta) \right]}_{\text{expected winner regret}} + \underbrace{E_{\mathbf{x}_{-i}} \left[\mathrm{LR}(b, x, \mathbf{x}_{-i}|\beta) \right]}_{\text{expected loser regret}} \right\}.$$

One can take a derivative to minimize her expected regret and, thus, solve following first order condition equation,

$$\underbrace{\mathrm{MELR}(b_i, x|\beta)}_{\text{marginal benefit of increasing bid}} \quad = - \quad \underbrace{\mathrm{MEWR}(b_i, x|\beta)}_{\text{marginal cost of increasing bid}}$$

where

$$\mathrm{MELR}(b, x|\beta) \equiv \frac{d}{db} E_{\mathbf{x}_{-i}} [\mathrm{LR}(b, x, \mathbf{x}_{-i}|\beta)] \quad \text{and} \quad \mathrm{MEWR}(b, x|\beta) \equiv \frac{d}{db} E_{\mathbf{x}_{-i}} [\mathrm{WR}(b, x, \mathbf{x}_{-i}|\beta)]$$

represent Marginal Expected Winner Regret (MEWR) and Marginal Expected Loser Regret (MELR) functions. Note that these expected benefits and costs are for the sake of minimizing regrets. By slightly increasing her bid, a bidder can reduce her expected loser regret (regret caused by underpricing) while she increases her expected winner regret (regret caused by overpricing). In equilibrium, such marginal benefit and cost are equated. Using above notations, we can rewrite the definition of BNE with regret criterion in the following concise way.

Definition 3 : Symmetric BNE with Regret Criterion (Marginal Benefit and Cost Notation)
A symmetric Bayesian Nash Equilibrium (BNE) strategy is a monotone increasing and continuously differentiable bidding function β that solves

$$\mathrm{MELR}(\beta(x_i), x_i|\beta) = -\mathrm{MEWR}(\beta(x_i), x_i|\beta) \tag{13}$$

for all $i \in (1, \ldots, n)$ and any $x_i \in [0, \bar{x}]$.

In the case of regret-neutrality $\gamma = 0$, the equation (13) collapses to a usual (expected payoff maximizing) standard auction's first order condition. In particular, in the case of first-price auction, the equation (13) becomes (11).

4.4 Empirical Hypotheses

Based on the theoretical definition above, we now consider empirics with the regret BNE concept. Although the theoretical definition provides us with the solid benchmark, sizable experimental literatures suggest the necessity of modifications to explain empirical phenomena. In this subsection, we introduce empirical hypotheses suggested by the experimental auction literature. Before moving to the detail of each hypothesis, we would like to emphasize that our hypothesis tests are structural, which means we jointly test hypotheses without making strong assumptions such as the risk-neutrality[25].

[25] In reduced form analysis, making strong assumptions before testing hypotheses is often inevitable. For example, an experimental researcher may claim that some experimental phenomena are statistically significant given the assumption of bidders' risk-neutrality.

4.4.1 Relaxing the Symmetry between Winner and Loser Regrets

The first empirical modification comes from the question about the symmetry between winner and loser regrets. In real-world auctions, bidders may or may not perceive winner and loser regrets equally, and one should empirically test the symmetry in regrets. In addition, Filiz-Ozbay and Ozbay (2007) [18] report asymmetry in regret-averse attitudes with their experimental private-value first-price auction data in the reduced form fashions[26]. Motivated by their findings, we modify the definition of BNE to make the asymmetry in regret-averse attitudes testable.

Definition 4 : *BNE with Winner and Loser Regrets*
A symmetric Bayesian Nash Equilibrium strategy is a monotone increasing and continuous bidding function β that satisfies

$$\beta(x) \quad = \quad \underset{b \in A}{argmin} \left\{ E_{\mathbf{x}_{-i}} \left[\left[\underset{\tilde{b} \in A}{sup} \left\{ u(\tilde{b}, x, \mathbf{x}_{-i}|\beta) \right\} - u(b, x, \mathbf{x}_{-i}|\beta) \right]^{1+\mathbf{1}_w \cdot \gamma_w + \cdot \mathbf{1}_l \gamma_l} \right] \right\} \tag{14}$$

where $\mathbf{1}_w$ and $\mathbf{1}_l$ are winning and losing indicator defined in the equation (12), γ_w is a winner-regret parameter, and γ_l is a losing-regret parameter.

The difference between equation (9) and (14) is that a regret is amplified by $1 + \gamma$ in equation (9), while it is amplified by $1 + \gamma_w \cdot \mathbf{1}_w + \gamma_l \cdot \mathbf{1}_l$ in equation (14)[27]. The equation (14) literally means that bidders in auctions may percept their regrets in winning and losing events differently, in terms of the regret amplification with homothetic index function[28]. Definition 4 provides an advantage in that symmetry in regret becomes testable by Hypothesis 2 below. Based on Definition 4, we now propose several empirically testable hypotheses that we will investigate in empirical sections. The first hypothesis tests the existence of bidders' regret-averse attitudes.

Hypothesis 1 : *Expected Utility Maximization*

$$\begin{aligned} H_{1n} &: \gamma_w + \gamma_l = 0 \quad \text{(Bidders are Expected Payoff Maximizer)} \\ H_{1a} &: \gamma_w + \gamma_l > 0 \quad \text{(Bidders are Regret Minimizer)} \end{aligned}$$

Hypothesis 1 is the amenity of the nesting property of the regret minimization problem. Next, we propose the second hypothesis for the asymmetry of regret-averse attitudes as we define in the equation (14).

Hypothesis 2 *Symmetricity in Regret-Averse Attitude*

$$\begin{aligned} H_{2n} &: \gamma_w = \gamma_l \quad \text{(Symmetric Regret)} \\ H_{2a} &: \gamma_w \neq \gamma_l \quad \text{(Asymmetric Regret)} \end{aligned}$$

The problem is that such a claimed conclusion depends on the assumption of risk-neutrality. Structural estimation researchers warn that such a conclusion suffers from "the Double-Hypotheses Problem," meaning that a testing result in one hypothesis depends on the assumption of another hypothesis. It is worth noting that seminal experimental auction papers such as Kagel and Levin (1993) [27] and Filiz-Ozbay and Ozbay (2007) [18] are avoiding such problems by carefully designing their experiments.

[26] We recognize our hypotheses are structurally testing and verifying Filiz-Ozbay and Ozbay (2007) [18]'s experimental findings.

[27] We assume an object auctioned is indivisible, so winning and losing status are mutual exclusive.

[28] In empirical sections, bidders in various experiments reveal significant loser regrets ($\gamma_l > 0$, statistically), while we cannot reject $\gamma_w = 0$.

The third hypothesis is testing the risk neutrality by investigating the concavity parameter in a payoff function $u(\cdot)$.

Hypothesis 3 : *Risk-Neutrality*
If we choose a CARA[29] $v(x) = \frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha x)$

$$\begin{aligned} H_{3n} &: \alpha = 0 \quad \text{(Risk-Neutral)} \\ H_{3a} &: \alpha > 0 \quad \text{(Risk-Averse)} \end{aligned}$$

4.4.2 Joy of Winning

The well-known overbidding phenomenon in second-price sealed-bid auctions motivates the next hypothesis. Experimental auction literature reports that bidders in private-value second-price auction experiments typically bid higher than their valuations, despite the strategy to bid their valuations serves as a weakly dominant strategy with the expected payoff maximizing criterion (as we will see soon, bidding their valuations also serves as a weakly dominant strategy with the regret minimizing criterion). Based on such experimental observations, experimental and empirical researchers takes stances of either (1) assuming a Joy of Winning[30] with perfectly rational bidders or (2) consider bounded-rationality models[31]. In this paper, we take the stance of (1). Nevertheless, we do believe our regret minimizing model is compatible with the literature derived from (2). Here, we would like to clarify the following point; we include a Joy of Winning parameter not to serve as a trivial explanation of overbidding across a variety of auctions, but rather to verify the fact that regret-averse attitudes still play important roles in explaining overbidding phenomena even after we include the Joy of Winning. In the empirical section of this paper, we will statistically verify existences of both regret-averse attitudes and Joy of Winning. The following is the definition of Joy of Winning.

Definition 5 : *Joy of Winning*
The payoff function takes a form of

$$u(b_i, x, \mathbf{x}_{-i}|\beta) = \begin{cases} v(x - P(b, \mathbf{x}_{-i}|\beta) + J) & : \text{ if bidder } i \text{ wins} \\ v(-P(b, \mathbf{x}_{-i}|\beta)) & : \text{ if bidder } i \text{ loses} \end{cases}$$

where $v(\cdot)$ is a vNM function, J is the Joy of Winning parameter, and $P(b, \mathbf{x}_{-i}|\beta)$ is a payment function that depends on an auction's rule.

The advantage of this specific form of payoff function is that one can interpret the Joy of Winning in monetary terms. We now propose the fourth hypothesis to test the existence of the Joy of Winning in the bidders' payoff function.

[29] We choose not CRRA but CARA payoff function to deal with negative payoffs in third-price and all-pay auctions. Note that the CARA payoff function $v(x) = \frac{1}{\alpha} - \frac{1}{\alpha}\exp(-\alpha x)$ gradually become linear as $\alpha \to 0$. In the limit, $v(x)$ becomes a 45 degree line.

[30] For example, see Ertac, Hortascu, and Roberts (2010) [13]. Some experimental researchers reports the existence of "spite," which is observationally equivalent to Job of Winning under many circumstances.

[31] For example, the logit distribution based quantile response model (McKelvey and Palfrey (1995) [30]) and the cognitive hierarchy model (Camerer, Ho, and Chong (2004) [5]). We strongly believe these models are compatible with our regret minimizing criterion.

Hypothesis 4 ; *Existence of the Joy of Winning*

$$\begin{aligned} H_{4n} &: J = 0 \quad \text{(No Joy of Winning)} \\ H_{4a} &: J > 0 \quad \text{(Joy of Winning)} \end{aligned}$$

We test Hypotheses 1–4 in the empirical sections of this paper.

5 Symmetric Private-Value Auctions

In this section, we investigate equilibrium characteristics of the risk- & regret-averse model among symmetric Independent Private Value (IPV) auctions. We explore this section in the order of second-price, first-price, all-pay, and third-price auctions[32].

In the remaining sections of this paper, we use the simplified notation $\beta_1 = \beta(y_1)$ and and $\beta_2 = \beta(y_2)$ where y_1 and y_2 are the highest and the second-highest signals among bidders excluding bidder i. In addition, we use the terminology "*affordable*" to describe a situation in which a bidder can win an auction with a positive payoff. In a similar manner, we define "*unaffordable*" as a situation in which it is impossible for a bidder to obtain a positive payoff. In addition, a generic payoff function $v(z)$ can be normalized as $v(0) = 0$, although we do not use such normalization to foster readers' intuitions. Moreover, we use the notation I, II, III, and AP to indicate First-, Second-, Third-, and All-Pay auctions.

5.1 Second-Price Auction (SPA)

We start with a second-price auction since one can easily derive the equilibrium bidding function by (weakly-) dominant strategy eliminations. Payoffs in a second-price auction is specified by

$$u(b, x, \mathbf{x}_{-i}|\beta) = \begin{cases} v(x - \beta_1 + J) & \text{if } \underbrace{\beta_1 < b}_{\text{win}} \\ v(0) & \text{if } \underbrace{b < \beta_1}_{\text{lose}} \end{cases}$$

where β_1 is a highest bid among bidders excluding bidder i (note that we use the short-hand notations of $b = b_i$ and $x = x_i$). Even with the regret minimizing criterion, a second-price auction keeps the simple equilibrium bidding function.

Proposition 1 In a second-price auction, a symmetric equilibrium bidding function β^{II} is

$$\beta^{\text{II}}(x) = x + J$$

for all $x \in [0, \bar{x}]$.

We can intuitively prove this proposition with Figure 6. The horizontal axis measures an amount of the highest of other bids, β_1, and the vertical axis measures amounts of ex-post best and realized payoffs. In Figure 6, the left (right) figure represents a case in which a bidder bids lower (higher) than $x + J$. The central figure represents a case in which a bidder bids $x + J$. We define regrets as the subtraction of thin-dashed line (which is realized payoffs)

[32] Third-price auctions are discussed in Appendix, as not all of readers are interested in third-price auctions.

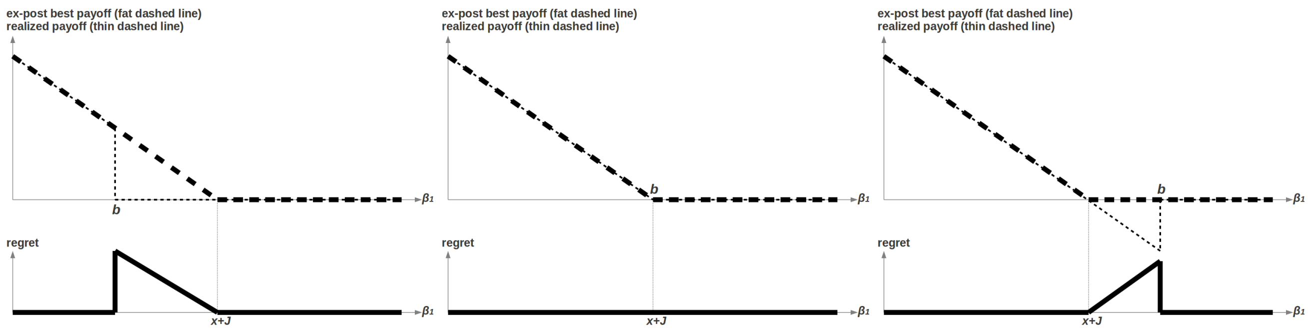


Figure 6: Regret in a Second-Price Auction: (1) $b < x+J$ (left figure), (2) $b = x+J$ (center figure),and (3) $b > x+J$ (right figure). Thin-dashed lines are realized payoffs, and thick-dashed lines are ex-post best payoffs . We define regrets as the differences between these two lines in each figure; we plot regrets as the solid lines.

from thick-dashed line (which is expost-beset payoffs). As we see, regrets in the central figure (in which regrets are zero over the entire region) weakly dominates those in the other two figures, in which regrets are positive in some regions[33].

5.2 First-Price Auction (FPA)

We investigate regrets in a first-price auction. Since a winner pays her bid in a first-price auction, payoffs are specified by

$$u(b, x, \mathbf{x}_{-i}|\beta) = \begin{cases} v(x - b + J) & \text{if} \quad \underbrace{\beta_1 < b}_{\text{win}} \\ v(0) & \text{if} \quad \underbrace{b < \beta_1}_{\text{lose}} \end{cases} \tag{15}$$

where $\beta_1 = \beta(y_1)$ and $y_1 = \max\{\mathbf{x}_{-i}\}$ is the highest signal among bidders excluding bidder i. We can define winner and loser regrets in the following ways, as discussed in Section 2.

FPA Winner Regret:

$$\mathrm{WR}^{\mathrm{I}}(b, x, \mathbf{x}_{-i}|\beta) = \left\{ \left[\underbrace{v(x - \beta_1 + J)}_{\text{ex-post best payoff}} - \underbrace{v(x - b + J)}_{\text{realized payoff}} \right]^{1+\gamma_w} \qquad \text{if} \quad \beta_1 < b \right. \tag{16}$$

Winner regret in a first-price auction occurs in a situation in which a bidder win an auction but she overprices. In such a situation, an ex-post best payoff is attained by bidding β_1.

[33] We borrow and modify these fascinating second-price auction figures from Martin J. Osborne's undergraduate textbook "An Introduction to Game Theory."

FPA Loser Regret:

$$\mathrm{LR}^{\mathrm{I}}(b,x,\mathbf{x}_{-i}|\beta)=\begin{cases}\Bigg[\underbrace{v(0)}_{\text{ex-post best payoff}}-\underbrace{v(0)}_{\text{realized payoff}}\Bigg]^{1+\gamma_l}=0 & \text{if } \underbrace{\beta_1>x+J}_{\text{unaffordable}}\\ \Bigg[\underbrace{v(x-\beta_1+J)}_{\text{ex-post best payoff}}-\underbrace{v(0)}_{\text{realized payoff}}\Bigg]^{1+\gamma_l} & \text{if } b<\underbrace{\beta_1<x+J}_{\text{affordable}}\end{cases} \tag{17}$$

The first row of equation (17) describes a situation in which the highest of other bids is above her valuation plus Joy of Winning $(x+J)$, thus, an object is unaffordable to her. In this situation, she has no regret. The second row of equation (17) is a case in which an object is affordable to her but she underprices and cannot obtain it.
In a first-price auction, expected regret takes a form

$$E_{\mathbf{x}_{-i}}[\mathrm{R}^{\mathrm{I}}(b,x,\mathbf{x}_{-i}|\beta)]=E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{I}}(b,x,\mathbf{x}_{-i}|\beta)]+E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{I}}(b,x,\mathbf{x}_{-i}|\beta)] \tag{18}$$

where

$$E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{I}}(b,x,\mathbf{x}_{-i}|\beta)] = \int_{\beta(0)}^{b}[v(x-\beta_1+J)-v(x-b+J)]^{1+\gamma_w}\,h_1(\beta_1)d\beta_1 \tag{19}$$

$$E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{I}}(b,x,\mathbf{x}_{-i}|\beta)] = \int_{b}^{x+J}[v(x-\beta_1+J)-v(0)]^{1+\gamma_l}\,h_1(\beta_1)d\beta_1 \tag{20}$$

where $h_1(\cdot)$ is the unconditional density of β_1. Note that, in the RHS of equation (19) and (20), we take integral over other bidders' bids[34] to calculate expected regret. This is equivalent to taking the integral over other bidders' signals after the changes of variable manipulations. By calculating the first order condition of (18) with respect to b, and by substituting the symmetric equilibrium condition $b=\beta(x)$, we obtain the following proposition (see the derivation in Appendix).

Proposition 2 *Symmetric IPV First-Price Auction Equilibrium*
The symmetric equilibrium bidding function β^{I} satisfies

$$\mathrm{MELR}^{\mathrm{I}}(\beta^{\mathrm{I}}(x),x|\beta^{\mathrm{I}})=-\mathrm{MEWR}^{\mathrm{I}}(\beta^{\mathrm{I}}(x),x|\beta^{\mathrm{I}})$$

for all $x\in[0,\bar{x}]$, where the precise form of the equation is

$$\begin{aligned}&[v(x-\beta^{\mathrm{I}}(x)+J)-v(0)]^{1+\gamma_l}g_1(x)\\ =\;&(1+\gamma_w)v'(x-\beta^{\mathrm{I}}(x)+J)\frac{d\beta^{\mathrm{I}}(x)}{dx}\int_0^x[v(x-\beta^{\mathrm{I}}(y_1)+J)-v(x-\beta^{\mathrm{I}}(x)+J)]^{\gamma_w}g_1(y_1)dy_1.\end{aligned}$$

The immediate consequence of Proposition 2 is the weak monotonicity property of β^{I}.

Corollary 1 If $\gamma_w,\gamma_l\geq 0$ and $v(\cdot)$ satisfies $v'(\cdot)>0$, β^{I} is weakly monotone increasing.

(proof) By manipulating the equation in Proposition 2, we obtain $\frac{d\beta^{\mathrm{I}}(x)}{dx}\geq 0$.

[34] Since this is a first-price auction with i.i.d. valuations, only the highest of the other bidders' bids matters to bidder i's payoffs and regrets.

Case 1 If $\gamma_w = \gamma_l = 0$ (regret-neutral), $J = 0$ (no joy of winning), and $v(z) = z$ (risk-neutral) the equation in proposition 2 becomes

$$\beta^{\mathrm{I}}(x) = x - \frac{\int_0^x F(x)^{n-1} dx}{F(x)^{n-1}}$$

which is the well-known risk-neutral BNE in a first-price auction.

5.3 All-Pay Auction (APA)

Next, we direct our attention to all-pay auctions that describe irretrievable costs such as research and development (R & D) patent races, political elections, and lobbying activities. Since a bidder has to pay her bid regardless of winning or losing, payoffs in an all-pay auction are specified as

$$u(b, x, \mathbf{x}_{-i}|\beta) = \begin{cases} v(x - b + J) & \text{if } \underbrace{\beta_1 < b}_{\text{win}} \\ v(-b) & \text{if } \underbrace{b < \beta_1}_{\text{lose}} \end{cases}.$$

We can define winner and loser regret in the following ways.

APA Winner Regret

$$\mathrm{WR}^{\mathrm{AP}}(b, x, \mathbf{x}_{-i}|\beta) = \left\{ \left[\underbrace{v(x - \beta_1 + J)}_{\text{ex-post best payoff}} - \underbrace{v(x - b + J)}_{\text{realized payoff}} \right]^{1+\gamma_w} \quad \text{if } \beta_1 < b \right. . \tag{21}$$

The above equation is bidder i's regret over overpricing when she wins. The winner regret in an all-pay auction is identical to that in a first-price auction. The ex-post best payoff is attained by bidding β_1.

APA Loser Regret

$$\mathrm{LR}^{\mathrm{AP}}(b, x, \mathbf{x}_{-i}|\beta) = \begin{cases} \left[\underbrace{v(x - \beta_1 + J)}_{\text{ex-post best payoff}} - \underbrace{v(-b)}_{\text{realized payoff}} \right]^{1+\gamma_l} & \text{if } \beta_1 > b \text{ and } \underbrace{\beta_1 < x + J}_{\text{affordable}} \text{ (Type A)} \\ \left[\underbrace{v(0)}_{\text{ex-post best payoff}} - \underbrace{v(-b)}_{\text{realized payoff}} \right]^{1+\gamma_l} & \text{if } \beta_1 > b \text{ and } \underbrace{\beta_1 > x + J}_{\text{unaffordable}} \text{ (Type B)} \end{cases} \tag{22}$$

There are two types of loser regrets in an all-pay auction. The first row is the case in which bidder i loses an auction although she ex-post realizes she can afford an object. By the all-pay auction rule, she incurs her bid b. The second row is the case in which bidder i loses an all-pay auction and she ex-post realizes an object is unaffordable to her, and she wastes her payment of b. In this case, since an object is unaffordable, bidding zero is her ex-post best action. To simplify discussion, we name the loser regrets in the first and second rows as Type A and Type B. Also, we denote Type A and Type B all-pay loser-regret functions as $\mathrm{LR}^{\mathrm{AP}}_{\mathrm{typeA}}(\cdot, \cdot, \cdot|\cdot)$ and $\mathrm{LR}^{\mathrm{AP}}_{\mathrm{typeB}}(\cdot, \cdot, \cdot|\cdot)$. Given that other bidders employ a bidding strategy β, expected regret in an all-pay auction takes a form

$$E_{\mathbf{x}_{-i}}[\mathrm{R}^{\mathrm{AP}}(b, x, \mathbf{x}_{-i}|\beta)] = E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{AP}}(b, x, \beta)] + E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{AP}}_{\mathrm{typeA}}(b, x, \mathbf{x}_{-i}|\beta)] + E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{AP}}_{\mathrm{typeB}}(b, x, \mathbf{x}_{-i}|\beta)] \tag{23}$$

where

$$E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{AP}}(b,x,\mathbf{x}_{-i}|\beta)] = \int_{\beta(0)}^{b}\left[v(x-\beta_1+J)-v(x-b+J)\right]^{1+\gamma_w} h_1(\beta_1)d\beta_1 \tag{24}$$

$$E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{AP}}_{\mathrm{typeA}}(b,x,\mathbf{x}_{-i}|\beta)] = \int_{b}^{x+J}\left[v(x-\beta_1+J)-v(-b)\right]^{1+\gamma_l} h_1(\beta_1)d\beta_1 \tag{25}$$

$$E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{AP}}_{\mathrm{typeB}}(b,x,\mathbf{x}_{-i}|\beta)] = \int_{x+J}^{\beta(\bar{x})}\left[v(0)-v(-b)\right]^{1+\gamma_l} h_1(\beta_1)d\beta_1 \tag{26}$$

where $h_1(\cdot)$ is the density of β_1 and $\bar{x}$ is the highest possible signal. In the right hand sides of the above equations, integrals are taken over the highest of the other bidders' bids, β_1. These integrals are equivalent to integrals other bidders' signals after the changes of variable manipulations. Taking the derivative of (23) with respect to b and substituting the symmetric equilibrium condition $b=\beta(x)$ provide the following proposition (see the derivation in Appendix).

Proposition 3 *All-Pay Auction Symmetric Equilibrium*
In an IPV all-pay auction, a symmetric equilibrium bidding function β^{AP} satisfies

$$\mathrm{MELR}^{\mathrm{AP}}_{\mathrm{TypeA}}(\beta^{\mathrm{AP}}(x),x|\beta^{\mathrm{AP}}) = -\mathrm{MEWR}^{\mathrm{AP}}(\beta^{\mathrm{AP}}(x),x|\beta^{\mathrm{AP}}) - \mathrm{MELR}^{\mathrm{AP}}_{\mathrm{TypeB}}(\beta^{\mathrm{AP}}(x),x|\beta^{\mathrm{AP}}) \tag{27}$$

for all $x\in[0,\bar{x}]$, where the precise form of the equation is

$$\begin{aligned}
& -[v(x-\beta^{\mathrm{AP}}(x)+J)-v(-\beta^{\mathrm{AP}}(x))]^{1+\gamma_l} g_1(x)\frac{1}{\frac{d\beta^{\mathrm{AP}}(x)}{dx}} \\
& +(1+\gamma_l)v^{'}(-\beta^{\mathrm{AP}}(x))\int_{x}^{\beta^{\mathrm{AP},-1}(x+J)}\Big[v(x-\beta^{\mathrm{AP}}(y_1)+J)-v(-\beta^{\mathrm{AP}}(x))\Big]^{\gamma_l} g_1(y_1)dy_1 \\
= \; & -(1+\gamma_w)v^{'}(x-\beta^{\mathrm{AP}}(x)+J)\int_{0}^{x}\Big[v(x-\beta^{\mathrm{AP}}(y_1)+J)-v(x-\beta^{\mathrm{AP}}(x)+J)\Big]^{\gamma_w} g_1(y_1)dy_1 \\
& -(1+\gamma_l)v^{'}(-\beta^{\mathrm{AP}}(x))[v(0)-v(-\beta^{\mathrm{AP}}(x))]^{\gamma_l}[1-G_1(\beta^{\mathrm{AP},\text{-}1}(x+J))].
\end{aligned}$$

The immediate consequence of the above proposition is the weak monotonicity of β^{AP}.

Corollary 2 If $\gamma_w,\gamma_l\geq 0$ and $v(\cdot)$ satisfies $v'(\cdot)>0$, β^{AP} is weakly monotone increasing.

(proof) By manipulating the equation in Proposition 3, we obtain $\frac{d\beta^{\mathrm{AP}}(x)}{dx}\geq 0$.

Case 2 If $\gamma_w=\gamma_l=0$ (regret-neutral), $J=0$ (no joy of winning), and $v(z)=z$ (risk-neutral) the equation in proposition 3 becomes

$$\beta^{\mathrm{AP}}(x)=\int_0^x y_1\cdot g_1(y_1)dy_1$$

which is the well-known risk-neutral BNE in an all-pay auction.

6 Regrets in Independent Private Value Auctions: Experimental Evidence

In this section, we structurally estimate regret and other parameters with experimental auction data. We first illustrate the dataset from the laboratory auction experiments. Second, we explain the structural estimation that is based on the widely used method proposed by Hotz and Miller (1993)[25] in the empirical industrial organization literature. Third, we report estimation results. Fourth, we test hypotheses 1–4 to review the significance of regrets.

6.1 Datasets

Kagel and Levin (1993)

The first dataset of Independent Private-Value (IPV) auctions is provided by Kagel and Levin (1993)[27][35][36]. Since we have plotted their experimental results in Figures 1, 3, and 4 in the introductory section (Section 1), we here shortly summarize their experiments, and readers who are interested in details are recommended to read the paper. In their series of auction experiments, they conducted first-, second-, and third-price auction experiments with two different number of bidders, five and ten. In each experiment, valuations are drawn from the uniform distribution $U[0, 28.3]$[37]. Since a bidder in a third-price auction can incur negative payments, the participation fee of 10 dollars were given to experiment participants before entering laboratory. Results of auctions are plotted in the introduction section (Section 1). Polynomial function estimations with order six to twelve with strict monotonicity restriction[38] functions are also plotted.

Noussair and Silver (2006)

The second dataset of IPV auctions is provided by Noussair and Silver (2006) [34], in which they implement all-pay auction experiments[39]. Noussair and Silver (2006)'s data are not used in our estimation, although they are used in comparison analysis (in Section 7). Noussair and Silver focus their research on all-pay auctions with six-bidders. Signals (valuations) are independently drawn from $U[0, 1000]$[40]. Since monetary gains generated by all-pay auctions can be negative, 20 dollars of participation fee were provided at the begging of the experiment[41]. Figure 2 in introductory section (Section 1) lists the results. We used polynomial of order twelve with the strict monotonicity restriction to estimate the bidding function.

[35] We appreciate John Kagel's and Dan Levin's generosity in providing the dataset for this project.

[36] In this research, we delete (1) the initial 4 rounds of experiments to eliminate the learning effects and (2) the final 2 rounds to eliminate the certain effects. We also exclude such outliers as bidding more than own valuations in first-price auctions and bidding irrationally high bids in third-price auctions with small valuations.

[37] The unit in their experiments is the dollar. In experimental auctions, if Bob (hypothetical name) is assigned a valuation of 25.42 dollars in a specific round of the auction experiment, and he won the first-price auction with his bid of 15.00 dollars, then he receives 25.24 - 15.00 = 10.24 dollars.

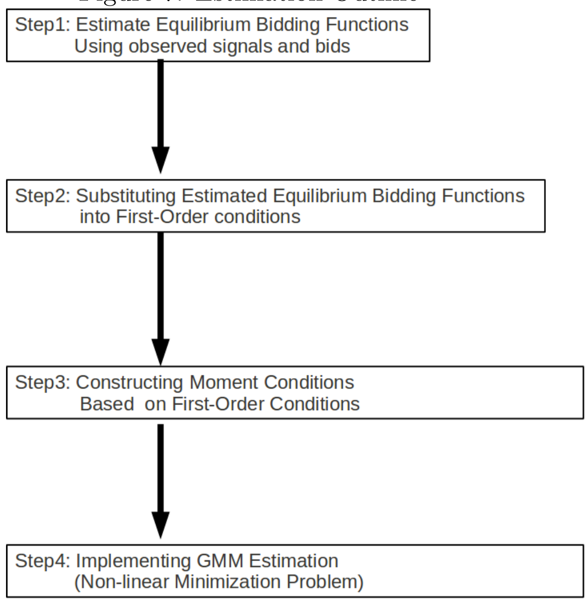
[38] For details about the strict monotonicity restriction in estimation, see Chernozhukov, Fernadez-Val, and Galichon (2009) [7].

[39] We thank Charles Noussair and Jonathon Silver for providing the dataset for this project. Since all-pay auctions have such important applications as research and development races, their experimental results have important implications in real world contest designs.

[40] Noussair and Silver (2006) used their own experimental monetary unit of 1000 = 4 dollars.

[41] Of interest, most of the participants left the experiment with fewer than 20 dollars. This means that participants, on average, lost money during series of all-pay auctions experiments.

Figure 7: Estimation Outline



6.2 Estimation Method

Our estimation is based on the Hotz and Miller (1993)[25] method[42], which is widely used in the empirical industrial organization literature. The econometric method proposed by Hotz and Miller (1993) has an advantage in that we can avoid computations to solve out equilibrium policy functions. Policy functions are biding functions in our auction setting. To the best of our knowledge, this paper is the first application of the Hotz and Miller (1993) method to experimental auction data. Here, we summarize the estimation procedure with first-, second-, and third-price auction data from Kagel and Levin (1993) [27] [43][44]. The estimation takes five steps:

Step 1:

We assume that the estimated bidding functions converge to the true equilibrium bidding functions. Then, we estimate the equilibrium bidding functions with high oder polynomials, $\hat{\beta}^{\mathrm{I}}_{n=5}$, $\hat{\beta}^{\mathrm{I}}_{n=10}$, $\hat{\beta}^{\mathrm{II}}_{n=5}$, $\hat{\beta}^{\mathrm{II}}_{n=10}$, $\hat{\beta}^{\mathrm{III}}_{n=5}$, and $\hat{\beta}^{\mathrm{III}}_{n=10}$. Note that these estimated biding functions are already plotted in Figures 1, 3, and 4 in the introductory section (Section 1).

Step 2:

We substitute estimated equilibrium bidding functions into the first-order conditions in Propositions 1, 2, and 4. We denote a first order condition function as ϕ. We interpret the first-order conditions as follows; bidders minimize their expected regret given that other bidders in experiments employ an estimated bidding function as an equilibrium bidding function.

[42] The empirical auction estimation method proposed by Guerre, Perrigne, Vuong (2000, 2009) also motivated our research.
[43] Yoshimoto (2011)[41] describes in detail the estimation method with experimental auction data.
[44] We use data from Noussair and Silver (2006) [34] for comparison purposes in the next section, and they are not used in estimation.

Step 3:

We assume bidders in auction experiments are homogeneous. However, they make mean zero optimization errors when they chose bids (or solve first-order conditions),

$$\phi\left(x, b|\hat{\beta};\theta\right) + \varepsilon = 0,$$

where $E[\varepsilon|x] = 0$[45]. We use the first order conditions as moment conditions, and estimate structural parameters. Denoting the vector of parameters $\theta = (\gamma_w, \gamma_l, \alpha, J)$. In addition, $\phi^K_{n=L}(x^K_{n=L,i}, b^K_{n=L,i}|\beta^K_{n=L} : \theta)$ represents a first-order condition in format K auction with $n = L$ bidders with observed data $(x^K_{n=L,i}, b^K_{n=L,i})$, given an equilibrium strategy $\hat{\beta}^K_{n=L}$. The vector moment function Φ is constructed as

$$\underbrace{\Phi}_{6\times 1 \text{ vector}} = \begin{bmatrix} \left(N^I_{n=5}\right)^{-1} \sum_{i=1}^{N^I_{n=5}} \phi^I_{n=5}\left(x^I_{n=5,i}, b^I_{n=5,i}|\hat{\beta}^I_{n=5};\theta\right) \\ \left(N^I_{n=10}\right)^{-1} \sum_{i=1}^{N^I_{n=10}} \phi^I_{n=10}\left(x^I_{n=10,i}, b^I_{n=10,i}|\hat{\beta}^I_{n=10};\theta\right) \\ \left(N^{II}_{n=5}\right)^{-1} \sum_{i=1}^{N^{II}_{n=5}} \phi^{II}_{n=5}\left(x^{II}_{n=5,i}, b^{II}_{n=5,i}|\hat{\beta}^{II}_{n=5};\theta\right) \\ \left(N^{II}_{n=10}\right)^{-1} \sum_{i=1}^{N^{II}_{n=10}} \phi^{II}_{n=10}\left(x^{II}_{n=10,i}, b^{II}_{n=10,i}|\hat{\beta}^{II}_{n=10};\theta\right) \\ \left(N^{III}_{n=5}\right)^{-1} \sum_{i=1}^{N^{III}_{n=5}} \phi^{III}_{n=5}\left(x^{III}_{n=5,i}, b^{III}_{n=5,i}|\hat{\beta}^{III}_{n=5};\theta\right) \\ \left(N^{III}_{n=10}\right)^{-1} \sum_{i=1}^{N^{III}_{n=10}} \phi^{III}_{n=10}\left(x^{III}_{n=10,i}, b^{III}_{n=10,i}|\hat{\beta}^{III}_{n=10};\theta\right) \end{bmatrix}$$

where $N^I_{n=5}$, $N^I_{n=10}$, $N^{II}_{n=5}$, $N^{II}_{n=10}$, $N^{III}_{n=5}$, and $N^{III}_{n=10}$ denotes number of observations in first-, second-, and third-aucton experiments[46]. We call the above object as pseudo-GMM moment function.

Step 4:

We apply the pseudo-GMM estimation to obtain the estimate of θ,

$$\hat{\theta} = \arg\min_{\theta} \left\{\Phi' V^{-1} \Phi\right\}$$

where V is the diagonal variance matrix obtained by a similar way as two-step GMM estimation method[47].

Step 5:

Bootstraps are implemented to obtain confidence interval of parameter θ

[45] Using first-order conditions as moment conditions is a commonly applied method in the Macro literature. Hansen and Singleton (1982) use the Euler condition, which is a dynamic optimization first order condition, as a GMM moment condition.

[46] This object is not a conventional Generalize Method of Moment (GMM) moment vector, since each element of the moment vector consists with the summation across a specific type of auction experiment. This unusual moment condition setting is due to the construction of the dataset. We do not observe bidders' identity across different types (rules) of auction experiments. For this reason, the standard GMM variance formula cannot be used in this research. See details in Yoshimoto (2011) [41].

[47] Note that we are unable to recover off-diagonal elements of V, since the dataset does not track identities across different types of auction experiments.

Table 1: Estimated parameters and bootstrapped confidence intervals

Parameter	Estimate (95% confidence interval)
α: Risk-Averse (CARA)	0.1624
	$(0.3459, 0.0001)$
γ_w: Winner-Regret	0.1329
	$(0.2941, 0.0000)$
γ_l: Loser-Regret	0.4912
	$(0.7539, 0.2383)$
J: Joy of Winning	0.9386
	$(0.0792, 0.0121)$

6.3 Estimation Results and Hypothesis Tests

We use the CARA payoff function $u(z) = \frac{1}{\alpha}[1 - \exp(-\alpha \cdot z)]$ and the index regret functions in our estimation. In the estimation, we use the scale of $U[0, 28.3]$ dollars, which is the scale used in Kagel and Levin (1993)[27]'s experiments, and normalizations are not applied. Estimation results are shown in Table 1[48]. We interpret these estimates by hypothesis tests in the next subsection.

6.4 Hypothesis Testings

We analyze each hypothesis step-by-step with interpretation of bidders' behaviors.

Hypothesis 1: Regret-Averse Attitude

The sum of the winner and loser regret-averse parameters is significantly larger than zero[49], and the non-existence of regret-averse attitudes is rejected. In other words, there exist regret-averse attitudes among bidders in Kagel and Levin (1993)[27]'s experiments.

Hypothesis 2: Symmetry in Winner and Loser Regret-Averse Attitudes

The difference between loser and winner regret-averse parameters is significantly larger than zero[50]. This means that bidders avoid loser regrets more than they avoid winner regrets. Note that this testing result is consistent with findings reported by Filiz-Ozbay and Ozbay (2007) [18].

Hypothesis 3: Risk-Averse Attitude

The CARA risk-averse parameter α is not significantly different from 0, which means bidders in Kagel-Levin (1993) experiments do not significantly exhibit risk-averse attitudes. This result meets expectations since bidders in third-price auctions behave opposite to the risk-averse preference theory's prediction.

Hypothesis 4: Joy of Winning

[48] In estimation and bootstrap, we put the restrictions $\alpha > 0$, $\gamma_w \geq 0$, and $\gamma_l \geq 0$ in the minimizing algorithm. We put these restrictions into matlab constrained minimization subroutines. These restrictions come from the common sense of real-world observations as we exclude the possibilities of risk-loving and regret-loving agents. Bootstrapped parameters $\hat{\alpha}$ and $\hat{\gamma_w}$, computed by minimization algorithm with non-negative restriction, tend to stack in zeros. This is why we have lower 5 percent of the bootstrapped quantile is 0.0000.

[49] We implement bootstraps 843 times to obtain the bootstrapped distribution of $(\hat{\alpha}_w + \hat{\alpha}_l)$. The lower 2.5 percent quantile is 0.2914.

[50] We compute the bootstrapped distribution of $(\hat{\alpha}_l - \hat{\alpha}_w)$. The lower 2.5 percent quantile is 0.0903.

Joy of Winning parameter J is significantly larger than 0, indicating that bidders in the experiment obtain payoffs from their winning status. We are not surprised by this testing result, considering that overbidding in the second-price auctions is commonly observed in the experimental auction literature.

These results from the hypothesis tests indicate the following conclusions. The loser regret-averse attitude, rather than risk-averse attitude, caused the overbidding observed across Independent Private Value (IPV) auctions. Bidders in Kagel and Levin (1993) auction experiments try to avoid their loser regrets, the avoidance of losing attainable positive payoffs. Compared to loser regret, winner regret plays a relatively small role when bidders make their bid choices. In addition, unlike the conventional theoretical and experimental auction literature, a risk-averse attitude does not have significant impact in the explanation of overbidding. Rather, bidders' risk-averse attitudes are close to risk-neutral. This insignificance of risk-averse attitude is expected because of the contradictions between observed overbidding in experiments and theoretical predictions in third-price and all-pay auctions.

7 Counterfactual Analysis: Calibrations and Revenue Rankings

In this section, we calibrate (approximate) equilibrium bidding functions and compute a seller's expected revenues in Independent Private Value (IPV) first-, second-, third-, and all-pay auctions. Then, we construct revenue rankings. Throughout this section, we use the estimated parameters in Table 1.

7.1 Calibrating Equilibrium Bidding Functions

In order to compute the seller's expected revenues, we need to obtain equilibrium bidding functions for each auction format. We apply the calibration (approximation) method with the parameters listed in Table 1. Bajari (2001) [1] proposed the methodology used in this section, and we follow his method. The calibration procedures are as follows:
Step 1:
we prepare a high order polynomial function[51] with the strict monotonicity assumption to approximate equilibrium bidding functions. We denote arbitrary coefficients of the Kth order polynomial as $a = (a_0, a_1, \dots, a_K)$. In addition, we denote the polynomial function as

$$f_k(x; a) = a_0 + a_1 x + a_2 x^2 + \cdots + a_k x^k \quad \text{(approximating an equilibirum bidding function).}$$

Step 2:
We assume valuations are drawn from $U[0, \bar{x}]$, and we define G+1 grid[52] on the $[0, \bar{x}]$.
Step 3:
We plug estimated parameters $\hat{\theta} = (\hat{\alpha}, \hat{\gamma}_w, \hat{\gamma}_l, \hat{J})$ in Table 1, and a polynomial (which approximates an equilibrium bidding function) into a first-order condition equation ϕ. Then, we evaluate first order condition values at each grid with values of $(x_g, f_k(x_g; a))$ where g indexes grids.

[51] We use 6th to 10th order polynomials in the calibration computation. For all-pay auctions, we exceptionally use 18th order polynomials.
[52] We set up $G = 1,000$. Computation time significantly increases if one uses a larger number of grids such as $G = 5,000$ or $10,000$.

Step 4:
Computing a following minimizing object, which is the summation of first order condition values at each grid point,

$$\sum_{g=0}^{G} \phi(x_g, f_K(x_g; a)) | f_k(a); \hat{\theta}).$$

Step 5:
We minimize the above object with respect to coefficients of polynomial, a. This means that we search over the coefficients $a = (a_1, a_2, \dots, a_K)$ of arbitrary polynomial function $f_K(x; a)$, in order to find an equilibrium bidding function that fits a first order condition ϕ.
Step 6:
We draw random numbers from the uniform distribution $U[0, \bar{x}]$, and compute an expected revenue using the calibrated bidding function in Step 5.

7.2 Calibration Results and Revenue Rankings

Figures 8, 9 and 10 depict calibration results with observed bids in experiments.

Figure 8 illustrates that, with our risk- & regret-averse criterion, overbidding happens in first- and second-price auctions. Figure 9, which contains the calibration results of third-price auctions, illustrates that bidders overbid in the area of low valuations (in the case of five bidders) and in the entire area of valuations (in the case of ten bidders). Such overbidding in third-price auctions is not expected if one adheres to the risk-averse preference. In Figure 10, we see the substantial overbidding in all-pay auctions among bidders who have high valuations. The strong loser regret, the avoidance of losing potentially positive payoffs when bidders have high valuations, causes this overbidding in all pay auctions. As every bidder pays the amount of her bid, the overbidding in all-pay auctions has significant effects on revenues.

Table 2 contains calibrated revenues generated with the bidder valuation distribution of $U[0, 28.3]$, as used in Kagel and Levin (1993) [27]. For comparison purposes, we also denote average of observed revenues in their experiments[53]. Furthermore, in order to make comparisons with other reported revenues in the literature, we normalize the revenues by simply dividing them by 28.3. Table 2 confirms that the all-pay auction provides highest revenue in both five- and ten-bidder auctions. The all-pay auction supremacy in revenues has been reported in earlier experiments conducted by Noussair and Silver (2006) [34] and Gneezy and Smorodinsky (2006) [22]. We believe our result structurally confirms their experimental findings. For five-bidder auctions, the calibrated revenue ranking is

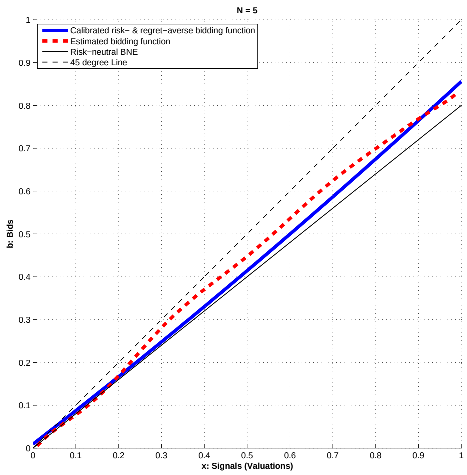
$$R^{\mathrm{AP}}_{\mathrm{r\&ra}} > R^{\mathrm{I}}_{\mathrm{r\&ra}} > R^{\mathrm{II}}_{\mathrm{r\&ra}} > R^{\mathrm{III}}_{\mathrm{r\&ra}} \qquad \text{(case of } n = 5)$$

that agrees with observed revenues in experiments[54]. For ten-bidder auctions, our numerical calibration results

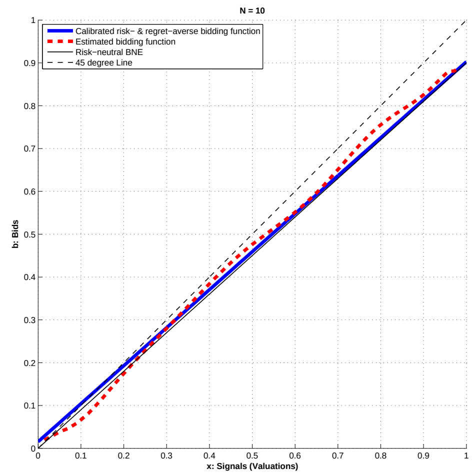
[53] In order to remove learning and certain effects, we removed data from initial 4 rounds and final 2 rounds from the dataset provided by Kagel and Levin (1993) [27]. Also, outliers, such as non-serious bids (bidding equal or close to zero), are removed.

[54] Due to the absence of all-pay auction experiments in Kagel and Levin (1993) [27], we cannot compare calibrated all-play auction revenues with observed revenue in experiments. For the reference value of revenue, Noussair and Silver (2006) [34] report that the average revenues in their six bidder (N=6) all-pay auction experiments was 1.0855. This means that bidders in the all-pay auction experiments, on average, obtained negative payment gains. Note that participation fees compensated for such negative gains.

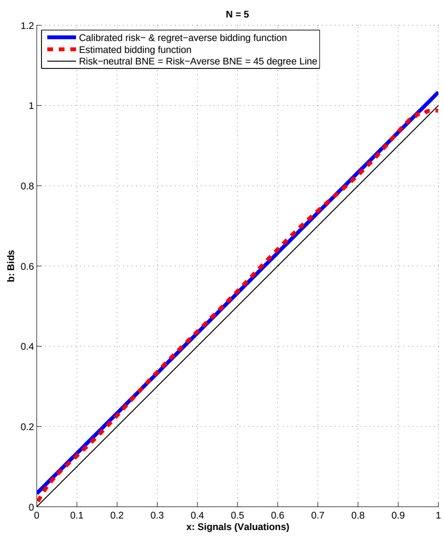
Figure 8:



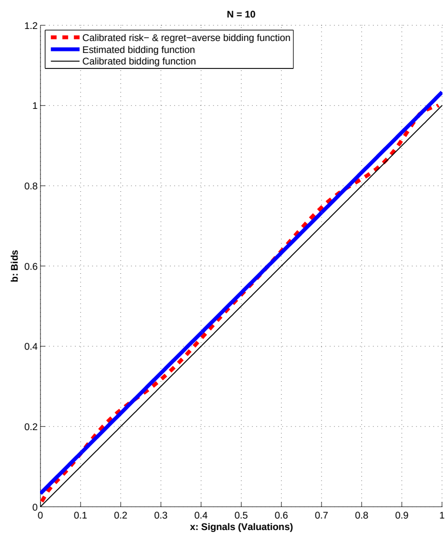
(a) First-Price Calibration: Five Bidders Case



(b) First-Price Calibration: with Ten Bidder Case



(c) Second-Price Calibration: Five Bidders Case



(d) Second-Price Calibration: with Ten Bidder Case

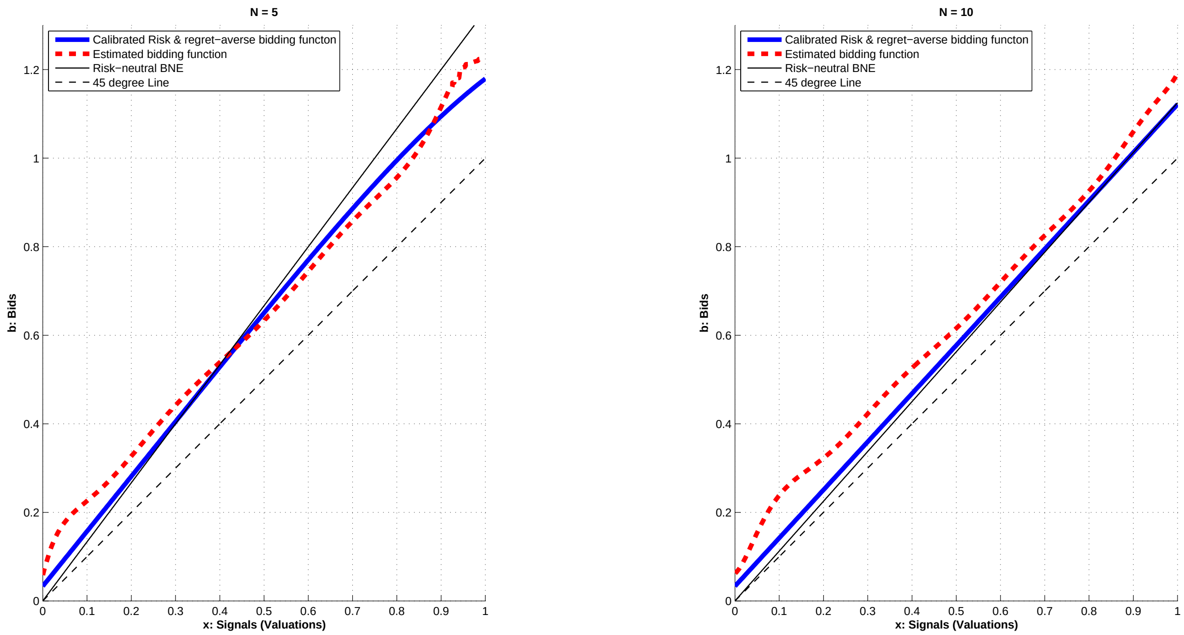
demonstrate that

$$R^{\mathrm{AP}}_{\mathrm{r\&ra}} > R^{\mathrm{II}}_{\mathrm{r\&ra}} > R^{\mathrm{I}}_{\mathrm{r\&ra}} > R^{\mathrm{III}}_{\mathrm{r\&ra}} \qquad \text{(case of } n = 10\text{)}.$$

This ten-bidder revenue ranking does not perfectly agree with the experimentally observed revenues in Kagel and Levin (1993) [27]. However, differences among experimentally observed revenues in ten-bidder auctions are quite small.

Last, our risk- & regret-averse model contrasts to the risk-averse model in all-pay auction revenues. With a

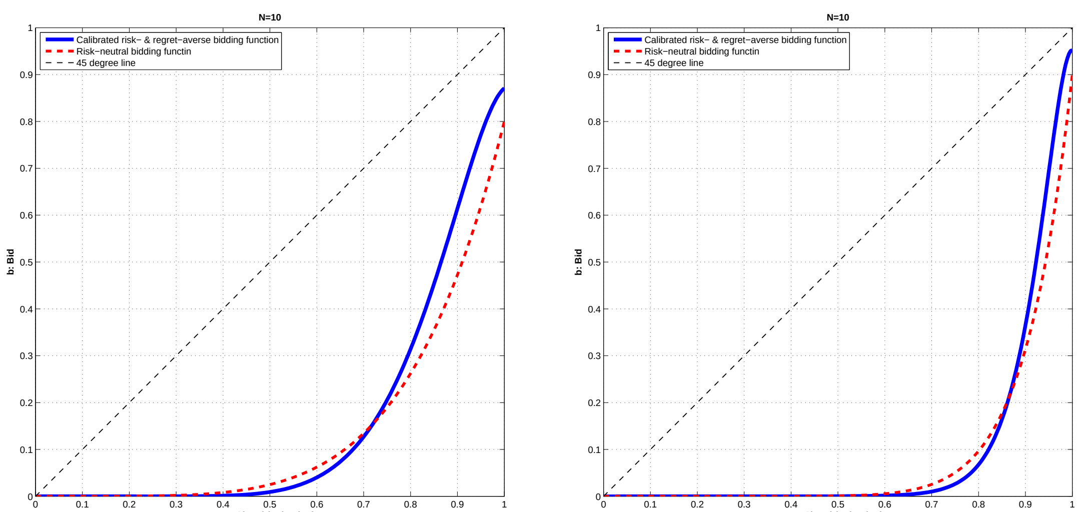
Figure 9:



(a) Third-Price Calibration: Five Bidders Case

(b) Third-Price Calibration: with Ten Bidder Case

Figure 10:



(a) All-Pay Calibration: Five Bidders Case

(b) All-Pay Calibration: with Ten Bidder Case

uniform distribution[55], the risk-averse preference model predicts (also computed in Table 2) a revenue ranking of

$$R_{\text{ra}}^{\text{I}} > R_{\text{ra}}^{\text{II}} > R_{\text{ra}}^{\text{III}} > R_{\text{ra}}^{\text{AP}},$$

[55] In general, revenue rankings depend on valuation distributions. We restrict our research to uniform distributions that are commonly assumed in the experimental auction literature.

Table 2: Expected Revenue in Independent Private Value Auctions with Five and Ten Bidders

Five Bidder Auctions (N=5)	Expected revenue risk- & regret-averse $R_{\text{r\&ra}}$	Theoretical expected revenue: risk-averse R_{ra}	Theoretical expected revenue: risk-neutral R_{rn}	Average of observed revenue in experiment $\hat{R}$
First-Price (I)	0.7124	0.7636	0.6666	0.7425
Second-Price (II)	0.6998	0.6666	0.6666	0.6879
Third-Price (III)	0.6459	0.6530	0.6666	0.6492
All-Pay (AP)	0.7592	0.6402	0.6666	N.A.

Ten Bidder Auctions (N = 10)	Expected revenue risk- & regret-averse $R_{\text{r\&ra}}$	Theoretical expected revenue: risk-averse R_{ra}	Theoretical expected revenue: risk-neutral R_{rn}	Average of observed revenue in experiment $\hat{R}$
First-Price (I)	0.8309	0.8613	0.8181	0.8312
Second-Price (II)	0.8523	0.8181	0.8181	0.8215
Third-Price (III)	0.8244	0.7676	0.8181	0.8378
All-Pay (AP)	0.9030	0.6779	0.8181	N.A.

Note: We first compute all revenue calculations with the $U[0, 28.3]$ distribution as in Kagel and Levin (1993), and then normalize them by dividing by 28.3.
Risk-averse revenues are calculated with CARA risk-averse parameter = 1.25.

and all-pay auctions generate the worst revenues. Such revenue predictions based on risk-averse preference are inconsistent with findings in experiments, as noted in the introductory section (Section 1).

8 Conclusions and Extensions

In this research, we propose and investigate the risk- & regret-averse model that nests the risk-averse and risk-neutral preference models. We found that the loser regret-averse attitude, rather than risk-averse attitude, caused the overbidding observed across Independent Private Value (IPV) auction experiments. The proposed model contributes to the literature in both qualitative and quantitative ways. In the qualitative sense, the model generates overbidding across a wide class of auctions. In the quantitative sense, the model confirms the revenue supremacy in all-pay auctions. These qualitative and quantitative results contrast with the risk-averse preference model that predicts underbidding in some auctions.

There are asymmetric and common-value extensions of this research (see the online Appendix for the derivations of first-order conditions). The experimental auction literature reports several interesting findings that may fit our risk- & regret-averse model. First, Chernomaz (2011) [6] reports the observance of overbidding in his asymmetric first-price auction experiments. Note that, in his experiment, weak bidders (whose valuation distribution is stochastically dominated by those of other bidders and who are more likely to lose) overbid more aggressively than other types of bidders do, even after accounting for risk aversions. Second, there is the well-known "winners' curse" problem in the experimental common-value auction literature. Kagel and Levin (1986) [26] report that bidders in common-value auction experiments overbid, and winners tend to receive negative payoffs[56]. Motivated by these experimental results, we are currently investigating the implications of our risk- & regret-averse model in asymmetric and common-value auctions.

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Appendix A: Derivations of First Order Necessary Conditions

First-Price Auction

In a first-price auction, a bidder i who has a signal x solves her regret minimization problem, which is

$$b^* = \underset{b \in \mathbb{R}_+}{argmin} \left\{ E_{\mathbf{x}_{-i}} [\mathrm{R^I}(b, x, \mathbf{x}_{-i}|\beta)] \right\}$$

where

$$\begin{aligned} E_{\mathbf{x}_{-i}}[\mathrm{R^I}(b,x,\mathbf{x}_{-i}|\beta)] &= E_{\mathbf{x}_{-i}}[\mathrm{WR^I}(b,x,\mathbf{x}_{-i}|\beta)] + E_{\mathbf{x}_{-i}}[\mathrm{LR^I}(b,x,\mathbf{x}_{-i}|\beta)] \\ &= \int_{\beta(0)}^{b} [v(x-\beta_1+J) - v(x-b+J)]^{1+\gamma_w} h_1(\beta_1) d\beta_1 \\ &\quad + \int_{b}^{x+J} [v(x-\beta_1+J) - v(0)]^{1+\gamma_l} h_1(\beta_1) d\beta_1 \end{aligned}$$

where $h_1(\cdot)$ is an unconditional density function of β_1, a highest bid among other bidders. By taking derivative with respect to b, we obtain

$$\begin{aligned} & (1+\gamma_w) v^{'}(x-b+J) \int_{\beta(0)}^{b} [v(x-\beta_1+J) - v(x-b+J)]^{\gamma_w} h_1(\beta_1) d\beta_1 \\ - \; & [v(x-b+J) - v(0)]^{1+\gamma_l} h_1(b) = 0 \end{aligned}$$

Next, we change variables. Since other bidders employ an equilibrium bidding strategy β, we have the relation $\beta_1 = \beta(y_1)$ and $d\beta_1 = \beta^{'}(y_1)dy_1$. Also $h_1(\cdot)$ is derived by (where H_1 is the distribution function of β_1)

$$\begin{aligned} h_1(\beta_1) &= \frac{dH_1(\beta_1)}{d\beta_1} = \frac{dH_1(\beta_1)}{dy_1}\frac{dy_1}{d\beta_1} = \frac{dF(y_1)^{n-1}}{dy_1}\frac{1}{\frac{d\beta_1}{dy_1}} \\ &= (n-1)f(y_1)F(y_1)^{n-2}\frac{1}{\beta^{'}(y_1)}, \end{aligned}$$

where we use the relation (assuming B_1 is a random variable of β_1)

$$H_1(\beta_1) = H_1(\beta(y_1)) = \Pr(B_1 < \beta(y_1)) = \Pr(\beta^{-1}(B_1) < y_1) = F(y_1)^{n-1}.$$

An integrating region changes from $\beta(0) \leftrightarrow b$ to $0 \leftrightarrow \beta^{-1}(b)$. By substituting and canceling out, we obtain

$$\underbrace{(1+\gamma_w)v'(x-b+J)\int_0^{\beta^{-1}(b)} [v(x-\beta(y_1)+J) - v(x-b+J)]^{\gamma_w} \overbrace{(n-1)f(y_1)F(y_1)^{n-2}(y_1)}^{=g_1(y_1)} dy_1}_{\text{marginal expected winner regret=MEWR}(b,x|\beta)}$$

$$\underbrace{-[v(x-b+J)-v(0)]^{1+\gamma_l} \overbrace{(n-1)f(\beta^{-1}(b))F(\beta^{-1}(b))^{n-2}}^{=g_1(\beta^{-1}(b))} \frac{1}{\beta'(\beta^{-1}(b))}}_{\text{marginal expected loser regret=MELR}(b,x|\beta)} = 0.$$

By replacing $(n-1)f(\cdot)F(\cdot)$ by $g_1(\cdot)$ and substituting $b = \beta(x)$, we obtain the equation in proposition 2.

All-Pay Auction

In an all-pay auction, a bidder i who has a signal x solves her regret minimization problem, which is

$$b^* = \underset{b \in \mathbb{R}_+}{argmin} \left\{ E_{\mathbf{x}_{-i}}[\mathrm{R}^{\mathrm{AP}}(b, x, \mathbf{x}_{-i}|\beta)] \right\}$$

where

$$\begin{aligned} E_{\mathbf{x}_{-i}}[\mathrm{R}^{\mathrm{AP}}(b, x, \mathbf{x}_{-i}|\beta)] &= E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{AP}}(b, x, \mathbf{x}_{-i}|\beta)] + E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{AP}}_{\mathrm{TypeA}}(b, x, \mathbf{x}_{-i}|\beta)] + E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{AP}}_{\mathrm{TypeB}}(b, x, \mathbf{x}_{-i}|\beta)] \\ &= \int_{\beta(0)}^{b} [v(x - \beta_1 + J) - v(x - b + J)]^{1+\gamma_w} h_1(\beta_1) d\beta_1 \\ &\quad + \int_{b}^{x+J} [v(x - \beta_1 + J) - v(-b)]^{1+\gamma_l} h_1(\beta_1) d\beta_1 \\ &\quad + \int_{x+J}^{\beta(\bar{x})} [v(0) - v(-b)]^{1+\gamma_l} h_1(\beta_1) d\beta_1 \\ &= \int_{\beta(0)}^{b} [v(x - \beta_1 + J) - v(x - b + J)]^{1+\gamma_w} h_1(\beta_1) d\beta_1 \\ &\quad - \int_{x+J}^{b} [v(x - \beta_1 + J) - v(-b)]^{1+\gamma_l} h_1(\beta_1) d\beta_1 \\ &\quad + [v(0) - v(-b)]^{1+\gamma_l} \int_{x+J}^{\beta(\bar{x})} h_1(\beta_1) d\beta_1 \end{aligned}$$

where $h_1(\cdot)$ is an unconditional density of β_1, a highest bid among other bidders. By taking derivative with respect to b, we obtain

$$(1 + \gamma_w) v'(x - b + J) \int_{\beta(0)}^{b} [v(x - \beta_1 + J) - v(x - b + J)]^{\gamma_w} h_1(\beta_1) d\beta_1 \tag{28}$$

$$- [v(x - b + J) - v(-b)]^{1+\gamma_l} h_1(b) \tag{29}$$

$$+ (1 + \gamma_l) v'(-b) \int_{b}^{x+J} [v(x - \beta_1 + J) - v(-b)]^{\gamma_l} h_1(\beta_1) d\beta_1 \tag{30}$$

$$+ (1 + \gamma_l) v'(-b) [v(0) - v(-b)]^{\gamma_l} \underbrace{\int_{x+J}^{\beta(\bar{x})} h_1(\beta_1) d\beta_1}_{=1 - H_1(x+J)} = 0 \tag{31}$$

Next, we change variables[57]. Since other bidders employ an equilibrium bidding strategy β, we have the relation $\beta_1 = \beta(y_1)$ and $d\beta_1 = \beta^{'}(y_1) dy_1$. Also $h_1(\cdot)$ is derived by (where H_1 is the distribution function of β_1)

$$\begin{aligned} h_1(\beta_1) &= \frac{dH_1(\beta_1)}{d\beta_1} = \frac{dH_1(\beta_1)}{dy_1} \frac{dy_1}{d\beta_1} = \frac{dF(y_1)^{n-1}}{dy_1} \frac{1}{\frac{d\beta_1}{dy_1}} \\ &= (n-1) f(y_1) F(y_1)^{n-2} \frac{1}{\beta'(y_1)}, \end{aligned}$$

where we use the relation (assuming B_1 is a random variable of β_1)

$$H_1(\beta_1) = H_1(\beta(y_1)) = \Pr(B_1 < \beta(y_1)) = \Pr(\beta^{-1}(B_1) < y_1) = F(y_1)^{n-1}.$$

Similarly, $H_1(x + J)$ is derived by (assuming Y_1 is a random variable of y_1)

$$H_1(x + J) = \Pr(B_1 < x + J) = \Pr(\beta^{-1}(B_1) < \beta^{-1}(x + J)) = \Pr(Y_1 < \beta^{-1}(x + J)) = F(\beta^{-1}(x + J))^{n-1}$$

[57] The change of variables discussion here is almost identical to the case of first-price auction first order condition derivation.

Also, integrating region changes from $\beta(0) \leftrightarrow b$ to $0 \leftrightarrow \beta^{-1}(b)$ for the first integral and $b \leftrightarrow x+J$ to $\beta^{-1}(b) \leftrightarrow \beta^{-1}(x+J)$ for the second integral. By substituting and canceling, we obtain.

$$
\begin{aligned}
&(1+\gamma_w)v'(x-b+J)\int_0^{\beta^{-1}(b)} [v(x-\beta(y_1)+J)-v(x-b+J)]^{\gamma_w} \underbrace{(n-1)f(y_1)F(y_1)^{n-2}}_{=g_1(y_1)} dy_1 \\
&-[v(x-b+J)-v(-b)]^{1+\gamma_l} \underbrace{(n-1)f(\beta^{-1}(b))F(\beta^{-1}(b))^{n-2}}_{=g_1(\beta^{-1}(b))} \frac{1}{\beta'(\beta^{-1}(b))} \\
&+(1+\gamma_l)v'(-b)\int_{\beta^{-1}(b)}^{\beta^{-1}(x+J)} [v(x-\beta(y_1)+J)-v(-b)]^{\gamma_l} \underbrace{(n-1)f(y_1)F(y_1)^{n-2}}_{=g_1(y_1)} dy_1 \\
&+(1+\gamma_l)v'(-b)\,[v(0)-v(-b)]^{\gamma_l} \underbrace{[1-F(\beta^{-1}(x+J))^{n-1}]}_{=1-G_1(\beta^{-1}(x+J))} = 0
\end{aligned}
$$

Furthermore, by introducing short hand notations $g_1(\cdot) = (n-1)f(\cdot)F(\cdot)^{n-1}$ and $G_1(\cdot) = F(\cdot)^{n-1}$, and by arranging, we have

$$
\begin{aligned}
&\underbrace{(1+\gamma_w)v'(x-b+J)\int_0^{\beta^{-1}(b)} [v(x-\beta(y_1)+J)-v(x-b+J)]^{\gamma_w} g_1(y_1)dy_1}_{\text{marginal expected winner regret}=\text{MEWR}^{\text{AP}}(b,x|\beta)} \\
&\underbrace{-[v(x-b+J)-v(-b)]^{1+\gamma_l} g_1(\beta^{-1}(b))\frac{1}{\beta'(\beta^{-1}(b))} + (1+\gamma_l)v'(-b)\int_{\beta^{-1}(b)}^{\beta^{-1}(x+J)} [v(x-\beta(y_1)+J)-v(-b)]^{\gamma_l} g_1(y_1)dy_1}_{\text{marginal expected Type I loser regret}=\text{MELR}^{\text{AP}}_{\text{Type I}}(b,x|\beta)} \\
&\underbrace{+(1+\gamma_l)v'(-b)\,[v(0)-v(-b)]^{\gamma_l}\,[1-G_1(\beta^{-1}(x+J))]}_{\text{marginal expected Type II loser regret}=\text{MELR}^{\text{AP}}_{\text{Type II}}(b,x|\beta)} = 0.
\end{aligned}
$$

By substituting $b = \beta(x)$, we obtain the equation in Proposition 3.

8.1 Third-Price Auction (TPA)

Finally[58], we analyze a third-price auction which is originally proposed for the theoretical investigation purposes[59]. Although a third-price auction only exists in theoretical and experimental environments, it gives us interesting equilibrium analysis. Since a winner pays a third highest bid, payoffs in the third-price auction are specified by

$$
u(b,x,\mathbf{x}_{-i}|\beta) = \begin{cases} v(x-\beta_2+J) & \text{if} \quad \underbrace{\beta_1 < b}_{\text{win}} \\ v(0) & \text{if} \quad \underbrace{b < \beta_1}_{\text{lose}} \end{cases}
$$

where $\beta_2 = \beta(y_2)$ and y_2 is the second highest signal among $\mathbf{x}_{-i}$. We can define wnning and loser regret in the following manners.

[58]This subsection can be skepped if a reader is not interested in a third-price auction.

[59] More generally, a k-th price auction where $k = 1, 2, 3, 4, 5, 6, \cdots, n-1, n$ are investigated by Monderer and Tennenholtz (2000)[31].

TPA Winner Regret

$$\mathrm{WR}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)=\begin{cases}\Bigg[\underbrace{v(x-\beta_2+J)}_{\text{ex-post best payoff}}-\underbrace{v(x-\beta_2+J)}_{\text{realized payoff}}\Bigg]^{1+\gamma_w}=0 & \text{if} \quad \beta_1<b \quad \text{and} \quad \underbrace{\beta_2<x+J}_{\text{affordable}}\\ \Bigg[\underbrace{v(0)}_{\text{ex-post best payoff}}-\underbrace{v(x-\beta_2+J)}_{\text{realized payoff}}\Bigg]^{1+\gamma_w} & \text{if} \quad \beta_1<b \quad \text{and} \quad \underbrace{\beta_2>x+J}_{\text{unaffordable}}\end{cases} \tag{32}$$

The first row is an affordable case in which a bidder i wins an auction. The payment, second highest bid among others, is less than her valuation plus joy of winning $(x+J)$. In this case, she has no regret since her ex-post best payoff is already attained. The second row is a winner regret which is specific to a third-price auction. It is an unaffordable case in which a bidder i wins auction but payment (which is β_2) exceeds $x+J$. In such a case, losing an auction is ex-post optimal to her and ex-post best payoff is $v(0)$.

TPA Loser Regret

$$\mathrm{LR}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)=\begin{cases}\Bigg[\underbrace{v(x-\beta_2+J)}_{\text{ex-post best payoff}}-\underbrace{v(0)}_{\text{realized payoff}}\Bigg]^{1+\gamma_l} & \text{if} \quad \beta_1>b \quad \text{and} \quad \underbrace{\beta_2<x+J}_{\text{affordable}}\\ \Bigg[\underbrace{v(0)}_{\text{ex-post best payoff}}-\underbrace{v(0)}_{\text{realized payoff}}\Bigg]^{1+\gamma_l}=0 & \text{if} \quad \beta_1>b \quad \text{and} \quad \underbrace{\beta_2>x+J}_{\text{unaffordable}}\end{cases} \tag{33}$$

The first row is the case of underpricing in which a bidder loses but she can obtain an object with a positive payoff. In this case, expost-best payoff is attained by bidding β_2. The second row is the case in which a bidder loses and an object is unaffordable to her afterall. Given other bidders employ bidding function β, expected regret in a third-price auction takes a form

$$E_{\mathbf{x}_{-i}}[\mathrm{R}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)]=E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)]+E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)] \tag{34}$$

where

$$\begin{aligned}E_{\mathbf{x}_{-i}}[\mathrm{WR}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)] &= \int_{x+J}^{b}\int_{\beta_2}^{b}[v(0)-v(x-\beta_2+J)]^{1+\gamma_w}\,h_{1,2}(\beta_1,\beta_2)\partial\beta_1\partial\beta_2\\ E_{\mathbf{x}_{-i}}[\mathrm{LR}^{\mathrm{III}}(b,x,\mathbf{x}_{-i}|\beta)] &= \int_{\beta(0)}^{x+J}\int_{b}^{\beta(\bar{x})}[v(x-\beta_2+J)-v(0)]^{1+\gamma_l}\,h_{1,2}(\beta_1,\beta_2)\partial\beta_1\partial\beta_2\end{aligned}$$

and $h_{1,2}(\beta_1,\beta_2)$ is the joint distribution of β_1 and β_2. Taking derivative of equation (34) and substituting the symmetric equilibrium condition $b=\beta(x)$ provide the following proposition.

Proposition 4 *Symmetric Third-Price Auction Equilibrium*
In the IPV third-price auction, the symmetric equilibrium bidding function β^{III} satisfies

$$\mathrm{MELR}^{\mathrm{III}}(\beta^{\mathrm{III}}(x),x|\beta^{\mathrm{III}})=-\mathrm{MEWR}^{\mathrm{III}}(\beta^{\mathrm{III}}(x),x|\beta^{\mathrm{III}}) \tag{35}$$

for all $x \in [0, \bar{x}]$[60], where the precise form of the equation is

$$\int_0^{\beta^{\text{III},-1}(x+J)} \left[v(x - \beta^{\text{III}}(y_2) + J) - v(0)\right]^{1+\gamma_l} f(y_2)F(y_2)^{n-3}\partial y_2$$
$$= \int_{\beta^{\text{III},-1}(x+J)}^{x} \left[v(0) - v(x - \beta^{\text{III}}(y_2) + J)\right]^{1+\gamma_w} f(y_2)F(y_2)^{n-3}\partial y_2$$

Case 3 If $\gamma_w = \gamma_l = 0$ (regret-neutral), $J = 0$ (no joy of winner), and $v(z) = z$ (risk-neutral) the equation in proposition 4 becomes

$$\beta^{\text{III}}(x) = x + \frac{1}{n-2}\frac{F(x)}{f(x)}$$

which is the well-known risk-neutral BNE in an third-price auction.

Third-Price Auction

In a third-price auction, a bidder i who has a signal x solves her regret minimization problem, which is

$$b^* = \underset{b \in \mathbb{R}_+}{argmin} \left\{E_{\mathbf{x}_{-i}}[\text{R}^{\text{III}}(b, x, \mathbf{x}_{-i}|\beta)]\right\}$$

where

$$\begin{aligned} E_{\mathbf{x}_{-i}}[\text{R}^{\text{III}}(b, x, \mathbf{x}_{-i}|\beta)] &= E_{\mathbf{x}_{-i}}[\text{WR}^{\text{III}}(b, x, \mathbf{x}_{-i}|\beta)] + E_{\mathbf{x}_{-i}}[\text{LR}^{\text{III}}(b, x, \mathbf{x}_{-i}|\beta)] \\ &= \int_{x+J}^{b}\int_{\beta_2}^{b} [v(0) - v(x - \beta_2 + J)]^{1+\gamma_w} \underbrace{h_{1,2}(\beta_1, \beta_2)}_{=h_{1|2}(\beta_1|\beta_2)h_2(\beta_2)} \partial\beta_1\partial\beta_2 \\ &\quad + \int_{\beta(0)}^{x+J}\int_{b}^{\beta(\bar{x})} [v(x - \beta_2 + J) - v(0)]^{1+\gamma_l} \underbrace{h_{1,2}(\beta_1, \beta_2)}_{=h_{1|2}(\beta_1|\beta_2)h_2(\beta_2)} \partial\beta_1\partial\beta_2 \end{aligned}$$

where $h_{1,2}(\cdot,\cdot)$ is an unconditional joint density of β_1 and β_2, a highest and a second highest bid among other bidders. Decomposing a joint density function as $h_{1,2}(\beta_1, \beta_2) = h_{1|2}(\beta_1|\beta_2)h_2(\beta_2)$, we obtain and by arranging

$$\begin{aligned} &= \int_{x+J}^{b} [v(0) - v(x - \beta_2 + J)]^{1+\gamma_w} \underbrace{\left[\int_{\beta_2}^{b} h_{1|2}(\beta_1|\beta_2)\partial\beta_1\right]}_{\text{integrating}} h_2(\beta_2)\partial\beta_2 \\ &\quad + \int_{\beta(0)}^{x+J} [v(x - \beta_2 + J) - v(0)]^{1+\gamma_l} \underbrace{\left[\int_{b}^{\beta(\bar{x})} h_{1|2}(\beta_1|\beta_2)\partial\beta_1\right]}_{\text{integrating}} h_2(\beta_2)\partial\beta_2 \\ &= \int_{x+J}^{b} [v(0) - v(x - \beta_2 + J)]^{1+\gamma_w} \left[H_{1|2}(b|\beta_2) - H_{1|2}(\beta_2|\beta_2)\right] h_2(\beta_2)\partial\beta_2 \\ &\quad + \int_{\beta(0)}^{x+J} [v(x - \beta_2 + J) - v(0)]^{1+\gamma_l} \left[1 - H_{1|2}(b|\beta_2)\right] h_2(\beta_2)\partial\beta_2. \end{aligned}$$

[60] We use the equation (35) as a moment condition in empirical section.

next, by taking derivative with respect to b, we obtain

$$\int_{x+J}^{b}[v(0)-v(x-\beta_2+J)]^{1+\gamma_w}\underbrace{h_{1|2}(b|\beta_2)h_2(\beta_2)}_{=h_{2|1}(\beta_2|b)h_1(b)}\partial\beta_2$$
$$-\int_{\beta(0)}^{x+J}[v(x-\beta_2+J)-v(0)]^{1+\gamma_l}\underbrace{h_{1|2}(b|\beta_2)h_2(\beta_2)}_{=h_{2|1}(\beta_2|b)h_1(b)}\partial\beta_2=0.$$

Now, by manipulating a conditional density function as $h_{1|2}(b|\beta_2)h_2(\beta_2)=h_{1,2}(b,\beta_2)=h_{2|1}(\beta_2|b)h_1(b)$, we have

$$\int_{x+J}^{b}[v(0)-v(x-\beta_2+J)]^{1+\gamma_w}h_{2|1}(\beta_2|b)\underbrace{h_1(b)}_{\text{cancel out}}\partial\beta_2$$
$$-\int_{\beta(0)}^{x+J}[v(x-\beta_2+J)-v(0)]^{1+\gamma_l}h_{2|1}(\beta_2|b)\underbrace{h_1(b)}_{\text{cancel out}}\partial\beta_2=0.$$

Next, we change variables. Since other bidders employ bidding function β, we have the relation $\beta_2=\beta(y_2)$ and $\partial\beta_2=\beta'(y_2)\partial y_2$. Also, $h_{2|1}(\cdot)$ is deived by (where denoting $H_{2|1}(\cdot|\cdot)$ is a corresponding conditional distribution function)

$$\begin{aligned}h_{2|1}(\beta_2|b) &= \frac{\partial H_{2|1}(\beta_2|b)}{\partial\beta_2}=\frac{\partial H_{2|1}(\beta_2|b)}{\partial y_2}\frac{\partial y_2}{\partial\beta_2}=\frac{\partial F_{2|1}(y_2|\beta^{-1}(b))}{\partial y_2}\frac{1}{\frac{\partial\beta}{\partial y_2}}=f_{2|1}(y_2|\beta^{-1}(b))\frac{1}{\frac{\partial\beta}{\partial y_2}}\\ &= \frac{(n-2)f(y_2)F(y_2)^{n-3}}{F(\beta^{-1}(b))^{n-2}}\frac{1}{\beta'(y_2)}\end{aligned}$$

where we use the relation (denoting B_2 as a random variable of β_2)

$$H_{2|1}(\beta_2|b)=\Pr(B_2<\beta_2|b)=\Pr(B_2<\beta(y_2)|b)=\Pr(\beta^{-1}(B_2)<y_2|\beta^{-1}(b))=F_{2|1}(y_2|\beta^{-1}(b)).$$

Integrating regions change from $x+J\leftrightarrow b$ to $\beta^{-1}(x+J)\leftrightarrow\beta^{-1}(b)$ for the first integral and from $\beta(0)\leftrightarrow x+J$ to $0\leftrightarrow\beta^{-1}(x+J)$ for the second integral. By substituting, we obtain

$$\int_{\beta^{-1}(x+J)}^{\beta^{-1}(b)}[v(0)-v(x-\beta(y_2)+J)]^{1+\gamma_w}\frac{(n-2)f(y_2)F(y_2)^{n-3}}{F(\beta^{-1}(b))^{n-2}}\frac{1}{\beta'(y_2)}\beta'(y_2)\partial y_2$$
$$-\int_{0}^{\beta^{-1}(x+J)}[v(x-\beta(y_2)+J)-v(0)]^{1+\gamma_l}\frac{(n-2)f(y_2)F(y_2)^{n-3}}{F(\beta^{-1}(b))^{n-2}}\frac{1}{\beta'(y_2)}\beta'(y_2)\partial y_2=0$$

By canceling outs, we have

$$\underbrace{\int_{\beta^{-1}(x+J)}^{\beta^{-1}(b)}[v(0)-v(x-\beta(y_2)+J)]^{1+\gamma_w}f(y_2)F(y_2)^{n-3}\partial y_2}_{\text{marginal expected winner regret }=\text{MEWR}^{\text{III}}(b,x|\beta)}$$
$$\underbrace{-\int_{0}^{\beta^{-1}(x+J)}[v(x-\beta(y_2)+J)-v(0)]^{1+\gamma_l}f(y_2)F(y_2)^{n-3}\partial y_2}_{\text{marginal expected loser regret }=\text{MELR}^{\text{III}}(b,x|\beta)}=0.$$

By substituting $b=\beta(x)$, we obtain the equation in Proposition 4.