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## UNCERTAIN FISCAL CONSOLIDATIONS\*

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This article explores the macroeconomic consequences of fiscal consolidations whose timing and composition – either tax- or spending- based – are uncertain. We find that the composition of the fiscal consolidation, its duration, the monetary policy stance, the level of government debt, and expectations over the likelihood and composition of fiscal consolidations all matter in determining the extent to which a given consolidation is expansionary or successful in stabilising government debt. We argue that the conditions that could render fiscal consolidation efforts expansionary are unlikely to apply in the current economic environment.

The financial crisis of 2007–9 left advanced economies with average levels of gross government debt breaching 100% of gross domestic product (GDP) for the first time since the aftermath of World War II, as IMF (2011) reports. The IMF now expects most governments in those economies, except for Japan and the US, to begin consolidation efforts by 2012. Politicians in some countries, most notably the UK, argue that fiscal consolidations will ultimately enhance growth but they cite the need to avoid rising debt costs as a key motivation in undertaking fiscal consolidations. Over the medium term, the dominant fiscal trend in advanced economies is a return to a position of fiscal sustainability, particularly when prompted to do so under financial market pressure.

Textbook Keynesian analysis suggests that fiscal consolidations inevitably contract aggregate demand, reducing output and consumption. Neoclassical and New Keynesian models, grounded in intertemporal consumption smoothing behaviour, also tend to suggest that temporary public expenditure cuts and distortionary tax increases reduce output, although with some crowding in of private sector consumption in the case of spending cuts.<sup>1</sup> Giavazzi and Pagano's (1990) analysis of fiscal consolidations in Denmark and Ireland in the 1980s, however, suggests that such fiscal actions could be expansionary, as output growth actually accelerated after these particular fiscal tightenings. Briotti's (2005) survey of empirical work considers a wider set of countries over a wider time period and also finds some evidence that fiscal consolidations can be expansionary. The persistence and composition of the consolidation often matter, with government spending cuts being thought to be pro-growth relative to tax increases.<sup>2</sup>

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<sup>1</sup> Linnemann and Schabert (2003) discuss the cyclical effects of fiscal policy in these models.

<sup>2</sup> Details appear in Alesina and Perotti (1995), Perotti (1996), Alesina and Ardagna (1998, 2010) and Ardagna (2004).

With standard theory unable to produce expansionary consolidations, emphasis has shifted to the role of expectations. Bertola and Drazen (1993) develop a model in which government spending is inherently unsustainable but the government periodically cuts spending to make policy sustainable. These consolidations may occur at a low threshold, but if not, they will definitely occur at a second, higher threshold. A worsening fiscal position raises the probability of soon entering a period of fiscal correction and, therefore, can lead to an expansion.<sup>3</sup> While Bertola and Drazen (1993) is often cited as an example of the importance of expectations when considering the impact of fiscal policy, it cannot address questions relating to the composition of consolidations, which the empirical literature often finds important. Our analysis begins by adding distorting taxes to Bertola and Drazen's (1993) model to explore whether a standard model, augmented with empirically motivated uncertainty over the timing and composition of fiscal consolidation, can plausibly explain the existence of the expansionary fiscal consolidations sometimes found in the empirical literature.<sup>4</sup>

Following this simple example, we develop a non-linear dynamic stochastic general equilibrium (DSGE) model, in which fiscal consolidations may occur with an increasing probability as government debt levels rise but the exact timing is uncertain. It is consistent with the empirical observation that sizeable consolidations can take place at low and high debt levels. We also introduce uncertainty over the composition of the fiscal consolidation, either tax based or spending based, building on the data set by Alesina and Ardagna (2010). We find that the nature of fiscal consolidation, its duration, expectations over its likelihood and composition, the monetary policy stance and the level of government debt all matter in determining the extent to which a given consolidation is expansionary and/or successful in stabilising government debt. When debt levels are high, the inflationary consequences of alternative fiscal instruments, conditional on the stance of monetary policy, are particularly important in determining the impact of alternative forms of fiscal consolidation. For example, when economic agents anticipate tax increases in an imminent fiscal consolidation package, they will suffer the ill-effects of distortionary taxation, including higher inflation and, when monetary policy is active, higher debt service costs, even if the realised consolidation is ultimately spending based. As a result, the resolution of the uncertainty associated with the composition or the timing can have a significant impact on the nature of the marginal economic response to the consolidation.

Such non-linear interactions among debt levels, the monetary policy stance, the compositions of consolidations and the expectations about the nature of consolidations are unlikely to be controlled for by adding individual variables to linear regressions or by sorting samples conditional on a single variable. This may explain why the empirical literature does not always fully agree on the relative importance of different factors in determining whether or not a consolidation is expansionary and/or

<sup>3</sup> Similarly, Sutherland (1997) suggests that there will be non-linearities in the economic impact of fiscal policy when debt levels affect the timing of fiscal consolidations in an overlapping generations economy. Alesina and Perotti (1997) also argue that the response to changes in tax rates may be quite different depending on the extent and nature of union wage bargaining.

<sup>4</sup> Fernández-Villaverde *et al.* (2011) and Born and Pfeifer (2011) consider fiscal policy uncertainty in the form of time-varying shock variances in fiscal policy rules. Our study differs from this line of work in that we focus on uncertainty about the systematic parts of fiscal rules to study the composition and timing of large-scale state-dependent fiscal consolidations.

successful. In many cases, one study finds a conditioning variable to be significant, while another study does not.<sup>5</sup>

The next Section discusses empirical evidence in Alesina and Ardagna (2010), who analyse large-scale fiscal consolidations within OECD countries between 1970 and 2007. Section 2 lays out a simple neoclassical model where uncertainty over the timing and the composition of fiscal consolidations can be expansionary. Section 3 outlines the richer New Keynesian model and the range of state-dependent fiscal consolidations that may occur. Section 4 describes the fiscal limit distribution that determines the state-dependent probability of observing a fiscal consolidation and Section 5 describes the calibration and solution for the non-linear model. Sections 6 and 7 present the model's implications for a wide range of fiscal consolidations. Section 8 concludes.

## 1. Fiscal Consolidations Data

Alesina and Ardagna (2010) (henceforth AA) analyse episodes of fiscal stimulus (rise in deficit/fall in surplus) and consolidation (fall in deficit/rise in surplus) of more than 1.5% of GDP, where the data are cyclically adjusted. They classify an episode as 'expansionary' if GDP growth in the 2 years following the stimulus/consolidation is greater than the 75th percentile of the empirical density in all episodes. They also define a 'successful' fiscal consolidation as one that reduces the debt-GDP ratio by 4.5% 3 years later. Based on a sample of developed economies between 1970 and 2007, there are 107 episodes of fiscal consolidation, 15.1% of the observations.

We follow AA in computing the average change in key fiscal variables in the 2 years following a fiscal consolidation relative to the 2 years prior to the adjustment.<sup>6</sup> Table 1 details the average change in fiscal variables under both types of consolidation, where all variables are measured relative to output. It reveals some striking differences between 'expansionary' and 'contractionary' consolidations that meet AA's definitions. 'Expansionary' consolidations feature a statistically significant fall in government spending of 2.19% of GDP and a statistically insignificant rise in tax revenues of 0.35% and fall in transfers of 0.58% of GDP. In contrast, contractionary consolidations entail a fall in government spending of only 0.8% and rises in tax revenues of 1.11% and in transfers of 0.47%, all of which are statistically significant.

The 'expansionary' fiscal consolidations appear to be driven by spending cuts with no significant increases in aggregate tax revenues, while the 'contractionary' episodes are far more heavily dependent on increases in taxation. AA also observe that one out of four fiscal consolidations are spending based and that, out of 107 fiscal consolidations, 65 last for 1 year, 13 last 2 years, four last 3 years and one lasts for 4 years. We use these observations to calibrate both the consolidation duration and the relative frequency of spending-based and tax-based consolidations in the numerical simulations below.

<sup>5</sup> For example, Lambertini and Tavares (2005) find that accompanying exchange rate devaluations help ensure fiscal consolidations are successful but Ardagna (2004) does not; while Alesina and Ardagna (2010) find that the composition of consolidations affects both how expansionary and successful a consolidation is, Ardagna (2004) argues that composition does not matter for success.

<sup>6</sup> Our numbers differ slightly from those in AA as we exclude consolidations that do not have observations either before or after the episode, because we wish to assess the statistical significance of the changes in fiscal variables over the course of a consolidation episode.

Table 1  
*Expansionary and Contractionary Fiscal Consolidations in AA  
 Data*

	Expansionary	Contractionary
Debt	-4.93* (1.69)	5.42* (1.41)
Change in debt	-0.54 (1.21)	-2.22* (0.53)
Total deficit	-3.05* (0.52)	-1.56* (0.33)
Primary deficit	-2.54* (0.58)	-1.91* (0.31)
Primary expenditures	-2.19* (0.65)	-0.80* (0.34)
Transfers	-0.58 (0.41)	0.47* (0.17)
Govt wage expenditures	-0.40* (0.17)	-0.40* (0.13)
Govt non-wage expenditures	-0.13 (0.12)	0.14 (0.08)
Subsidies	-0.32* (0.11)	-0.16* (0.05)
Govt investment	-0.76* (0.25)	-0.83* (0.15)
Total rev	0.35 (0.42)	1.11* (0.24)
Income tax	0.16 (0.33)	0.27 (0.17)
Business tax	0.81* (0.36)	0.39* (0.14)
Indirect tax	0.01 (0.15)	0.27* (0.12)
Social security contributions	-0.06 (0.22)	0.14 (0.13)

*Notes.* \*Statistical significance at the 5% level, all variables are the average changes in the variable relative to GDP in the 2 years preceding and following a fiscal consolidation. The standard errors are in brackets.

## 2. Simple Model of Fiscal Consolidation

In this Section, we use a small, open economy to highlight the role that expectations may play in determining whether a fiscal consolidation is expansionary. We augment Bertola and Drazen's (1993) model with distortionary taxation. The small, open economy assumption allows us to generate analytical results in an endowment economy in which households still face meaningful consumption/saving decisions. Uncertainty over both the composition and the timing of fiscal consolidations generates expectation effects that have implications for the existence of expansionary consolidations.

A representative household chooses consumption,  $c_t$ , and financial assets,  $a_t$ , to maximise utility

$$E_t \sum_{s=0}^{\infty} \beta^s (\eta_0 c_{t+s} - \eta_1 c_{t+s}^2) \quad (1)$$

$$\text{s.t. } \beta a_t = a_{t-1} + y[1 - \tau_t - \psi(\tau_t)^2] - c_t, \quad (2)$$

where  $y$  is the household's endowment income and utility parameters  $\eta_0, \eta_1 > 0$  are assumed to be consistent with a positive but declining marginal utility of consumption over the relevant range. The holdings of financial assets at the start of period,  $a_{t-1}$ , earn a world interest rate of  $1/\beta$ .  $\tau_t$  is the tax rate on endowment income, which carries deadweight losses of  $y\psi(\tau_t)^2$ . Deadweight losses can be motivated by tax avoidance activities, but more generally, they capture the costs of distortionary taxation in economies with a more sophisticated supply side.<sup>7</sup> The household's intertemporal budget constraint, after imposing its transversality condition, is

$$\sum_{s=0}^{\infty} \beta^s \mathbb{E}_t c_{t+s} = a_{t-1} + \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s y[1 - \tau_{t+s} - \psi(\tau_{t+s})^2]. \quad (3)$$

The household's first-order condition delivers pure consumption smoothing

$$c_t = \mathbb{E}_t c_{t+s}. \quad (4)$$

Only surprises in the either the composition or the timing of fiscal consolidations induce jumps in consumption, while anticipated cuts in government spending and/or tax rises affect consumption only at the time when they are news.

The government's flow budget constraint is

$$\beta b_t = b_{t-1} - y\tau_t + g_t, \quad (5)$$

implying the intertemporal condition

$$b_{t-1} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s y\tau_{t+s} - \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s g_{t+s}. \quad (6)$$

Imposing equilibrium – (4) and (6) – the household's intertemporal budget constraint implies

$$\frac{c_t}{1 - \beta} = (a_{t-1} - b_{t-1}) + \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s y[1 - \psi(\tau_{t+s})^2] - \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s g_{t+s}, \quad (7)$$

where  $a_{t-1} - b_{t-1}$  are the net foreign assets held by households. At time  $t$ , the right hand side of (7) is predetermined or exogenous to the household, so this expression maps alternative compositions and timings of fiscal consolidations into equilibrium consumption.

Assume that the initial levels of government spending,  $g^0$ , and tax rates,  $\tau^0$ , are insufficient to ensure government solvency. Then debt is increasing and government spending or taxes must change in the future. After  $n$  periods, debt reaches a level  $b_{t+n-1}$ , found by accumulating the government's flow budget constraint forwards  $n$  periods.

<sup>7</sup> In the New Keynesian DSGE model in Section 3, there are two distortions: the standard mechanism of taxes distorting labour supply decisions and sticky prices, which create additional distortions caused by the inflationary consequences of changes in distortionary taxation.

$$b_{t+n-1} = \beta^{-n} b_{t-1} - \beta^{-n} \sum_{s=0}^{n-1} \beta^s y \tau^0 + \beta^{-n} \sum_{s=0}^{n-1} \beta^s g^0. \quad (8)$$

We now consider two types of uncertainty: uncertainty in the timing of the fiscal consolidation and uncertainty in its composition.

### 2.1. *The Timing of Consolidations*

The timing of fiscal consolidation can affect the likelihood of an expansionary consolidation only through the non-linear deadweight losses associated with distortionary taxation. To ensure government solvency, spending cuts or tax increases must stabilise debt. In the absence of deadweight losses, the timing of tax and spending changes cannot matter in this simple endowment economy: unexpected delays in fiscal consolidation would have no effect, so long as fiscal policy ultimately adjusts to satisfy (6). In the presence of deadweight losses, however, the discounted value of these losses erodes the resources available to the household for consumption. If a tax-based consolidation is delayed, the required tax increase rises, and the associated deadweight losses rise even faster.

Consider the household's consumption decision, (7), when only taxes adjust to stabilise debt and  $g_t \equiv g^0$ . Using  $b_{t-1} + g^0/(1 - \beta) = E_t \sum_s \beta^s y \tau_{t+s}$  from (6), the consumption decision becomes

$$\frac{c_t}{1 - \beta} = a_{t-1} + E_t \sum_{s=0}^{\infty} \beta^s y \left[ 1 - \tau_{t+s} - \psi(\tau_{t+s})^2 \right]. \quad (9)$$

Altering the timing of a tax-based consolidation does not affect the size of the discounted tax revenues needed to maintain fiscal solvency, but does affect the expected discounted sum of the deadweight losses

$$E_t \sum_{s=0}^{\infty} \beta^s y \left[ \psi(\tau_{t+s})^2 \right]. \quad (10)$$

From familiar tax smoothing arguments, the discounted sum of these deadweight losses is minimised by an immediate one-off increase in the tax rate to a level sufficient to satisfy the government's budget. Any delay in the implementation of the consolidation deviates from tax smoothing, raising the discounted value of deadweight losses and reducing consumption. News of a speedy consolidation that brings forward the expected date of a tax-based consolidation increases consumption, while news of a delayed consolidation that raises deadweight losses reduces consumption.

### 2.2. *Composition Uncertainty*

To illustrate composition uncertainty, we assume that households expect a fiscal consolidation  $n$  periods from now, with fiscal policy changing taxes or government spending to new levels that satisfy (6) at period  $t + n$ . Households expect the consolidation to be spending based with probability  $1 - \omega$ , and tax based with probability  $\omega$ . To stabilise debt at  $b_{t+n-1}$ , a spending-based consolidation sets  $g^1$  from period  $t + n$  onwards to satisfy

$$g^1 = y\tau^0 - (1 - \beta)b_{t+n-1}, \quad (11)$$

where tax rate remains at  $\tau^0$ . In the case of a tax-based consolidation, the new tax rate,  $\tau^1$ , solves

$$y\tau^1 = g^0 + (1 - \beta)b_{t+n-1}. \quad (12)$$

Spending-based consolidation requires a cut in spending and a tax-based consolidation requires an increase in tax revenues of an equal amount to ensure that debt is stabilised at the level  $b_{t+n-1}$  from that point onwards. Consumption under each type of consolidation, from period  $t + n$  onwards, is

$$c^s = (1 - \beta)(a_{t+n-1} - b_{t+n-1}) + y[1 - \psi(\tau^1)^2] - g^0, \quad (13)$$

$$c^g = (1 - \beta)(a_{t+n-1} - b_{t+n-1}) + y[1 - \psi(\tau^0)^2] - g^1. \quad (14)$$

Before consolidation, consumption lies between these two cases, so that there will be a positive (negative) jump in consumption at the point when the consolidation is revealed to be spending (tax) based. The exact size of the jump depends on expectations of the consolidation. Consumption before the consolidation is

$$\begin{aligned} c^0 &= (1 - \beta)(a_{t-1} - b_{t-1}) + (1 - \beta) \sum_{s=0}^{n-1} \beta^s y[1 - \psi(\tau^0)^2] - (1 - \beta) \sum_{s=0}^{n-1} \beta^s g^0 \\ &\quad + \beta^n \{(1 - \omega)y[1 - \psi(\tau^0)^2] + \omega y[1 - \psi(\tau^1)^2]\} - \beta^n [(1 - \omega)g^1 + \omega g^0] \\ &= (1 - \beta)(a_{t-1} - b_{t-1}) + y[1 - \psi(\tau^0)^2] - g^0 \\ &\quad - \beta^n \{\omega y[\psi(\tau^1)^2 - \psi(\tau^0)^2]\} + \beta^n [(1 - \omega)(g^0 - g^1)]. \end{aligned} \quad (15)$$

Pre-consolidation consumption takes account of the accumulation of government debt in the  $n$  periods before consolidation and also attaches probability weights to the types of consolidation that will ultimately emerge. The current consumption gain (loss) from an anticipated government spending (tax) based consolidation is clear. These expectations drive current consumption and saving behaviour: current consumption rises if agents anticipate a future cut in spending but falls if they fear a future rise in taxes. While the magnitude of the realised spending cuts or tax increases is unaffected by these expectations – since they do not affect debt dynamics prior to the consolidation – the accumulation of net foreign assets changes. Combining the government's and households' flow budget constraints, prior to the fiscal consolidation, net foreign assets evolve according to

$$\beta(a_t - b_t) = a_{t-1} - b_{t-1} + y[1 - \psi(\tau^0)^2] - c^0 - g^0. \quad (16)$$

Substituting for the pre-consolidation level of consumption implies that for any period prior to the consolidation,  $n \geq s$ ,



$$(a_{t+s-1} - b_{t+s-1}) - (a_{t+s-2} - b_{t+s-2}) = \beta^{n-s} \left\{ \omega y \psi [(\tau^1)^2 - (\tau^0)^2] - (1 - \omega)(g^0 - g^1) \right\}, \quad (17)$$

and the accumulated change in net foreign assets between  $t$  and  $t + n$  is

$$(a_{t+n-1} - b_{t+n-1}) - (a_{t-1} - b_{t-1}) = \frac{(1 - \beta^n)}{1 - \beta} \left\{ \omega y \psi [(\tau^1)^2 - (\tau^0)^2] - (1 - \omega)(g^0 - g^1) \right\}. \quad (18)$$

When the expected deadweight losses from the tax increase,  $\omega y \psi [(\tau^1)^2 - (\tau^0)^2]$ , are greater than the expected cut in government spending,  $(1 - \omega)(g^0 - g^1)$ , households accumulate net foreign assets in anticipation of the deadweight losses to come. Since these expectations are formed over the relative probabilities of each type of consolidation, households will accumulate more (fewer) net foreign assets when they anticipate that the consolidation will be tax (spending) based.

When a spending-based consolidation is realised, the jump in consumption is

$$c^g - c^0 = (1 - \beta)[(a_{t+n-1} - b_{t+n-1}) - (a_{t-1} - b_{t-1})] + g^0 - g^1, \quad (19)$$

$$\begin{aligned} &+ \beta^n \{ \omega y [\psi(\tau^1)^2 - \psi(\tau^0)^2] - (1 - \omega)(g^0 - g^1) \} \\ &= \omega y [\psi(\tau^1)^2 - \psi(\tau^0)^2] - (1 - \omega)(g^0 - g^1) + g^0 - g^1. \end{aligned} \quad (20)$$

The consolidation is classified as expansionary if the jump in consumption exceeds the cut in government spending

$$c^g - c^0 > g^0 - g^1, \quad (21)$$

which requires

$$\omega y [\psi(\tau^1)^2 - \psi(\tau^0)^2] > (1 - \omega)(g^0 - g^1). \quad (22)$$

To achieve an expansionary fiscal consolidation, the expected size of tax distortions (not the tax revenues themselves) needs to exceed the expected size of the government expenditure cut, both of which reflect economic agents' views about the relative probability of each type of consolidation. Any delay in consolidation raises the required increases in tax revenues or cuts in expenditure because initially the government's finances are on an unsustainable path. With deadweight losses increasing non-linearly in the tax rate, the losses associated with tax increases will be rising faster than the equivalent cuts in expenditure. This means that unexpected delays in consolidation efforts will reduce current consumption at the moment the delay is revealed but are more likely to support an expansionary consolidation should the ultimate fiscal consolidation be spending based.

### 2.3. Anticipated Consolidations

In the previous scenario, the uncertainty over the fiscal consolidation was resolved only when the consolidation actually occurred in period  $t + n$ . However, the nature of the consolidation could be revealed at an earlier date, say  $t + m$ ,  $m \leq n$ . The level of

consumption prior to the news will be the same as given in (15). If it is then revealed in period  $t + m$  that the fiscal consolidation will be spending based, then from that period on consumption will be given by

$$\begin{aligned} c^g &= (1 - \beta)(a_{t+m-1} - b_{t+m-1}) + y[1 - \psi(\tau^0)^2] - g^0 + \beta^{n-m}(g^0 - g^1) \\ &= (1 - \beta)(a_{t+n-1} - b_{t+n-1}) + y[1 - \psi(\tau^0)^2] - g^1, \end{aligned} \quad (23)$$

and will not change when the actual consolidation is realised. Consumption will jump only in period  $t + m$  when the news of the type of consolidation is revealed

$$c^g - c^0 = (1 - \beta)[(a_{t+m-1} - b_{t+m-1}) - (a_{t-1} - b_{t-1})] + \beta^{n-m}(g^0 - g^1), \quad (24)$$

$$+ \beta^n \{ \omega y [\psi(\tau^1)^2 - \psi(\tau^0)^2] - (1 - \omega)(g^0 - g^1) \}. \quad (25)$$

Accumulating the change in net foreign assets between period  $t$  and the date of the announcement of the consolidation type,  $t + m$

$$\begin{aligned} (a_{t+m-1} - b_{t+m-1}) - (a_{t-1} - b_{t-1}) &= \frac{\beta^{n-m}(1 - \beta^m)}{1 - \beta} t \{ \omega y \psi [(\tau^1)^2 - (\tau^0)^2] \\ &\quad - (1 - \omega)(g^0 - g^1) \} \end{aligned} \quad (26)$$

allows us to rewrite the consumption jump as

$$c^g - c^0 = \beta^{n-m} \omega \{ (g^0 - g^1) + y [\psi(\tau^1)^2 - \psi(\tau^0)^2] \}. \quad (27)$$

When the fiscal consolidation at time  $t + n$  is known to be spending based at time  $t + m$ ,  $m \leq n$ , there is an immediate positive jump in private consumption. The size of the jump rises with the weight attached to the tax-based consolidation,  $\omega$ , but falls with the time between the announcement and realisation of the consolidation,  $n - m$ . Unless the consolidation was always known to be spending based, there is a positive jump in private consumption upon the announcement at time  $t + m$  that the consolidation will be spending based. When the consolidation is realised, there is no further jump in private consumption but there is a decline in public consumption that contracts aggregate demand. This is because the realisation of the consolidation no longer provides any additional information. This leads to an additional condition for observing an expansionary fiscal consolidation: the realisation of the consolidation must contain the new information required to boost private sector consumption.

In sum, fiscal consolidations are more likely to be expansionary when economic agents were expecting them to be tax based with a high associated deadweight loss but the realised consolidation is spending-based. Conversely, the biggest consumption decline occurs when the consolidation is tax based, but economic agents were expecting cuts in government spending. To observe an expansionary fiscal consolidation, the realisation of the consolidation must contain the positive information which induces households to significantly increase private consumption.

We explore the quantitative importance of uncertainty over the timing and composition of fiscal consolidations in a full DSGE model below. Our experiments in that model differ from this simple example in a crucial respect: in line with the data, we consider temporary, rather than permanent, consolidations.

### 3. Quantitative Model of Fiscal Consolidation

We now turn to study the macroeconomic consequences of uncertain fiscal consolidations in a richer and more plausible environment. Since debt service costs are particularly important in determining debt dynamics at high debt levels, we use a conventional New Keynesian model modified to allow occasional fiscal consolidations. Consolidations are triggered after debt rises to a level that breaches a stochastic ‘fiscal limit’. The fiscal limit is the maximum level of debt the government is able to support, which is constrained by the tax Laffer curve and the realisations of shocks. Households anticipate that the government will attempt to stabilise debt through fiscal consolidations in advance of reaching this limit. Political factors such as a war of attrition over who bears the costs of a particular consolidation, however, may induce the government to leave consolidation to the last minute.<sup>8</sup> To accord with this evidence, the probability of a fiscal consolidation rises with the level of government debt.

We also allow periodically explosive lump-sum transfers. In addition to being a feature of the data, temporarily explosive transfers produce a plausible transition from relatively low to very high debt levels. This assumption also changes the distribution of fiscal limits and, therefore, the likelihood of fiscal consolidation at a given debt level. Bi (2012) shows that the possibility of explosive transfers can significantly lower expected future fiscal surpluses and generate a more dispersed distribution of fiscal limits, making it more likely that the economy will hit its fiscal limit at relatively low levels of debt like those observed in countries’ experiences.

Households supply labour to intermediate goods producing firms with Rotemberg-style price adjustment. Their labour and profit income are taxed. The setup delivers a rich set of monetary and fiscal policy interactions. Sticky price adjustment gives monetary policy real effects that affect both the tax base – labour income – and real debt service costs. Changes in taxes or government spending not only have the usual fiscal consequences but also influence inflation through either the labour supply response to distortionary taxation or the aggregate demand effect of changes in government spending. These inflationary consequences of fiscal consolidations generate resource costs that go beyond the usual deadweight losses of distortionary taxation.

#### 3.1. Households

The cashless economy is populated by a large number of identical households of size 1 with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t),$$

where  $\beta \in (0, 1)$  is the household’s subjective discount factor,  $c_t$  is consumption and  $n_t$  the household’s labour supply. The household receives nominal wages,  $W_t$ , and monopoly profits,  $Y_t$ , from the firm, both of which are taxed at the rate  $\tau_t$ , and lump-sum transfers,  $z_t$ , from the government. The household chooses consumption,

<sup>8</sup> Alesina *et al.* (2006) find that political factors play a significant role in determining when a consolidation is implemented, consistent with war-of-attrition effects.

hours worked and nominal bond holdings,  $B_t$ , to maximise utility subject to the budget constraint

$$P_t c_t + \frac{B_t}{R_t} = B_{t-1} + (1 - \tau_t)(W_t n_t + P_t Y_t) + P_t z_t. \quad (28)$$

The maximisation problem yields the typical first-order conditions

$$\frac{1}{R_t} = \beta E_t \frac{u_c(t+1)}{u_c(t)} \frac{1}{\pi_{t+1}}, \quad (29)$$

$$-\frac{u_n(t)}{u_c(t)} = w_t(1 - \tau_t), \quad (30)$$

where  $\pi_t \equiv P_t/P_{t-1}$  is the inflation rate and  $w_t \equiv W_t/P_t$  is the real wage.

### 3.2. Final Goods Production

Final goods are used for private and public consumption. Competitive final goods firms buy the differentiated products produced by intermediate goods producers to construct consumption aggregates, which have the CES (constant elasticity of substitution) form

$$y_t = \left[ \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad (31)$$

where  $y_t$  is aggregate output,  $y_t(i)$  is the output of intermediate good firm  $i$  and  $\theta > 1$  is the elasticity of demand for each firm's product. Cost minimisation for final goods producers results in the demand curve for intermediate good  $i$

$$y_t(i) = \left[ \frac{p_t(i)}{P_t} \right]^{-\theta} y_t \quad (32)$$

and an associated price index for final goods

$$P_t = \left[ \int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}. \quad (33)$$

### 3.3. Intermediate Goods Production

The imperfectly competitive intermediate goods firms are subject to Rotemberg adjustment costs that penalise large price changes in excess of steady-state inflation rates. Price adjustment costs make the firm's problem dynamic

$$\max \sum_{t=0}^{\infty} R_{0,t} \left\{ p_t(i) y_t(i) - m c_t P_t y_t(i) - \frac{\phi}{2} \left[ \frac{p_t(i)}{p_{t-1}(i)} \frac{1}{\pi} - 1 \right]^2 P_t y_t \right\}, \quad (34)$$

$$s.t. \quad y_t(i) = \left[ \frac{p_t(i)}{P_t} \right]^{-\theta} y_t, \quad (35)$$

where  $mc_t = w_t/A_t$  is the real marginal cost implied by a linear production function and  $y_t(i) = A_t n_t(i)$ . Productivity,  $A_t$ , is common to all firms. The first-order condition, after imposing symmetry across firms, is

$$(1 - \theta) + \theta mc_t - \phi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} + \beta \phi E_t \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{y_{t+1}}{y_t} = 0,$$

which represents the non-linear New Keynesian Phillips curve under Rotemberg pricing.<sup>9</sup>

Monopoly profits, which the government taxes when households receive them, are

$$\Upsilon_t = y_t - mc_t y_t - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t. \tag{36}$$

The aggregate resource constraint is

$$c_t + g_t = A_t n_t \left[ 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 \right]$$

making clear the resource losses that rapid price adjustment produces.

### 3.4. Monetary and Fiscal Policy

Monetary policy follows a simple inflation-targeting rule

$$R_t - R = \alpha(\pi_t - \pi), \tag{37}$$

where  $\pi$  is the target inflation rate. In a deterministic steady state,  $R_t = R$  and  $\pi_t = \pi$ .

Fiscal transfers evolve exogenously, but their process depends on a regime-switching index  $x_t^z$

$$z(x_t^z) = \begin{cases} (1 - \rho^z)z + \rho^z z_{t-1} & \text{if } x_t^z = 1 \quad (\rho^z < 1) \\ \zeta^z z_{t-1} & \text{if } x_t^z = 2 \quad (\zeta^z > 1) \end{cases}$$

with  $x_t^z$  following a transition matrix of

$$\begin{pmatrix} p_1^z & 1 - p_1^z \\ 1 - p_2^z & p_2^z \end{pmatrix}.$$

The Markov regime-switching process moves from a stationary process with  $\rho^z < 1$  to one where transfers explode with  $\zeta^z > 1$ . There can be prolonged periods when growing transfers produce sustained increases in government debt, which can prompt attempts at fiscal consolidation. Periodic instability in transfers is common to many advanced economies and, as the IMF (2009) reports, is likely to become more widespread as populations age.

Monetary and fiscal policies must satisfy the government's flow budget constraint

$$\frac{B_t}{R_t} + \tau_t(W_t n_t + P_t \Upsilon_t) = B_{t-1} + P_t g_t + P_t z_t. \tag{38}$$

While fiscal policy has obvious effects on debt dynamics, monetary policy will also have a role to play, especially when debt stocks are large.

<sup>9</sup> To a first order, Rotemberg and Calvo pricing deliver identical Phillips curves.

### 4. Fiscal Limit and Fiscal Consolidations

#### 4.1. Distribution of the Fiscal Limit

Laffer curves provide a natural starting point for quantifying the fiscal limit from the tax revenue side of the government’s budget constraint. At the peak of the Laffer curve, tax revenues reach their maximum and, for a given level of total government expenditures, the present value of primary surpluses is maximised. Revenues, expenditures and discount rates, of course, vary with the shocks hitting the economy, generating a distribution for the maximum debt-GDP level that can be supported.

To compute the distribution, we assume that the monetary authority keeps the inflation rate at its target ( $\pi_t = \pi$ ),<sup>10</sup> so the peak of the Laffer curve is a function of the exogenous state of the economy ( $A_t, g_t$ ). At the Laffer curve peak, define

$$\tau_t^{\max} = \tau^{\max}(A_t, g_t), \tag{39}$$

$$T_t^{\max} = \mathcal{T}^{\max}(A_t, g_t), \tag{40}$$

where the function  $\tau^{\max}$  ( $\mathcal{T}^{\max}$ ) maps the state into the tax rate (revenues) at the peak. Evidently, the stochastic processes governing the exogenous states induce stochastic processes for both the tax rate that maximises revenues and the maximum level of revenues.

The fiscal limit is defined, following Bi (2012), as the discounted sum of expected maximum primary surpluses in all future periods.

$$B^* = E \sum_{t=0}^{\infty} \beta^t \underbrace{\beta_p}_{\text{political factor}} \frac{u_c^{\max}(A_t, g_t)}{u_c^{\max}(A_0, g_0)} \underbrace{[T_t^{\max}(A_t, g_t, z_t, x_t^z) - g_t - z_t]}_{s_t^{\max}}. \tag{41}$$

The government spending,  $g_t$ , follows an AR(1) process that is calibrated to data, see Table 2.<sup>11</sup>

The stochastic discount factor is obtained when tax rates are at the peak of the Laffer curve,  $\beta^t u_c^{\max}(A_t, g_t) / u_c^{\max}(A_0, g_0)$ , but modified to allow for a political risk parameter  $\beta_p$ . Higher political risk – lower  $\beta_p$  – lends itself to multiple interpretations that reflect the private sector’s beliefs about policy. Most straightforward is the idea that policy makers have effectively shorter planning horizons than the private sector; see Acemoglu *et al.* (2008). To see this, rewrite the discount factor in (41) as  $(\beta_p \beta)^t / (\beta_p)^{t-1}$ , so that a lower value of  $\beta_p$  reduces the present value of maximum surpluses. An alternative interpretation is that private agents place probability mass on both the maximum surpluses ( $s^{\max}$ ) and on zero primary surpluses. Rewrite the surpluses as  $\beta_p s^{\max} + (1 - \beta_p) \cdot 0$  for this interpretation. Nothing we do hinges on the

<sup>10</sup> Fiscal consolidations can and do affect equilibrium inflation rates. By fixing inflation in computing the fiscal limit, we are assuming that seigniorage revenues are not an important source of fiscal financing in the long run, a plausible assumption for advanced economies.

<sup>11</sup> When computing the fiscal limits, it is necessary to assume government spending follows an exogenous process. Endogenising spending through a fiscal rule or state-dependent fiscal consolidations is a daunting task as it involves solving a fixed-point problem by computing the fiscal limit and solving the non-linear model simultaneously. More importantly, Bi (2012) shows, in a similar setup to this article, that current government spending has a negligible impact on the fiscal limit distribution. Instead, it is the variation in the potential paths of transfers that drive the distribution of the fiscal limit.

Table 2  
*Model Calibration*

	Parameter	Calibration
Discount factor	$\beta$	0.99
Elasticity of substitution	$\theta$	11
Rotemberg adjustment parameter	$\phi$	100
Inflation rate	$\pi$	1.03 (annual)
Technology	$A$	1
Labour supply	$n$	0.25
Government spending/GDP	$g/y$	0.21
Government transfer/GDP	$z/y$	0.18
Government debt/GDP	$b/y$	0.50 (annual)
Tax rate	$\tau$	0.41
Fiscal rule parameter	$\gamma_\tau$	0.5/4
Taylor rule parameter	$\alpha$	1.5
Political factor	$\beta^p$	0.85
Spending shock persistence	$\rho^g$	0.9
Spending shock variance	$\sigma_g^2$	$(0.03g)^2$
Tax shock variance	$\sigma_\tau^2$	$(0.03\tau)^2$
Transfer persistence	$\rho_z$	0.8
	$\zeta^z$	1.003
Transfer regime parameter	$\beta^z$	0.975
Length of consolidations	$h$	4
Tax-type consolidation	$m^\tau$	0.01
Spending-type consolidation	$m^g$	0.01y
Probability of tax-type FC	$\omega$	0.75

precise interpretation attached to  $\beta_p$ . As a practical matter, setting  $\beta_p < 1$  serves to shift down the distribution of the fiscal limit, which generates occurrences of fiscal consolidations at lower levels of debt similar to those observed in the data. Moreover, as discussed in Section 3, the possibility of temporarily explosive transfers leads to a wider dispersion of the fiscal limit, which also creates the possibility of consolidations at relatively low debt levels.

We compute the unconditional distribution of the fiscal limit,  $f(\mathcal{B}^*)$ , using Markov Chain Monte Carlo simulation, as Appendix A describes.

#### 4.2. State-dependent Fiscal Consolidations

For the state-dependent fiscal consolidations, the government spending process and the tax rule follow

$$g_t - g = -m_t^g, \quad (42)$$

$$\tau_t - \tau = m_t^\tau + \gamma^\tau(b_{t-1} - b). \quad (43)$$

Fiscal consolidations take the form of positive values for the intercept terms,  $m_t^g$  and  $m_t^\tau$ , implying reductions in government spending and increases in taxation.

At each period  $t$ , the effective fiscal limit,  $b_t^*$ , is drawn from the distribution of the fiscal limit. We treat the choice of  $b_t^*$  as random, being driven by policy makers' perceived costs of fiscal consolidation. If the existing debt level,  $b_{t-1}$ , surpasses the effective fiscal limit, the government undertakes a consolidation that lasts for four

periods, in line with AA’s data. We consider three models of state-dependent fiscal consolidations, which we denote  $x^i$  ( $i = \tau, g, m$ ). For the  $x^\tau$  ( $x^g$ ) model, we consider the case where state-dependent fiscal consolidations will always be tax (spending) based. Economic agents know this to be the case: there is only timing uncertainty, no composition uncertainty. In the  $x^m$  model, economic agents attach a probability ( $\omega$ ) to the possibility that the realised fiscal consolidations will be tax based, denoted as  $x^\tau$ , and the complementary probability ( $1 - \omega$ ) that it will be spending based, denoted as  $x^g$ . Outside of periods of fiscal consolidation, the government sets  $m_t^\tau = m_t^g = 0$ .

We use a state variable  $x_t$  to track the path of fiscal consolidations: in normal times – no consolidation – it equals 1; in a tax-based consolidation,  $x_t$  switches to 2 and the consolidation lasts for another three periods, so  $x_{t+1} = 3, x_{t+2} = 4, x_{t+3} = 5$ , before returning to the normal no-consolidation state; in a spending-based consolidation that lasts four periods,  $x_t = 6, x_{t+1} = 7, x_{t+2} = 8, x_{t+3} = 9$ , before exiting.<sup>12</sup> These policy dynamics are summarised by

$$\left\{ \begin{array}{ll} \text{if } b_{t-1} < b_t^* : & \text{no consolidation } (x_t = 1, m_t^\tau = m_t^g = 0) \\ \text{otherwise} & \\ \quad \text{with prob } \omega : & \text{tax-based consolidation } (x_t \dots x_{t+3} = 2, \dots, 5) \\ & (m_t^\tau \dots m_{t+3}^\tau = m^\tau, m_t^g \dots m_{t+3}^g = 0) \\ \quad \text{with prob } 1 - \omega : & \text{spending-based consolidation } (x_t \dots x_{t+3} = 6, \dots, 9) \\ & (m_t^\tau \dots m_{t+3}^\tau = 0, m_t^g \dots m_{t+3}^g = m^g) \end{array} \right.$$

Even though the households know the distribution of the fiscal limit, both the timing and the composition of consolidation are uncertain. The  $x^\tau$  and  $x^g$  models follow the same structure except that there is no composition uncertainty and the probability  $\omega$  is set equal to one for model  $x^\tau$  and zero for the case of  $x^g$ .

#### 4.3. Unanticipated Independent and Identically Distributed (i.i.d.) Fiscal Consolidations

To draw out the role of expectations, we contrast state-dependent fiscal consolidations,  $x^i$  ( $i = \tau, g, m$ ), with the same-sized consolidations implemented through a sequence of unanticipated i.i.d. policy shocks on government spending and tax, labelled as  $s^i$  ( $i = \tau, g$ ).<sup>13</sup> Fiscal behaviour obeys

$$g_t - g = \varepsilon_t^g \quad \varepsilon_t^g \sim \text{i.i.d. } \mathcal{N}(0, \sigma_g^2), \tag{44}$$

$$\tau_t - \tau = \gamma^\tau (b_{t-1} - b) + \varepsilon_t^\tau \quad \varepsilon_t^\tau \sim \text{i.i.d. } \mathcal{N}(0, \sigma_\tau^2). \tag{45}$$

We simulate a series of shocks that mimic the size of the state-dependent fiscal consolidations and then isolate the effects of timing uncertainty on the marginal impact of a fiscal consolidation. Because the expected value of the i.i.d. shocks is zero, this device removes the expectations effects associated with state-dependent fiscal consolidations.

<sup>12</sup> After a consolidation, policy stays in the no-consolidation state for at least one period.

<sup>13</sup> Persistent shocks do not change the qualitative results.



## 5. Calibration and Solution

The model is calibrated at quarterly frequency to EU-14 data. We focus on those economies because they feature heavily in the AA data set: those countries have undertaken sizeable consolidations and they have occasionally enjoyed consolidations that AA label as ‘expansionary’.

We calibrate fiscal parameters to match average EU-14 data from 1971 to 2007.<sup>14</sup> In steady state, government purchases are 21% of GDP, lump-sum transfers are 18% of GDP, and the tax rate is 0.41, implying a steady-state government debt-GDP ratio of 50.38% when the discount factor,  $\beta$ , is chosen to deliver an annual real interest rate of 4.1%. The tax adjustment parameter,  $\gamma$ , is 0.5 at an annual rate, which is close to the average of estimates in EU-14. The regime-switching parameters  $p_1^z$  and  $p_2^z$  are calibrated to 0.975, so that the average length of each regime is 10 years. A higher  $p^z$  leads to a more dispersed distribution of fiscal limits.  $\zeta^z$  is set at 1.003, implying an increase of 12.75% in transfers in 10 years. As summarised in Table 2, the calibrations for the shock processes for tax and spending follow Traum and Yang (2010), among others.

Consistent with data, consolidations last 1 year and are calibrated, through the  $m^r$  and  $m^g$  terms, to 1% of steady-state GDP. The International Country Risk Guide’s index of political risk offers one way to calibrate the political factor,  $\beta_p$ , as Arteta and Galina (2008) discuss. The average of that index across EU-14 countries was 85 of 100 during the period 1984–2009.

Utility is given by  $u(c, n) = \log c + \chi_n \log(1 - n)$ .  $\chi_n$  is set to imply that the household spends 25% of its time working in steady-state and the Frisch elasticity of labour supply is three. Time endowment and steady-state productivity are normalised to one. For simplicity, we keep productivity at its steady state level but none of the results below hinge on this assumption. The demand elasticity,  $\theta$ , is 11 and the Rotemberg adjustment parameter,  $\phi$ , is 100, which is equivalent to 26.7% of the firms re-optimising each quarter in a Calvo-type overlapping contracts model, as in Keen and Wang (2007) Gross inflation is 1.03 at an annual rate and the Taylor rule parameter is set to 1.5 in the benchmark case.

Given this calibration, the distribution of the fiscal limit can be simulated by drawing from the distributions of the exogenous shocks. Figure 1 reports the kernel-estimated cumulative distribution of the fiscal limit. As the debt rises, so does the probability that debt will exceed the effective fiscal limit,  $b_t^*$ , drawn from the distribution. The fat tail is generated by the possibility of entering the explosive transfers regime.

We solve the full non-linear model in Section 3, coupled with the fiscal limit described in Section 4, using the monotone mapping method. The solution method, based on Coleman (1991) and Davig (2004), discretises the state space and conjectures candidate decision rules that reduce the system to a set of first-order expectational difference equations. Decision rules map the state at period  $t$  into the stock of government debt, the real wage and the inflation rate in the same period, denoted as  $b_t = f^b(\psi_t)$ ,  $w_t = f^w(\psi_t)$ ,  $\pi_t = f^\pi(\psi_t)$  with  $\psi_t$  being the vector of states that Appendix

<sup>14</sup> Appendix B describes the data.

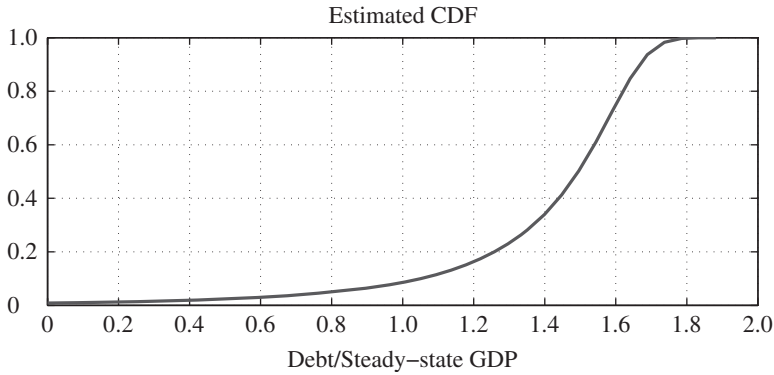


Fig. 1. *Cumulative Distribution of the Fiscal Limit*

C describes. After finding the decision rules, we solve for the bond-pricing rule,  $q_t = f^q(\psi_t)$ , using the government budget constraint. The interest rate on government bonds can also be solved using  $R_t = 1/q_t$ , denoted as  $f^R(\psi_t)$ . Appendix C describes the non-linear solution method, and Appendix D assesses the accuracy of that solution using the dynamic Euler equation accuracy test of Den Haan (2010).

**6. Fiscal Consolidation: Timing Uncertainty Only**

Fiscal consolidations can occur across a wide range of debt levels, but it is reasonable to posit that the probability of a fiscal consolidation is rising in the debt-GDP ratio. Consolidations at low debt levels are more surprising than those that follow sustained increases in debt. Using the policy rules from subsection 4.2, the consolidation intercepts,  $m_t^\tau$  and  $m_t^g$ , depend on the state variable  $x_t$ ; whenever government debt exceeds the stochastic effective fiscal limit, a fiscal consolidation occurs and lasts for 1 year. This section focuses on uncertainty over the timing and the duration of consolidations.

6.1. *Tax-based Fiscal Consolidation*

Tax-based consolidations, labelled as  $x^\tau$ , follow the rule

$$x^\tau : \quad \tau_t - \tau = m^\tau(x_t) + \gamma^\tau(b_{t-1} - b).$$

The size of the consolidation,  $m^\tau$ , depends on the state-dependent variable  $x_t$ , which in turn hinges on government liabilities,  $b_{t-1}$ , and the stochastic fiscal limit,  $b_t^*$ . With consolidation lasting four periods, regime change is governed by

$$\begin{cases} \text{if } b_{t-1} < b_t^* : & x_t = 1; m_t^\tau = 0 \\ \text{otherwise:} & x_t \dots x_{t+3} = 2, \dots, 5; m_t^\tau \dots m_{t+3}^\tau = m^\tau. \end{cases}$$

If government debt exceeds the stochastic fiscal limit,  $b_t^*$ , fiscal policy implements a 1-year consolidation by raising taxes beyond the level implied by the usual fiscal rule – when  $m^\tau(x_t) \equiv 0$  – in an attempt to reduce government debt.

We contrast the  $x^\tau$  model with the same-sized tax consolidation implemented through a sequence of unanticipated i.i.d. policy shocks, labelled as  $s^\tau$ , using the tax rule<sup>15</sup>

$$s^\tau: \quad \tau_t - \tau = \gamma^\tau(b_{t-1} - b) + \varepsilon_t^\tau.$$

Expectations play a central role in determining the macroeconomic impacts of a consolidation. When a consolidation changes the policy regime, agents know the new policy rules remain in effect for four periods and adjust their expectations accordingly. A successful consolidation lowers the probability of hitting the fiscal limit in the future, reducing the likelihood of further consolidations. A sequence of surprise policies, in contrast, has no such effect, as it does not affect the likelihood of future consolidations. Figure 2 compares the impulse responses from the  $s^\tau$  (dotted lines) and the  $x^\tau$  (solid lines) cases when the initial expected probability of fiscal consolidation is only 0.05 and the consolidation occurs in period 5. The Figure plots the variable differences between their values under a fiscal consolidation and those without consolidation. With a low probability of consolidation, the realised consolidation comes as a surprise in both cases.

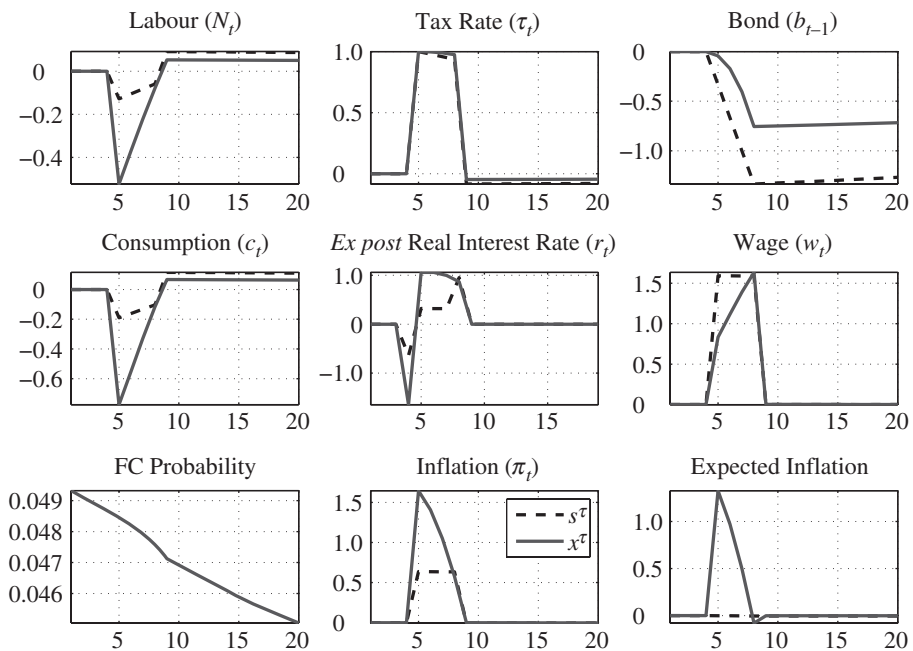


Fig. 2. Tax-based Consolidations at a Low Debt Level

Notes. i.i.d. consolidation,  $s^\tau$ , and state-dependent consolidations,  $x^\tau$ , when the initial probability of consolidation is 0.05. Plots variable differences between their values under consolidation and those without consolidation. Tax, interest and inflation rates are in percentage points, while the other variables are in terms of percentage of their steady-state values.

<sup>15</sup> Fiscal rules of this form have been used extensively in the literature – see, for example, Leeper (1991) and Leith and Wren-Lewis (2000).

In the  $x^\tau$  case, once the fiscal consolidation begins, economic agents know that taxes will remain high for four quarters, raising real wages and marginal costs. Firms raise prices in anticipation of this sustained rise in marginal costs; inflation jumps up and gradually declines over the course of the consolidation. While the initial jump helps deflate the real value of government debt, the active monetary policy raises real interest rates in response to the rise in inflation, offsetting some of the debt reduction.<sup>16</sup> In the  $s^\tau$  case, consolidations arrive as i.i.d. shocks. Price setters are repeatedly surprised by the tax hikes, which raise marginal costs and inflation, though by less than when regime changes. Active monetary policy does not raise real interest rates by as much and the repeated inflation surprises drive a wedge between *ex ante* and *ex post* real interest rates, making the consolidation more effective in stabilising debt.

Since debt levels are low in this case, there is little of the expectation effects highlighted in the simple model: fiscal consolidations were seen as remote prior to the consolidation, and remain so afterwards. High debt levels, on the other hand, elevate the probabilities of hitting the fiscal limit and of consolidation. When agents anticipate consolidation, they alter their behaviour in pre-consolidation periods, and the consolidation itself can have smaller effects when it is finally realised.

Figure 3 repeats the same experiment as in Figure 2 but with the initial debt-GDP ratio at 160%, which raises the probability of fiscal consolidation to 0.75. When a

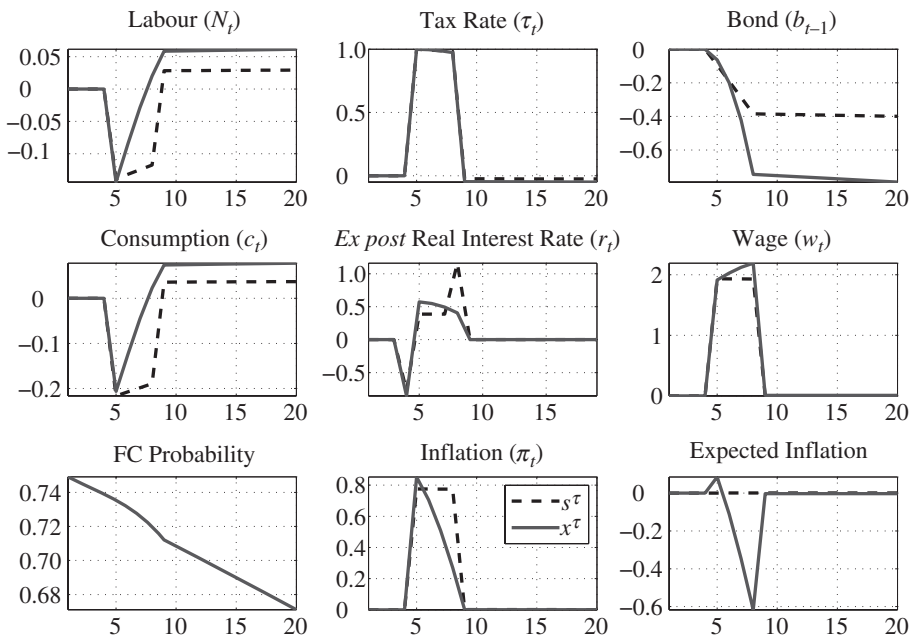


Fig. 3. Tax-based Consolidations at a High Debt Level

Notes. i.i.d. consolidation,  $s^\tau$ , and state-dependent consolidations,  $x^\tau$ , when the initial probability of consolidation is 0.75.

<sup>16</sup> Defining the *ex post* real rate at  $t$  as  $r_t \equiv R_t - \pi_{t+1}$ , the consolidation in period nine generates surprise inflation that reduces the realised return on debt sold in period 8.

consolidation is expected but has not yet arrived, it generates negative inflation surprises, which worsen debt dynamics under an active monetary policy. As a result, when the fiscal consolidation is realised, its negative impact is not as great as it would have been if the consolidation had been unanticipated. Relative to the case where the consolidation was not perceived to be imminent, the marginal impact on debt is now reversed: removing the uncertainty of the consolidation duration removes the large negative inflation surprises that come with the  $x^c$  case. Since these surprises are acting on a very large stock of debt, removing the uncertainty stabilises the debt. This reversal is consistent with the analysis of the simple model above, highlighting the importance of expectations over the likelihood and the duration of fiscal consolidations.

To understand inflation dynamics and the nature of the surprises induced by state-dependent fiscal consolidations, we plot the *level* of inflation and expected inflation for the  $x^c$  model in Figure 4. The top panel shows the case when the initial probability of fiscal consolidation is 0.05. The triangle dash-dotted line shows the path of  $\pi_t$  and the square solid line shows that of  $E_{t-1}\pi_t$  when a fiscal consolidation occurs at period 5. The tax rate rises, labour supply contracts and consumption falls. Higher marginal costs further raise inflation and, since the consolidation was unexpected, there is an inflation surprise in the first period of the consolidation. There is no inflation surprise during the consolidation or in the period immediately following the exit.

If the probability of fiscal consolidation is 0.75, shown in the bottom panel, inflationary expectations are significantly higher; actual inflation, on the other hand, mimics the path in the top panel. When consolidation does occur at period 5, taxes

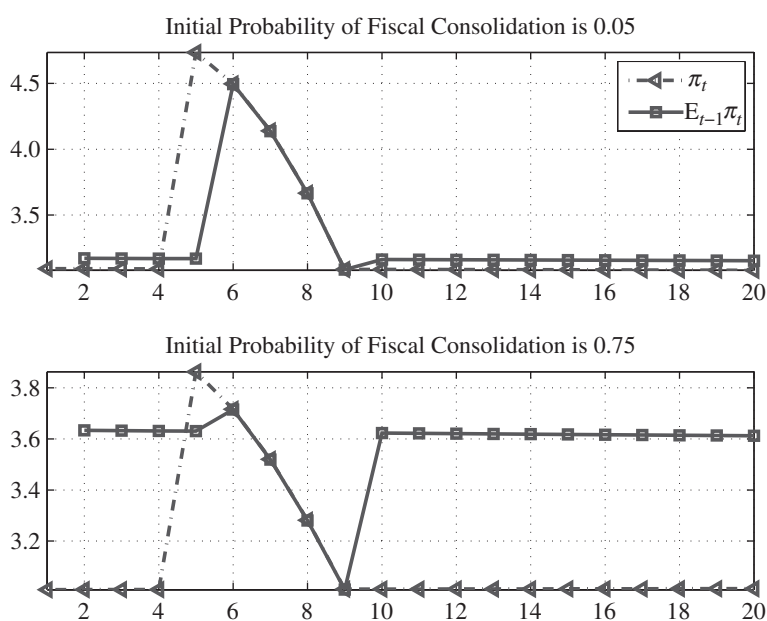


Fig. 4. Inflation Dynamics Comparison in the  $x^c$  Model

Notes. Consolidation occurs at period 5. Inflation rates are in percentage points.

and inflation rise, creating a positive inflation surprise. In all other periods, there is a non-zero probability attached to consolidation, creating an ongoing inflation surprise.

6.2. *Spending-Based Fiscal Consolidations*

We now consider government spending-based consolidations, labelled  $x^g$ . Spending policy obeys

$$x^g: \quad g_t - g = -m^g(x_t). \tag{46}$$

When government debt exceeds the stochastic fiscal limit,  $b_t^*$ , the government cuts its spending by  $m^g$  for 1 year. We contrast this  $x^g$  model with the same-sized spending consolidations implemented through a sequence of unanticipated i.i.d. policy shocks

$$s^g: \quad g_t - g = \varepsilon_t^g,$$

which effectively shuts down the expectations effects associated with the state-dependent fiscal consolidations in the  $x^g$  model.

Figure 5 compares the impulse responses from  $s^g$  and  $x^g$  models when the expected probability of fiscal consolidation is low. Once a consolidation begins, price-setters expect it to last for a year in the  $x^g$  model. Inflation falls immediately and then slowly returns to steady state. With an active monetary policy, lower inflation lowers real interest rates, reducing debt service costs and maintaining the size of the tax base. In contrast, in the  $s^g$  model, price-setters fail to anticipate the subsequent decreases in

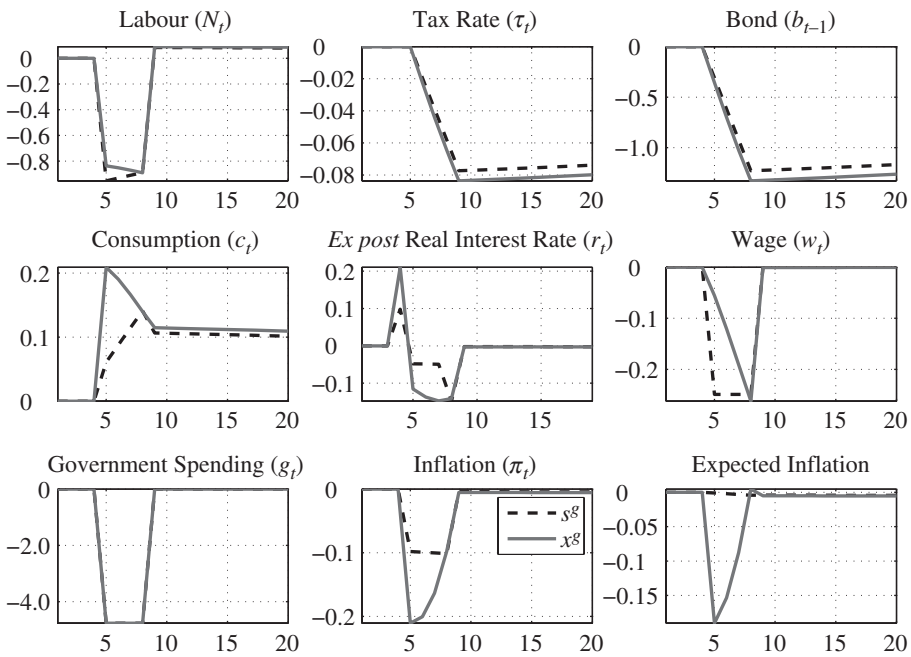


Fig. 5. *Spending-based Consolidations at a Low Debt Level*

Notes. i.i.d. consolidation,  $s^g$ , and state-dependent consolidations,  $x^g$ , when the initial probability of consolidation is 0.05.

government spending and inflation does not fall by as much on impact. Uncertainty over the duration of a spending-based consolidation reduces its deflationary consequences, in contrast to tax-based consolidations.

Figure 6 considers the same experiments except that the probability of consolidation is high. In the  $x^g$  model, economic agents anticipate that government spending cuts are imminent and the no-consolidation case contains positive inflation surprises as consolidations are expected but not realised. Outcomes are quite similar to those under lower debt levels. One noticeable difference is that there is a smaller increase in consumption when the consolidation is realised, as households were already expecting government spending to be cut. Similarly, the initial deflation is smaller as it was already factored into inflation expectations.

6.3. Key Message of Timing Uncertainty

Output multipliers are a convenient way to summarise differences across the  $s^i$  and  $x^i$  ( $i = \tau, g$ ) policy scenarios. The multipliers are computed as

$$\Gamma_{t+k}^y = \frac{\sum_{j=0}^k \left( \prod_{i=0}^j r_{t+i}^{-1} \right) (y_{t+j}^{shock} - y_{t+j}^{no})}{\sum_{j=0}^k \left( \prod_{i=0}^j r_{t+i}^{-1} \right) (f_{t+j}^{shock} - f_{t+j}^{no})}, \tag{47}$$

where the ‘shock’ superscript indicates that the consolidation has been realised and the ‘no’ superscript that it has not.  $r_t$  is the real interest rate and  $f$  denotes the type of

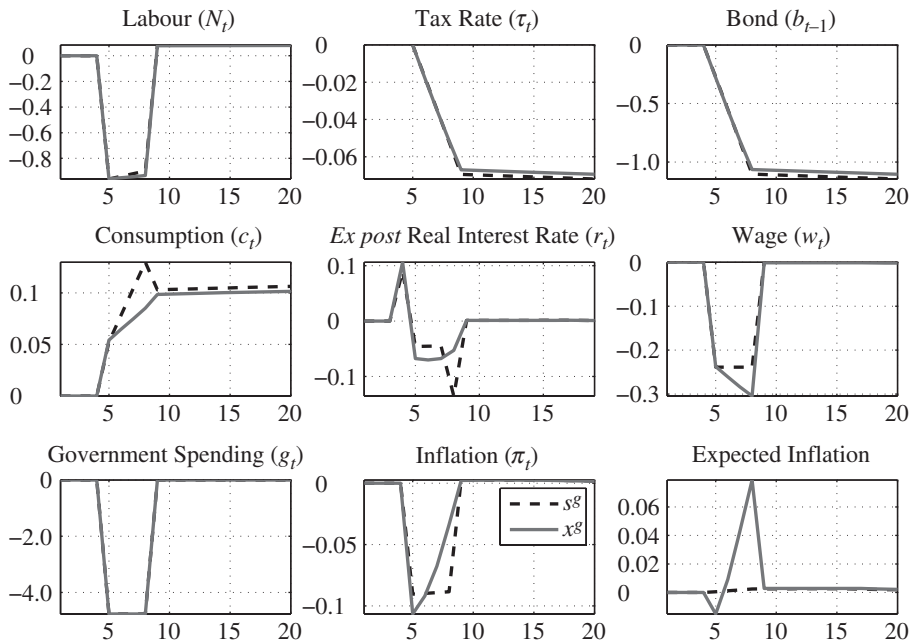


Fig. 6. Spending-based Consolidations at a High Debt Level

Notes. i.i.d. consolidation,  $s^g$ , and state-dependent consolidations,  $x^g$ , when the initial probability of consolidation is 0.75.

fiscal adjustment:  $f_i$  is  $(\tau_{t,y})$  for tax-based and  $(-g_t)$  for spending-based consolidations. The multiplier measures the discounted percentage change in cumulative output for one discounted unit of fiscal consolidation measure.

Figure 7 shows that at relatively low levels of initial debt, i.i.d. tax and government spending consolidations – labelled  $s^\tau$  and  $s^g$  – provide upper and lower bounds for the same-sized consolidations of a known duration. Not knowing the duration limits, the inflationary (deflationary) response to the tax (spending)-based fiscal consolidation which, in turn, affects the extent to which monetary policy raises (reduces) real interest rates during the consolidation. At high debt levels, however, tax-based consolidations of known duration outperform those of uncertain duration, while government spending-based consolidations perform in a similar way regardless of the duration uncertainty. The expansionary effect from the  $x^\tau$  model is due to the fact that the tax increase today reduces the need for future tax increases, which would otherwise have negative effects on current debt service costs and the tax base. This model retains the key message from the simple model of Section 3 over the timing of fiscal consolidations: a realised consolidation that reduces the expectation of higher future tax distortions mitigates the negative impact of the consolidation; such effects are associated with tax-based consolidations rather than spending-based ones.

### 7. Fiscal Consolidation: Timing and Composition Uncertainty

In practice, fiscal consolidations are uncertain both in their timing and their composition. We now consider the two sources of uncertainty jointly – a fiscal

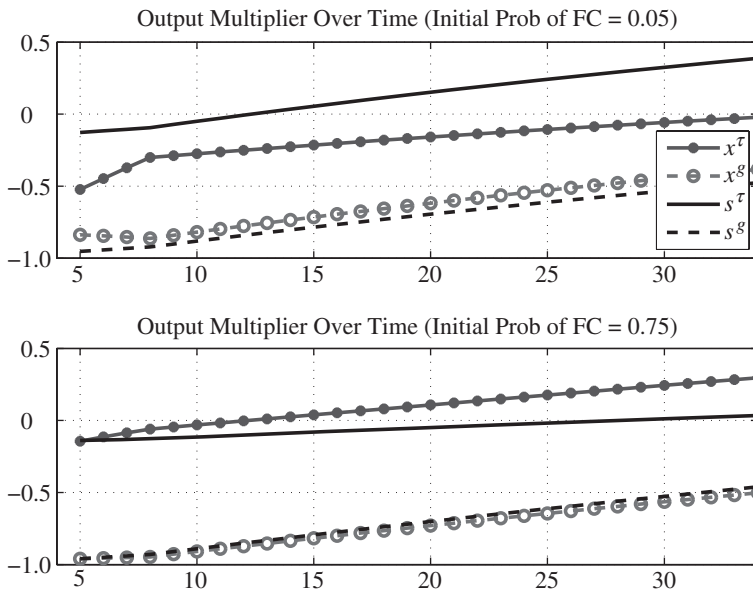


Fig. 7. Output Multiplier

Notes. i.i.d. consolidations,  $s^i$ , and state-dependent consolidations,  $x^i$ , for  $i = \tau, g$ , under different initial probabilities of consolidation.



consolidation can be based on tax increases with probability  $\omega$  and spending cuts with probability  $1 - \omega$ .

7.1. *Benchmark Case:  $\omega = 0.75$  and  $\alpha = 1.5$*

In line with the AA data, the probability  $\omega$  is calibrated to 0.75, so that a tax-based consolidation is three times more likely than a spending-based consolidation. Setting  $\alpha = 1.5$  makes monetary policy actively combat inflation in the manner that Taylor (1993) suggests.

Figure 8 compares the impulse responses for the two types of consolidations,  $x_\tau^m$  and  $x_g^m$ , when the initial probability of fiscal consolidation is low. There are few expectation effects beyond the fact that when a consolidation occurs, economic agents know it will last for 1 year. If the fiscal consolidation turns out to be tax based,  $x_\tau^m$ , the impulse responses are very similar to those observed when tax-based consolidations are the only possible type, the  $x^\tau$  model in Figure 2. Similarly, if the realised consolidation is spending based,  $x_g^m$ , then the impulse responses are very similar to the outcomes when spending is the only possible instrument, the  $x^g$  model in Figure 5. When the probability of fiscal consolidation is low, economic agents do not expect a consolidation of any kind, so uncertainty over the composition is not important.

In Figure 9, government debt is high and agents believe a fiscal consolidation is imminent. Now the composition uncertainty matters. Agents place a probability of 0.75

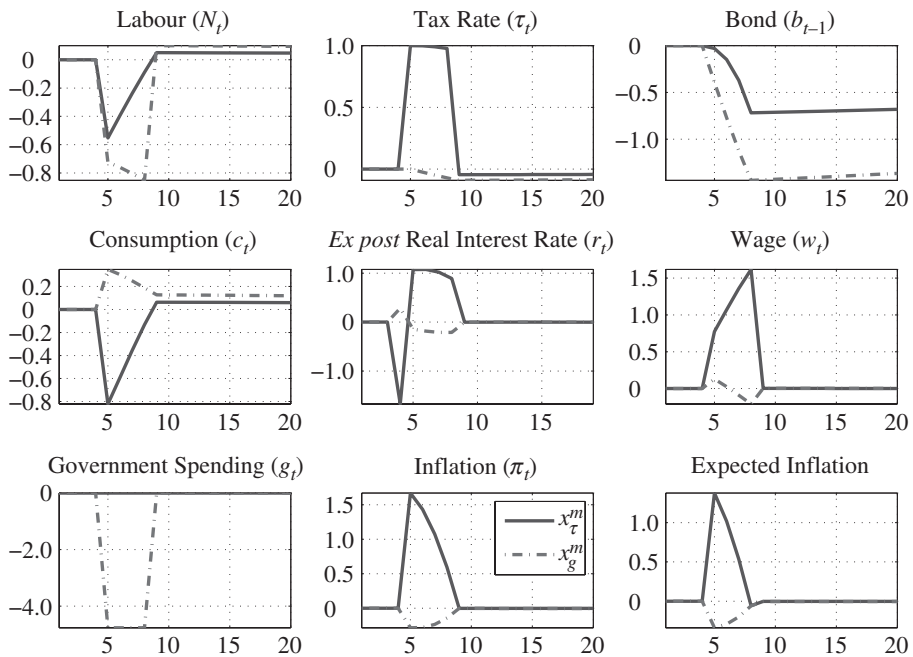


Fig. 8. *Uncertain-Composition Consolidations at a Low Debt Level*

Notes. Tax-based consolidation,  $x_\tau^m$ , and spending-based consolidation,  $x_g^m$ , when the initial probability of consolidation is 0.05.

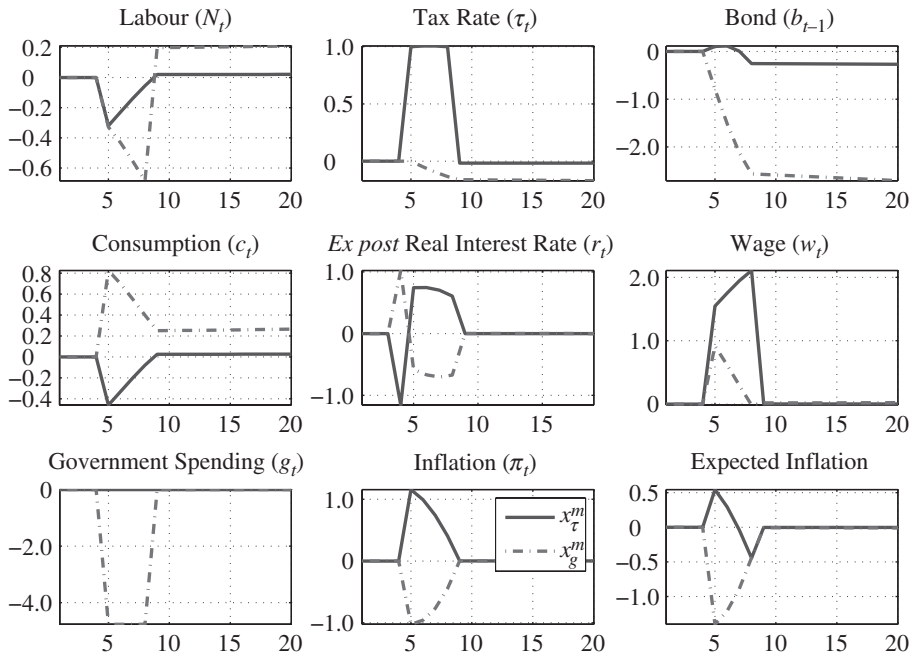


Fig. 9. *Uncertain-Composition Consolidations at a High Debt Level*

Notes. Tax-based consolidation,  $x_{\tau}^m$ , and spending-based consolidation,  $x_g^m$ , when the initial probability of consolidation is 0.75.

on tax increases, anchoring their expectations on inflationary increases in distortionary taxation prior to the consolidation. If a spending-based consolidation actually occurs,  $x_g^m$ , it surprises agents and reduces inflation relative to the no-consolidation case. The deflationary spending-based consolidation, together with active monetary policy, reduces real interest rates, raising the tax base and reducing debt service costs. Real wages rise relative to the no-consolidation case, and consumption rises significantly.

When the realised consolidations are the tax-based type,  $x_{\tau}^m$ , the results are qualitatively similar to the case without composition uncertainty,  $x^{\tau}$ , since tax increases were largely anticipated. During the fiscal consolidation, higher tax rates raise marginal costs and inflation, and active monetary policy raises real interest rates. This accounts for the relatively poor performance of the tax-based consolidations in stabilising debt when debt levels are high.

Figure 10 compares the output multiplier under the state-dependent consolidations with composition uncertainty,  $x_{\tau}^m$  and  $x_g^m$ , and tax increases and spending cuts in the  $x^{\tau}$  and  $x^g$  models. At low levels of debt, the two types of consolidations without composition uncertainty,  $x^{\tau}$  and  $x^g$ , provide bounds for the model with composition uncertainty,  $x_{\tau}^m$  and  $x_g^m$ . When debt levels are high, spending-based consolidations in the model with composition uncertainty,  $x_g^m$ , significantly outperform the same-sized consolidations in the  $x^g$  model. On the other hand, tax-based consolidations in the model with composition uncertainty,  $x_{\tau}^m$ , underperform tax increases in the  $x^{\tau}$  model. This is due to the expectation spill-over effect, as explained in the analytical model in Section 2. When economic agents fear that a consolidation is imminent and are

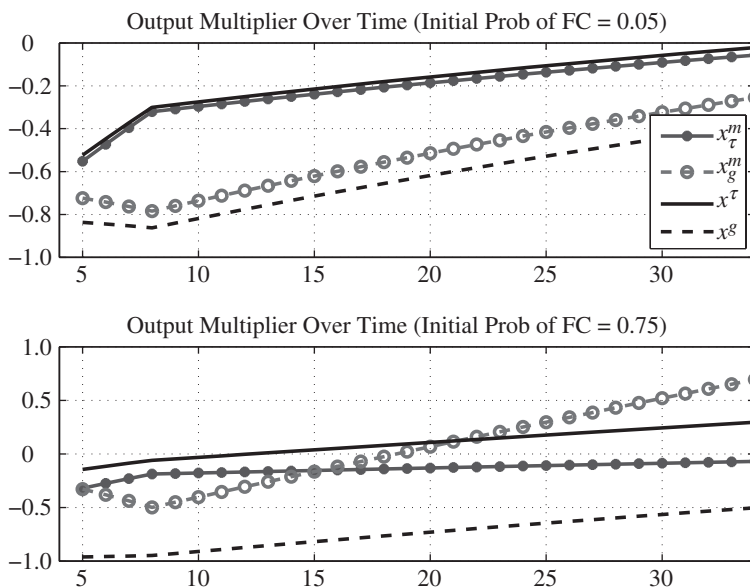


Fig. 10. Output Multipliers with Uncertain Composition

Notes. State-dependent consolidations,  $x^i$ , and state-dependent consolidations with composition uncertainty,  $x^m_i$ , for  $i = \tau, g$ , under different initial probabilities of consolidation.

expecting it to be tax-based, they are relieved to find it to be spending-based. While the spending cuts do not lead to an immediate increase in output, they significantly reduce the short-run costs and raise the medium-term to long-term benefits. In this sense, a spending-based consolidation can be expansionary.

If a tax-based consolidation is never expected, these expectation effects would not apply and the output multiplier from a spending-based consolidation would always be negative. In contrast, when there was some possibility that it could be spending-based but the realised consolidation is tax-based, the output costs rise. As we now discuss, this ranking could depend on the monetary policy stance (via  $\alpha$ ) and economic agents' expectations about the composition (via  $\omega$ ). These experiments are also informative about the likelihood of observing an expansionary fiscal consolidation as part of ongoing fiscal adjustments in developed economies, as the conclusion addresses.

7.2. Less Active Monetary Policy

In Figure 9, deflationary spending cuts facilitate relaxing monetary policy, which stabilises debt through its impact on the tax base and debt service costs. But when facing the higher inflation generated by tax-based consolidations, monetary policy raises the interest rates on government debt, which is particularly destabilising when debt levels are high. This reasoning suggests that the responsiveness of monetary policy to inflation is critical in determining the relative efficacy of the alternative types of fiscal consolidation.

If the initial probability of fiscal consolidation is 0.75, Figure 11 shows the impulse responses for the two types of fiscal consolidation when monetary policy is less active

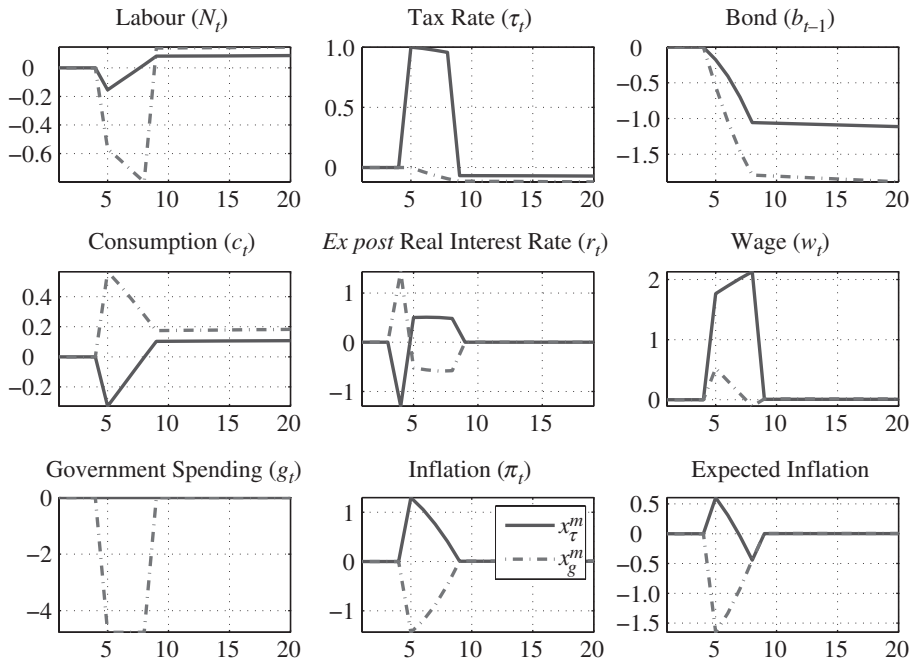


Fig. 11. *Less Active Monetary Policy*

Notes. Tax-based consolidation,  $x_{\tau}^m$ , and spending-based consolidation,  $x_g^m$ , when the initial probability of consolidation is 0.75.

( $\alpha = 1.2$ ). Comparing to the benchmark ( $\alpha = 1.5$ ) in Figure 9, a less active monetary policy deepens the recession under spending-based consolidations, reducing its ability to stabilise debt. Tax-based consolidations, though, are no longer thwarted by monetary policy: there is a more pronounced decline in debt following the tax-based consolidation. Nevertheless, spending-based consolidations remain relatively more effective in reducing the debt burden and this relative efficacy at high debt levels is likely to exist as long as monetary policy is active.<sup>17</sup>

Figure 12 plots the multipliers under the less active monetary policy. Tax increases become more expansionary, as the output multiplier turns positive upon the exit of fiscal consolidation, while spending cuts become more contractionary. In an environment when nominal interest rates are close to, or at, the zero lower bound, we are far more likely to observe economic expansions following tax-based rather than spending-based consolidations.

### 7.3. Lower Probability of Tax-based Consolidation

In our final experiment, we return to our benchmark monetary policy of  $\alpha = 1.5$ , but reverse the relative likelihood of tax-based and spending-based consolidations by

<sup>17</sup> Giavazzi and Pagano's (1990) case studies of Ireland and Denmark suggest, consistent with our mechanism, that there was a significant fall in inflation in the expansionary consolidations considered, which was not the case in the initial unsuccessful consolidation undertaken in Ireland.

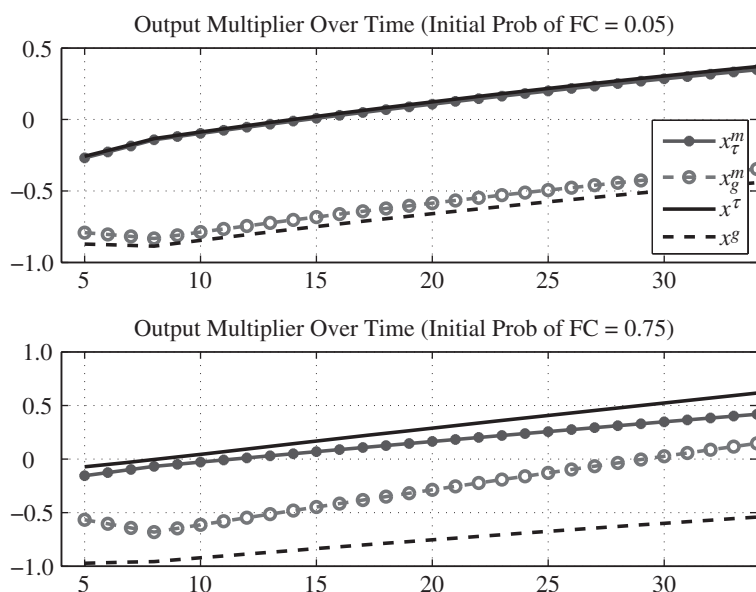


Fig. 12. *Output Multipliers with Less Active Monetary Policy*

Notes. State-dependent consolidations,  $x^i$ , and state-dependent consolidations with composition uncertainty,  $x_i^m$ , for  $i = \tau, g$ , under different initial probabilities of consolidation.

setting  $\omega = 0.25$ . Spending cuts are now three-times more likely than tax increases. This reversal makes negligible difference at low-debt levels since neither kind of consolidations is expected, but will matter at high-debt levels.

Figure 13 shows that when the relatively low probability tax-based consolidation is realised, inflation rises relative to the no-consolidation case and monetary policy raises real interest rates, reducing the tax base and fuelling debt service costs. Government debt rises relative to the no-consolidation case, undermining the stabilising effects in Figure 9. Spending-based consolidations remain relatively effective in stabilising debt, but become less expansionary than those observed in Figure 9.

## 8. Conclusions

We explored the non-linearities and expectation effects inherent in state-dependent fiscal consolidations. Three main policy implications emerge. First, quite restrictive conditions are required to generate expansionary fiscal consolidations in the medium term: a highly indebted economy operating under an active monetary policy, unexpectedly undertakes a spending-based fiscal consolidation when economic agents were confident that consolidations were going to be tax based. The nine large-scale fiscal consolidations contained in the IMF (2012) suggest that the current consolidation measures are predominantly spending based and that any revisions to consolidation plans have tended to shift the burden even further away from revenue raising measures, as electorates resist tax increases. The uniformity in the broad composition of current consolidation efforts negates the first condition for observing an expansionary fiscal consolidation.

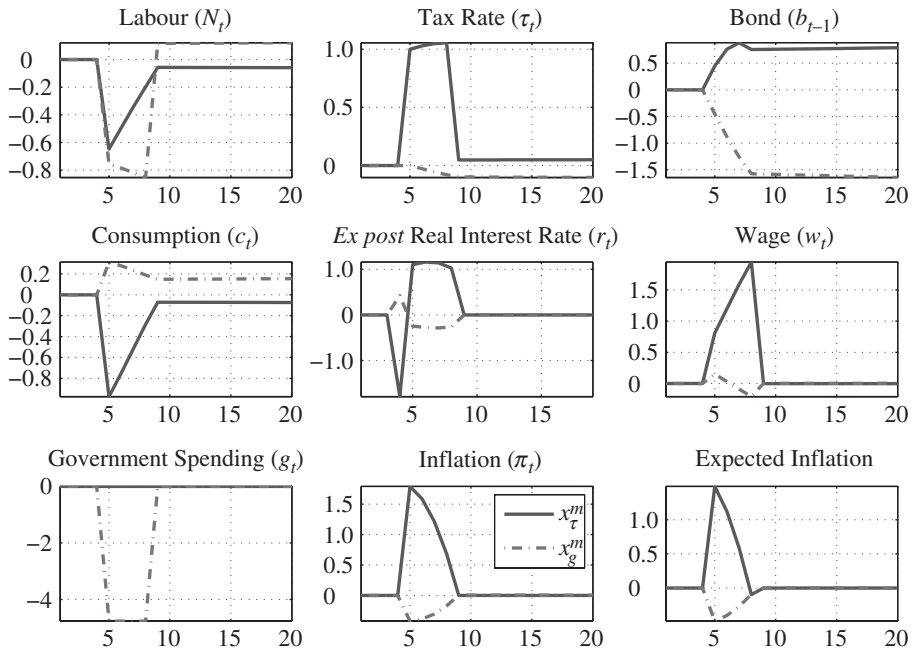


Fig. 13. Lower Probability of Tax-based versus Spending-based Consolidations

Notes. Tax-based consolidation,  $x_{\tau}^m$ , and spending-based consolidation,  $x_g^m$ , when the initial probability of consolidation is 0.75.BB.

Second, the possibility of observing ‘expansionary’ fiscal consolidations is driven by the favourable resolution of uncertainty associated with undesirable types of consolidation, either in terms of their composition or timing. Although we do not undertake a formal welfare analysis, we can conjecture that it is likely to be desirable for governments to remove such uncertainty as early as possible, since the possibility of undesirable consolidations acts as a drag on economic activity. As soon as the uncertainty is removed, however, the realised consolidation contains no new information, and an expansionary consolidation would not follow. In our model, ‘expansionary’ fiscal consolidations reflect a failure to rule out undesirable policy options sooner, rather than the adoption of an inherently expansionary policy.

Third, the inflationary consequences of alternative fiscal instruments and the monetary policy response to the inflation are very important in determining the outcomes. Tax-based and spending-based consolidations are fundamentally different in the inflation consequences in a sticky-price economy: distortionary taxation raises marginal costs and fuels inflation, while spending cuts are typically deflationary. How these different inflation responses affect debt service costs depends on the monetary policy response to inflation. Although we do not formally consider policy at the zero lower bound, that bound is a limiting case of the reduction in monetary policy activism we do consider. The fact that actual monetary policy is currently constrained at the zero lower bound means that it is far more difficult for monetary policy to offset the deflationary effects of a spending-based consolidation. This further decreases the likelihood of observing an expansionary consolidation.

Taken together, these points suggest that expansionary fiscal consolidations are unlikely to accompany ongoing consolidation efforts.

### Appendix A. Simulating the Fiscal Limit

The utility function is  $u(c_t, n_t) = \log c_t + \chi_n \log(1 - n_t)$ . Assuming the inflation rate is at its target, labour supply can be solved analytically as a function of  $(\tau_t, g_t)$  using the first-order conditions.

$$n_t = \frac{w_t(1 - \tau_t) + \chi_n g_t}{w_t(1 - \tau_t) + \chi_n A},$$

where  $w_t = (\theta - 1)/\theta A$ . The peak of Laffer curve,  $\tau_t^{max}$ , can be solved as,

$$\tau_t^{max}(g_t) = 1 + \chi_n \frac{A}{w_t} - \frac{\sqrt{\chi_n(w_t + \chi_n A)(A - g_t)}}{w_t}. \quad (\text{A.1})$$

The fiscal limit  $\mathcal{B}^*$  can be obtained using Markov chain Monte Carlo simulation:

- (i) for each simulation, we randomly draw the shocks of government purchases, and transfers for 1,500 periods. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables using the household first-order conditions and the budget constraints. According to (41), we compute the discounted sum of maximum fiscal surplus by discarding the first 500 draws as a burn-in period.
- (ii) we repeat the simulation for 100,000 times and obtain the distribution of the fiscal limit, which is then approximated through kernel density estimation.
- (iii) at each period of time, the effective fiscal limit,  $b_t^*$ , is a random draw from the distribution.

### Appendix B. Data

The fiscal data are from the OECD Economic Outlook No. 84 (2009) for the period between 1971 and 2007. The sample includes Austria, Belgium, Germany, Denmark, Spain, Finland, France, Greece, Ireland, Italy, the Netherlands, Norway, Sweden and the UK. The average tax rate is defined as the ratio of the total tax revenue over GDP, including social security, indirect and direct taxes. The government purchases are government final consumption of expenditures. Lump-sum transfers are defined as the sum of social security payments, net capital transfers and subsidies.

### Appendix C. Solving the Non-linear Model

The decision rules for government debt  $b_t = f^b(\psi_t)$ , real wage  $w_t = f^w(\psi_t)$  and inflation rate  $\pi_t = f^\pi(\psi_t)$ , are solved in the following steps:

- (i) discretise the state space  $\psi_t = \{b_{t-1}, z_t, x_t, x_t^z\}$  for  $x^i$  ( $i = \tau, g, m$ ) models, and  $\psi_t = \{b_{t-1}, \tau_t, g_t, z_t, x_t^z\}$  for  $s^i$  ( $i = \tau, g$ ) model, with grid points of  $n_b = 26, n_\tau = 17, n_g = 11, n_z = 11, n_x = 9, n_{xz} = 2$ . Make an initial guess of the decision rules  $(f_0^b, f_0^w, f_0^\pi)$  over the state space.
- (ii) at each grid point, solve the model and obtain the updated rule  $(f_t^b, f_t^w, f_t^\pi)$  using the given rule  $(f_{t-1}^b, f_{t-1}^w, f_{t-1}^\pi)$ . Other than the monetary and fiscal policy rules, the optimisation equations can be summarised:

$$\frac{1}{R_t} = \beta E_t \frac{u_c(t+1)}{u_c(t)} \frac{1}{\pi_{t+1}}, \quad (\text{C.1})$$

$$-\frac{u_n(t)}{u_c(t)} = w_t(1 - \tau_t), \quad (\text{C.2})$$

$$c_t + \frac{b_t}{R_t} = \frac{b_{t-1}}{\pi_t} + (1 - \tau_t)(w_t n_t + Y_t) + z_t, \quad (\text{C.3})$$

$$c_t + g_t = A n_t \left[ 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 \right], \quad (\text{C.4})$$

$$(1 - \theta) + \theta m c_t = \phi \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} - \beta \phi E_t \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{y_{t+1}}{y_t}. \quad (\text{C.5})$$

The integrals implied by the expectation terms on the right-hand side are evaluated using numerical quadratures.

- (iii) check convergence of the decision rules. If  $|f_i^b - f_{i-1}^b|$  or  $|f_i^w - f_{i-1}^w|$  or  $|f_i^\pi - f_{i-1}^\pi|$  is above the desired tolerance (set to  $1e - 7$ ), go back to step 2; otherwise,  $f_i^b$ ,  $f_i^w$  and  $f_i^\pi$  are the decision rules.

#### Appendix D. Dynamic Euler equation Accuracy Test

We evaluate the accuracy of numerical solutions using the dynamic Euler equation test proposed by Den Haan (2010). The idea is to compare a time series for consumption,  $c_t$ , that is constructed using the decision rule directly, with an alternative series,  $\tilde{c}_t$ , that is implied by the Euler equation and the budget constraint.

Take the  $s^t$  model for example. The construction of  $c_t$  is straightforward:

- (i) draw shocks on  $\tau_t$ ,  $g_t$  and  $x_t^z$  for  $T$  periods:  $\varepsilon_t^\tau \sim \mathcal{N}(0, \sigma_\tau^2)$ ,  $\varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$ , and  $u_t^{xz} \sim \mathcal{U}(0, 1)$  with  $t = 1, \dots, T$ .
- (ii) at each period  $t$ , construct the state variable at period  $t$ ,  $\psi_t = \{b_{t-1}, \tau_t, g_t, z_t, x_t^z\}$ , for the given shocks  $(\varepsilon_t^\tau, \varepsilon_t^g, u_t^{xz})$ , and the state variable at previous period,  $\psi_{t-1}$ .
- (iii) then use the decision rules to construct  $c_t = f^c(\psi_t)$  for the given state  $\psi_t$ , and also update the endogenous state,  $b_t = f^b(\psi_t)$ .
- (iv) repeat steps 2 and 3 until  $t = T$ .

For comparison, we use the same initial state  $b_0$  and shocks  $\varepsilon_t^\tau, \varepsilon_t^g, u_t^{xz}$  ( $t = 1, \dots, T$ ) to construct  $\tilde{c}_t$ :

- (i)  $\tilde{b}_0 = b_0, \tilde{z}_0 = z_0$ .
- (ii) at each period  $t$ , construct the state variable at period  $t$ ,  $\tilde{\psi}_t = \{\tilde{b}_{t-1}, \tilde{\tau}_t, \tilde{g}_t, \tilde{z}_t, \tilde{x}_t^z\}$ , for the given shocks  $(\varepsilon_t^\tau, \varepsilon_t^g, u_t^{xz})$ , and the state variable at previous period,  $\tilde{\psi}_{t-1}$ .
- (iii) use the decision rules to construct some temporary variables,  $\hat{b}_t, \hat{R}_t$  and  $\hat{\pi}_t$ ,

$$\hat{b}_t = f^b(\tilde{\psi}_t) \quad \hat{R}_t = f^R(\tilde{\psi}_t) \quad \hat{\pi}_t = f^\pi(\tilde{\psi}_t). \quad (\text{D.1})$$



(iv) also use the decision rule to construct  $\hat{c}_{t+1}$  for possible realisations for  $\varepsilon_{t+1}^\tau, \varepsilon_{t+1}^g, u_{t+1}^{xz}$ ,

$$\hat{c}_{t+1} = f^c(\tilde{\psi}_{t+1}).$$

(v) then compute the consumption  $\tilde{c}_t$  using the Euler equation,

$$\frac{1}{\tilde{c}_t} = \beta E_t \frac{1}{\hat{c}_{t+1}} \frac{\hat{R}_t}{\tilde{\pi}_t}. \tag{D.2}$$

(vi) use  $\tilde{c}_t$  and the government budget constraint to construct  $\tilde{b}_t$ ,

$$\tilde{b}_t = \hat{R}_t \left[ \frac{\tilde{b}_{t-1}}{\tilde{\pi}_t} + \tilde{g}_t + \tilde{z}_t - \tilde{\tau}_t(\tilde{g}_t + \tilde{c}_t) \right]. \tag{D.3}$$

(vii) go back to steps 2 and continue until  $t = T$ .

The  $x^i$  models follow the similar procedure, except the state space is  $\psi_t = \{b_{t-1}, z_t, x_t, x_t^s\}$ . The dynamic Euler equation error is then given by.

$$100 \left| \frac{c_t - \tilde{c}_t}{c_t} \right|, \tag{D.4}$$

and Table 3 reports the test results for all scenarios with  $T = 500$ . The errors, even the maximum errors, are low. For instance, the mean error for  $x^\tau$  case is 0.015%, implying that households make a 1.5 cent mistake for each \$100 dollars spent. It is interesting to observe that the endogenous regime-switching cases  $x^i$  ( $i = \tau, g, m$ ) feature larger errors than the i.i.d. shock case  $s^i$  ( $i = \tau, g$ ), even though  $x^i$  has one fewer state variables than  $s^i$ .

Table D3  
*Dynamic Euler Equation Tests*

	$x^\tau$	$x^g$	$x^m$	$s$
Average errors (%)	0.015	0.018	0.016	0.003
Maximum errors (%)	0.057	0.065	0.061	0.007

*Bank of Canada  
Indiana University, Monash University and NBER  
University of Glasgow*

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