Mixed L_-/L_1 fault detection filter design for fuzzy positive linear systems with time-varying delays

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1 Introduction

Over the past few decades, model-based fault detection and isolation have been of considerable interest [1–4]. The basic idea is to construct a residual signal and compare it with a predefined threshold. If the residual exceeds the threshold, an alarm is generated. It is well known that unknown inputs and control inputs are coupled in many industrial systems and are potential sources of false alarm. Thus fault detection and isolation systems have to be robust to unknown inputs and control inputs. Several approaches using the H_∞ norm techniques have been developed for the design of robust fault detection observers or filters [5–9]. It should be pointed out that the H_∞ norm measures the maximum effect of an input on an output, contrary to the main objective of fault detection.

Generally speaking, high sensitivity of the residual signal to faults (i.e., high fault sensitivity) is preferred. To ensure the detection of the worst possible faults, the minimum fault sensitivity must be maximised. Recently, the study on the smallest singular value of a transfer function matrix has attracted much attention and various H_- 'norms' have been defined by using the minimum 'non-zero' singular value, taken either at $\omega = 0$ [10], or over non-zero frequency ranges [11, 12]. The exclusion of possible zero singular values in the definition prevents it from being a true worst-case sensitivity measure. In [13–15], the definition of H_- 'norm' is extended to what is called the H_- index, which is defined as the minimum singular value of the transfer function matrix over a given frequency range. The inclusion of possible zero singular values in the definition renders the H_- index of a true worst-case sensitivity measure. In addition, mixed-norm fault detection problems have attracted a great deal of attention and various approaches and schemes have been proposed in the literature [16–19]. For example, Wang *et al.* [16] proposed a suboptimal solution to the H_-/H_∞ fault detection observer design problem. In [17], the H_-/H_∞ fault detection problem was considered and formulated as a quasi-linear matrix inequality formulation. Linear matrix inequality based sufficient conditions for the existence of the mixed H_-/H_∞ fault detection observers were proposed in [18, 19].

On another research front line, there has been increasing interest in the stability analysis and control problems of positive systems because of their significance both in theory and applications (see, for instance, [20–23] and references therein). Furthermore, because of the fact that time delays are one of the main causes of instability and poor performance of systems, some researchers have devoted their efforts to the study of positive systems with time delays [24, 25]. Some results on positive systems are based on linear Lyapunov functions. The motivation for using a linear Lyapunov function is that the state of a positive system is non-negative and hence a linear Lyapunov function serves as a valid candidate. As stated in [26], the results obtained with the use of linear Lyapunov functions are easier to analyse than the ones based on quadratic Lyapunov functions. Since the Lyapunov function is linear, there is no more relationship with the vector 2-norm and the L_2-norm as in the quadratic case, but rather with the vector 1-norm and the L_1-norm. This framework is then more suitable for the L_1-gain analysis of positive systems. Very recently, an L_1-induced performance index [26–29] has been proposed to characterise the disturbance attenuation property of positive systems.

It is well known that non-linearities exist widely in practical systems. Owing to the difficulty of non-linearity, some key results in linear positive systems cannot be applied to non-linear positive systems [30]. One of the main reasons might be the difficulty in modeling of the non-linearity. It is noted that the Takagi–Sugeno (T–S) fuzzy model [31, 32] has shed some light on this difficult problem, based on the fact that the T–S fuzzy model can approximate the smooth non-linear system on a compact set. This model formulates the non-linear systems into a framework consisting of a set of local models, which are smoothly connected by some membership functions. Based on the local linearity, the stability and performance analysis approaches for linear systems can be fully developed for non-linear systems in this framework [33]. Recently, many authors have focused their interest on fuzzy positive systems, and some results on the stability and stabilisation of fuzzy positive systems with and without delays have appeared in [34–39]. However, to the best of our knowledge, the fault detection problem of fuzzy positive systems has not been fully investigated to date, which constitutes the main motivation of the present study.

In this paper, we are interested in dealing with the fault detection filter design problem of fuzzy positive systems with time-varying delays by utilising the co-positive type Lyapunov–Krasovskii functional method. The main contributions of this paper can be summarised as follows: (i) a novel residual generator is constructed based on the filter, and an L_- index that fits well into a linear Lyapunov functional is proposed to measure the sensitivity of the residual signal to faults; (ii) based on the proposed L_- index, we give a sufficient condition under which the L_1-gain from faults to residuals is not less than a prescribed level; and (iii) by using the L_- index, we design a mixed L_-/L_1 fault detection filter such that the effect of disturbances on the residual output is minimised and the effect of faults on the residual output is maximised.

The rest of this paper is organised as follows. In Section 2, problem formulation and some necessary lemmas are given. In Section 3, robustness conditions on fault detection filter and L_- index fault sensitivity conditions are provided, respectively. Then based on the results above, the problems of the L_- fault detection filter design and the multi-objective L_-/L_1 fault detection filter design are solved. Two examples are provided to illustrate the proposed results in Section 4. Concluding remarks are given in Section 5.

Notations: In this paper, $A \succeq 0 (\preceq 0)$ means that all entries of matrix A are non-negative (non-positive); $A \succ 0 (\prec 0)$ means that all entries of matrix A are positive (negative); $A \succ B (A \succeq B)$ means that $A - B \succ 0 (A - B \succeq 0)$. A^{T} means the transpose of matrix A; $R (R_+)$ is the set of all real (positive real) numbers; $R^n (R^n_+)$ is the n-dimensional real (positive real) vector space; $R^{n\times m}$ is the set of all real matrices of dimension $n \times m$. The 1-norm of a vector $x \in R^n$ is defined as $\|x\|_1 = \sum_{k=1}^{n} |x_k|$, where x_k is the kth element of x. $L_1[t_0, \infty)$ is the space of absolute integrable vector-valued functions on $[t_0, \infty)$, that is, we say $z : [t_0, \infty) \to R^p$ is in $L_1[t_0, \infty)$ if $\int_{t_0}^{\infty} \|z(t)\|_1 \mathrm{d}t < \infty$. We denote $\mathbf{1} = [1, 1, \ldots, 1]^{\mathrm{T}}$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2 Problem formulation and preliminaries

Consider the following fuzzy model system described by the ith rule as follows:

Plant Rule i: **IF** $\theta_1(t)$ is M_{i1} and ... and $\theta_g(t)$ is M_{ig}, **THEN**

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + E_i w(t) + G_i f(t) \\ y(t) = C_i x(t) + C_{di} x(t - d(t)) + D_i u(t) + F_i w(t) + H_i f(t) \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^q$ is the measured output; and $u(t) \in R^l$, $w(t) \in R^p$, $f(t) \in R^z$ are the control input, disturbance input and the fault input, respectively, which belong to $L_1[0, \infty)$; r is the number of fuzzy IF–THEN rulers. $\theta_1(t), \theta_2(t), \ldots, \theta_g(t)$ are the premise variables. $M_{ik} (i = 1, 2, \ldots, r;\ k = 1, 2, \ldots, g)$ are the fuzzy sets. A_i, A_{di}, B_i, E_i, G_i, C_i, C_{di}, D_i, F_i and H_i, $i = 1, 2, \ldots, r$, are constant matrices with appropriate dimensions; $\varphi(t)$ is a vector-valued initial function defined on interval $[-\tau, 0]$, $\tau > 0$; $d(t)$ is the interval time-varying delay satisfying

$$0 \le d(t) \le \tau, \quad \dot{d}(t) \le \mu < 1 \tag{2}$$

where τ and μ are real positive constants.

Through the use of ‘fuzzy blending’, the final fuzzy system is inferred as follows (see (3))

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t)) \{A_i x(t) + A_{di} x(t - d(t)) + B_i u(t) + E_i w(t) + G_i f(t)\} \\ y(t) = \sum_{i=1}^{r} h_i(\theta(t)) \{C_i x(t) + C_{di} x(t - d(t)) + D_i u(t) + F_i w(t) + H_i f(t)\} \\ x(t) = \varphi(t), \quad t \in [-\tau, 0] \end{cases} \tag{3}$$

where $h_i(\theta(t)) = \mu_i(\theta(t)) / \sum_{i=1}^{r} \mu_i(\theta(t))$, $\mu_i(\theta(t)) = \prod_{k=1}^{g} M_{ik}(\theta_k(t))$, and $M_{ik}(\theta_k(t))$ is the degree of the membership of $\theta_k(t)$ in M_{ik}. $\mu_i(\theta(t)) \ge 0$ for $i = 1, 2, \ldots, r$, and $\sum_{i=1}^{r} \mu_i(\theta(t)) > 0$ for all t. Then $h_i(\theta(t)) \ge 0$ $(i = 1, 2, \ldots, r)$ and $\sum_{i=1}^{r} h_i(\theta(t)) = 1$.

Definition 1: System (3) is said to be positive if, for all $\varphi(t) \succeq 0$, $t \in [-\tau, 0]$, $u(t) \succeq 0$, $w(t) \succeq 0$ and $f(t) \succeq 0$, the state $x(t) \succeq 0$ and the output $y(t) \succeq 0$ for all $t \ge 0$.

Definition 2 [20]: A is called a Metzler matrix, if its off-diagonal entries are non-negative.

Lemma 1 [37]: System (3) is positive if A_i, $i = 1, 2, \ldots, r$, are Metzler matrices, and $A_{di} \succeq 0$, $B_i \succeq 0$, $E_i \succeq 0$, $G_i \succeq 0$, $C_i \succeq 0$, $C_{di} \succeq 0$, $D_i \succeq 0$, $F_i \succeq 0$ and $H_i \succeq 0$, $i = 1, 2, \ldots, r$.

Model-based fault detection relies on the generation of a residual, which must be sensitive to failures and able to distinguish failures from other unknown disturbance inputs. The design must ensure that residuals are close to zero in fault-free situations while clearly deviating from zero in the presence of faults. For the purpose of residual generation, the following fault detection filter is constructed as a residual generator.

Filter ruler i: **IF** $\theta_1(t)$ is M_{i1} and ... and $\theta_g(t)$ is M_{ig}, **THEN**

$$\begin{cases} \dot{\hat{x}}(t) = A_{fi}\hat{x}(t) + B_{fi}y(t) \\ r(t) = C_{fi}\hat{x}(t) + D_{fi}y(t) \end{cases} \tag{4}$$

where $\hat{x}(t) \in R^{n_f}$ and $r(t) \in R^m$ are the state and the residual output, respectively. A_{fi}, B_{fi}, C_{fi} and D_{fi}, $i = 1, 2, \ldots, r$, are the parameterised filter matrices to be determined, such that the following requirements are guaranteed:

(i) System (4) is asymptotically stable under fault-free conditions;
(ii) The effect of disturbances on the residual output is minimised;
(iii) The effect of faults on the residual output is maximised.

Remark 1: As a matter of fact, there exist two kinds of filters, that is, fuzzy-rule-independent filter and fuzzy-rule-dependent one. Generally speaking, the fuzzy-rule-dependent filter, because of the fact that it takes the fuzzy rules into account, is less conservative than the fuzzy-rule-independent one. In this paper, we consider the fuzzy-rule-dependent fault detection filter in the form of (4). Moreover, in the framework of positive fuzzy systems, it is necessary to construct a positive fuzzy-rule-dependent fault detection filter to generate a positive residual. Therefore it is also required that the designed filter (4) is positive, that is, A_{fi} is a Metzler matrix, $B_{fi} \succeq 0$, $C_{fi} \succeq 0$ and $D_{fi} \succeq 0$, $i = 1, 2, \ldots, r$.

Augmenting the model of positive system (3) to include the states of positive system (4), we can obtain the following augmented positive system (see (5))

$$\begin{cases} \dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\tilde{A}_{ij}\tilde{x}(t) + \tilde{A}_{dij}\tilde{x}(t-d(t)) + \tilde{E}_{ij}\tilde{w}(t) + \tilde{G}_{ij}f(t)\} \\ r(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{dij}\tilde{x}(t-d(t)) + \tilde{F}_{ij}\tilde{w}(t) + \tilde{H}_{ij}f(t)\} \end{cases} \tag{5}$$

where

$$\tilde{x}(t) = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}, \quad \tilde{w}(t) = \begin{bmatrix} u(t) \\ w(t) \end{bmatrix}$$

$$\tilde{A}_{ij} = \begin{bmatrix} A_i & 0 \\ B_{fj}C_i & A_{fj} \end{bmatrix}, \quad \tilde{A}_{dij} = \begin{bmatrix} A_{di} & 0 \\ B_{fj}C_{di} & 0 \end{bmatrix}$$

$$\tilde{E}_{ij} = \begin{bmatrix} B_i & E_i \\ B_{fj}D_i & B_{fj}F_i \end{bmatrix}, \quad \tilde{G}_{ij} = \begin{bmatrix} G_i \\ B_{fj}H_i \end{bmatrix}$$

$$\tilde{C}_{ij} = \begin{bmatrix} D_{fj}C_i & C_{fj} \end{bmatrix}$$

$$\tilde{C}_{dij} = \begin{bmatrix} D_{fj}C_{di} & 0 \end{bmatrix}, \quad \tilde{F}_{ij} = \begin{bmatrix} D_{fj}D_i & D_{fj}F_i \end{bmatrix}$$

$$\tilde{H}_{ij} = D_{fj}H_i, \quad i,j = 1, 2, \ldots, r$$

Definition 3 [28]: Given a positive scalar γ, system (5) is said to have an L_1-gain performance index γ, if under zero-initial condition, that is, $\varphi(t) = 0$, $t \in [-\tau, 0]$, it holds that

$$\sup_{\tilde{w}(t)\neq 0, f(t)=0} \frac{\int_0^\infty \|r(t)\|_1 \mathrm{d}t}{\int_0^\infty \|\tilde{w}(t)\|_1 \mathrm{d}t} < \gamma, \quad \tilde{w}(t) \in L_1[0, \infty) \tag{6}$$

Remark 2: It is interesting to point out that despite of being computed with the assumption of non-negative input signals belonging to $L_1[0, \infty)$ and non-negative state values, the determined L_1-gain index is valid for any input signals in $L_1[0, \infty)$ and any non-negative initial state.

Definition 4: Given a positive scalar β, system (5) is said to have an L_- performance index β, if under zero-initial condition, that is, $\varphi(t) = 0$, $t \in [-\tau, 0]$, the following inequality holds

$$\inf_{\tilde{w}(t)=0, f(t)\neq 0} \frac{\int_0^\infty \|r(t)\|_1 \mathrm{d}t}{\int_0^\infty \|f(t)\|_1 \mathrm{d}t} > \beta, \quad f(t) \in L_1[0, \infty) \tag{7}$$

Remark 3: Unlike the H_- index proposed in the literature [13–15], the L_- index is defined based on the L_1 signal spaces, and (7) means that the lower bound of the L_1-gain from faults to residuals for any fault signals in $L_1[0, \infty)$ is greater than β, which is contrary to L_1-gain index. Hence, the L_- index can be regarded as a measure of the fault sensitivity.

Definition 5: Given positive system (3), for two positive scalars γ and β, the filter (4) is said to be an L_-/L_1 fault detection filter if

1) System (5) is asymptotically stable when $\tilde{w}(t) = 0$ and $f(t) = 0$;
2) Under zero-initial condition, (6) and (7) hold.

The objectives considered in this paper are to find an admissible filter (4) to minimise γ and to maximise β.

L_- fault detection filter design: Given positive system (3) and a performance bound $\beta > 0$, find a stable fault detection filter in the form of (4), if exists, such that (7) is satisfied. Then the filter (4) is called an L_- fault detection filter.

Multi-objective L_-/L_1 fault detection filter design: Given positive system (3), find a stable L_-/L_1 fault detection filter, if exists, such that (6) and (7) hold and $\gamma - \beta$ is minimised.

Remark 4: Various mixed H_-/H_∞ performance ($\gamma^2 - \beta^2$, γ/β, etc.) criteria were proposed in [18, 19] using the H_- index. Here in this paper, we adopt the $\gamma - \beta$ criterion, using the L_- index, for ease of comparison.

After designing the residual generator, the remaining important task is to evaluate the generated residual. One of the widely adopted approaches is to select a threshold and a residual evaluation function. In this paper, the residual evaluation function is chosen as

$$J_r(T) = \int_0^T \|r(t)\|_1 \mathrm{d}t \tag{8}$$

where T is the evaluation time window.

Once the evaluation function has been selected, we are able to determine the threshold. It is reasonable to choose the threshold as

$$J_{\mathrm{th}} = \sup_{f(t)=0} J_{\mathrm{r}}(T) \tag{9}$$

Based on this, the faults can be detected by using the following logical relationship

$$J_{\mathrm{r}}(T) > J_{\mathrm{th}} \Rightarrow \text{with faults} \Rightarrow \text{alarm}$$
$$J_{\mathrm{r}}(T) \le J_{\mathrm{th}} \Rightarrow \text{no faults}$$

3 Main results

In this section, we will focus on the design of fault detection filter. In order to obtain the main results, we firstly consider the stability of system (5) with $\tilde{w}(t) = 0$ and $f(t) = 0$, that is

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\left\{\tilde{A}_{ij}\tilde{x}(t) + \tilde{A}_{dij}\tilde{x}(t-d(t))\right\} \tag{10}$$

Lemma 2: If there exist vectors v, $\upsilon \in R_+^{n+n_f}$ such that, for $i, j = 1, 2, \ldots, r$

$$\tilde{A}_{ij}^T v + \upsilon \prec 0 \tag{11}$$
$$\tilde{A}_{dij}^T v - (1-\mu)\upsilon \prec 0 \tag{12}$$

then system (10) is asymptotically stable.

Proof: Consider the following co-positive type Lyapunov–Krasovskii functional candidate

$$V(t) = \tilde{x}^T(t)v + \int_{t-d(t)}^{t} \tilde{x}^T(s)\upsilon \mathrm{d}s \tag{13}$$

where v, $\upsilon \in R_+^{n+n_f}$ are vectors to be determined.

Along the trajectory of system (10), we have

$$\dot{V}(t) \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\left\{\tilde{x}^T(t)\left(\tilde{A}_{ij}^T v + \upsilon\right) + \tilde{x}^T(t-\mathrm{d}(t))\left(\tilde{A}_{dij}^T v - (1-\mu)\upsilon\right)\right\} \tag{14}$$

It follows from (11), (12) and (14) that

$$\dot{V}(t) < 0 \tag{15}$$

Thus system (10) is asymptotically stable. The proof is completed.

3.1 Robustness conditions

In this subsection, the robustness requirement (6) of system (5) is considered. Let $f(t) = 0$ in (5), we have (see (16))

Based on Lemma 2, the following theorem presents a sufficient condition for the existence of L_1-gain performance for system (16).

Theorem 1: Given a positive constant γ, if there exist vectors v, $\upsilon \in R_+^{n+n_f}$ such that, for $i, j = 1, 2, \ldots, r$

$$\tilde{A}_{ij}^T v + \upsilon + \tilde{C}_{ij}^T \mathbf{1} \prec 0 \tag{17}$$
$$\tilde{A}_{dij}^T v - (1-\mu)\upsilon + \tilde{C}_{dij}^T \mathbf{1} \prec 0 \tag{18}$$
$$\tilde{E}_{ij}^T v + \tilde{F}_{ij}^T \mathbf{1} - \gamma \mathbf{1} \prec 0 \tag{19}$$

then system (16) is asymptotically stable with a prescribed L_1-gain performance index γ.

Proof: According to Lemma 2, we obtain from (17) and (18) that system (16) is asymptotically stable when $\tilde{w}(t) \equiv 0$. In the sequel, we shall prove that the L_1-gain performance of system (16) is satisfied for all non-zero $\tilde{w}(t) \in L_1[0, \infty)$ under zero-initial condition (see (20))

$$\begin{cases} \dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\tilde{A}_{ij}\tilde{x}(t) + \tilde{A}_{dij}\tilde{x}(t-d(t)) + \tilde{E}_{ij}\tilde{w}(t)\} \\ r(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{dij}\tilde{x}(t-d(t)) + \tilde{F}_{ij}\tilde{w}(t)\} \end{cases} \tag{16}$$

$$\begin{aligned} \|r(t)\|_1 - \gamma\|\tilde{w}(t)\|_1 &= \left\| \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\tilde{C}_{ij}\tilde{x}(t) + \tilde{C}_{dij}\tilde{x}(t-d(t)) + \tilde{F}_{ij}\tilde{w}(t)\} \right\|_1 - \gamma\|\tilde{w}(t)\|_1 \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\|\tilde{C}_{ij}\tilde{x}(t)\|_1 + \|\tilde{C}_{dij}\tilde{x}(t-d(t))\|_1 + \|\tilde{F}_{ij}\tilde{w}(t)\|_1 - \gamma\|\tilde{w}(t)\|_1\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\{\tilde{x}^T(t)\tilde{C}_{ij}^T\mathbf{1} + \tilde{x}^T(t-d(t))\tilde{C}_{dij}^T\mathbf{1} + \tilde{w}^T(t)(\tilde{F}_{ij}^T\mathbf{1} - \gamma\mathbf{1})\} \end{aligned} \tag{20}$$

Consider the Lyapunov–Krasovskii functional candidate (13), then along the trajectory of system (16), one obtains

$$\begin{aligned}&\dot{V}(t)+\|r(t)\|_1-\gamma\|\tilde{w}(t)\|_1\\&\quad\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t))\{\tilde{x}^T(t)(\tilde{A}_{ij}^T v+\upsilon+\tilde{C}_{ij}^T\mathbf{1})\\&\qquad+\tilde{x}^T(t-d(t))(\tilde{A}_{dij}^T v-(1-\mu)\upsilon+\tilde{C}_{dij}^T\mathbf{1})\\&\qquad+\tilde{w}^T(t)(\tilde{E}_{ij}^T v+\tilde{F}_{ij}^T\mathbf{1}-\gamma\mathbf{1})\}\end{aligned}\tag{21}$$

From (17)–(19), we have

$$\dot{V}(t)+\|r(t)\|_1-\gamma\|\tilde{w}(t)\|_1\le 0\tag{22}$$

Under zero-initial condition, integrating both sides of (22) from 0 to ∞ leads to

$$\int_0^{\infty}\|r(t)\|_1\mathrm{d}t\le\gamma\int_0^{\infty}\|\tilde{w}(t)\|_1\mathrm{d}t\tag{23}$$

Thus, (6) in Definition 3 is satisfied.

This completes the proof.

Remark 5: It should be noticed that the stability conditions (11) and (12) are implied in (17)–(19). Thus the filter is stable if (17)–(19) are satisfied. Moreover, in the derivation of Theorem 1, the co-positive type Lyapunov–Krasovskii functional is employed for the robustness performance analysis, which makes it easier to analyse the obtained results.

3.2 L_- index fault sensitivity conditions

Here, we study the fault sensitivity condition (7). Let $\tilde{w}(t)=0$ in (5), one has (see (24))

In the following, a sufficient condition is provided for system (24) to have a prescribed L_- fault sensitivity index β.

Theorem 2: Given a positive scalar β, if there exist vectors v_s, $\upsilon_s\in R_+^{n+n_f}$ such that, for $i,j=1,2,\dots,r$

$$\tilde{A}_{ij}^T v_s+\upsilon_s-\tilde{C}_{ij}^T\mathbf{1}\prec 0\tag{25}$$

$$\tilde{A}_{dij}^T v_s-(1-\mu)\upsilon_s-\tilde{C}_{dij}^T\mathbf{1}\prec 0\tag{26}$$

$$\tilde{G}_{ij}^T v_s-\tilde{H}_{ij}^T\mathbf{1}+\beta\mathbf{1}\prec 0\tag{27}$$

then system (24) has a prescribed L_- fault sensitivity index β.

Proof: Consider the following Lyapunov–Krasovskii functional candidate

$$V(t)=\tilde{x}^T(t)v_s+\int_{t-d(t)}^{t}\tilde{x}^T(s)\upsilon_s\mathrm{d}s$$

where v_s, $\upsilon_s\in R_+^{n+n_f}$ are vectors to be determined.

Similar to the proof of Theorem 1, we can obtain

$$\begin{aligned}&\|r(t)\|_1-\beta\|f(t)\|_1\\&\quad\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t))\{\tilde{x}^T(t)\tilde{C}_{ij}^T\mathbf{1}\\&\qquad+\tilde{x}^T(t-d(t))\tilde{C}_{dij}^T\mathbf{1}+f^T(t)(\tilde{H}_{ij}^T\mathbf{1}-\beta\mathbf{1})\}\end{aligned}\tag{28}$$

Combining (14) and (28) yields

$$\begin{aligned}&\beta\|f(t)\|_1-\|r(t)\|_1+\dot{V}(t)\\&\quad\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t))\{\tilde{x}^T(t)(\tilde{A}_{ij}^T v_s+\upsilon_s-\tilde{C}_{ij}^T\mathbf{1})\\&\qquad+\tilde{x}^T(t-d(t))(\tilde{A}_{dij}^T v_s-(1-\mu)\upsilon_s-\tilde{C}_{dij}^T\mathbf{1})\\&\qquad+f^T(t)(\tilde{G}_{ij}^T v_s-\tilde{H}_{ij}^T\mathbf{1}+\beta\mathbf{1})\}\end{aligned}\tag{29}$$

It can be obtained from (25)–(27) that

$$\beta\|f(t)\|_1-\|r(t)\|_1+\dot{V}(t)<0\tag{30}$$

Integrating both sides of (30) from 0 to ∞ leads to

$$\beta\int_0^{\infty}\|f(t)\|_1\mathrm{d}t-\int_0^{\infty}\|r(t)\|_1\mathrm{d}t<V(0)-V(\infty)\tag{31}$$

Noting that $V(\infty)\ge 0$ and $V(0)=0$, (7) is directly obtained.

The proof is completed.

Remark 6: Unlike the L_1-gain performance analysis problem, although vectors v_s and υ_s are positive, these two vectors do not guarantee the negativeness of the chosen Lyapunov–Krasovskii functional. Hence, the conditions in (25)–(27) do not ensure a stable filter.

3.3 Design of L_- fault detection filter

Let us consider the L_- fault detection filter design problem. Because the L_- index measure requires no stability and (25)–(27) do not always provide a stable solution, we should consider the stability of a proposed fault detection filter in the design process, that is, the existence of $v_s=v\in R_+^{n+n_f}$ and $\upsilon_s=\upsilon\in R_+^{n+n_f}$ such that (11) and (12) hold. Note that (25) and (26) hold if (11) and (12) are satisfied. Then (11), (12) and (27) provide a solution to the L_- fault detection filter design problem.

Theorem 3: Consider positive system (3), for a given positive scalar β, if there exist vectors $v_s=v\in R_+^{n+n_f}$ and $\upsilon_s=\upsilon\in R_+^{n+n_f}$ such that (11), (12) and (27) hold, then there exists a stable L_- fault detection filter in the form of (4).

In what follows, the L_- fault detection filter design procedure is provided.

$$\begin{cases}\dot{\tilde{x}}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t))\{\tilde{A}_{ij}\tilde{x}(t)+\tilde{A}_{dij}\tilde{x}(t-d(t))+\tilde{G}_{ij}f(t)\}\\ r(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(t))\{\tilde{C}_{ij}\tilde{x}(t)+\tilde{C}_{dij}\tilde{x}(t-d(t))+\tilde{H}_{ij}f(t)\}\end{cases}\tag{24}$$

Theorem 4: Consider positive system (3), for a given positive scalar β, there exists a stable L_- fault detection filter in the form of (4) if there exist vectors $\nu_1 \in R^n_+$, $\nu_2 \in R^{n_f}_+$, $\upsilon_1 \in R^n_+$, $\upsilon_2 \in R^{n_f}_+$, $\rho_{1j} \in R^q_+$ and $\rho_{2j} \in R^{n_f}$, and matrices $K_j \succeq 0$ with appropriate dimensions, such that, for $i, j = 1, 2, \ldots, r$

$$A_i^T \nu_1 + C_i^T \rho_{1j} + \upsilon_1 \prec 0 \tag{32}$$

$$A_{di}^T \nu_1 + C_{di}^T \rho_{1j} - (1-\mu)\upsilon_1 \prec 0 \tag{33}$$

$$\rho_{2j} + \upsilon_2 \prec 0 \tag{34}$$

$$G_i^T \nu_1 + H_i^T \rho_{1j} - H_i^T K_j^T \mathbf{1} + \beta \mathbf{1} \prec 0 \tag{35}$$

Moreover, if the conditions above have a feasible solution, the filter parameters can be constructed by

$$\rho_{2j} = A_{fj}^T \nu_2, \quad \rho_{1j} = B_{fj}^T \nu_2, \quad D_{fj} = K_j \tag{36}$$

Proof: Denote $\nu = \begin{bmatrix} \nu_1^T & \nu_2^T \end{bmatrix}^T$ and $\upsilon = \begin{bmatrix} \upsilon_1^T & \upsilon_2^T \end{bmatrix}^T$, then substituting these vectors and the parameters of system (5) into (11), (12) and (27), we can obtain from Theorem 3 that the theorem is true.

Remark 7: It can be seen that C_{fj} can be any vectors satisfying $C_{fj} \succeq 0$ because of the fact that there is no additional constraint on C_{fj} in the conditions (32)–(35). Furthermore, (32)–(34) ensure the stability of the filter.

3.4 Design of L_-/L_1 fault detection filter

In this subsection, we study the mixed L_-/L_1 fault detection filter design. The following theorem gives a solution to the multi-objective design problem.

Theorem 5: Consider positive system (3), for given positive scalars β and γ, if there exist vectors $\nu_s = \nu \in R_+^{n+n_f}$ and $\upsilon_s = \upsilon \in R_+^{n+n_f}$ such that (17)–(19) and (27) hold, then there exists a stable L_-/L_1 fault detection filter in the form of (4).

Proof: Note that (25) and (26) can be directly obtained from (17) and (18) for $\nu_s = \nu \in R_+^{n+n_f}$ and $\upsilon_s = \upsilon \in R_+^{n+n_f}$. Thus the theorem holds.

The proof is completed.

The L_-/L_1 fault detection filter design procedure is given in the following.

Theorem 6: Consider positive system (3), for given positive scalars β and γ, there exists a stable L_-/L_1 fault detection filter in the form of (4) if there exist vectors $\nu_1 \in R^n_+$, $\nu_2 \in R^{n_f}_+$, $\upsilon_1 \in R^n_+$, $\upsilon_2 \in R^{n_f}_+$, $\rho_{1j} \in R^q_+$ and $\rho_{2j} \in R^{n_f}$, and matrices $K_{1j} \succeq 0$ and $K_{2j} \succeq 0$ with appropriate dimensions, such that, for $i, j = 1, 2, \ldots, r$

$$A_i^T \nu_1 + C_i^T \rho_{1j} + \upsilon_1 + C_i^T K_{2j}^T \mathbf{1} \prec 0 \tag{37}$$

$$A_{di}^T \nu_1 + C_{di}^T \rho_{1j} - (1-\mu)\upsilon_1 + C_{di}^T K_{2j}^T \mathbf{1} \prec 0 \tag{38}$$

$$\rho_{2j} + \upsilon_2 + K_{1j}^T \mathbf{1} \prec 0 \tag{39}$$

$$B_i^T \nu_1 + D_i^T \rho_{1j} + D_i^T K_{2j}^T \mathbf{1} - \gamma \mathbf{1} \prec 0 \tag{40}$$

$$E_i^T \nu_1 + F_i^T \rho_{1j} + F_i^T K_{2j}^T \mathbf{1} - \gamma \mathbf{1} \prec 0 \tag{41}$$

$$G_i^T \nu_1 + H_i^T \rho_{1j} - H_i^T K_{2j}^T \mathbf{1} + \beta \mathbf{1} \prec 0 \tag{42}$$

Moreover, if the conditions above have a feasible solution, the filter parameters can be constructed by

$$\rho_{2j} = A_{fj}^T \nu_2, \quad \rho_{1j} = B_{fj}^T \nu_2, \quad C_{fj} = K_{1j}, \text{ and } D_{fj} = K_{2j} \tag{43}$$

Proof: Denote $\nu = \begin{bmatrix} \nu_1^T & \nu_2^T \end{bmatrix}^T$ and $\upsilon = \begin{bmatrix} \upsilon_1^T & \upsilon_2^T \end{bmatrix}^T$, then substituting these vectors and the parameters of system (5) into (17)–(19) and (27), it can be obtained that the theorem is true.

A solution to the mixed L_-/L_1 fault detection filter design problem can be obtained by solving the following optimisation problem

$$\textbf{Problem 1} \quad \min_{\substack{\nu_1,\nu_2,\upsilon_1,\upsilon_2,\\ \rho_{1j},\rho_{2j},K_{1j},K_{2j}}} \gamma - \beta$$

$$\text{s.t. (37)–(42)}, \quad i, j = 1, 2, \ldots, r$$

then the optimal filter can be obtained from (43).

4 Examples

In this section, two examples are presented to check the validity of the proposed results.

Example 1: Consider system (1) with parameters as follows

$$A_1 = \begin{bmatrix} -4 & 3 \\ 2.5 & -3 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.14 & 0 \\ 0.1 & 0.12 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.3 \\ 0.14 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.24 \\ 0.12 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.5 \\ 0.4 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.12 & 0.13 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.2 & 0.15 \end{bmatrix}$$

$$D_1 = 0.12, \quad F_1 = 0.24, \quad H_1 = 0.35$$

$$A_2 = \begin{bmatrix} -6 & 1 \\ 2.4 & -5 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.12 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.15 \\ 0.16 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.4 \\ 0.23 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.3 & 0.25 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0.14 & 0.24 \end{bmatrix}, \quad D_2 = 0.5$$

$$F_2 = 0.25, \quad H_2 = 0.2, \quad \mu = 0.2, \quad \tau = 0.5$$

Take $\beta = 1$, then solving (32)–(35) in Theorem 4 gives rise to

$$\nu_1 = \begin{bmatrix} 1.2634 \\ 1.6063 \end{bmatrix}, \quad \nu_2 = \begin{bmatrix} 2.6331 \\ 2.6331 \end{bmatrix}, \quad \upsilon_1 = \begin{bmatrix} 0.9602 \\ 0.9317 \end{bmatrix}$$

$$\upsilon_2 = \begin{bmatrix} 2.6331 \\ 2.6331 \end{bmatrix}$$

$$\rho_{11} = 0.4206, \quad \rho_{21} = \begin{bmatrix} -5.2662 \\ -5.2662 \end{bmatrix}, \quad \rho_{12} = 0.4206$$

$$\rho_{22} = \begin{bmatrix} -5.2662 \\ -5.2662 \end{bmatrix}, \quad K_1 = 10.4314, \quad K_2 = 13.9762$$

From (36), the desired L_- fault detection filter can be obtained with the parameterised matrices as follows

$$\left[\begin{array}{c|c} A_{f1} & B_{f1} \\ \hline C_{f1} & D_{f1} \end{array}\right] = \left[\begin{array}{cc|c} -4 & 1.5 & 0.0799 \\ 2 & -3.5 & 0.0799 \\ \hline / & / & 10.4314 \end{array}\right]$$

$$\left[\begin{array}{c|c} A_{f2} & B_{f2} \\ \hline C_{f2} & D_{f2} \end{array}\right] = \left[\begin{array}{cc|c} -4.5 & 2 & 0.0799 \\ 2.5 & -4 & 0.0799 \\ \hline / & / & 13.9762 \end{array}\right]$$

where C_{f1} and C_{f2} can be designed as any vectors satisfying $C_{f1} \succeq 0$ and $C_{f2} \succeq 0$.

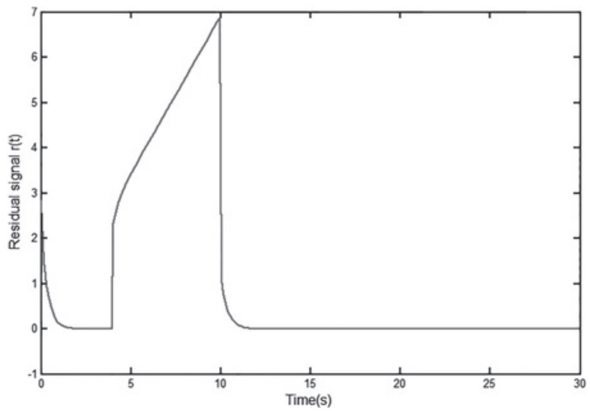

Fig. 1 *Residual signal in Example 1*

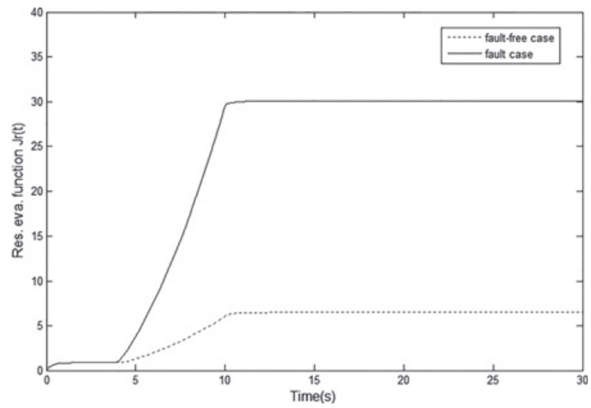

Fig. 2 *Evolution of residual evaluation function in Example 1*

In this example, the initial conditions are as follows: $x(0) = [0.3\ 0.5\ 0\ 0]^{\mathrm{T}}$, $x(t) = [0\ 0\ 0\ 0]^{\mathrm{T}}$, $t \in [-0.5, 0)$.

The control input $u(t)$ and the external disturbance are zeros. The fault signal $f(t)$ is set up as

$$f(t) = \begin{cases} 0.2t, & 4 \leq t \leq 10 \\ 0, & \text{others} \end{cases}$$

The generated residual $r(t)$ is shown in Fig. 1. The threshold can be determined as $J_{th} = 6.5$ for $t = 30$ s. Fig. 2 shows the evolution of residual evaluation function $J_r(t)$, in which the dashed line is fault-free case, and the solid line is the case with the fault $f(t)$. The simulation results show that $J_r(t) = 7 > 6.5$ when $t = 6$ s, which means that the fault $f(t)$ can be detected 2 s after its occurrence.

Example 2: Consider the following continuous-time nonlinear positive system with delay (see equation at the bottom of the page)

where $0 \leq d(t) \leq 0.4$ and $\dot{d}(t) \leq 0.4$.

Let $z(t) = \sin^2(x_1(t))$ and $x(t) = \left[x_1^T(t) \quad x_2^T(t)\right]^{\mathrm{T}}$. Its fuzzy model can be represented as follows:

Rule 1: IF $z(t)$ **is about** 0, **THEN**

$$\dot{x}(t) = A_1x(t) + A_{d1}x(t - d(t)) + B_1u(t) + E_1w(t) + G_1f(t)$$
$$y(t) = C_1x(t) + C_{d1}x(t - d(t)) + D_1u(t) + F_1w(t) + H_1f(t)$$

Rule 2: IF $z(t)$ **is about** 1, **THEN**

$$\dot{x}(t) = A_2x(t) + A_{d2}x(t - d(t)) + B_2u(t) + E_2w(t) + G_2f(t)$$
$$y(t) = C_2x(t) + C_{d2}x(t - d(t)) + D_2u(t) + F_2w(t) + H_2f(t)$$

For the convenience of simulation, the normalised membership functions $h_1(t) = \sin^2(x_1(t))$ and $h_2(t) = 1 - \sin^2(x_1(t))$ are used for Rules 1 and 2 in this example. Then state-space matrices of system (3) in form of fuzzy model are given as follows

$$A_1 = \begin{bmatrix} -4 & 0.3 \\ 0.5 & -3 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.14 & 0 \\ 0.1 & 0.12 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.3 \\ 0.14 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.24 \\ 0.12 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.12 & 0.13 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.2 & 0.15 \end{bmatrix}$$

$$\begin{aligned}
\dot{x}_1(t) = {} & -4x_1(t) - 2\sin^2(x_1(t))x_1(t) + 0.3x_2(t) + 0.7\sin^2(x_1(t))x_2(t) \\
& + 0.14x_1(t - d(t)) - 0.04\sin^2(x_1(t))x_1(t - d(t)) + 0.1\sin^2(x_1(t))x_2(t - d(t)) \\
& + 0.3u(t) + 0.05\sin^2(x_1(t))u(t) + 0.4w(t) \\
& - 0.16\sin^2(x_1(t))w(t) + 0.1f(t) + 0.02\sin^2(x_1(t))f(t) \\
\dot{x}_2(t) = {} & 0.5x_1(t) + 1.9\sin^2(x_1(t))x_1(t) - 3x_2(t) - 2\sin^2(x_1(t))x_2(t) \\
& + 0.1x_1(t - d(t)) - 0.1\sin^2(x_1(t))x_1(t - d(t)) + 0.12x_2(t - d(t)) \\
& + 0.14u(t) + 0.02\sin^2(x_1(t))u(t) + 0.12w(t) \\
& + 0.11\sin^2(x_1(t))w(t) + 0.2f(t) - 0.1\sin^2(x_1(t))f(t) \\
y(t) = {} & 0.12x_1(t) + 0.18\sin^2(x_1(t))x_1(t) + 0.13x_2(t) + 0.12\sin^2(x_1(t))x_2(t) \\
& + 0.2x_1(t - d(t)) - 0.06\sin^2(x_1(t))x_1(t - d(t)) + 0.15x_2(t - d(t)) \\
& + 0.09\sin^2(x_1(t))x_2(t - d(t)) + 0.12u(t) + 0.38\sin^2(x_1(t))u(t) + 0.24w(t) \\
& + 0.01\sin^2(x_1(t))w(t) + 0.35f(t) - 0.15\sin^2(x_1(t))f(t)
\end{aligned}$$

$$D_1 = 0.12, \quad F_1 = 0.24, \quad H_1 = 0.35$$

$$A_2 = \begin{bmatrix} -6 & 1 \\ 2.4 & -5 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.12 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0.35 \\ 0.16 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.4 \\ 0.23 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.12 \\ 0.1 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 0.3 & 0.25 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0.14 & 0.24 \end{bmatrix}, \quad D_2 = 0.5$$

$$F_2 = 0.25, \quad H_2 = 0.2, \quad \mu = 0.4, \quad \tau = 0.4$$

Solving the optimisation Problem 1 gives rise to

$$v_1 = \begin{bmatrix} 0.9056 \\ 0.9386 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0.7025 \\ 0.7025 \end{bmatrix}, \quad \upsilon_1 = \begin{bmatrix} 2.1543 \\ 2.0704 \end{bmatrix}$$

$$\upsilon_2 = \begin{bmatrix} 0.7025 \\ 0.7025 \end{bmatrix}$$

$$\rho_{11} = 0.0639, \quad \rho_{21} = \begin{bmatrix} -2.1076 \\ -2.1076 \end{bmatrix}, \quad \rho_{12} = 0.0639$$

$$\rho_{22} = \begin{bmatrix} -2.1076 \\ -2.1076 \end{bmatrix}, \quad K_{11} = 2.1306, \quad K_{12} = 2.1306$$

$$K_{21} = \begin{bmatrix} 0.7025 & 0.7025 \end{bmatrix}, \quad K_{22} = \begin{bmatrix} 0.7025 & 0.7025 \end{bmatrix}$$

$$\beta = 0.2066, \quad \gamma = 1.7107$$

From (43), the parameters of the designed optimal filter can be obtained

$$\left[\begin{array}{c|c} A_{f1} & B_{f1} \\ \hline C_{f1} & D_{f1} \end{array}\right] = \left[\begin{array}{cc|c} -4 & 2 & 0.0454 \\ 1 & -5 & 0.0454 \\ \hline 0.7025 & 0.7025 & 2.1306 \end{array}\right]$$

$$\left[\begin{array}{c|c} A_{f2} & B_{f2} \\ \hline C_{f2} & D_{f2} \end{array}\right] = \left[\begin{array}{cc|c} -3.8 & 1.3 & 0.0227 \\ 0.8 & -4.3 & 0.0681 \\ \hline 0.7025 & 0.7025 & 2.1306 \end{array}\right]$$

In this example, the external disturbance and the initial state are as follows

$$w(t) = 0.1e^{-0.04t}, \quad x(0) = \begin{bmatrix} 0.3 & 0.5 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$x(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \quad t \in [-0.4, 0)$$

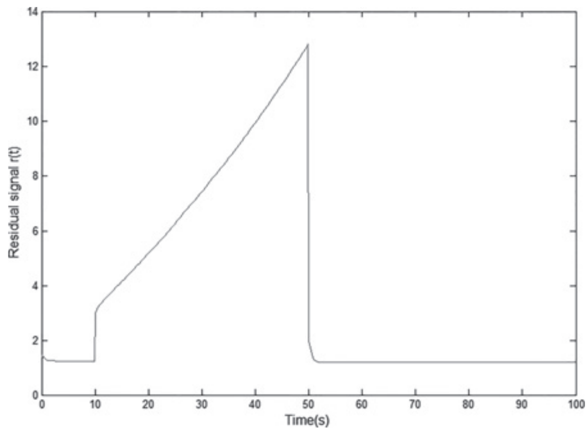

Fig. 3 *Residual signal in Example 2*

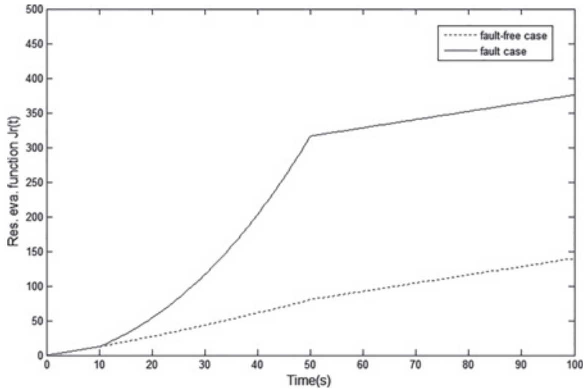

Fig. 4 *Evolution of residual evaluation function in Example 2*

The control input $u(t)$ is the unit step function, and the fault signal $f(t)$ is set up as

$$f(t) = \begin{cases} 0.4t, & 10 \le t \le 50 \\ 0, & \text{others} \end{cases}$$

The generated residual $r(t)$ is shown in Fig. 3. The threshold can be determined as $J_{\mathrm{th}} = 140$ for $t = 100$ s. Fig. 4 shows the evolution of residual evaluation function $J_{\mathrm{r}}(t)$, in which the dashed line is fault-free case, and the solid line is the case with the fault $f(t)$. The simulation results show that $J_{\mathrm{r}}(t) = 142 > 140$ when $t = 33$ s, which means that the fault $f(t)$ can be detected 23 s after its occurrence.

5 Conclusions

In this paper, we have presented a solution to the multi-objective L_-/L_1 fault detection filter design problem for fuzzy positive systems with time-varying delays. The L_- index is proposed as a fault-sensitivity measure. By constructing a co-positive type Lyapunov–Krasovskii functional, sufficient conditions for the existence of such a filer are given. Finally, two examples are provided to show the effectiveness and applicability of the proposed method. Our future work will focus on the design of L_-/L_1 fault detection filter for switched positive systems.

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