Use of data assimilation methods in coastal sediment models

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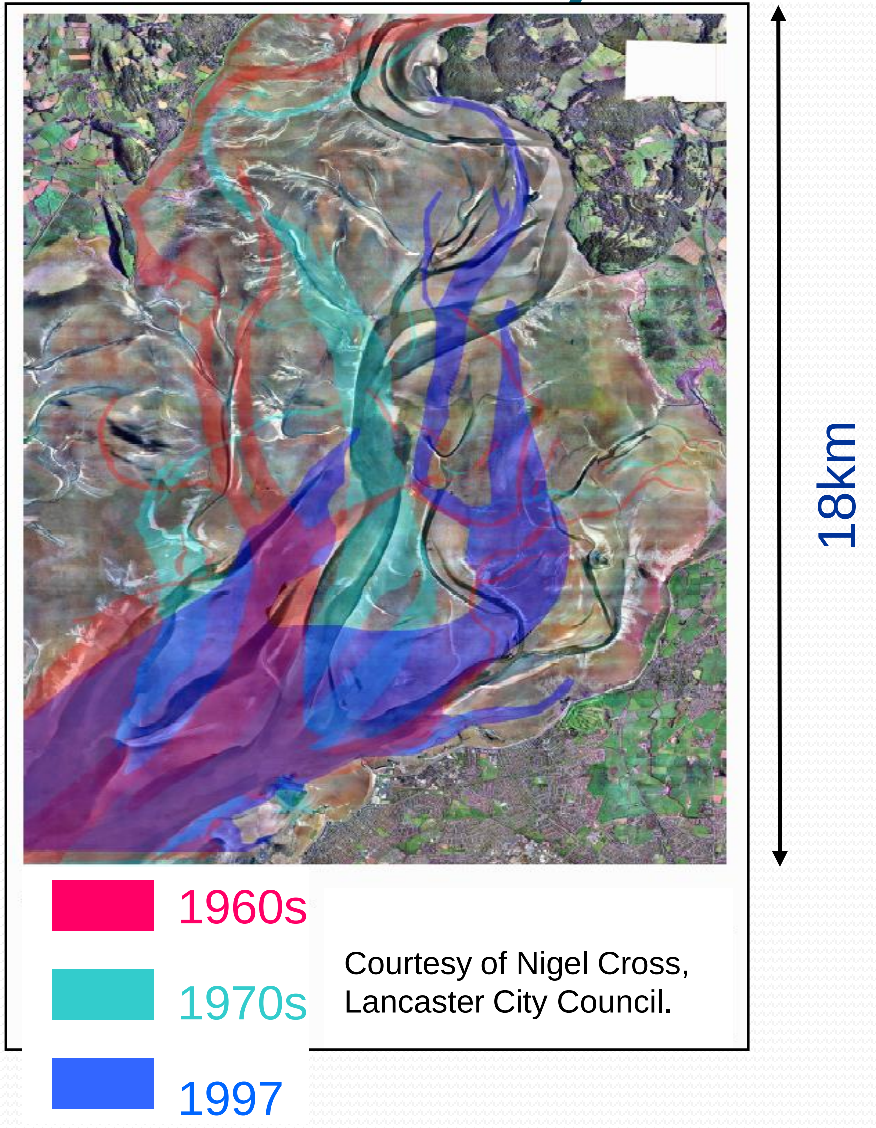
Outline

- Motivation: morphodynamics and flooding
- Observational data and morphodynamic model
- Data assimilation
- Parameter estimation
- Conclusions

Case study: Morecambe Bay

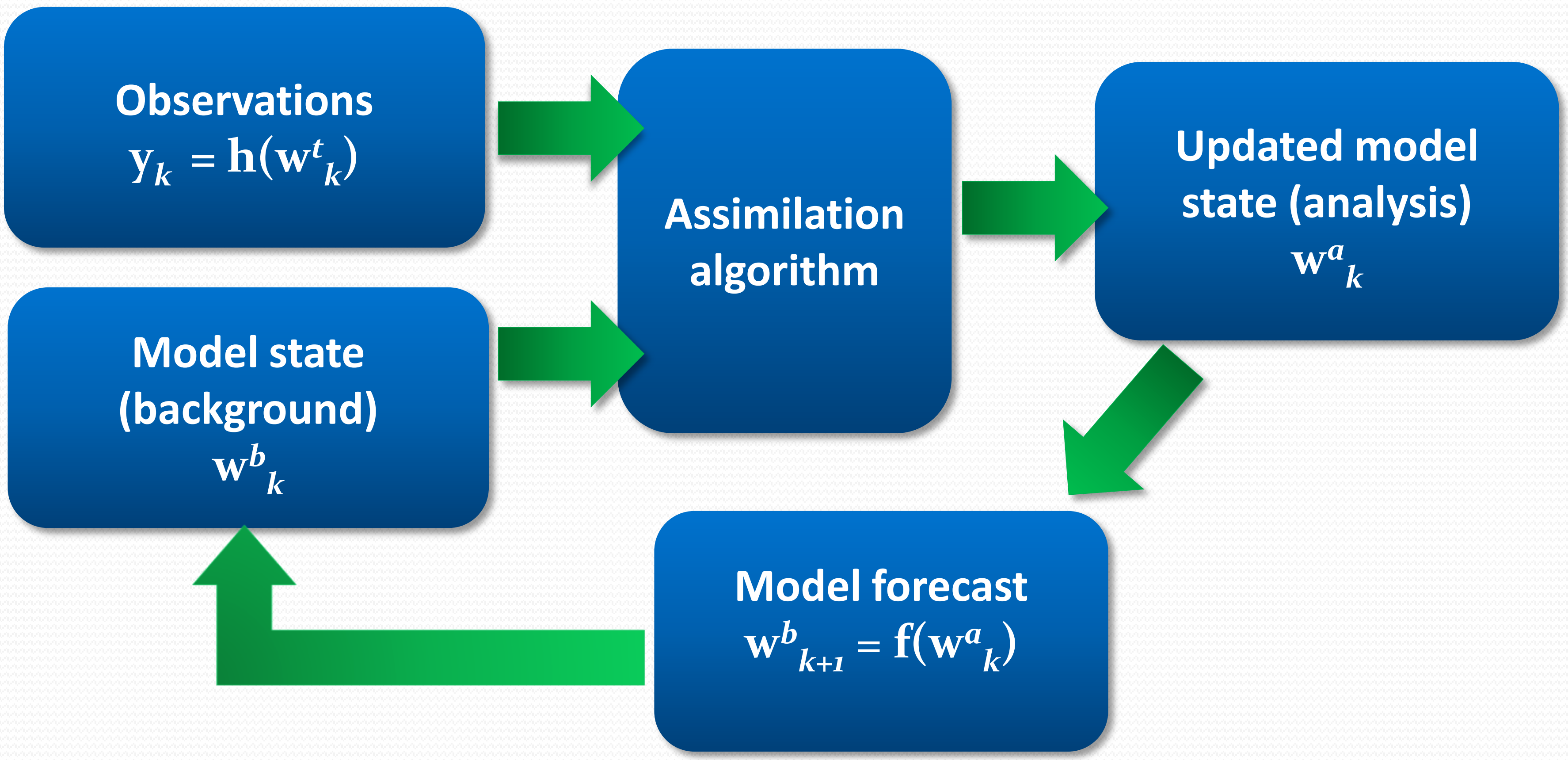
- Channels in Morecambe Bay can move by several kilometres in just a few years
- Channel movement
  - impacts on habitats in the bay
  - affects access to ports
  - has implications for flooding during storm events
- For example, Morecambe can be flooded by storm waves propagating up the deep-water channels



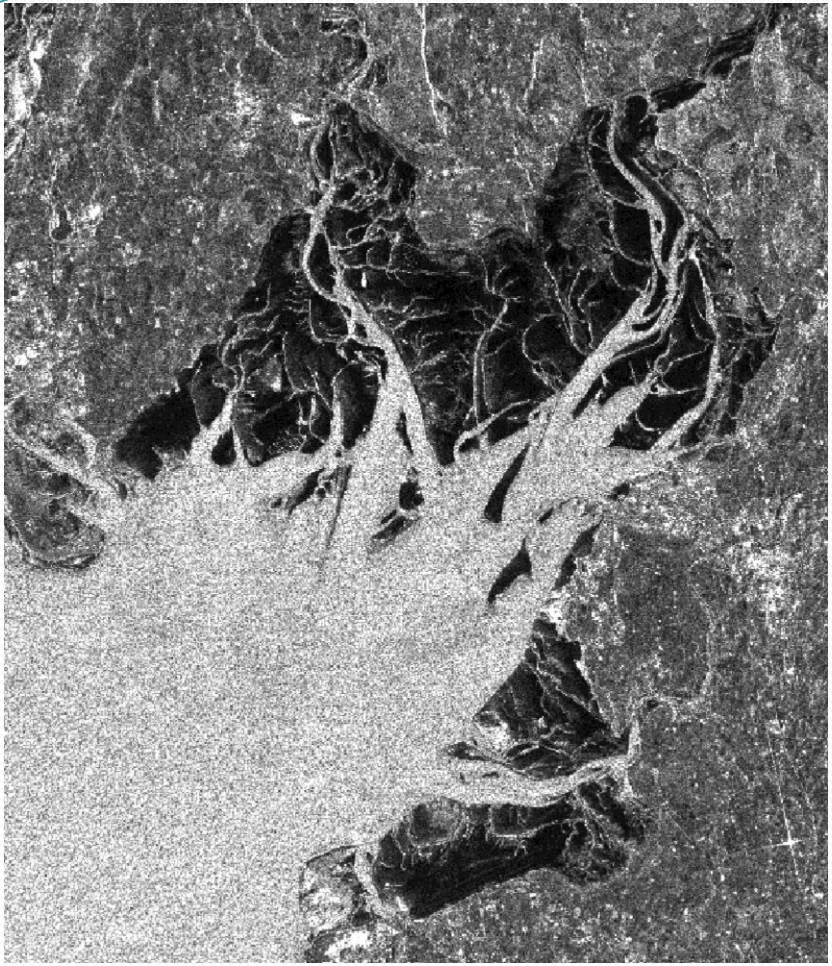
The general idea

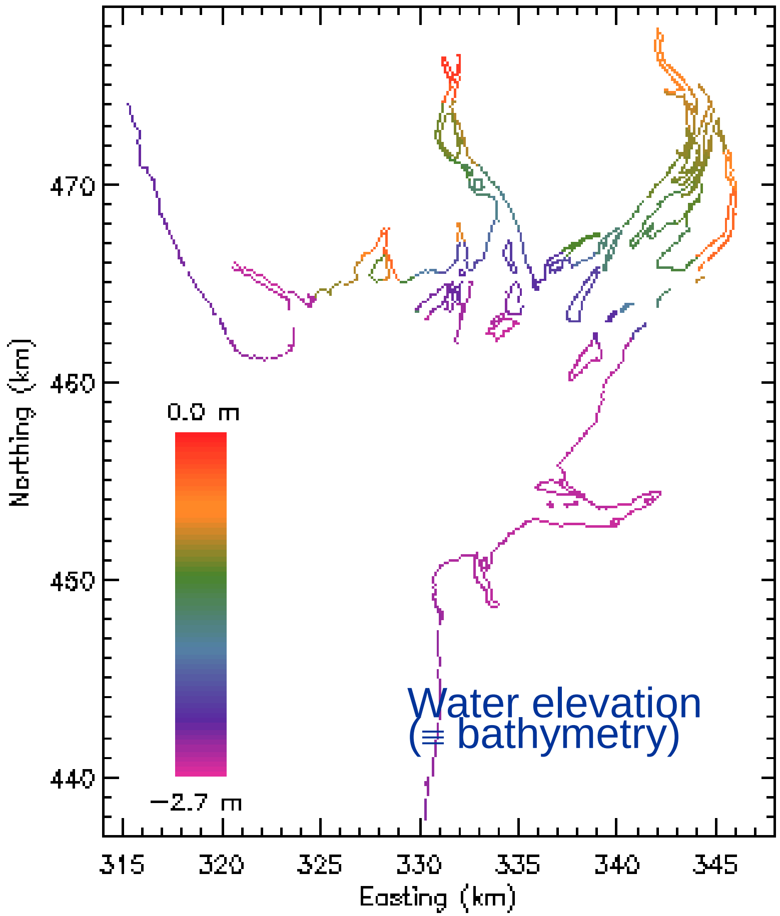
- Coastal bathymetry is dynamic and evolves with time
  - water action erodes, transports, and deposits sediment, which changes the bathymetry, which alters the water action, and so on
- Accurate knowledge of coastal bathymetry at the time of a storm event would allow improved flood forecasting using coastal flood inundation models
  - but it is impractical to continually monitor large coastal areas in anticipation of a storm
- A solution may be to run an operational coastal area morphodynamic model
  - and keep the model on track using data assimilation
- As observations become available they can be used to nudge model bathymetry back towards true bathymetry
  - these observations may be infrequent and only sample a small part of the model domain
- Data assimilation can also be used for parameter estimation

Sequential data assimilation



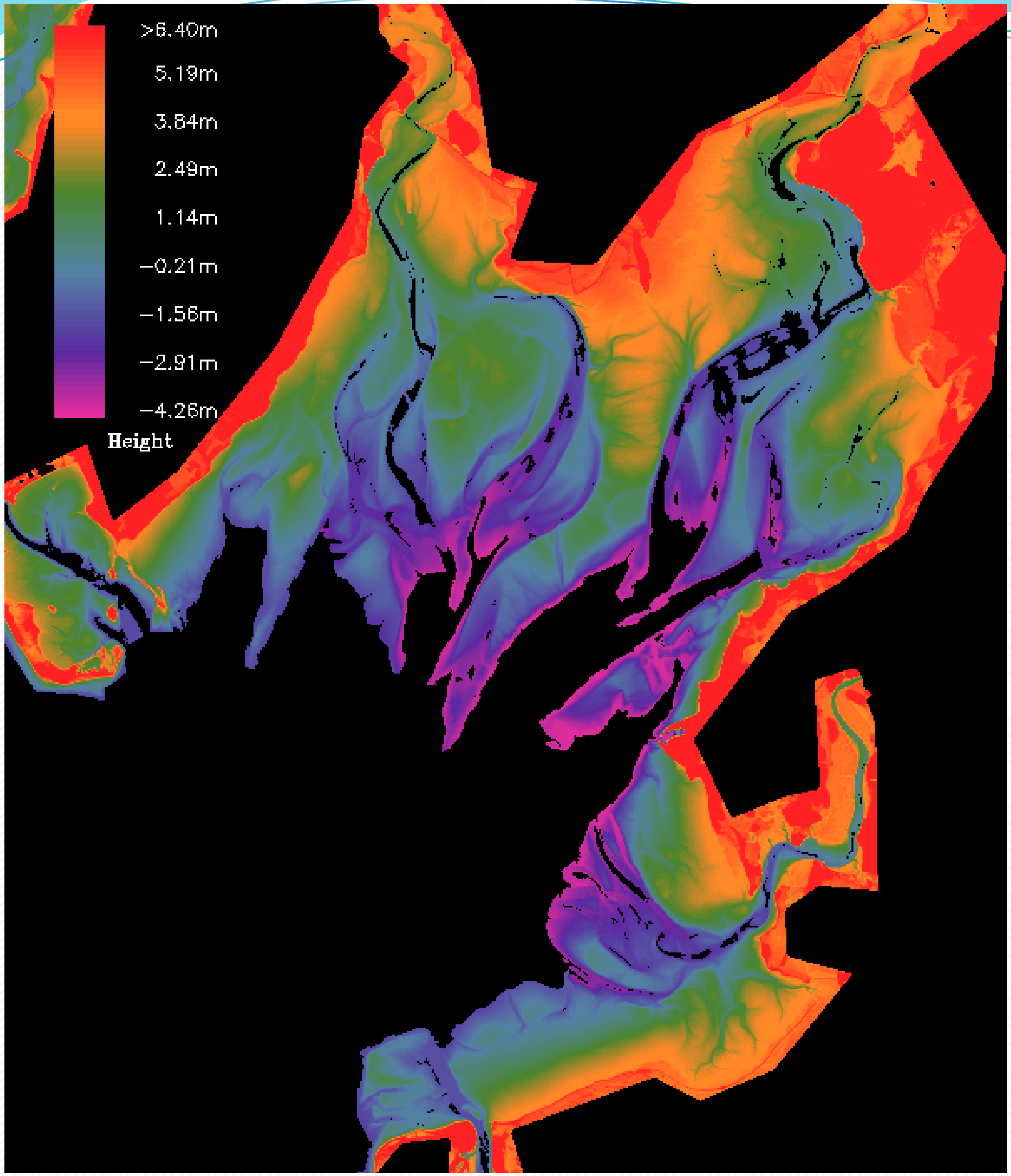
Waterline data for Morecambe Bay





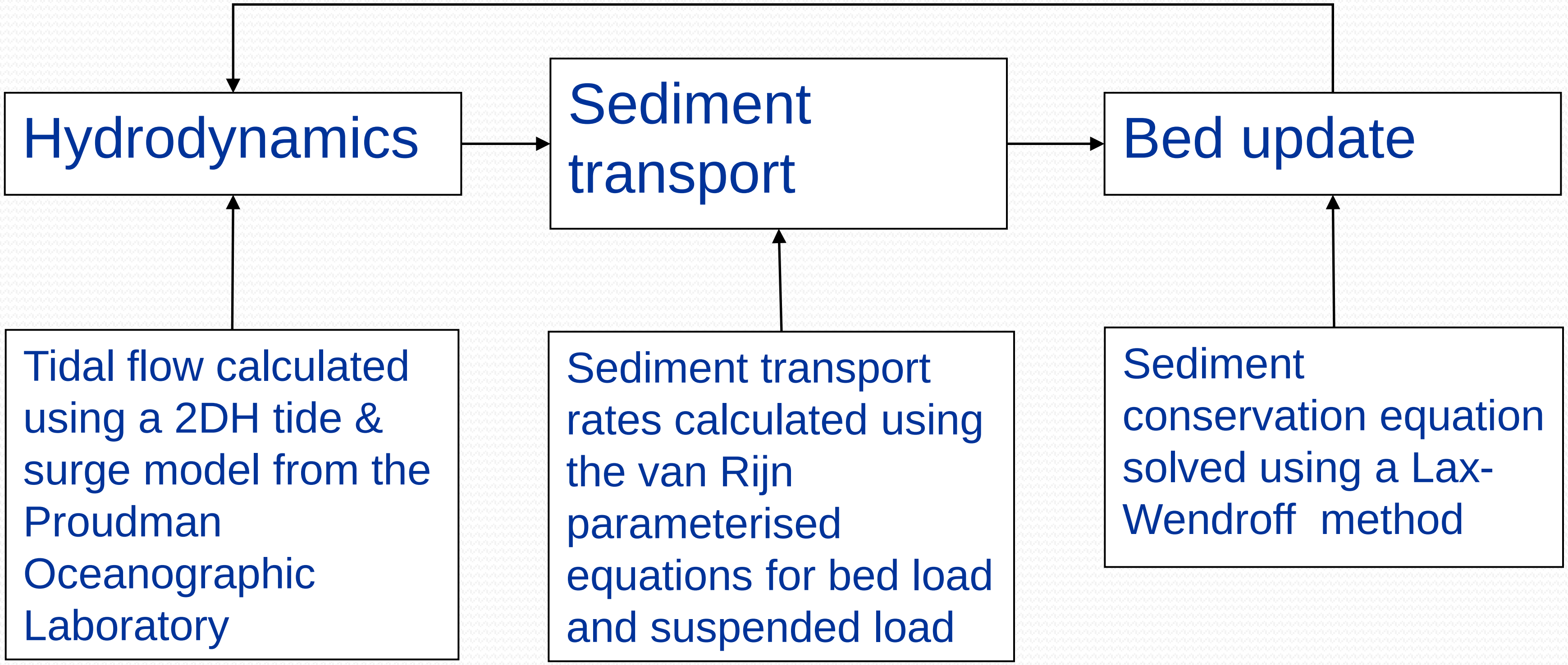
An image of the land-sea boundary
(satellite SAR image © ESA)

Water elevation predicted along the boundary by a hydrodynamic model with surge component



LiDAR data from 15/11/05 to be used for validation.

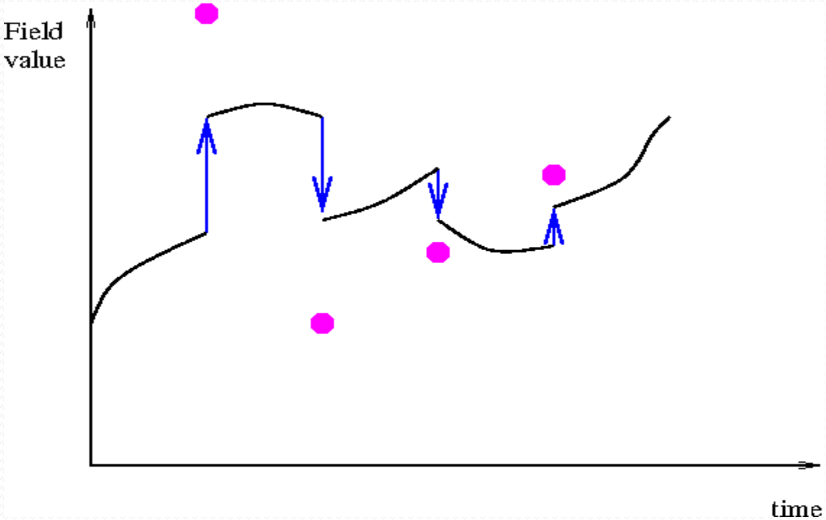
Morphodynamic model



Spatial resolution ~ 240m

A simple model, relative to state-of-the-art engineering models, but adequate for assessing the benefits of data assimilation

3D Var data assimilation



- Analysis state is found by minimising a cost function

$$J(\mathbf{w}_k) = (\mathbf{w}_k - \mathbf{w}_k^b)^{\mathbf{T}}\mathbf{B}_k^{-1}(\mathbf{w}_k - \mathbf{w}_k^b)$$

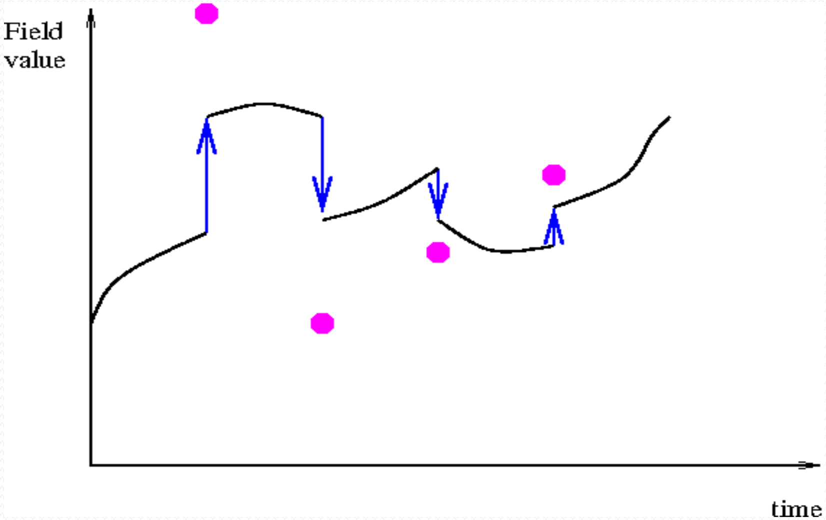
$$+ (\mathbf{y}_k - \tilde{\mathbf{h}}_k(\mathbf{w}_k))^T\mathbf{R}_k^{-1}(\mathbf{y}_k - \tilde{\mathbf{h}}_k(\mathbf{w}_k))$$

background term

observation term

B and **R** are the covariance matrices of the background and observation errors

3D Var data assimilation



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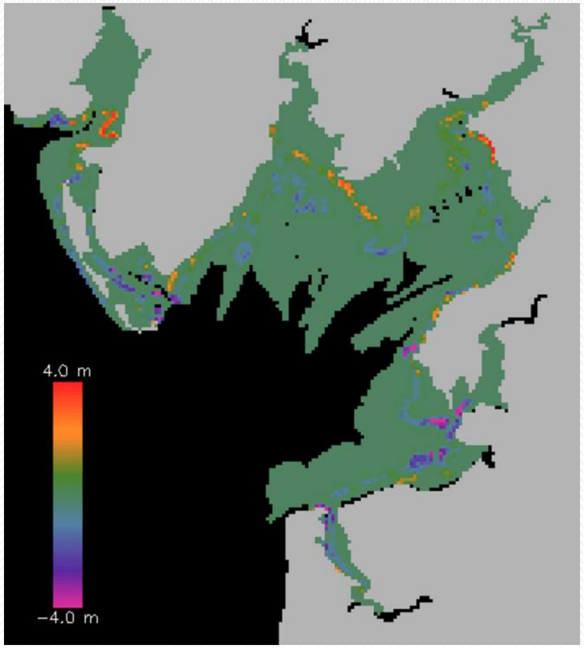
observation term

B and **R** are the covariance matrices of the background and observation errors

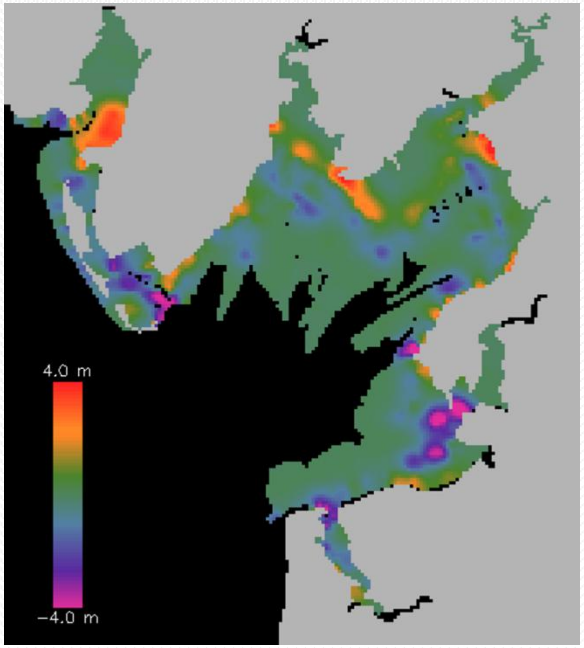
Choice of matrix is crucial to success of assimilation scheme

Choosing the lengthscale for B

Increment (Analysis-Background) for assimilation of 1 waterline

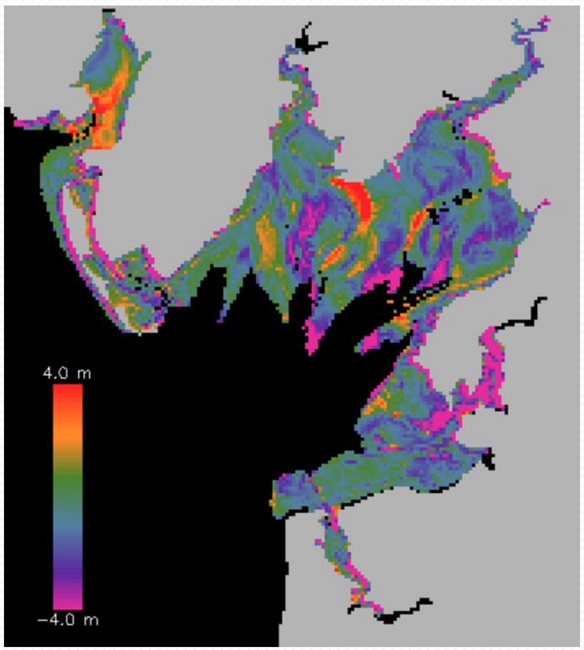


L = 0.5

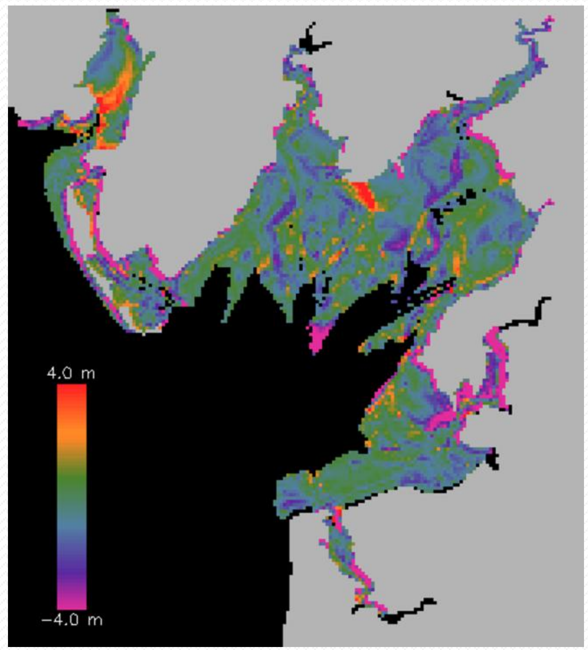


L = 3.0

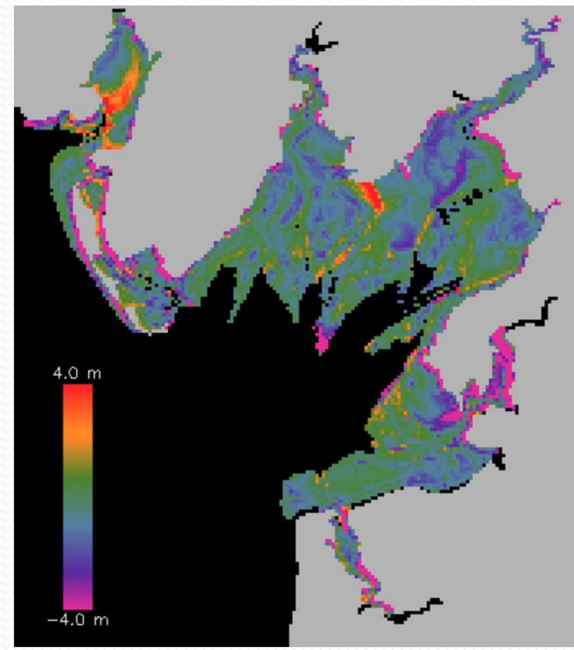
Comparison with lidar validation data



No assimilation



With assimilation
Calibrated params



With assimilation
Weighted ensemble mean
Parameter ensemble

Parameter estimation

- Numerical models suffer from errors in their initial conditions and parameters.
- Initial conditions are often estimated by data assimilation; combining model predictions with observational data to produce an updated model state (the analysis) whilst keeping the model parameters fixed.
- Even with perfect initial data, inaccurate model parameters will lead to the growth of prediction errors.
- Using *state augmentation* in data assimilation we can estimate model parameters concurrently with the state.

Joint state-parameter estimation

- Model state & parameter evolution

$$\mathbf{z}_{k+1} = \mathbf{f}(\mathbf{z}_k, \mathbf{p}) \qquad \mathbf{p}_{k+1} = \mathbf{p}_k.$$

- Augmented system model

$$\mathbf{w}_{k+1} = \begin{pmatrix} \mathbf{z}_{k+1} \\ \mathbf{p}_{k+1} \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{z}_k, \mathbf{p}_k) \\ \mathbf{p}_k \end{pmatrix} = \tilde{\mathbf{f}}(\mathbf{w}_k)$$

- Observations

$$\mathbf{y}_k = \tilde{\mathbf{h}}(\mathbf{w}_k) + \boldsymbol{\delta}_k = \tilde{\mathbf{h}} \begin{pmatrix} \mathbf{z} \\ \mathbf{p} \end{pmatrix} + \boldsymbol{\delta}_k = \mathbf{h}(\mathbf{z}_k) + \boldsymbol{\delta}_k$$

Background error covariance

$$J(\mathbf{w}_k) = (\mathbf{w}_k - \mathbf{w}_k^b)^{\mathbf{T}}\mathbf{B}_k^{-1}(\mathbf{w}_k - \mathbf{w}_k^b)$$

$$+ (\mathbf{y}_k - \tilde{\mathbf{h}}_k(\mathbf{w}_k))^T\mathbf{R}_k^{-1}(\mathbf{y}_k - \tilde{\mathbf{h}}_k(\mathbf{w}_k))$$

background term

observation term

$$\mathbf{B}_k = \begin{pmatrix} \mathbf{B}_{\mathbf{zz}_k} & \mathbf{B}_{\mathbf{zp}_k} \\ (\mathbf{B}_{\mathbf{zp}_k})^{\mathbf{T}} & \mathbf{B}_{\mathbf{pp}_k} \end{pmatrix}$$

State background error covariance

state-parameter cross covariance

parameter background error covariance

For joint state-parameter estimation, it is important that the a priori cross-covariances between the parameters and the state are well specified.

A hybrid approach

Combines ideas from 3D-Var and the extended Kalman filter (EKF)

- assumes $\mathbf{B}_{\mathbf{zz}}$ and $\mathbf{B}_{\mathbf{pp}}$ fixed
- uses a flow dependent state-parameter cross covariance $\mathbf{B}_{\mathbf{zp}_k}$

$$\mathbf{B}_{k+1} = \begin{pmatrix} \mathbf{B}_{\mathbf{zz}} & \mathbf{N}_k \mathbf{B}_{\mathbf{pp}} \\ \mathbf{B}_{\mathbf{pp}} \mathbf{N}_k^T & \mathbf{B}_{\mathbf{pp}} \end{pmatrix}$$

where $\mathbf{N}_k = \left. \frac{\partial \mathbf{f}(\mathbf{z},\mathbf{p})}{\partial \mathbf{p}} \right|_{\mathbf{z}_k^a, \mathbf{p}_k^a}$

A simple sediment transport model

Based on the sediment conservation equation

$$\frac{\partial z}{\partial t} = -\left(\frac{1}{1-\varepsilon}\right)\frac{\partial q}{\partial x} \quad \text{with} \quad q = Au^n$$

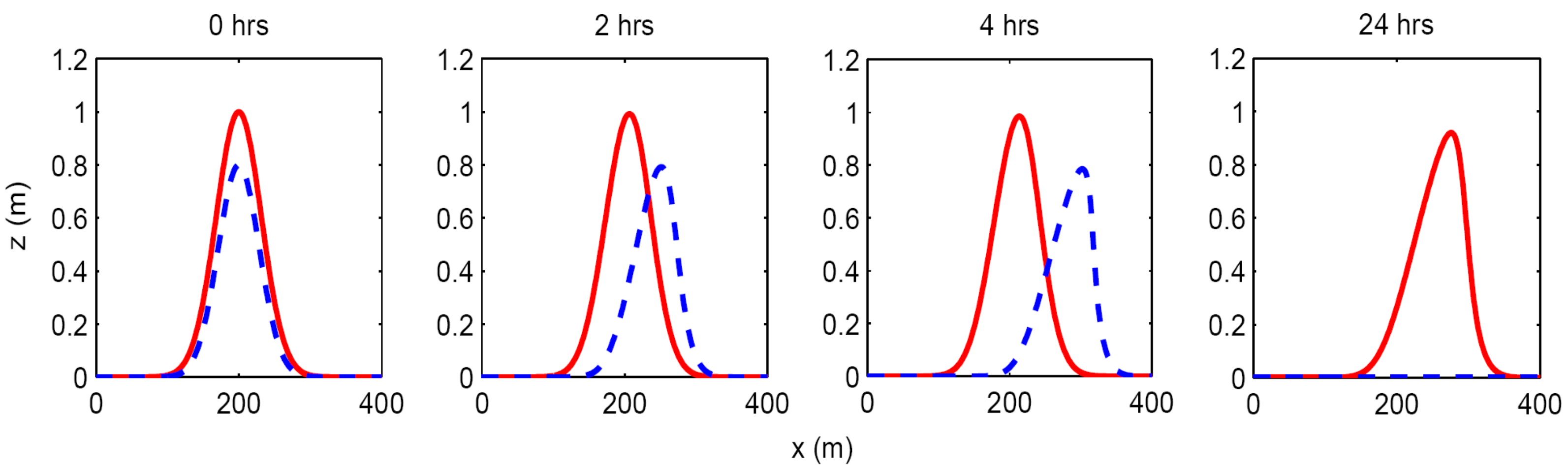
where $z(x,t)$ is the bathymetry, t is time, ε is the sediment porosity, q is the sediment transport rate, $u(x,t)$ is the depth averaged current and A and n are constant parameters.

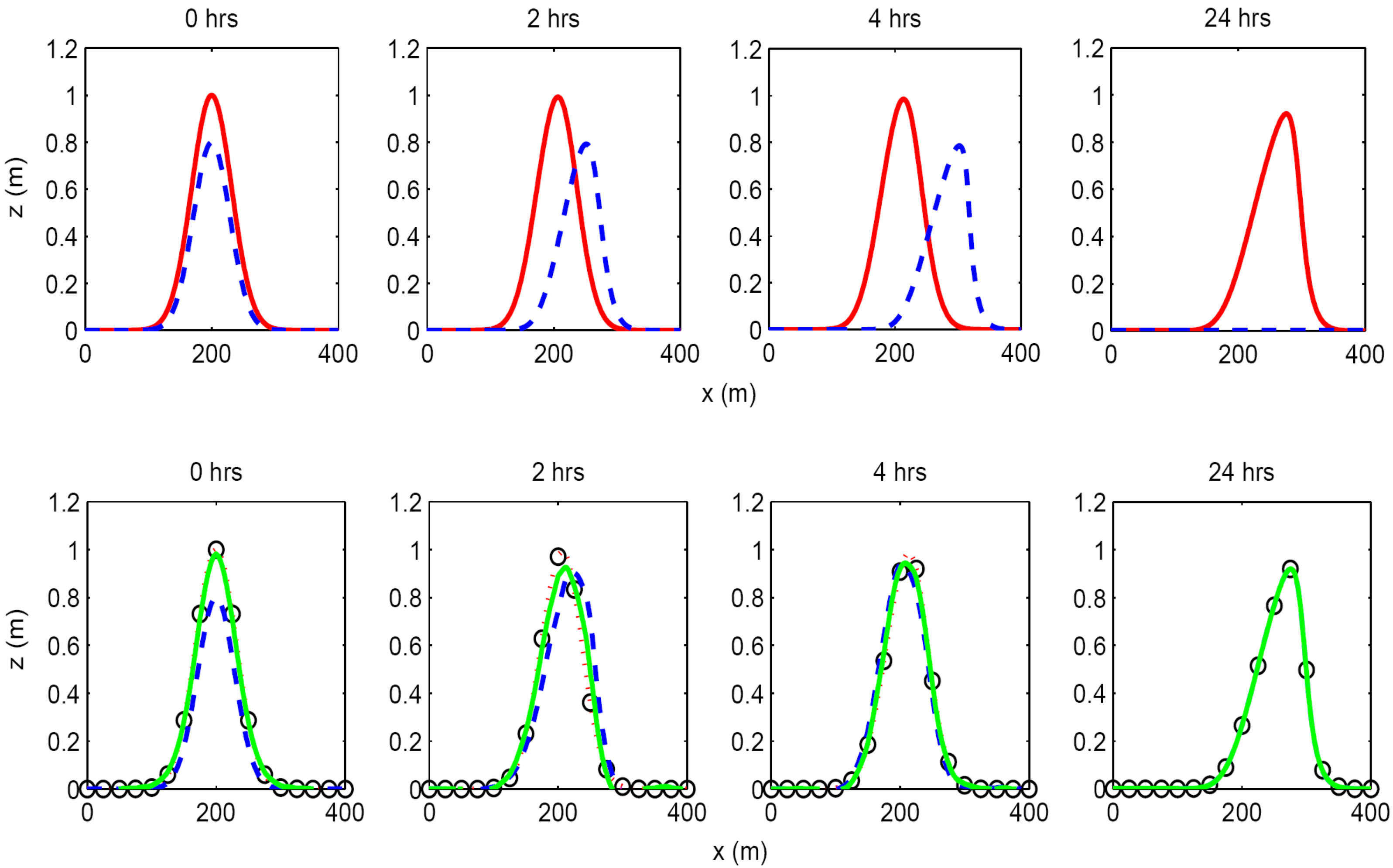
Can we use data assimilation to estimate the parameters A and n?

Experiments

- Identical twin:
  - reference solution generated using 'true' parameter values A = 0.002 ms^{-1} and n = 3.4
  - model then re-run with incorrect initial bathymetry and parameter values
- Observations assimilated sequentially at regular time intervals
  - taken from reference solution & assumed perfect
  - the 3D-Var cost function is minimized iteratively using a quasi-Newton descent algorithm
- Background error covariances
  - $\mathbf{B}_{\mathbf{zz}}$ fixed
  - $\mathbf{B}_{\mathbf{zp}_k}$ time varying

without data assimilation …

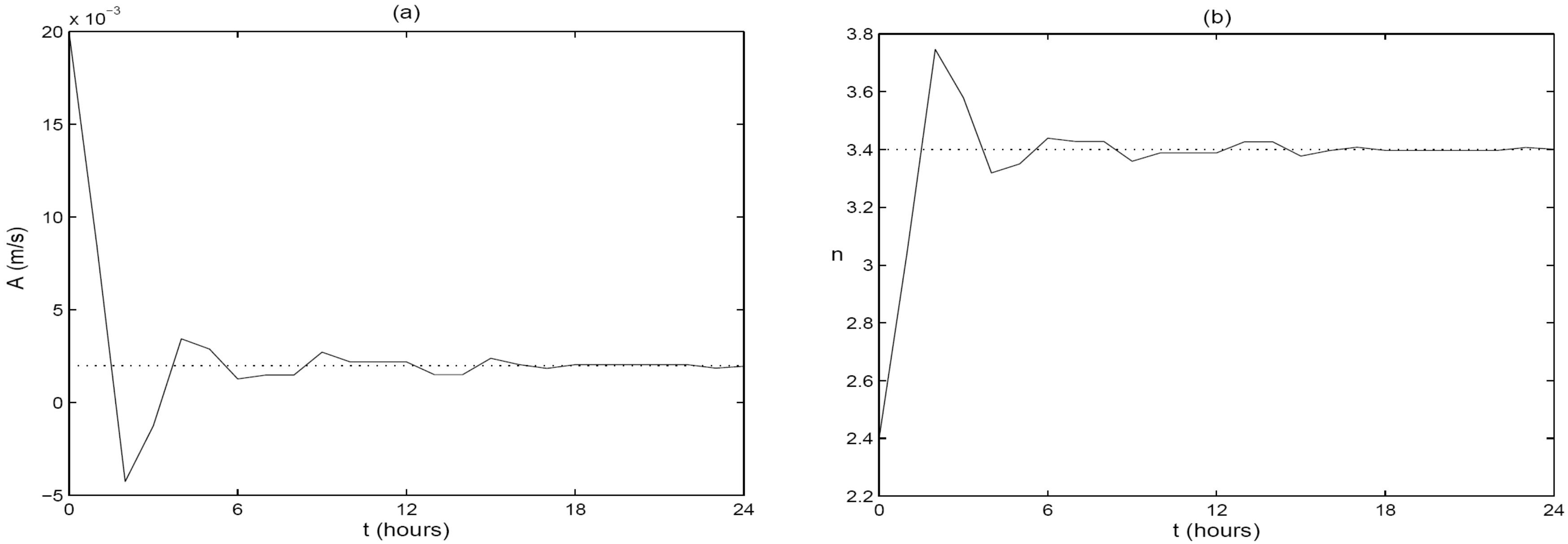




with data assimilation

Parameter estimates

Initial estimates (a) $A_o = 0.02\ ms^{-1}$ (b) $n_o = 2.4$



($A_{true} = 0.002\ ms^{-1}$, $n_{true} = 3.4$)

Summary

- Up-to-date knowledge of near-shore coastal bathymetry is important in flood prediction and risk management
- Assimilated SAR waterline data into a model of Morecambe Bay to keep the model on track
- Best results are obtained using an ensemble of parameters
- Developed a new hybrid data assimilation scheme for joint estimation of model parameters and state.
- Recovers the true parameter values to a good level of accuracy, even when observations are noisy.
- Relatively simple to implement and computationally inexpensive to run
- Method also successfully applied to a range of simple dynamical system models.
- Expect this new technique to be easily transferable to more complex models.