Research and Productivity Growth Across Industries*

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Abstract

What factors underlie industry differences in research intensity and productivity growth? We develop a multisector growth model using standard parameters to capture the main factors considered in the empirical R&D and productivity growth literature. Along the balanced growth path, we find that the primary factor behind industry differences in productivity growth is the extent to which new knowledge builds upon prior knowledge. In contrast, R&D intensity also depends upon the relative importance of different sources of prior knowledge. Quantitatively, we find that the key factor behind industry differences in both productivity growth and R&D intensity is the extent to which new knowledge builds upon prior knowledge, regardless of the source.

JEL Codes: D24, O3, O41 .

Keywords: Multisector growth, total factor productivity, R&D intensity, technological opportunity.

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"If the auto industry had done what the computer has done in the past 30 years, a Rolls Royce would cost \$2.50 and get 2 million miles to the gallon." Computerworld (1980).

1 Introduction

Total factor productivity (TFP) growth rates differ widely across industries, and these differences have been linked to persistent cross-industry variation in R&D intensity – see Figure 1. The correlation in Figure 1 is sometimes interpreted as causation from R&D intensity to TFP growth, leading to the policy recommendation that R&D should be subsidized. However, both R&D and productivity change reflect the responses of firms to deeper industry characteristics. This paper develops a general equilibrium model in which both research activity and productivity growth vary endogenously across industries – to study these factors from the perspective of growth theory, and to provide a rich framework for policy analysis.

We build the model according to criteria that we believe define a natural benchmark. First, industries differ in terms of factors commonly identified in the empirical literature as being potential determinants of research intensity: technological opportunity (factors that affect the efficiency of research), appropriability (the extent to which R&D benefits the innovator) and demand (the magnitude and sensitivity of the potential returns to research). Second, these factors are implemented in the model using standard preference and technology parameters drawn from the growth literature. Third, to discipline our analysis, we study the behavior of the model along an aggregate balanced growth path – consistent with our use of data from the United States, where GDP has grown at a stable rate for over a century.

As in Jones (1995), the aggregate growth rate along the balanced growth path is pinned down by the population growth rate. Comparing across industries, we find that differences in TFP growth rates depend primarily on one factor of technological opportunity – the extent to which each industry is able to generate new knowledge by drawing on prior knowledge. We call this ability *receptivity*, modeled as the elasticity of the ideas production function with respect to all sources of prior knowledge. By contrast, differences in R&D intensities also depend on the fraction of receptivity that accrues from the firm's own stock of knowledge. We use this fraction to capture the notion of appropriability.

Our results are consistent with the claim of Nelson (1988), Klevorick et al (1995) and Nelson and Wolff (1997) that the extent to which knowledge spills from a firm

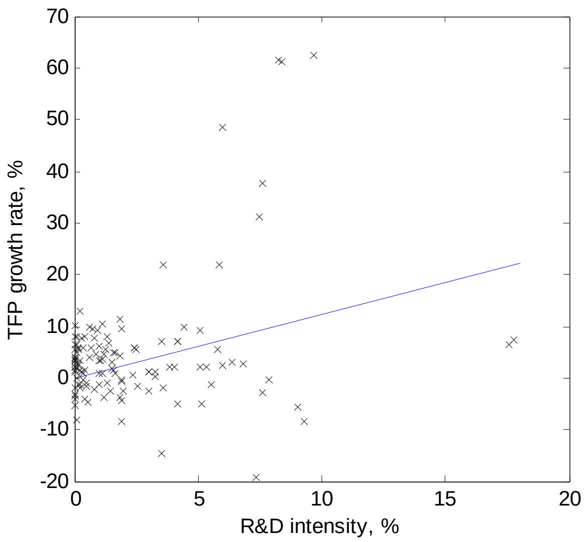

Figure 1: R&D intensity and TFP growth across manufacturing industries. R&D intensity is measured using the median ratio of R&D spending to sales in Compustat, 1950-2000. TFP growth rates are from the NBER manufacturing productivity database – see Bartelsman et al (2000). The correlation is 33%, P-value 0.01%. Data include all 133 industries for which Compustat contained at least one firm. We exclude an outlier (Biological products excluding diagnostics) which has R&D intensity of 77% – 10 standard deviations from the mean. Including it reduces the correlation to 15%, P-value 8%. Other authors find a similar relationship: see Terleckyj (1980) for an early survey.

to its competitors affects R&D intensity, but not TFP growth rates.[1] Using the NBER patent citation database as an indicator of knowledge flows, we find that cross-industry spillovers are relatively weak – whereas spillovers *within* industries are large. Moreover, within-industry spillovers are mainly due to knowledge flows across firms, so that appropriability is quite low. As a result, our model predicts that differences in R&D intensities are mainly driven by differences in receptivity, in the form of large knowledge spillovers across firms in the same industry. When we calibrate the model economy to match changes in the relative prices of different capital goods over time (adjusted for quality), we find that the correlation between R&D in the model and in the data is close to 80%.

Along a balanced growth path, neither differences in TFP growth rates nor differences in R&D intensity turn out to be related to demand factors. This is consistent with the finding that a robust relationship between demand factors and R&D intensity is hard to pin down – see the survey of Cohen and Levin (1989) – and with a pervading sense among historians of technical change that the pace and direction of technical progress is primarily supply-driven. A well-known example of this phenomenon is "Moore's Law", a prediction of stable decline in the price of computing efficiency which has held for about 40 years.[2] In the model, while product demand provides an incentive to perform research, innovations follow a primarily technological rationale, leading to stable rates of technological progress in the long run.

As an application of the model, we solve the planner's problem and derive industry-specific tax and subsidy schemes that allow the decentralized equilibrium to replicate the planner's solution. While a variety of policies and institutions may impact the incentives to perform R&D, we focus on R&D subsidies because they are fairly common (in the form of tax exemptions) and because they are easily interpretable in quantitative terms. We find that optimal R&D subsidies are not uniform: industries with higher receptivity or lower appropriability should receive higher subsidies. Thus, R&D should be subsidized more in industries in which R&D intensity is low relative to its rate of TFP growth. In a one-sector model, Jones and Williams (1998) find that R&D intensity is between half and a quarter of its optimal level: we find that the size of the wedge between the equilibrium and the optimum R&D intensity varies within this range, depending positively on industry receptivity.

[1] "Appropriability conditions, through their influence on R&D intensity, affect the position at any time along the productivity track, but not the slope of that track." Klevorick et al (1995).

[2] The original statement of Moore (1965) is "the complexity for minimum component costs [of an integrated circuit] has increased at a rate of roughly a factor of two per year... There is no reason to believe [this rate] will not remain nearly constant for at least 10 years." However, the *costs* of a transistor and of hedonic computing performance measures such as processor speed and memory capacity have also experienced steady declines, and it is common to cite the "law" in those terms.

A related paper is Klenow (1996), which studies the determinants of cross-industry differences in TFP growth and R&D intensity in a 2-sector version of the Romer (1990) model. We confirm his finding that industries which are more R&D-intensive because of better appropriability should receive a lower R&D subsidy. However, by allowing for a broader set of parameters, we also find that industries which are more R&D-intensive because of higher receptivity should receive larger R&D subsidies – an effect that turns out to be quantitatively dominant.

Also related is Krusell (1998), who develops a 2-sector framework to endogenize the gap in TFP growth between capital good and consumption good industries documented by Greenwood et al (1997). Vourvachaki (2006) and Acemoglu and Guerrieri (2006) feature two-sector endogenous growth models: however, in all these papers, either there is only research in one sector, or the focus is not on the factors that determine sectorial TFP growth rates.

Finally, Jones (1995, 1999) finds that magnitude of the elasticity with respect to all prior knowledge in the aggregate idea production function is crucial for a balanced growth path to exist in R&D-based growth models. In a multisector model, we find that the corresponding elasticity at the industry level is crucial for comparing research and productivity growth across industries.

Section 2 provides an overview of the related empirical literature. Section 3 describes the structure of the model, and Section 4 studies its long run behavior. Section 5 applies the model to the problem of optimal research policy, and Section 6 uses patent citations to determine the relative importance of different kinds of spillovers and derive the quantitative implications the model by calibrating to the US economy. Section 7 summarizes the results. All proofs are collected in Appendix *A*.

2 Factors of R&D Intensity

Numerous empirical studies have attempted to find the determinants of industry variation in innovative activity. While some studies assume that R&D activity causes TFP growth, others take our view that both may be determined by deeper "fundamentals" of each industry. Consistent with our view, Nelson and Wolff (1998) are able to identify factors that explain R&D intensity that do not account for TFP growth rates.

The literature has focused on three sets of fundamental factors that might drive research activity and TFP growth: product demand factors, technological opportunity, and appropriability.

Demand factors affect the returns to R&D. In Schmookler (1966), large product markets are thought to encourage innovation by offering relatively large returns to innovators. Kamien and Schwartz (1970) argue that the gains from reducing the cost of production may be larger when demand is more elastic. However, the survey of Cohen and Levin (1989) suggests that the evidence concerning demand factors is weak. Studies often rely on categorical or dummy variables to stand in for demand factors but, even using a more structural approach to estimate demand size and elasticity, Cohen et al (1985) find that demand factors lose significance in cross-industry R&D regressions when indicators of opportunity and appropriability are included. Independently, case-based and historical studies suggest that technical change appears driven by scientific or engineering considerations rather than by demand conditions.[3]

Technological opportunity encompasses factors that lead research to be more productive in some industries than others. Opportunity has been modeled in different ways – for example, in Klenow (1996) it is a constant Z_i in the production function for knowledge of industry i. Nelson (1988) and Klevorick et al (1995) list three sources of technological opportunity, all of which are inherently *dynamic*: the advance of scientific understanding (modeled as an exogenous rate of increase in Z_i),[4] technological advances outside the industry that may "spill over", and the influence of pre-existing ideas on the ability to generate new ones – which we call *receptivity.*

Identifying all these factors empirically is difficult. Using surveys of R&D managers, Cohen et al (1985), Cohen et al (1987) and Klevorick et al (1995) try to identify all three, and relate them to R&D activity as well as to technical change. Using a different approach, Bernstein and Nadiri (1988) estimate cost functions for a set of five "high-tech" industries, including the R&D stock of other industries in each one, and find some evidence of cross-industry spillovers.

Appropriability relates to the extent that an innovating firm (as opposed to its competitors) benefits from its own newly generated knowledge. Cohen et al (1987),

[3] "In some of the writing on technological advance, there is a sense that innovation has a certain inner logic of its own....– particularly in industries where technological advance is very rapid, advances seem to follow advances in a way that appears somewhat 'inevitable' and certainly not fine tuned to the changing demand and cost conditions." Nelson and Winter (1977), on 'natural trajectories.'

[4] Since the trademark of R&D-based growth models is that technical progress is endogenous, our model does not feature exogenously growing factors other than the population. However, it is not clear that academic research is best thought of as being exogenous: it benefits from spillovers from commercial research, and it is also conducted in response to economic incentives. Thus, an interpretation of academic research within our model is simply that it is research conducted by a sector (for instance, educational services, perhaps disaggregated by field), the outcome of which may spill over to other sectors.

Klevorick et al (1995) and Nelson and Wolff (1997) find evidence that appropriability is related to R&D intensity and, interestingly, Klevorick et al (1995) and Nelson and Wolff (1997) argue that the survey data are consistent with an influence of opportunity factors on both R&D intensity and technical change, whereas appropriability is only related to R&D intensity. Cohen et al (1987) do find a positive link between appropriability and an indicator of innovation, also using survey data. What may cloud these results is that the appropriability measure in all these papers may not distinguish clearly between appropriability and opportunity. The measure is based on the response to the question "in this line of business, how much time would a capable firm typically require to effectively duplicate and introduce a new or improved product developed by a competitor?" This may not distinguish between (a) the ease with which a competitor might access a firm's knowledge, and (b) the ease in general with which preexisting knowledge can be used to generate new knowledge. In particular, if appropriability itself is generally small, then the measure may reflect primarily differences in receptivity.

The following stylized facts emerge from the literature.

1. the link between *demand factors* and research intensity (or rates of TFP growth) is not robust;

2. There is some evidence that *opportunity* affects both variables of interest;

3. *Appropriability* is easier to relate to R&D intensity than to TFP growth rates.

We wish to articulate these factors within a general equilibrium growth model, based on primitives of preferences and technology drawn from the growth literature. Given the measurement difficulties inherent in studying the role of knowledge in technical progress, we use the structure of the model to inform us regarding the long run relationships that may hold between R&D, TFP growth, and each of these factors, from the perspective of growth theory. We use a model of firm level R&D that is intentionally close to the production function approach common in the empirical literature, with the aim of providing a benchmark to help organize our understanding of how different industry characteristics may be related to long-run research intensity and TFP growth.

3 Model Economy

The economy consists of $z \geq 2$ sectors. Firms in sectors $i \in \{1, ..m-1\}$ produce consumption goods, whereas firms in sectors $j \in \{m, ...z\}$ produce investment goods.

Each firm in sector i produces a differentiated variety $h \in [0,1]$ of good i, using capital and labor as physical inputs. The firm's productivity depends upon the quantity of technical knowledge at its disposal. New knowledge is produced as a result of individual firm activity, and of spillovers from other firms. We first consider spillovers within sectors, and later allow for spillovers across sectors too. We defer a more detailed discussion of our modeling choices until Section 4.

3.1 Firms

Time is discrete and indexed by $t \in \mathbb{N}$. Output of variety h of good i is

$$Y_{iht} = T_{iht} K_{iht}^{\alpha} N_{iht}^{1-\alpha}, \ \alpha \in (0,1) \tag{1}$$

where Y_{iht} is output, T_{iht} is knowledge, K_{iht} is capital and N_{iht} is labor. Knowledge accumulates over time according to the function

$$T_{ih,t+1} = F_{iht} + (1-\delta_T)\, T_{iht} \tag{2}$$

where new knowledge F_{iht} is produced according to[5]

$$F_{iht} = Z_i T_{iht}^{\kappa_i} T_{it}^{\sigma_i} \left(Q_{iht}^{\alpha} L_{iht}^{1-\alpha}\right)^{\psi}, \ \psi \in (0,1)\,. \tag{3}$$

Q_{iht} and L_{iht} are capital and labor used in production of knowledge, and $T_{it} = \int_0^1 T_{iht} dh$. Let $\gamma_{iht} \equiv T_{iht+1}/T_{iht}$ be the growth factor of T_{ih}.

The firm's profits are

$$\Pi_{iht} = p_{iht} Y_{iht} - w_t \left(N_{iht} + L_{iht}\right) - R_t \left(K_{iht} + Q_{iht}\right). \tag{4}$$

Each sector $i \leq z$ is monopolistically competitive, so that p_{iht} is a function of Y_{iht}.

Taking its demand function as given, firm h in sector i chooses its level of output and R&D inputs in order to maximize the discounted stream of real profits,

$$\sum_{t=0}^{\infty} \lambda_t \frac{\Pi_{iht}}{p_{ct}} \tag{5}$$

where λ_t is the discount factor at time t, with $\lambda_0 = 1$, $\lambda_t = \prod_{s=1}^{t} \frac{1}{1+r_t}$ for $t \geq 1$, and r_t is the real interest rate.[6]

[5] The empirical literature focuses on the case $\psi = 1$ so that the stock of knowledge is proportional to the stock of R&D spending. However, it is not uncommon in the growth literature to allow for diminishing returns ($\psi < 1$).

[6] The transversality condition is $\lim_{t \to \infty} \chi_{iht} T_{iht+1} = 0$, where χ_{iht} is the shadow price of T_{iht+1}.

Z_i, κ_i, and σ_i are parameters of opportunity, as they affect the productivity of research. κ_i represents the effect of in-house knowledge, and is known in the growth literature as the intertemporal knowledge spillover. σ_i represents spillovers from other firms. We refer to their combined effect $\rho_i \equiv \kappa_i + \sigma_i$ as the *receptivity* of sector i: the extent to which the production of new knowledge benefits from prior knowledge in sector i.

Conditional on receptivity, industries may differ in the importance of in-house knowledge relative to knowledge acquired through spillovers from its competitors. We define *appropriability* as the share of receptivity accounted for by in-house knowledge $A_i \equiv \kappa_i/\rho_i$.

The last set of factors considered by the empirical literature relates to demand, which we model through the household preference structure below.

3.2 Households

There is a continuum of households, each of measure $N_t = g_N^t$. In what follows, we use lower case letters to denote per-capita variables. The life-time utility of a household is

$$\sum_{t=0}^{\infty} (\beta g_N)^t \frac{c_t^{1-\theta} - 1}{1-\theta} \tag{6}$$

$$c_t = \prod_{i=1}^{m-1} \left(\frac{c_{it}}{\omega_i}\right)^{\omega_i}, \; c_{it} = \left(\int_0^1 c_{iht}^{\frac{\mu_i - 1}{\mu_i}} dh\right)^{\frac{\mu_i}{\mu_i - 1}} \qquad i \in \{1, ..., m-1\} \tag{7}$$

where β is the discount factor, and $1/\theta$ is the intertemporal elasticity of substitution. We assume that $\beta g_N < 1, \theta > 0, \mu_i > 1, \omega_i > 0$ and $\sum_{i=1}^{m-1} \omega_i = 1$.

Each household member is endowed with one unit of labor and k_t units of capital, and receives income by renting capital and labor to firms, and by earning profits from the firms. Her budget constraint is

$$\sum_{i=1}^{m-1} \int p_{iht} c_{iht} dh + \sum_{j=m}^{z} \int p_{jht} x_{jht} dh \leq w_t + R_t k_t + \pi_t \tag{8}$$

where x_{jht} is investment in variety h of capital good j, p_{iht} is the price of variety h of good i, w_t and R_t are rental prices of labor and capital, and $N_t \pi_t \equiv \sum_{i=1}^{z} \int_0^1 \Pi_{iht} dh$ equals total profits from firms.

The capital accumulation equation is $g_N k_{t+1} = x_t + (1 - \delta_k) k_t$. The composite investment good x_t is produced via a Cobb-Douglas function of all capital types j, while the elasticity of substitution across different varieties of capital good j is equal to $\mu_j > 1$, so

$$x_t = \prod_{j=m}^{z} \left(\frac{x_{jt}}{\omega_j} \right)^{\omega_j}, \; x_{jt} = \left[\int x_{jht}^{(\mu_j - 1)/\mu_j} dh \right]^{\mu_j/(\mu_j-1)} \qquad j \in \{m, ..., z\} \tag{9}$$

where $\omega_j > 0$ and $\sum_{j=m}^{z} \omega_j = 1$. Finally, the transversality condition for capital is $\lim_{t\to\infty} \zeta_t k_t = 0$, where ζ_t is the shadow price of capital.

Define the price index for the consumption composite c_t and the investment composite x_t respectively as:

$$p_{ct} \equiv \frac{\sum_{i=1}^{m-1} \int_0^1 p_{iht} c_{iht} dh}{c_t}; \qquad p_{xt} \equiv \frac{\sum_{j=m}^{z} \int_0^1 p_{jht} x_{jht} dh}{x_t}. \tag{10}$$

Parameters μ_i and ω_i capture the industry-specific demand factors considered in the literature. μ_i is the elasticity of substitution across different varieties of good i which, in equilibrium, determines the price elasticity of demand, while ω_i determines the spending share of each good (market size).

4 Decentralized Equilibrium

In this section, we define the equilibrium concept and characterize conditions for the equilibrium to display a balanced growth path. Then, we discuss the determinants of TFP growth rates and R&D intensity in such an equilibrium. Our equilibrium concept is decentralized, so that there is a potential role to play for externalities related to appropriability, market rivalry effects, etc. in the determination of outcomes. Given these externalities, it is of interest to contrast equilibrium outcomes with efficient outcomes, which we do later in Section 5.

Definition 1 *A decentralized equilibrium consists of*

allocations of final output $\left\{ \left\{ (c_{iht})_{h\in[0,1]} \right\}_{i=1}^{m-1}, \left\{ (x_{jht})_{h\in[0,1]} \right\}_{j=m}^{z} \right\}_{t=0,1,...}$

allocations of inputs $\left\{ \left\{ (K_{iht}, N_{iht}, Q_{iht}, L_{iht})_{h\in[0,1]} \right\}_{i=1}^{z} \right\}_{t=0,1...}$

and sequences of prices $\left\{ \left\{ (p_{iht})_{h\in[0,1]} \right\}_{i=1}^{z}, R_t, w_t \right\}_{t=0,1...}$ *such that:*

1. *Given the sequence of prices, households choose investment and consumption to maximize their discounted stream of utility* (6)*;*

2. *Given the sequence of input prices, and taking their demand functions as given, firms choose input allocations to maximize* (5)*;*

3. *The sequence of input prices, satisfies the capital and labor market clearing conditions in all periods:*

$$K_t = \sum_{i=1}^{z} \int_0^1 (K_{iht} + Q_{iht})\, dh,\ N_t = \sum_{i=1}^{z} \int_0^1 (N_{iht} + L_{iht})\, dh \qquad (11)$$

Our aim is to understand productivity dynamics across industries, and not across different varieties of any given good. Therefore, we focus on symmetric equilibria across varieties within each sector i, and suppress the firm index h henceforth. Later we discuss the implications of symmetry. Technical details of the following discussion are reported in the Appendix, in Lemmata $1 - 4$.

In equilibrium, our assumption of Cobb-Douglas production functions with equal (relative) input shares across sectors and activities implies that:

$$\frac{K_{it}}{N_{it}} = \frac{Q_{jt}}{L_{jt}} \qquad \forall i, j, \text{ and } \frac{p_{it}}{p_{jt}} = \frac{T_{jt}\left(1 - 1/\mu_j\right)}{T_{it}\left(1 - 1/\mu_i\right)}. \qquad (12)$$

The mapping between relative prices and relative TFP will be useful in our quantitative exercises.

Given (12), the aggregate capital-labor ratio k is also the capital-labor ratio for production and R&D activities. This allows us to aggregate industries $j \in \{m, ..., z\}$ into a single investment sector x, where the knowledge index T_{xt} equals

$$T_{xt} = \left[\prod_{j=m}^{z} \left(\frac{1 - 1/\mu_j}{1 - 1/\mu_x}\right)\right] \prod_{j=m}^{z} T_{jt}^{\omega_j}. \qquad (13)$$

and $\mu_x = \left(\sum_{i=m}^{z} \omega_i \mu_i^{-1}\right)^{-1}$. Define $\gamma_{xt} = T_{x,t+1}/T_{xt}$, so that

$$\gamma_{xt} = \prod_{j=m}^{z} \gamma_{jt}^{\omega_j}. \qquad (14)$$

The firm's dynamic optimization condition implies

$$\chi_{iht} = \underbrace{\left[\frac{\lambda_{t+1}}{p_{ct+1}} \frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}}\right]}_{(a)\text{ production}} + \chi_{iht+1} \left[\underbrace{\frac{\partial F_{iht+1}}{\partial T_{iht+1}}}_{(b)\text{ research}} + \underbrace{(1 - \delta_T)}_{(c)\text{ future knowledge}}\right], \forall i \leq z. \qquad (15)$$

where χ_{iht} is the shadow price of knowledge T_{iht+1}, which in equilibrium is determined by the optimal allocation of inputs across activities.

Equation (15) reflects three benefits to the firm of producing more knowledge: (a) more efficient *production of goods and services*, (b) more efficient *production of knowledge*, and (c) a larger stock of future knowledge. The equilibrium shadow price of knowledge is determined by the arbitrage condition for allocating inputs across activities. In equilibrium, the R&D expenditure share is the same as the R&D employment share within firm, so we define R&D intensity as $L_{iht}/\left(N_{iht}+L_{iht}\right)$. Combined with the conditions for optimal input allocation, (15) implies

$$\frac{N_{iht}}{L_{iht}}=\frac{1}{\psi}\left(\frac{\chi_{iht-1}/\chi_{iht}-(1-\delta_T)}{\gamma_{iht}-(1-\delta_T)}-A_i\rho_i\right). \tag{16}$$

4.1 Aggregate Balanced growth path

We look for a balanced growth path equilibrium (BGP), along which aggregate variables are growing at constant rates although industry TFP growth rates may be different.[7] Conditions under which a BGP exist in a multisector endogenous growth model are of independent interest. Such a BGP requires a constant ratio of consumption to capital; if q is the relative price of capital, this ratio is the expression $c/\left(qk\right)$.

Define ρ_c and ρ_x as weighted averages of the receptivity parameters for aggregate consumption and aggregate capital,[8] and define Φ as:

$$\Phi\equiv\left(\frac{1-\rho_x}{\psi}-\frac{\alpha}{1-\alpha}\right)^{-1}. \tag{17}$$

Proposition 1 *Suppose there exists an equilibrium with $l_i, n_i>0$ that satisfies the transversality conditions for T_i and k. If $\Phi>0$, then there exists a unique aggregate balanced growth path. Along this path c/q and k grow by a constant factor $\left(\gamma_x^*\right)^{1/(1-\alpha)}$ where*

$$\gamma_x^*=g_N^{\Phi}. \tag{18}$$

[7] Ngai and Pissarides (2007) show that balanced growth with different values of γ_i is possible in an exogenous growth setting.

[8] Specifically, $\left(1-\rho_x\right)^{-1}=\sum_{j=m}^{z}\omega_j\left(1-\rho_j\right)^{-1}$ and $\left(1-\rho_c\right)^{-1}=\sum_{i=1}^{m-1}\omega_i\left(1-\rho_i\right)^{-1}$.

and knowledge T_i grows by a factor γ_i^* where[9]

$$(\gamma_i^*)^{1-\rho_i} = (\gamma_x^*)^{1-\rho_x} \qquad \forall i. \tag{19}$$

The proof observes that the return to investment is constant if k grows by a factor $\gamma_{xt}^{1/(1-\alpha)}$, which by (14) is constant if TFP growth is constant in all the capital good sectors. The restriction for constant sectorial TFP growth follows from the firm's dynamic optimization condition (15).

From the household's Euler condition, consumption growth is constant over time if the return to saving in terms of consumption goods is constant. In this model, however, there are $z+1$ ways of saving – carrying resources from one period to another. Agents may invest in physical capital, or in knowledge in any of z industries. For physical capital, both the return to investment and the investment rate are constants along the BGP. The analogous condition for knowledge is that the growth rate of the shadow price of knowledge χ_{it+1}/χ_{it} and the "yield" of knowledge F_{it}/T_{it} are constant over time. Proposition 1 emerges from these conditions. For capital goods industries, the constancy of F_{it}/T_{it} implies equation (18) whereas the equivalence of F_{it}/T_{it} and F_{jt}/T_{jt} across any industries i and j implies equation (19).

Proposition 1 contrasts with the behavior of the one-sector model of Jones (1995). In Jones (1995), the condition for balanced growth is similar to (18), replacing Φ with the expression $\psi/(1-\rho)$, where ρ is the receptivity parameter for the aggregate economy. Note that this is the same as requiring $\Phi > 0$ when $\alpha \to 0$. Thus, the Jones (1995) restriction is not sufficient when capital is used in the production for knowledge, as productivity improvements targeting capital goods become a factor of aggregate productivity growth. In addition, Jones (1995) requires $\rho < 1$, whereas our multi-sector model restricts only the *weighted average* of receptivity parameters across capital goods, not for the economy as a whole nor for any particular sector.[10]

4.2 Comparing industries

In the remainder of the paper, we focus on the relationship between equilibrium TFP growth, R&D intensity and industry parameters.[11] From (19), we immediately conclude that:

[9] Proposition 8 in the Appendix reports *sufficient* conditions for the existence of a BGP with R&D activity in all sectors.

[10] Some aggregate estimates of ρ are larger than unity and in a one-sector context this poses a potential puzzle – see Samaniego (2007) for a discussion. This need not pose a puzzle in a multisector context.

[11] We show in Appendix that the BGP satisfies the Kaldor (1961) stylized facts of a constant consumption-output ratio and a constant real interest rate.

Proposition 2 *Along the BGP, consider two sectors i, j such that $\gamma_i, \gamma_j > 1$. Then, $\gamma_i \geqslant \gamma_j$ if and only if $\rho_i \geqslant \rho_j$.*

Along the BGP, the shadow price of knowledge grows by a constant factor :

$$\frac{\chi_{it+1}}{\chi_{it}} = \frac{\gamma_x}{\gamma_i} \left(\frac{\gamma_x^{(1-\rho_x)/\psi}}{G} \right) \tag{20}$$

where $G \equiv 1 - \delta_k + \frac{R_t}{p_{xt}}$ is the gross return on capital. It follows from (16) that R&D intensity in each sector is constant.

The equilibrium value of $\frac{\chi_{it+1}}{\chi_{it}}$ depends only on one industry parameter, ρ_i. Still, equations (16) and (20) imply that industries with the same level of receptivity (ρ_i) but different appropriability (A_i) will have different R&D intensity even if they have the same equilibrium TFP growth rate.

Proposition 3 *Along the BGP, for any sectors with positive TFP growth rates, R&D intensity is increasing in receptivity ρ_i and in appropriability A_i.*

4.3 Cross-industry spillovers

Suppose that it is possible for knowledge in any sector i to influence knowledge of type $j \neq i$. Let the knowledge production function be:

$$F_{iht} = Z_i T_{iht}^{\kappa_i} T_{it}^{\sigma_i} \left(\prod_{j \neq i} T_{jt}^{\rho_{ij}} \right) \left(Q_{iht}^{\alpha} L_{iht}^{1-\alpha} \right)^{\psi} \tag{21}$$

where ρ_{ij} is the extent to which sector i benefits from knowledge produced in sector j. Equation (3) is the special case in which $\rho_{ij} = 0 \; \forall i \neq j$. Recalling that $\rho_i = \kappa_i + \sigma_i$ and letting $\rho_{ii} \equiv \rho_i$, define the *total receptivity* of industry i as $\sum_j \rho_{ij}$: the total spillovers received by firms in industry i. An industry is more receptive than another if total receptivity is larger.

Along the BGP, sectorial TFP growth rates depend on the full matrix of spillovers ρ_{ij}:

$$g_i - \sum_j \rho_{ij} g_j = \log \left(\gamma_x^{\alpha/(1-\alpha)} \gamma_N \right)^{\psi} \tag{22}$$

However, as in the case without cross-industry spillovers, it does not depend on appropriability shares A_i nor on demand parameters ω_i and μ_i. To proceed further, we examine two special cases:

Case 1 *For all* j *and* $i \neq j$, $\rho_{ij} = \tilde{\rho}_j$.

Case 2 *If* $\rho_{ij} \neq 0$, *then* $\rho_{ik} = 0$ *and* $\rho_{kj} = 0$ *for* $k \neq i, j$.

Under Case 1, industries generate knowledge that spills over in the same fashion to all other industries. For example, the Computer industry generates knowledge that is equally useful for generating new knowledge in Communications and in Aircraft, and the Communications industry generates knowledge that is equally useful for generating new knowledge in Computing and in Aircraft. On the other hand, the spillover that Aircraft receives from Communications may be different from the spillover it received from Computing.

Under Case 2, industries are in spillover "pairs." For example, if Communications and Computing receive spillovers from each other, they do not receive spillovers from other industries. Note that it is not required that $\rho_{ij} = \rho_{ji}$, nor that $\rho_i = \rho_j$.

Proposition 4 *Along the BGP for Cases* 1 *and* 2, *if* $\gamma_i, \gamma_j \geqslant 1$, *then* $\gamma_i \geq \gamma_j$ *if and only if sector* i *is more receptive than* j.

4.4 Discussion

How do our results compare to the empirical literature? First, the model ranking of TFP and R&D intensity is stable along a BGP, which allows us to make meaningful comparisons across industries. However, is this consistent with the data? We computed TFP growth rates for durable goods over non-overlapping 5-year periods, using the procedure applied later in Section 6 to account for quality improvements. We found that the correlations among cross sections were always 80% or higher. Ilyina and Samaniego (2007) find that the decade-to-decade correlation of R&D intensity across US manufacturing industries is over 90%.

Second, Proposition 2 states that the ranking of TFP growth rates depends on one parameter – ρ_i – whereas Proposition 3 implies that the ranking of R&D intensities depends on two parameters – ρ_i and A_i. Thus, consistent with the findings reviewed in Section 2, TFP growth depends on factors of technological opportunity, whereas R&D intensity also depends upon appropriability. As a result, TFP growth rates and R&D intensity may or may not be correlated in the model, depending on the quantitative impact of A_i. In particular, industries with rapid TFP growth will be relatively R&D intensive, provided that inter-firm spillovers are small or vary little across industries. Thus, a third prediction is that there should be a *negative* relationship between measures of *intra*-industry spillovers and R&D intensity, controlling for other variables. This is exactly what Nelson and Wolff (1997) find.

Third, Klevorick et al (1995) identify two effects of appropriability on R&D intensity. First, in their terminology, there is an "incentive effect" whereby large, uninternalized spillovers reduce R&D activity, causing the negative relationship between appropriability A_i and R&D intensity in Proposition 3. Second, there is an "efficiency" effect, whereby larger spillovers may *encourage* R&D at other firms. The efficiency effect is seen in that, conditional on κ_i, a larger value of σ_i raises ρ_i while leaving $A_i\rho_i$ constant, so that R&D intensity rises. However, in our model, the "efficiency" effect is related to the magnitude of spillovers, not to appropriability *per se* and, as suggested by Klevorick et al (1995), this effect disappears once opportunity (ρ_i) is kept constant.[12]

Fourth, note that demand parameters ω_i and μ_i affect neither TFP growth rates nor R&D intensity along the BGP. General equilibrium mechanisms play a key role in this result. The relative price levels of different goods depend on ω_i, and the elasticity of a firm's demand function depends on μ_i. Since ω_i affects the level of returns to production at all dates, but not their growth rate, it does not affect the decision of whether to use resources for current production or for investment in knowledge. As for μ_i, the reason it may matter in a partial equilibrium framework is that elastic demand allows an innovator to increase market share without having to lower her price to the same extent as the cost reduction. However, in equilibrium, all firms are performing research: R&D by the firm's *competitors* results in a commensurate fall in the relative price of *their* goods, so that this partial equilibrium benefit of research need not materialize in general equilibrium.

4.5 Model Assumptions and Extensions

In this paper we make several assumptions about functional forms, which we now discuss. For example, we allow industries to differ in terms of all the factors raised in the empirical literature that studies the determinants of R&D intensity. However, other parameters could vary across industries too.

So far, we have assumed that capital shares are the same across industries and activities. However, an important channel leading to the determination of equilibrium TFP growth rates is the "price mechanism" whereby the price of capital declines as a result of productivity change. This encourages R&D, and explains why γ_x enters the equilibrium TFP growth rate of each industry. If we allow capital shares

[12] As they put it, "given demand and opportunity, stronger appropriability enhances the private incentive to engage in R&D, but weaker appropriability lowers the cost of research (increases opportunity) for others." Thus, their terminology does not distinguish between the magnitude of spillovers (which is a factor of opportunity) and appropriability (which holds opportunity constant).

to vary across industries and activities, this price mechanism also contributes to cross-industry TFP growth rate *differences*: capital-intensive industries may enjoy inherently high TFP growth, as suggested by Rosenberg (1969) and Nelson and Winter (1977) inter alia. However, what matters is not capital intensity per se, but the *capital-intensity of research activity.* This is because the flow of capital into research in response to productivity improvements in the capital goods sector depends on this industry parameter. We are not aware of a precedent to this result. See Appendix B for details.[13]

These results are also informative as to how our results would be affected if we were to allow for intermediate goods. To the extent that intermediates benefit from productivity improvements, their price would affect growth rates in much the same way as the price of capital. Thus, intermediates only affect our theoretical results on cross-industry productivity growth comparisons to the extent that the intermediate share *in the production of knowledge* varies across industries.

Ngai and Samaniego (2007) allow for cross industry differences in ψ, the returns to inputs in the knowledge production function. In this case, the industry value of ψ may affect both TFP growth rates and R&D intensity. However, variation in ψ turns out to be incapable of reproducing the range of TFP growth rates in the data.

We show that, along a balanced growth path, demand factors do not matter for industry differences in R&D intensity and productivity growth. In future work, it would be interesting to examine whether demand factors play any role in transition dynamics. We use Cobb-Douglas aggregation for deriving the balanced growth path in this context, and a study of transition dynamics would also allow for more general aggregation with an elasticity of substitution across goods that need not equal one.

The literature on appropriability distinguishes between two channels whereby research by a firm might affect its competitors. The first is the spillover of knowledge, or σ_i in our model. The second is the "business stealing" or "product rivalry" effect whereby innovations by one's competitors decreases one's market share. In our model, the severity of this rivalry depends on μ_i. Even so, this does not imply that μ_i affects equilibrium TFP growth rates since, in equilibrium, all firms perform R&D. Symmetry within industries is not responsible for this result: in notes available upon request, we prove that Propositions 1 and 2 continue to hold in asymmetric equilibria

[13] This is worth underlining. It is well known that differences in factor shares in *output production* affect the expression for relative prices in (12) and the *measurement* of productivity change: when the relative price of capital declines over time, relative prices fall faster than relative TFP in sectors with higher capital shares in output production. However, they do not determine differences in TFP growth rates: only differences in capital shares in the *knowledge production function* do so. We leave an assessment of the quantitative impact of this channel for future work.

such that the distribution of productivity is stable over time within industries, and show that such equilibria exist. Consistent with our results, Bloom et al (2007) estimate that the rivalry effect is quantitatively dominated by technological spillovers.

It is worth commenting further on our approach to appropriability. In general there are three ways for a firm to acquire knowledge for use in production. First, firms may produce knowledge by investing in R&D,as in our model. Second, knowledge that spills over between firms may be used as an input into R&D. This activity is free in the sense that, for example, if one patent cites another, there is no requirement that any payments be made between patent holders. While our model allows for such spillovers, function (2) implies that a firm can only receive spillovers from other firms if it is also carrying out research, as argued by Cohen and Levinthal (1990). Third, firms may employ the knowledge produced by other firms *in production,* by means of a license payment – as in Klenow (1996). However, Arora et al (2002) find that revenues from licensing equal about 4% of R&D expenditure, suggesting that licensing is not a major incentive behind R&D activity in general. We abstract from this third form of knowledge transfers, as the other two appear to be more quantitatively important.[14]

Our model does not distinguish between product and process innovation, for several reasons. First, much (although by no means all) of the related empirical literature neglects the distinction. Second, it is rare that a "truly new" product is introduced. Rather, thinking of industries as being defined at the 2- or 3-digit SIC level, both product and process innovations may result in improved (or cheaper) consumer (or capital) services of a given type. Third, although one-sector growth models that distinguish between product and process innovation sometimes have different properties, such as Young (1998), Jones (1999) argues that these properties are not generic in the sense that they require a "knife-edge" condition on the parameter linking the rate of product innovation to the scale of the economy. Fourth, given that our focus is on industry differences, the exact manner in which we avoid scale effects *on aggregate* is of little consequence. Still, it would be interesting in future work to perform our analysis in a model that allows for product innovation also.

[14] Another potential form of knowledge transfer is a merger. We abstract from mergers for three reasons. First, M&A activity tends to occur in waves, often due to regulatory change – see Andrade el al (2001). Second, since the acquiring firm becomes the owner of the technology and (effectively) pays for the costs of R&D upon acquisition of the target firm, in the final analysis it is as though it had performed its own R&D. This might affect our quantitative results if a lot of mergers are across industries: however, Andrade et al (2001) find that merger activity is under 1% of firms in CRSP by value, and that under half of the mergers in their study are across industries.

5 Research subsidies

As an application of the model, this section studies the planner's problem, and the taxes and subsidies that can replicate optimal allocations. The planner chooses a distribution of capital and labor across sectors at each date.[15] Given the variety of externalities in the model, it is of interest to see how optimal and competitive allocations differ.

In the US and in many countries, R&D is subsidized by means of a tax write-off – equivalent to a uniform subsidy if tax rates on corporate income are constant across sectors. This "one size fits all" policy is built on the assumption of causation between research and productivity growth, rather than an analysis of the underlying causes of research activity – for example, the U.S. Chamber of Commerce (2008) advocates an R&D tax credit on the basis that "research ... promotes both job creation and economic expansion." On the other hand, R&D policy discussions sometimes raise the profile of one sector over another. Nelson and Winter (1977) observe that high productivity growth and the possibility of positive spillovers are raised in policy circles as reasons to subsidize R&D in particular industries. OECD (2001) suggests subsidizing innovation in the service sector, due to its dominant size in most OECD economies and its low TFP growth relative to the manufacturing sector. It is interesting to see how these views contrast with optimal policy in the model economy.

In the model with taxes, we allow the government to assess an industry-specific tax $\tau_i \in \mathbb{R}$ on the sales of industry i, and to apply a subsidy rate $h_i \in \mathbb{R}$ on any R&D expenditures. Proceeds are redistributed via a lump sum T_t to the firms.[16] The setup remains essentially as before (allowing for cross-industry spillovers), except that the profit function becomes:

$$\Pi_{iht} = (1-\tau_i)\, p_{iht} Y_{iht} - w_t\,(N_{iht} + (1-h_i)\, L_{iht}) - R_t\,(K_{iht} + (1-h_i)\, Q_{iht}) + T_t. \quad (23)$$

Proposition 5 *Along a BGP, TFP growth rates in the decentralized economy are the same as in the planner's problem. The allocation of resources in production is efficient in the decentralized problem if and only if*

$$(1-\tau_i)\left(1-\frac{1}{\mu_i}\right) = 1 \quad \forall i = 1, ..z. \quad (24)$$

[15] See Romer (1990) and Krusell (1998) for a discussion of some technical issues that arise in environments with a continuum of choice variables.

[16] In our model, a research subsidy is equivalent to an industry specific R&D tax credit funded out of a tax on profits.

Proposition 6 *When there are no cross-industry spillovers ($\rho_{ij} = 0$ for $i \neq j$), the optimal research subsidy is*

$$h_i^* = (1 - A_i)\,\rho_i \left[\frac{\chi_{it}/\chi_{it+1} - (1 - \delta_T)}{\gamma_i - (1 - \delta_T)} - A_i \rho_i\right]^{-1} \tag{25}$$

Equation (25) has several implications for research policy. The denominator is always positive in an interior solution. Hence, R&D subsidies are positive if and only if spillovers are positive. On the other hand, in the case of "fishing out" ($\rho_i < 0$) whereby new discoveries are progressively more difficult, $h_i^* < 0$ so that R&D should be *taxed*.

Also, conditional on TFP growth rates, industries that perform relatively less R&D should receive higher subsidies. In the model, given γ_i, low R&D intensity is indicative of large, uninternalized spillovers. Still, if appropriability is generally small or varies little across industries, then industries with rapid TFP growth rates deserve higher subsidies.[17]

When we allow for cross-industry spillovers, the R&D intensity in the planner's problem must be determined simultaneously from a system of equations.[18] The optimal R&D subsidy now satisfies:

$$h_i^* = \left[(1 - A_i)\,\rho_i + \textstyle\sum_{s \neq i} \rho_{si} \frac{l_s}{l_i}\right] \left[\frac{\chi_{it}/\chi_{it+1} - (1 - \delta_T)}{\gamma_i - (1 - \delta_T)} - A_i \rho_i\right]^{-1} \tag{26}$$

Two new factors affect the magnitude of optimal research subsidies h_i^*. The first is the magnitude of its spillovers to *other* sectors: h_i^* is increasing in ρ_{si}, $s \neq i$. As in the case without cross-industry spillovers, the optimal R&D subsidy is increasing in γ_i, as it is positively related to receptivity: however, industries may have rapid TFP growth because they receive large spillovers from other industries: whether or not they *provide* spillovers is not reflected in their own value of γ_i. By contrast, industries that *provide* a lot but *receive* little will have low TFP growth, yet should get R&D subsidies nonetheless – as an indirect way to foster knowledge production in *other* sectors. For example, although the service sector is known to have very

[17] We show in the Appendix that χ_{it}/χ_{it+1} for the Planner is the same as in (49), so $\frac{\chi_{it}/\chi_{it+1}-(1-\delta_T)}{\gamma_i-(1-\delta_T)}$ is decreasing in γ_i (hence in ρ_i) and independent of A_i. So h_i is increasing in ρ_i given A_i, and h_i is decreasing in A_i given ρ_i.

[18] Once $\{l_i/n_i\}_i$ in the planner's problem is solved from the system of first order conditions similar to (16), and $\{n_i\}_i$ is solved using the market clearing condition, we can derive $\{l_i\}_i$ which then implies the level of h_i^* for each industry.

low TFP growth (which is due to low receptivity, according to our model), it should receive R&D subsidies if it provides large, positive spillovers to other sectors. Thus, whether productivity growth is a criterion for subsidies depends on whether cross-industry spillovers are significant.

The second new factor is the *size* of the sectors to which an industry provides spillovers. To see this, consider two industries i and j that provide positive spillovers to other industries, and that have identical technological parameters but different demand parameters.

Proposition 7 *Suppose sectors i and j have identical parameters except for ω_i and μ_i. If $\rho_{si}, \rho_{sj} \geq 0$ $\forall s$, then we have $h_i^* > h_j^*$ if and only if $n_i < n_j$.*

For example, if industries i and j are either both consumption industries or both capital industries, then $n_i/n_j = \omega_i/\omega_j$, so that industries with a lower weight in the utility function receive higher subsidies – because they provide spillovers to industries with a larger weight in the utility function.

6 Quantitative implications

6.1 Cross-industry Knowledge Spillovers

Our theory has different implications depending on whether there are significant cross-industry knowledge spillovers. Knowledge spillovers are difficult to measure: however, previous papers such as Jaffe et al (2000) have shown that patent citations appear to represent an indicator of knowledge spillovers, albeit with some degree of noise. Following this work, we draw on the NBER patent citation database described in Hall et al (2001). For each patent granted over the period 1975-1999, the database mentions every patent that it cites – its bibliography. The database also includes patent categories for patents granted 1963-1999, at the 2-digit SIC level and also more finely. As discussed in Hall et al (2006), industries seem to vary in their propensity to patent. We handle this by normalizing cross-citations by the total number of patents in the citing industry. Thus, the citation matrix we construct reflects the average rate at which patents in industry i cite patents in any industry j.[19]

[19] This is analogous to classifying all Economics papers by field, and looking at the rates at which papers in any given field cite papers in any other given field. At the United States Patent and Trademark Office, one role of the patent examiner is to determine that the applicant has cited all relevant "prior art," and the presumption is that this mechanism ensures that patent citations accurately report the intellectual precursors of the patent under review as not doing so would risk

		Spillover source															Total
Code	Spillover recipient	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	CIT
1	Comp. & Office	5.30	0.55	0.01	0.26	0.03	0.05	0.32	0.13	0.00	0.35	0.14	0.00	0.01	0.00	0.29	7.43
2	Commun.	0.49	4.17	0.01	0.27	0.01	0.02	0.36	0.05	0.01	0.23	0.04	0.00	0.01	0.00	0.19	5.85
3	Aircraft	0.10	0.11	2.11	0.07	0.01	0.04	0.19	0.22	0.04	0.06	0.12	0.00	0.06	0.03	0.48	3.64
4	Instr. & Photocop	0.15	0.16	0.00	4.69	0.03	0.00	0.21	0.07	0.00	0.22	0.13	0.00	0.03	0.00	0.69	6.39
5	Fab. Met. Prod	0.05	0.02	0.00	0.07	2.06	0.01	0.29	0.15	0.00	0.07	0.25	0.00	0.02	0.02	0.64	3.62
6	Autos and Trucks	0.06	0.03	0.01	0.02	0.01	2.99	0.11	0.18	0.03	0.02	0.28	0.00	0.13	0.01	0.44	4.32
7	Electrical transm.	0.14	0.11	0.01	0.10	0.09	0.01	3.46	0.05	0.00	0.12	0.09	0.00	0.01	0.00	0.35	4.54
8	Other Durables	0.10	0.04	0.01	0.09	0.07	0.04	0.09	2.59	0.01	0.04	0.31	0.00	0.04	0.02	0.62	4.07
9	Ships and boats	0.01	0.04	0.03	0.02	0.01	0.06	0.08	0.11	2.32	0.01	0.16	0.00	0.04	0.02	0.46	3.36
10	Electrical eq. n.e.c.	0.30	0.23	0.00	0.36	0.07	0.01	0.38	0.05	0.00	2.86	0.07	0.00	0.01	0.00	0.40	4.75
11	Machinery	0.06	0.02	0.00	0.08	0.07	0.04	0.10	0.18	0.01	0.03	2.86	0.00	0.02	0.01	0.72	4.19
12	Mining and oilfield	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.04	0.00	0.00	0.06	0.65	0.00	0.00	0.29	1.09
13	Furniture and fixt	0.01	0.02	0.01	0.08	0.02	0.07	0.06	0.14	0.01	0.03	0.11	0.00	3.00	0.05	0.77	4.37
14	Structures	0.00	0.01	0.01	0.02	0.05	0.01	0.09	0.23	0.01	0.01	0.12	0.00	0.12	3.16	1.08	4.92
15	Other	0.05	0.03	0.00	0.17	0.08	0.02	0.14	0.14	0.01	0.06	0.26	0.01	0.06	0.03	6.66	7.72

Table 1 – Patent citation matrix derived from the NBER patent citation database. We focus on 14 durable goods sectors to match between our patent citation data and the data we use to calibrate the model subsequently.

Table 1 reports the patent citation matrix. Each row corresponds to the average number of citations made by a given industry. Numbers on the diagonal represent within-industry citations. CIT is the sum of each row, the average number of citations per patent in each industry. For all industries, citations are dominated by within-industry citations, suggesting that cross-industry spillovers are relatively small. We therefore proceed with our quantitative applications assuming away cross-industry spillovers. For instance, in the absence of cross-industry spillovers, we can compute receptivity parameters ρ_i given TFP growth rates using equation (19).[20]

6.2 Calibration

We now calibrate the model without cross-industry spillovers, to provide some quantitative applications. First, using relative price data from Cummins and Violante

delaying the approval of the patent. The examiner's name is reported on the patent, so the examiner is responsible for any mis-attributions. Since the bibliography does not include knowledge that is not patented, the presumption is also that the extent to which different sectors rely on each other's knowledge is roughly similar regardless of whether the knowledge concerned is patented or not. In this, our results are conservative: if non-patented knowledge is more likely to remain in-house, then our cross-industry spillovers are upper bounds.

[20] We show in Appendix E how one might compute the receptivity parameters ρ_{ij} in the presence of cross-industry spillovers.

CIT, the average citations per patent in each industry, may indicate the extent to which knowledge in any given sector builds on prior knowledge. Ngai and Samaniego (2007) show that CIT is correlated with TFP growth and R&D intensity across these 14 durables industries.

(2002), we show that industry TFP growth rates can differ substantially even while aggregate growth is constant. Second, to compute industry research intensity, we derive A_i using the proportion of own-industry citations that are *self*-citations. We find that R&D in the model is highly correlated with R&D in the data. Finally, using these parameters, we solve for the optimal R&D subsidy in each sector.

We calibrate the model to US data. To begin, we assume that $m = 2$, so that there is only one sector producing non-durables. We set $z = 15$, so that there are 14 capital producing industries. This partition was the finest that allowed us to use all our data sources, some of which use different industry classification systems.

We set $\alpha = 0.3$ as in Greenwood et al (1997). Samaniego (2007) surveys values of ψ in the range 0.3 to 0.6. We select $\psi = 0.3$: higher values lead to higher R&D intensity, but do not affect results otherwise.

Lemma 2 shows that the model can be aggregated into a 2-sector economy with an investment sector x and consumption sector c. The US National Income and Product Accounts indicate that $g_y = 1.022$ in consumption units, and the US Census Bureau that $g_N = 1.012$. In the model, g_y also represents the growth of real consumption, so $g_y = \gamma_x^{1/(1-\alpha)} g_q$ where $g_q = \gamma_c/\gamma_x$ is growth in the relative price of capital. Cummins and Violante (2002) report that $g_q = 1.026^{-1}$, so that $\gamma_x = 1.0338$ and $\gamma_c = 1.0076$. Equation (18) then implies that $\rho_x = 0.76$. This suggests that knowledge in the capital sector generally "stands on the shoulders" of pre-existing knowledge. On the other hand, $\rho_c = -0.04$, so that knowledge in non-durables is subject to very mild "fishing out," whereby new knowledge becomes progressively harder to generate.

6.3 TFP growth rates and Receptivity

Equation (12) implies a relationship between relative rates of price decline and TFP growth, which we use to compute TFP growth rates $\left(\gamma_i^{CV}\right)$ using the quality-adjusted relative price of capital provided by Cummins and Violante (2002).[21] Equation (19) yields the implied value of ρ_i, given the values of γ_x and ρ_x computed above. Results

[21] We calibrate the model using relative price data instead of TFP growth rates for several reasons. First, the TFP measures do not map directly into the model, as their construction allows for factors that are not present such as intermediate goods, energy, etc. Second, we take seriously the view of Greenwood et al (1997) among others that quality improvements are an important source of prodcuvity change. Quality adjusted prices are mostly available only for durable goods. Third, given that several authors have found TFP growth rates to be closely related to R&D intensity, it would be no surprise to find a strong correlation between R&D intensity in the model and in the data. Instead, we calibrate TFP growth rates in the model using quality-adjusted price data – so there is no a-priori reason to expect a positive link between R&D intensity in the data and in the model.

are reported in Table 2.

Capital good sector	γ_i^{CV}	ρ_i
Computers and office equipment	20.48	0.96
Communication equipment	10.21	0.92
Aircraft	9.36	0.91
Instruments and photocopiers	6.81	0.88
Fabricated metal products	3.81	0.79
Autos and trucks	3.76	0.79
Electrical transm. distrib. and industrial appl.	3.72	0.79
Other durables	3.48	0.77
Ships and boats	3.16	0.75
Electrical equipment, n.e.c.	3.00	0.73
Machinery	2.82	0.72
Mining and oilfield machinery	2.38	0.67
Furniture and fixtures	2.15	0.63
Structures	1.82	0.57

Table 2 – TFP growth rates across capital goods, based on the quality-adjusted relative price of capital from Cummins and Violante (2002) $\left(\gamma_i^{CV}\right)$. Values of ρ_i are based on γ_i^{CV}, using equation (19), assuming no cross-industry spillovers, and assuming benchmark values of parameters.

Based on relative prices, TFP growth rates across capital types range from 20% for Computers and Office equipment to about 2% for Structures. The model is consistent with a wide dispersion of TFP growth rates and suggests a wide distribution of values of ρ_i across different types of capital good.

6.4 Appropriability

The citation data may also be used to construct an estimate of appropriability. The data report the assignee of each patent awarded since 1969. As a result, we can establish what proportion of own-industry citations are in fact self-citations. We define appropriability A_i as this ratio. Combining with values of ρ_i in Table 3, we can compute κ_i and σ_i ($\sigma_i = \rho_i - \kappa_i$). The required assumption is that κ_i and σ_i do not differ significantly depending on whether or not knowledge is patented. If unpatented knowledge flows across firms more easily than patented knowledge, then the measure of spillovers implied by the patent data is an upper bound on A_i. On the other hand, if ideas that flow most easily across firms are the ones patented, then our numbers represent a lower bound on A_i. The patent data represent an unusually rich source of information on knowledge spillovers, so we proceed while keeping these caveats in mind. Nonetheless, as we shall see, the difference between patented and unpatented knowledge must be quite drastic to affect our results.

Capital good sector	ρ_i	A_i	κ_i
Computers and office equipment	0.96	0.16	0.15
Communication equipment	0.92	0.16	0.15
Aircraft	0.91	0.19	0.17
Instruments and photocopiers	0.88	0.17	0.15
Fabricated metal products	0.79	0.21	0.17
Autos and trucks	0.79	0.19	0.15
Electrical transm. distrib. and industrial appl.	0.79	0.16	0.13
Other Durables	0.77	0.19	0.15
Ships and boats	0.75	0.16	0.12
Electrical equipment, n.e.c.	0.73	0.22	0.16
Machinery	0.72	0.18	0.13
Mining and oilfield machinery	0.67	0.34	0.22
Furniture and fixtures	0.63	0.13	0.08
Structures	0.57	0.12	0.07

Table 3 – Receptivity ρ_i from Table 2 and appropriability A_i based on the NBER patent citation database.

Table 3 finds that appropriability A_i is generally quite low – 18.5% on average.[22] Consequently, R&D intensity in equation (16) will be mainly determined by receptivity. As we will see, this affects the pattern of optimal R&D intensity and optimal R&D subsidies in the model. It also suggests that receptivity ρ_i should account for both R&D intensity and TFP growth.

6.5 Optimal R&D subsidies

We now look at the optimal research subsidy for each of these industries. Given A_i and ρ_i, equations (16) and (20) imply that we require values of G and δ_T to

[22] If we measure A_i using the proportion of self citations out of all citations, the average is even lower, about 13%.

derive R&D intensities.[23] We match the real rate of return to capital to be 7% as in Greenwood et al (1997). Hence the gross return in terms of capital goods is $G = 1.07/g_q$, where $g_q = 1.026^{-1}$ as before. We choose $\delta_T = 0$ as a benchmark.[24]

Figure 2 displays R&D intensity in the model, assuming the average level of appropriability and the values of ρ_i in Table 1. Values are higher than in the data, as the model concept of knowledge is probably broader than simply scientific R&D. However, the correlation between the two series is striking. Notably, the results are insensitive to variations in appropriability: as A_i is small, R&D in the model is mainly determined by receptivity.

It is notable that this correlation is not simply due to the link between TFP and R&D intensity in Figure 1. The receptivity values used to compute R&D intensity are based on quality adjusted relative prices. Figure 3 shows that these too correlate highly with R&D in the data. Hence, while the mapping between TFP and relative prices in equation (12) rests on assumptions about input shares across sectors, those assumptions do not seem too far off the mark.

In Table 4, the mean optimal subsidy rate for capital goods industries is 38%, but ranges up to over 75% for the fastest growing sectors. The optimal subsidy rate is highly correlated with ρ_i across capital goods (82%). Thus, the model suggests subsidizing the *fastest-growing industries.* This is because appropriability A_i varies a lot less across industries than the magnitude of spillovers σ_i themselves – and, in Table 3, σ_i accounts for the bulk of receptivity. Industries deserve subsidies in the model when they provide large knowledge spillovers: however, since the main beneficiary of the knowledge of any given industry seems to be that industry itself, allowing for cross industry spillovers would not change this conclusion.

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[23] We also require information on R&D subsidies. In the US, mostly this is done through R&D tax credits. In practice the credit rate is about 13% of expenditures: see Wilson (2005). Only expenditures above a certain limit count towards the credit, which is 3% of sales for new firms or a 3-year moving average of past R&D spending otherwise. Our R&D intensity measure in Table (5) is mostly lower, and Wilson (2005) notes that federal R&D tax credits are in fact "recaptured" (i.e. taxed back). All this suggests that the effective subsidy is very small. Hence, we assume $h_i = 0\ \forall i$ in the benchmark economy.

[24] Samaniego (2007) surveys values up to 25%, but these are all measures of the *economic* depreciation of ideas, whereas the "physical" rate at which ideas cease to be altogether useful in production or in research is likely very small. Ngai and Samaniego (2007) find that large values of δ_T generate high values for R&D intensity. The average ratio of R&D to GDP in our economy is about 5%, which is larger than the value reported by the National Science Foundation but is in line with estimates that include "non-scientific" R&D by Corrado et al (2006).

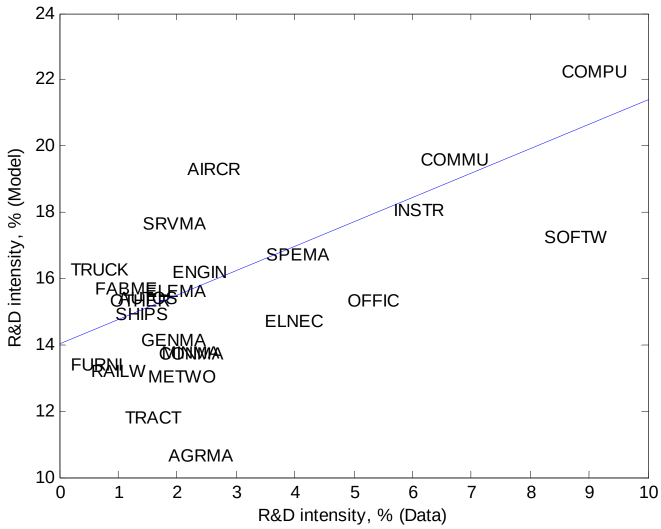

Figure 2: R&D intensity in the model and the data. R&D in the model is derived using equation (16), assuming appropriability of 18.5% in all industries. R&D intensity in US data is measured using the ratio of R&D spending to sales at the median firm in Compustat, 1950-2000. Industries included are the 25 categories reported in Cummins and Violante (2002). The correlation between R&D in the model and the data is 64.7%, P-value = 0.06%. Using instead values of appropriability computed from the citation data, and imputing the average value for software (which is not in the citation data), the correlation rises to 65.8%, P-value 0.05%.

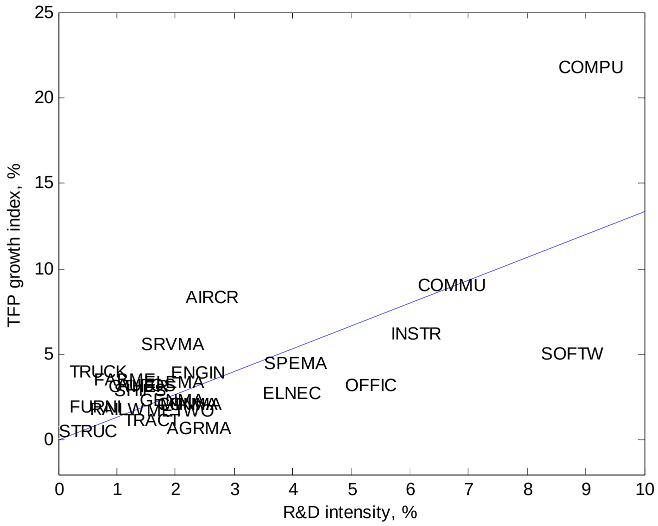

Figure 3: TFP in the model and R&D intensity. TFP growth is derived from equation (19), using the quality-adjusted relative price of capital goods in Cummins and Violante (2002), 1947-2000. R&D intensity in US data is measured using the ratio of R&D spending to sales at the median firm in Compustat, 1950-2000. Industries included are the 25 categories reported in Cummins and Violante (2002). The correlation between γ_i and R&D in the data is 69%, P-value = 0.01%. If the outlier (Computers and peripherals) is deleted, the correlation drops to 56%, P-value 0.4%.

Capital good sector	Model	Planner	Ratio	Subs.
Computers and office equipment	22.2	55.0	40.4	76.6
Communication equipment	19.7	39.8	49.4	63.0
Aircraft	19.6	37.9	51.7	60.0
Instruments and photocopiers	17.7	31.2	56.8	52.5
Fabricated metal products	14.3	20.7	79.4	35.7
Autos and trucks	14.4	20.5	70.2	34.8
Electrical transm. distrib. and industrial appl.	14.0	20.3	69.2	35.9
Other durables	13.8	19.3	71.4	33.2
Ships and boats	13.0	17.9	72.8	31.3
Electrical equipment, n.e.c.	12.8	17.1	75.4	29.4
Machinery	12.5	16.3	76.7	26.4
Mining and oilfield machinery	11.7	14.1	82.8	19.5
Furniture and fixtures	10.5	13.0	80.8	21.4
Structures	9.5	11.3	84.2	17.4

Table 4 – R&D intensity in the decentralized model and the planner's solution. The third column is the ratio of model R&D to the planner's. The fourth is the subsidy rate h_i. All values are percentages.

Equilibrium R&D intensity ranges from $40-84\%$ of the planner's value, depending mainly on ρ_i – since A_i is too small to be of quantitative importance. In a one-sector model, Jones and Williams (1998) find that R&D intensity is between half and a quarter of its optimal level, suggesting that our measures of appropriability are more likely to be upper than lower bounds. Unlike them, however, we find a wide variety of "wedges" between actual and optimal R&D across industries: there is no "one size fits all" research policy, primarily because of significant differences in receptivity across sectors.

7 Concluding remarks

We develop a multi-sector, general equilibrium model of endogenous growth, incorporating a number of factors identified in the literature as potential determinants of the costs and benefits of research, based on preference and technological primitives drawn from the growth literature. In the model, we find that the main long-run determinant of productivity growth differences across sectors is the extent to which pre-existing knowledge is useful for producing new ideas – *receptivity*. Although this parameter has not been identified as a potentially important source of cross-industry differences in the related literature, it turns out to play a pivotal role in a growth model that is consistent with stable growth over the long run.

In addition, the fraction of receptivity that accrues from the firm's own stock of knowledge affects research intensity but not TFP growth, whereas demand factors affect neither. This is consistent with the lack of robustness in the empirical literature on the role of demand, and is also in line with a sense in the technology literature that technical change is primarily supply-driven. Nelson and Winter (1977) argue that innovations follow "natural trajectories" that have a technological or scientific rationale rather than being driven by movements in demand and, similarly, Rosenberg (1969) writes of innovation following a "compulsive sequence." In our model, the incentives to conduct research depend very much on demand-side factors: nonetheless, *in equilibrium*, the primary determinant of differences in long run productivity growth rates is receptivity. Thus, in the model, "natural trajectories" are an *equilibrium outcome*, as long-run TFP growth rates are determined by technological factors.

We do find that demand parameters may matter for the planner's allocation of R&D activity, and hence for optimal R&D subsidies. This result depends on whether there are cross-industry knowledge spillovers, which the patent citation data suggest are weak. Nonetheless, the broader point is that whether an industry should optimally receive subsidies may depend not just on its own characteristics but on those of the industries that benefit from the knowledge it produces. It would be useful in future work to develop microfoundations for these spillover parameters, which could suggest a richer set of policies and mechanisms that might achieve a more efficient allocation of research activity.

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A Derivations and Proofs

A.1 Household maximization

We first determine the optimal spending across different goods taking as given the total per capita spending on consumption s_c and total spending on investment s_x. Omitting time subscripts, the maximization problems across goods are

$$\begin{aligned} \max_{\{c_{ih}\}} c \qquad s.t. \qquad s_c &= \sum\nolimits_{i=1}^{m-1} \int_0^1 p_{ih} c_{ih} dh, \qquad \text{and} \\ \max_{\{x_{jh}\}} x \qquad s.t. \qquad s_x &= \sum\nolimits_{j=m}^{z} \int p_{jh} x_{jh} dh \end{aligned}$$

where c and x are defined in the household problem. The optimal spending within sectors $i = 1, ..m-1$, across different varieties h, is

$$(c_{ih}/c_{ih'})^{\frac{-1}{\mu_i}} = p_{ih}/p_{ih'} \Longrightarrow c_{ih'} = c_{ih} \left(p_{ih}/p_{ih'}\right)^{\mu_i} \tag{27}$$

which implies

$$c_i = \left(\int_0^1 c_{ih'}^{\frac{\mu_i - 1}{\mu_i}} dh'\right)^{\frac{\mu_i}{\mu_i - 1}} = c_{ih} \left[\int \left(p_{ih}/p_{ih'}\right)^{\mu_i - 1} dh'\right]^{\frac{\mu_i}{\mu_i - 1}} \tag{28}$$

Using (28), define $p_i \equiv \left[\int p_{ih} c_{ih} dh\right] / c_i = \left[\int p_{ih}^{1-\mu_i} dh\right]^{1/(1-\mu_i)}$, we can rewrite (28) as $c_i = c_{ih} \left(p_{ih}/p_i\right)^{\mu_i}$. Across good i, Cobb-Douglas utility yields $p_i c_i / \left(p_j c_j\right) = \omega_i / \omega_j$, so $p_i c_i = \omega_i s_c$, together with the utility function,

$$p_c \equiv s_c/c = \prod_{i=1}^{m-1} p_i^{\omega_i} \tag{29}$$

and the demand for good ih is

$$c_{ih} = s_c \left(p_i/p_{ih}\right)^{\mu} \omega_i/p_i \tag{30}$$

The result follows analogously for investment,

$$x_{jh} = s_x \left(p_j/p_{jh}\right)^{\mu_j} \left(\omega_j/p_j\right) \text{ and } x_j = s_x \left(\omega_j/p_j\right), \tag{31}$$

where p_j is analogously defined as before, and

$$p_x \equiv s_x/x = \prod_{j=m}^{z} p_j^{\omega_j} \tag{32}$$

Given the solution of the static maximization, the dynamic problem is

$$\begin{aligned} &\max_{\{c_t, x_t\}} \textstyle\sum_{t=0}^{\infty} \left(\beta g_N\right)^t u\left(c_t\right) \qquad s.t. \\ p_{ct} c_t + p_{xt} x_t &= w_t + R_t k_t + \pi_t \\ g_N k_{t+1} &= x_t + \left(1 - \delta_k\right) k_t \end{aligned}$$

The solution implies

$$\frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)} = \frac{p_{xt+1}/p_{ct+1}}{p_{xt}/p_{ct}} \left(1 - \delta_k + \frac{R_{t+1}}{p_{xt}}\right) \tag{33}$$

A.2 Firm's maximization

The firm's maximization problem is

$$\begin{aligned} &\max_{\{N_{it}, K_{it}, Q_{it}, L_{it}\}} \textstyle\sum_{t=0}^{\infty} \lambda_t \Pi_{iht}/p_{ct} \qquad s.t. \\ T_{iht+1} &= F_{iht} + \left(1 - \delta_T\right) T_{iht} \\ Y_{iht} &= N_t c_{iht} \qquad if \qquad i = 1, ..m-1 \\ Y_{iht} &= N_t x_{jht} \qquad if \qquad i = m, ...z \end{aligned}$$

Given Π_{iht} in (4), the static efficiency conditions are standard: $\frac{\partial Y_{iht}/\partial N_{iht}}{\partial Y_{iht}/\partial K_{iht}} = \frac{w_t}{R_t} = \frac{\partial F_{iht}/\partial L_{iht}}{\partial F_{iht}/\partial Q_{iht}}$. The assumptions on production functions imply

$$\frac{K_{iht}}{N_{iht}} = \frac{Q_{iht}}{L_{iht}}; \quad p_{iht} \left(1 + \frac{Y_{jht}}{p_{jht}} \frac{\partial p_{jht}}{\partial Y_{jht}}\right) T_{iht} = p_{jht} \left(1 + \frac{Y_{jht}}{p_{jht}} \frac{\partial p_{jht}}{\partial Y_{jht}}\right) T_{jht} \tag{34}$$

Using the demand function, relative prices are

$$\frac{p_{iht}}{p_{jht}} = \frac{T_{jht}\left(1 - 1/\mu_j\right)}{T_{iht}\left(1 - 1/\mu_i\right)} \tag{35}$$

The dynamic efficiency condition involves the optimal R&D decision. The first order condition of the Lagrangian with respect to T_{iht+1} is

$$\frac{\lambda_{t+1}}{p_{ct+1}} \frac{\partial \Pi_{iht+1}}{\partial T_{iht+1}} - \chi_{iht} + \chi_{iht+1}\left(\frac{\partial F_{iht+1}}{\partial T_{iht+1}} + 1 - \delta_T\right) = 0 \tag{36}$$

where the shadow price for T_{iht+1} is:

$$\chi_{iht} = \left(\frac{\lambda_t}{p_{ct}}\right) \frac{R_t}{\partial F_{iht}/\partial Q_{iht}}. \tag{37}$$

A.3 Market Equilibrium

The capital market clearing condition (11) and equal capital-labor ratios (34) imply

$$K_{iht}/N_{iht} = Q_{iht}/L_{iht} = K/N = k, \qquad \text{and} \tag{38}$$

$$R_t = \alpha p_{iht} T_{iht} k_t^{\alpha-1}\left(1 - 1/\mu_i\right); \quad w_t = \left(1 - \alpha\right) p_{iht} T_{iht} k_t^{\alpha}\left(1 - 1/\mu_i\right). \tag{39}$$

We now focus on the symmetric equilibrium across h within i.

A.4 Proofs

The equilibrium structure is summarized in the following claims.

Lemma 1 *In equilibrium,* (12) *holds.*

Proof of Lemma 1. See the derivation of (35) and (38) in the Firm's maximization. ■

Lemma 2 *Let* $1/\mu_x = \sum_{i=m}^{z} \omega_i \left(1/\mu_i\right)$. *Investment industries* $j \in \{m, ..., z\}$ *can be aggregated into one sector with a production function*

$$N_t x_t \equiv T_{xt}\left(\sum_{j=m}^{z} K_{jt}\right)^{\alpha}\left(\sum_{j=m}^{z} N_{jt}\right)^{1-\alpha} = T_x k^{\alpha} \sum_{i=m}^{z} N_{it} \tag{40}$$

where the knowledge index T_{xt} *equals* (13).

Proof of Lemma 2. Define T_x as in (40), where the second equality follows from (38). To determine T_x, (31) implies $\frac{p_j T_j k^\alpha n_j}{p_i T_i k^\alpha n_i} = \frac{\omega_j}{\omega_i}$ and, by (35), $\frac{n_i}{\omega_i} = \left(\frac{1-1/\mu_i}{1-1/\mu_j}\right)\frac{n_j}{\omega_j}$, so

$$\sum_{i=m}^{z} n_{it} = \frac{n_j\left(1-1/\mu_x\right)}{\omega_j\left(1-1/\mu_j\right)} \tag{41}$$

where we define μ_x such that

$$1-1/\mu_x = \sum_{i=m}^{z}\omega_i\left(1-1/\mu_i\right) \Leftrightarrow 1/\mu_x = \sum_{i=m}^{z}\omega_i/\mu_i \tag{42}$$

By definition, $x = \prod_{j=m}^{z}\left(x_j/\omega_j\right)^{\omega_j} = \prod_{j=m}^{z}\left(T_j k^\alpha n_j/\omega_j\right)^{\omega_j}$, so using (41) and (42), we obtain

$$x = k^\alpha\left(\sum_{i=m}^{z} n_{it}\right)\prod_{j=m}^{z}\left[T_j\left(\frac{1-1/\mu_j}{1-1/\mu_x}\right)\right]^{\omega_j}$$

so the index of knowledge is (13). ■

Let q_t be the relative price of capital and G_t the gross return on investment in terms of capital goods. Then,

$$q_t = \frac{p_{xt}}{p_{ct}} \qquad G_t \equiv 1-\delta_k+\frac{R_t}{p_{xt}}. \tag{43}$$

Lemma 3 *The Euler condition for the consumer satisfies*

$$\frac{1}{\beta}\left(\frac{c_{t+1}}{c_t}\right)^\theta = \frac{q_{t+1}}{q_t}G_{t+1} \tag{44}$$

where the equilibrium physical gross return of investment is:

$$G_t = 1-\delta_k+\alpha T_{xt}k_t^{\alpha-1}\left(1-\frac{1}{\mu_x}\right). \tag{45}$$

Proof of Lemma 3. The Euler condition follows from (33) in the Consumer's Maximization. Using (32) and (35),

$$\begin{aligned}\frac{p_x}{p_i} &= \prod_{j=m}^{z}\left(\frac{p_j}{p_i}\right)^{\omega_j} = \prod_{j=m}^{z}\left(\frac{T_i\left(1-1/\mu_i\right)}{T_j\left(1-1/\mu_j\right)}\right)^{\omega_j} \\ &= T_i\left(1-1/\mu_i\right)\prod_{j=m}^{z}\left[T_j\left(1-1/\mu_j\right)\right]^{-\omega_j} \quad \forall i,\end{aligned}$$

so by (13), we have

$$\frac{p_x}{p_i} = \frac{T_i\left(1-1/\mu_i\right)}{T_x\left(1-1/\mu_x\right)} \quad \forall i, \tag{46}$$

together with (39), we have

$$R/p_x = \alpha p_i T_i k^{\alpha-1} \left(1 - 1/\mu_i\right)/p_x = \alpha k^{\alpha-1} T_x \left(1 - 1/\mu_x\right),$$

so the expression for G follows from its definition. ■

Lemma 4 *If positive, the firm's R&D intensity satisfies* (16). *Equilibrium employment shares for production satisfy*

$$\begin{aligned} n_{it} &= \omega_i \frac{c_t/q_t}{T_{xt} k_t^{\alpha}} \left(\frac{1 - 1/\mu_i}{1 - 1/\mu_x}\right) \qquad \forall i < m, & (47) \\ n_{jt} &= \frac{1 - 1/\mu_j}{1 - 1/\mu_x} \omega_j \left(\sum_{s=m}^{z} n_{st}\right) \qquad \forall j \geq m. & (48) \end{aligned}$$

Proof of Lemma 4. Equation (15) is derived in the firm's optimization (36), n_{iht}/l_{iht} in (16) follows from (2) and (37). To obtain n_i, use market clearing and the expenditure share of good $i = 1, ..m - 1$, $p_i T_i k^{\alpha} n_i = p_i c_i = \omega_i p_c c$, and so $n_i = \omega_i \frac{c/q}{k^{\alpha}} \left(\frac{p_x}{T_i p_i}\right)$ where $q = p_x/p_c$. The result for n_i follows from (46). The result for n_j, $j = m, ...z$, follows from (41). ■

Lemma 5 *The shadow price* χ_{iht} *satisfy:*

$$\frac{\chi_{iht}}{\chi_{iht+1}} = \left(g_k^{\alpha} g_N \frac{l_{it+1}}{l_{it}}\right)^{\psi-1} \frac{G_{t+1} \gamma_{iht}^{\kappa_i} \gamma_{it}^{\sigma_i}}{\gamma_{xt}}. \qquad (49)$$

Proof. Note that

$$\begin{aligned} \frac{\partial F_{iht+1}/\partial Q_{iht+1}}{\partial F_{iht}/\partial Q_{iht}} &= \frac{F_{iht+1}}{F_{iht}} \frac{Q_{iht}}{Q_{iht+1}} = \gamma_{iht}^{\kappa_i} \gamma_i^{\sigma_i} \left(\frac{Q_{iht+1}}{Q_{iht}}\right)^{\alpha\psi-1} \left(\frac{L_{iht+1}}{L_{iht}}\right)^{(1-\alpha)\psi} \\ &= \gamma_{iht}^{\kappa_i} \gamma_i^{\sigma_i} \left(k_{t+1}/k_t\right)^{\alpha\psi-1} \left(L_{iht+1}/L_{iht}\right)^{\psi-1} \end{aligned}$$

so using the $F(.)$ function and R from (39),

$$\begin{aligned} \frac{\chi_{iht}}{\chi_{iht+1}} &= \left(\frac{\lambda_t/p_{ct}}{\lambda_{t+1}/p_{ct+1}}\right) \frac{R_t}{R_{t+1}} \left(\frac{\partial F_{iht+1}/\partial Q_{iht+1}}{\partial F_{iht}/\partial Q_{iht}}\right) \\ &= \left(\frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1} p_{iht+1}/p_{ct+1}}\right) \left(\frac{T_{iht} k_t^{\alpha-1}}{T_{iht+1} k_{t+1}^{\alpha-1}}\right) \gamma_{iht}^{\kappa_i} \gamma_i^{\sigma_i} \left(\frac{k_{t+1}}{k_t}\right)^{\alpha\psi-1} \left(\frac{L_{iht+1}}{L_{iht}}\right)^{\psi-1} \\ &= \left(\frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1} p_{iht+1}/p_{ct+1}}\right) \gamma_{iht}^{\kappa_i - 1} \gamma_{it}^{\sigma_i} \left(g_k^{\alpha} g_N \frac{l_{it+1}}{l_{it}}\right)^{\psi-1} \end{aligned}$$

Euler equation (33) implies

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{1+r_{t+1}} = \frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{p_{xt}/p_{ct}}{G_{t+1}p_{xt+1}/p_{ct+1}} \tag{50}$$

Using (46), $\frac{p_{xt+1}/p_{it+1}}{p_{xt}/p_{it}} = \frac{\gamma_i}{\gamma_x}$, so

$$\left(\frac{\lambda_t p_{iht}/p_{ct}}{\lambda_{t+1}p_{iht+1}/p_{ct+1}}\right) = \left(\frac{\lambda_t p_{xt}/p_{ct}}{\lambda_{t+1}p_{ixt+1}/p_{ct+1}}\right)\left(\frac{p_{iht}/p_{xt}}{p_{iht+1}/p_{xt+1}}\right) = G_{t+1}\frac{\gamma_{iht}}{\gamma_{xt}}.$$

Result follows. ■

Proof of Proposition 1. Define $k_{et} = k_t T_{xt}^{-1/(1-\alpha)}$. Let $g_x \equiv x_{t+1}/x_t$ for all variables x. From the Euler condition (44), g_c is constant if g_q and G are constants. From (45), G is constant if and only if k_e is constant. Use Lemma 2 and (38) to rewrite the capital accumulation equation as $g_N g_k = k_{et}^{\alpha-1}\sum_{j=m}^{z} n_{jt} + 1 - \delta_k$, it follows that g_k is constant if and only if $\sum_{j=m}^{z} n_{jt}$ is constant. So (48) and (16) imply n_j and l_j are constants for $j = m, ..z$. By definition, k_e is constant if and only if $g_k = \gamma_x^{-1/(1-\alpha)}$, which by (14) is constant if and only if γ_j is constant for all $j = m, ..z$. Constant γ_j requires constant

$$F_{jt+1}/T_{jt+1} = Z_j T_{jt+1}^{\rho_j - 1}\left(Q_{jt}^{\alpha}L_{jt}^{1-\alpha}\right)^{\psi} = Z_j T_{jt+1}^{\rho_j-1} k_t^{\alpha\psi} l_{jt}^{\psi} N_t^{\psi}$$

to be constant, i.e. $\gamma_j^{\rho_j-1} g_k^{\alpha\psi} g_N^{\psi} = 1$, since $g_k = \gamma_x^{-1/(1-\alpha)}$, so $\forall j \geq m$,

$$1 = \left(\gamma_x^{\alpha/(1-\alpha)} g_N\right)^{\psi}\gamma_j^{\rho_j-1} \tag{51}$$

which implies $\forall i, j \geq m$,

$$\gamma_j^{\rho_j-1} = \gamma_i^{\rho_i-1}, \tag{52}$$

so

$$\gamma_x = \prod_{j=m}^{z}\gamma_j^{\omega_j} = \prod_{j=m}^{z}\left[\gamma_i^{\rho_i-1}\right]^{\frac{\omega_j}{\rho_j-1}} = \left\{\gamma_i^{\rho_i-1}\right\}^{\frac{1}{\rho_x-1}}$$

where $\frac{1}{1-\rho_x} = \sum_{j=m}^{z}\frac{\omega_j}{1-\rho_j}$. Using (51),

$$\gamma_j^{\rho_j-1} = \gamma_x^{\rho_x-1}, \tag{53}$$

then sub. into (51) to obtain

$$\gamma_x^* = g_N^{\Phi}, \qquad \Phi = \left(\frac{1-\rho_x}{\psi} - \frac{\alpha}{1-\alpha}\right)^{-1}. \tag{54}$$

Finally we need to show g_q is constant, using (29),

$$q^{-1} = \prod_{i=1}^{m-1} \left(\frac{p_x}{p_i}\right)^{\omega_i} = \prod_{i=1}^{m-1} \left(\frac{T_i\left(1-1/\mu_i\right)}{T_x\left(1-1/\mu_x\right)}\right)^{\omega_i}, \tag{55}$$

So g_q is constant if and only if γ_i are constants. From (47), n_i are constants for $i < m$. Using (15) for $i < m$, γ_i is constant if and only if (51) holds for $i < m$ as well. Therefore, (52) holds for $i < m$ as well. Finally we verify the results are consistent with (15) by showing both χ_{jht+1}/χ_{jht} and $\frac{1}{\chi_{jht}}\left(\frac{\lambda_{t+1}p_{jht+1}}{p_{ct+1}}\frac{\partial Y_{jht+1}}{\partial T_{jht+1}}\right)$ are constants. Constant χ_{jht+1}/χ_{jht} follows immediately from (49). The second term,

$$\frac{1}{\chi_{jht}}\left(\frac{\lambda_{t+1}p_{jht+1}}{p_{ct+1}}\frac{\partial Y_{jht+1}}{\partial T_{jht+1}}\right) = \frac{\lambda_{t+1}p_{xt+1}/p_{ct+1}}{\lambda_t p_{xt}/p_{ct}}\frac{p_{jt+1}/p_{xt+1}}{p_{jt}/p_{xt}}\frac{\partial F_{jht}/\partial Q_{jht}}{R_t/p_{jt}}\frac{\partial Y_{jht+1}}{\partial T_{jht+1}} \tag{56}$$

using (39), (46), (50), and the production functions,

$$\frac{1}{\chi_{jht}}\frac{\lambda_{t+1}p_{jht+1}}{p_{ct+1}}\frac{\partial Y_{jht+1}}{\partial T_{jht+1}} = \frac{1}{G}\frac{\gamma_x}{\gamma_j}\frac{\alpha\psi T_j^{\rho_j}k_t^{\alpha\psi-1}L_j^{\psi-1}}{\alpha T_{jt}k_t^{\alpha-1}\left(1-1/\mu_j\right)}k_{t+1}^{\alpha}N_{jt+1}$$

which is constant given (51). ■

Proposition 8 *Along the BGP, the non-negativity constraints on l_i and n_i do not bind and the transversality conditions for T_i and k are satisfied if*

$$g_N^{\Phi\left(\frac{1-\rho_x}{1-\rho_i}\right)} \geq 1-\delta_T, \qquad \kappa_i < 1, \quad \forall i. \tag{57}$$

and $\beta < (1/g_N)^{1+(1-\theta)\Phi\Upsilon}$, where Υ is defined below.

Proof of Proposition 8. The transversality conditions are: $\lim_{t\to\infty}\zeta_t k_{t+1} = \lim_{t\to\infty}\chi_{it}T_{it+1} = 0$, $\forall i$. χ_{it} and ζ_t are the corresponding shadow values. Substituting (19) into (49),

$$\begin{aligned}\chi_{it}/\chi_{it+1} &= (g_k^{\alpha}g_N)^{\psi-1}G\gamma_i^{\rho_i}/\gamma_x = \gamma_i^{(1-\rho_i)(\psi-1)/\psi}G\gamma_i\gamma_i^{\rho_i-1}/\gamma_x \\ &= \gamma_i^{(\rho_i-1)/\psi}G\gamma_i/\gamma_x = \gamma_x^{(\rho_x-1)/\psi}G\gamma_i/\gamma_x,\end{aligned}$$

which implies (20) in text. So $\frac{\chi_{it}T_{it}}{\chi_{it-1}T_{it-1}} = \left(\frac{\gamma_x}{G}\right)\gamma_x^{(1-\rho_x)/\psi}$. Using (44), $1/G = \beta g_c^{-\theta}g_q = \beta g_c^{1-\theta}\gamma_x^{-1/(1-\alpha)}$, together with (18)

$$\frac{\chi_{it}T_{it}}{\chi_{it-1}T_{it-1}} = \gamma_x^{(1-\rho_x)/\psi}\beta g_c^{1-\theta}\gamma_x^{-\alpha/(1-\alpha)} = \beta g_N g_c^{1-\theta}. \tag{58}$$

To solve for g_c, use (52) and (55),

$$g_q = \gamma_x^{\left(\frac{1-\rho_x}{1-\rho_c}\right)-1} \tag{59}$$

where we define $(1-\rho_c)^{-1} = \sum_{i=1}^{m-1} \omega_i (1-\rho_i)^{-1}$ and $\psi(1-\rho_c)^{-1} = \sum_{i=1}^{m-1} \omega_i \psi (1-\rho_i)^{-1}$, so

$$g_c = g_q \gamma_x^{1/(1-\alpha)} = \gamma_x^{\Upsilon} \qquad \Upsilon \equiv \frac{(1-\rho_x)}{(1-\rho_c)} + \frac{\alpha}{1-\alpha} \tag{60}$$

Sub. (54) and (60) into (58), we have

$$\lim_{t\to\infty} \chi_{it} T_{it+1} = \chi_{i0} T_{i0} \lim_{t\to\infty} \left(\beta g_N^{(1-\theta)\Phi\Upsilon+1}\right)^t$$

So TVC for T_i holds if $\beta < \min\left(1/g_N, \bar{\beta}\right)$, where $\bar{\beta} \equiv (1/g_N)^{1+(1-\theta)\Phi\Upsilon}$. The Lagrangian multiplier for k is the discounted marginal utility, $\zeta_t = (\beta g_N)^t (p_{xt}/p_{ct}) u'(c_t) = (\beta g_N)^t (q_t/c_t) c_t^{1-\theta}$, since qk/c is constant,

$$\frac{\zeta_t k_t}{\zeta_{t-1} k_{t-1}} = \beta g_N g_c^{1-\theta}$$

So TVC for k holds when TVC for T_i holds.

Conditions for $l_i, n_i > 0$: From (2), $l_i > 0 \Leftrightarrow \gamma_i > 1 - \delta_T$, using (53) and (54), $l_i > 0 \Leftrightarrow g_N^{\Phi\frac{1-\rho_x}{1-\rho_i}} > 1 - \delta_T$. We now find the condition for $n_i/l_i > 0$. From (58)

$$\frac{\chi_{it}/\chi_{it+1}}{\gamma_i} = \frac{\chi_{it} T_{it}}{\chi_{it+1} T_{it+1}} = \left(\beta g_N g_c^{1-\theta}\right)^{-1} > 1 \quad for\ \beta < \bar{\beta} \tag{61}$$

So from (16), a sufficient condition for $n_i > 0$ is $\kappa_i < 1$. ■

A.5 Properties along BGP

We next verify the Kaldor stylized facts. Using (12) and (47),

$$\frac{y}{c} = \frac{p_x T_x k^\alpha}{c p_c} \sum_{i=1}^{z} \frac{1-1/\mu_x}{1-1/\mu_i} n_i = \frac{qk}{c} k_e^{\alpha-1} \sum_{i=1}^{z} \frac{1-1/\mu_x}{1-1/\mu_i} n_i \Longrightarrow \frac{c}{y} = \frac{\sum_{i=1}^{m-1} n_i/(1-1/\mu_i)}{\sum_{i=1}^{z} n_i/(1-1/\mu_i)}$$

which is constant given n_i are constants. The real interest rate is constant from (50). Using (39), the R&D expenditure share is

$$\frac{\sum_{i=1}^{z} (L_i w + Q_i R)}{\sum_{i=1}^{z} p_i T_i K_i^\alpha N_i^{1-\alpha}} = \frac{(1-1/\mu_x) p_x T_x k^\alpha \sum_{i=1}^{z} L_i}{p_x T_x k^\alpha \sum_{i=1}^{z} \frac{1-1/\mu_x}{1-1/\mu_i} N_i} = \frac{\sum_{i=1}^{z} l_i}{\sum_{i=1}^{z} n_i/(1-1/\mu_i)}.$$

Proof of Proposition 3. It is straightforward to show that

$$\psi\left(\frac{n_i}{l_i}-\frac{n_j}{l_j}\right)=\kappa_j-\kappa_i+\left(\frac{G\gamma_i\gamma_x^{(\rho_x-1)/\psi}/\gamma_x-1+\delta_T}{\gamma_i-1+\delta_T}-\frac{G\gamma_j\gamma_x^{(\rho_x-1)/\psi}/\gamma_x-1+\delta_T}{\gamma_j-1+\delta_T}\right), \tag{62}$$

the bracket term becomes

$$\begin{aligned}&\frac{G\gamma_j\gamma_x^{(\rho_x-1)/\psi}/\gamma_x+\gamma_i-G\gamma_i\gamma_x^{(\rho_x-1)/\psi}/\gamma_x-\gamma_j}{[\gamma_i-(1-\delta_T)]\left[\gamma_j-(1-\delta_T)\right]}(1-\delta_T)\\ =\;&\frac{\left(G\gamma_x^{(\rho_x-1)/\psi}/\gamma_x-1\right)\left(\gamma_j-\gamma_i\right)}{[\gamma_i-(1-\delta_T)]\left[\gamma_j-(1-\delta_T)\right]}(1-\delta_T)\end{aligned}$$

Using (58)

$$G\gamma_x^{(\rho_x-1)/\psi}/\gamma_x=\frac{\chi_{it}/\chi_{it+1}}{\gamma_i}=\frac{\chi_{it}T_{it}}{\chi_{it+1}T_{it+1}}=\left(\beta g_N g_c^{1-\theta}\right)^{-1}>1 \quad for\ \beta<\bar{\beta}.$$

So the bracket term in (62) is positive if and only if $\gamma_j>\gamma_i$. Result follows from Proposition (1). ■

A.6 Cross-industry spillovers

Proof of Proposition 4. Since T_j is taken as given by firm ih, in terms of the firm's dynamic optimization, we just need to replace previous Z_i with the term $\left(\prod_{j\neq i}T_{jt}^{\rho_{ij}}\right)Z_i$. Similar to the proof of Proposition 1, we require

$$\begin{aligned}&\left(\prod_{j\neq i}T_{jt}^{\rho_{ij}}\right)Z_i k_{t+1}^{\alpha}N_{it+1}T_{it+1}^{\rho_i-1}k_t^{(1-\alpha)(1-\psi)}Q_{it}^{\psi-1}\\ =\;&\left(\prod_{j\neq i}T_{jt}^{\rho_{ij}}\right)Z_i k_{et+1}^{\alpha}T_{xt+1}^{\alpha/(1-\alpha)}n_{it+1}N_{t+1}T_{it+1}^{\rho_i-1}k_{et}^{\alpha(\psi-1)}T_{xt}^{\alpha(\psi-1)/(1-\alpha)}l_{it}^{\psi-1}N_t^{\psi-1}\end{aligned}$$

to be constant. So the restriction for BGP (51) is modified to

$$1=\left(\prod_{j\neq i}\gamma_j^{\rho_{ij}}\right)\left(\gamma_x^{\alpha/(1-\alpha)}g_N\right)^{\psi}\gamma_i^{\rho_i-1} \tag{63}$$

Case 1 $\rho_{ij}=\tilde{\rho}_j,\, i\neq j$: the restriction (63) becomes $1=\left(\prod_{j\neq i}\gamma_j^{\tilde{\rho}_j}\right)\left(\gamma_x^{\alpha/(1-\alpha)}g_N\right)^{\psi}\gamma_i^{\rho_i-1}$, which implies $\forall i\neq s$,

$$\begin{aligned}\gamma_i^{\tilde{\rho}_i}\gamma_s^{\rho_s-1}&=\gamma_s^{\tilde{\rho}_s}\gamma_i^{\rho_i-1}\Leftrightarrow\gamma_i^{1-\rho_i+\tilde{\rho}_i}=\gamma_s^{1-\rho_s+\tilde{\rho}_s}\\ &\Longrightarrow\gamma_i\geqslant\gamma_s\geqslant 1\Longleftrightarrow\rho_i+\tilde{\rho}_s\geqslant\rho_s+\tilde{\rho}_i.\end{aligned}$$

Case 2: if $\rho_{ij} \neq 0$, then $\rho_{ik} = 0$ and $\rho_{kj} = 0$ for $k \neq i, j$. The restriction (63) implies

$$\begin{aligned} \gamma_i^{\rho_{ji}} \gamma_j^{\rho_j - 1} &= \gamma_j^{\rho_{ij}} \gamma_i^{\rho_i - 1} \Leftrightarrow \gamma_i^{1-\rho_i+\rho_{ji}} = \gamma_s^{1-\rho_s+\rho_{ij}}. \\ &\Longrightarrow \gamma_i \geqslant \gamma_j \geqslant 1 \Longleftrightarrow \rho_i + \rho_{ij} \geqslant \rho_j + \rho_{ji}. \end{aligned}$$

■

B Capital Intensity and Productivity

Assume $\psi = 1$ for simplicity. The consumer's problem is the same as the baseline model. The firm's problem is modified so that $Y_{iht} = T_{iht} K_{iht}^{\alpha_i} N_{iht}^{1-\alpha_i}$ and $F_{iht} = A_i T_{iht}^{\kappa_i} T_{it}^{\sigma_i} \left(Q_{iht}^{\eta_i} L_{iht}^{1-\eta_i} \right)^{\psi}$. Static efficiency implies equal marginal rates of substitution across sectors and activities. So across sectors, we have

$$\frac{K_{iht}}{N_{iht}} = \frac{\alpha_i}{\alpha_j} \left(\frac{1-\alpha_j}{1-\alpha_i} \right) \frac{K_{jht}}{N_{jht}} \tag{64}$$

across activities, we have

$$\frac{Q_{iht}}{L_{iht}} = \frac{\eta_i}{\alpha_i} \left(\frac{1-\alpha_i}{1-\eta_i} \right) \frac{K_{iht}}{N_{iht}} \tag{65}$$

Equal value of marginal product across sectors implies the relative price:

$$\frac{p_{iht}}{p_{jht}} = \frac{\left(1 - \frac{1}{\mu_j}\right) T_{jht}}{\left(1 - \frac{1}{\mu_i}\right) T_{iht}} \left(\frac{K_{iht}}{N_{iht}} \right)^{\alpha_j - \alpha_i} \left(\frac{\alpha_j}{\alpha_i} \right)^{\alpha_j} \left(\frac{1-\alpha_j}{1-\alpha_i} \right)^{1-\alpha_j} \tag{66}$$

and its growth rate

$$\frac{p_{iht+1}/p_{iht}}{p_{jht+1}/p_{jht+1}} = \frac{\gamma_{jht}}{\gamma_{iht}} \left(\frac{K_{iht+1}/N_{iht+1}}{K_{iht}/N_{iht}} \right)^{\alpha_j - \alpha_i} \tag{67}$$

As in the baseline, we focus on symmetric equilibrium. Sub. (64) and (65) into capital capital market clearing condition

$$K_t = \sum_i \left(\frac{K_{it}}{N_{it}} N_{it} + \frac{Q_{it}}{L_{it}} L_{it} \right) = \frac{K_{jt}}{N_{jt}} \Psi_{jt}^{-1} \tag{68}$$

where $\Psi_{jt}^{-1} = \sum_i \left(\frac{\alpha_i}{\alpha_j} \frac{1-\alpha_j}{1-\alpha_i} \frac{N_{it}}{N_t} + \left(\frac{\eta_i}{\alpha_j}\right) \left(\frac{1-\alpha_j}{1-\eta_i}\right) \frac{L_{it}}{N_t} \right)$. So (66) becomes

$$\frac{p_{it}}{p_{jt}} = \frac{\left(1 - \frac{1}{\mu_j}\right) T_{jt}}{\left(1 - \frac{1}{\mu_i}\right) T_{it}} \left(k_t \Psi_{it}\right)^{\alpha_j - \alpha_i} \left(\frac{\alpha_j}{\alpha_i}\right)^{\alpha_j} \left(\frac{1-\alpha_j}{1-\alpha_i}\right)^{1-\alpha_j} \tag{69}$$

So falling p_i/p_j can be due to $\gamma_i > \gamma_j$ or $\alpha_i > \alpha_j$.

We now look for BGP as before. Using (68),

$$G_{t+1} = 1 - \delta_K + \alpha_m \left(1 - \frac{1}{\mu_m}\right) T_{mt+1} k_t^{\alpha_m - 1} \Psi_{mt}^{\alpha_m - 1} \tag{70}$$

Along BGP, n_{it} and l_{it} are constants, Ψ_{mt} is constant, G_{t+1} is constant if $g_k = \gamma_m^{-1/(1-\alpha_m)}$, i.e. γ_m to be constant. As before, the dynamic efficiency condition (36) requires the shadow price χ_{jht} and $\frac{\lambda_{t+1} p_{jht+1}}{p_{ct+1}} \frac{\partial Y_{jht+1}}{\partial T_{jht+1}}$ to grow at the same constant growth rate. From the production function, $\frac{\partial F_{jht}}{\partial Q_{jht}} = \eta_j T_{jt}^{\rho_i} \left(Q_{jt}/L_{jt}\right)^{\eta_{jt}-1}$, which is growing at constant rate if γ_j is constant, so from (20), χ_{jht} is growing at constant rate. Together with (69) the ratio (56) simplifies to

$$\frac{1}{\chi_{jht}} \frac{\lambda_{t+1} p_{jht+1}}{p_{ct+1}} \frac{\partial Y_{jht+1}}{\partial T_{jht+1}} = B \frac{1}{G} \frac{\gamma_x g_k^{\alpha_m - \alpha_j}}{\gamma_j} \frac{T_j^{\rho_j} k_t^{\eta_j - 1}}{T_{jt} k_t^{\alpha_j - 1}} k_{t+1}^{\alpha_j} N_{jt+1}$$

where B is a constant, so it requires

$$1 = \gamma_j^{\rho_j - 1} g_k^{\eta_j - \alpha_j} g_k^{\alpha_j} g_N = \gamma_j^{\rho_j - 1} {\gamma_m}^{\eta_j/(1-\alpha_m)} g_N.$$

Finally combine the dynamic efficiency condition with the knowledge accumulation equation to solve for n_i/l_i

$$\frac{n_i}{l_i} = \frac{G \frac{\gamma_i^{\rho_i}}{\gamma_m} \gamma_m^{(\eta_i - \alpha_m)/(1-\alpha_m)} - (1 - \delta_T)}{\gamma_{it} - (1 - \delta_T)} \left(\frac{1-\alpha_i}{1-\eta_i}\right) - \kappa_i. \tag{71}$$

C Planner's Problem and Optimal Subsidies

Taking $\{N_t\}_{t=0}^{\infty}, k_t, T_{iht}, T_i = \int T_{iht} dh$ as given, the planner chooses $\{N_{iht}, L_{iht}, K_{iht}, Q_{iht}\}$ to maximize (6) subject to (1)-(2), (7), (9), (11), and

$$\begin{aligned}
g_N k_{t+1} &= x_t + (1-\delta_k) k_t \\
N_t c_{iht} &= Y_{iht} = T_{iht} K_{iht}^{\alpha} N_{iht}^{1-\alpha} \qquad i \in \{1, ..., m-1\} \\
N_t x_{jht} &= Y_{jht} = T_{jht} K_{jht}^{\alpha} N_{jht}^{1-\alpha} \qquad j \in \{m, ..., z\}. \\
T_{iht+1} &= F_{iht} + (1-\delta_T) T_{iht} \qquad i \in \{1, ..., z\} \\
F_{iht} &= Z_i T_{iht}^{\kappa_i} T_{it}^{\sigma_i} \left(\prod_{j \neq i} T_{jt}^{\rho_{ij}} \right) \left(Q_{iht}^{\alpha} L_{iht}^{1-\alpha} \right)^{\psi} \qquad i \in \{1, ..., z\}
\end{aligned}$$

The complete derivation for the planner and decentralized economy with taxes and subsidies is available from the authors. Here we report the key steps of comparing the two economies.

Proof of Proposition 5. Restrictions for the BGP in both economies continue to be (18) and (19) because they both boil down to restricting F_{it}/T_{it} to be constant. The consumption growth rates are the same for both economies when gross return on capital G is equal under (24). The LHS of (24) is from the solution of the decentralized economy. Given the Cobb-Douglas production with equal α, equating marginal rate of substitution and marginal rate of technical substitution between goods in the planner's problem implies:

$$\frac{\partial U / \partial C_{st}}{\partial U / \partial C_{it}} = \frac{T_i}{T_s}$$

the corresponding condition for the decentralized economy is:

$$\frac{p_{iht}}{p_{sht}} = \frac{T_{sht} (1 - 1/\mu_s)(1 - \tau_s)}{T_{iht} (1 - 1/\mu_i)(1 - \tau_i)}. \tag{72}$$

So allocations across goods are the same when (24) holds. ■

Proof of Proposition 6. The planner's solution implies

$$\frac{n_i}{l_i} = \frac{1}{\psi} \left[\frac{\chi_{it}/\chi_{it+1} - (1-\delta_T) - \sum_{s \neq i} \frac{\chi_{st+1}}{\chi_{it+1}} \frac{\partial F_{st+1}}{\partial T_{it+1}}}{\gamma_{it+1} - (1-\delta_T)} - \rho_i \right] \tag{73}$$

The decentralized economy implies

$$\frac{n_i}{l_i} = \frac{1}{\psi} \left[\frac{\chi_{it}/\chi_{it+1} - (1-\delta_T)}{\gamma_{it+1} - (1-\delta_T)} - \kappa_i \right] (1 - h_i) \tag{74}$$

Since χ_{it}/χ_{it+1} for both economies has the same expression as follows:

$$\frac{\chi_{it}}{\chi_{it+1}} = \frac{\gamma_i}{\gamma_x}\left(\prod_{j\neq i}\gamma_j^{\rho_{ij}}\right) G\gamma_x^{(\rho_x-1)/\psi}, \tag{75}$$

which is the same in both economies because G is the same. The term related to cross-industry spillover for the planner's solution is

$$\sum_{s\neq i}\frac{\chi_{st+1}}{\chi_{it+1}}\frac{\partial F_{st+1}}{\partial T_{it+1}} = \left[\gamma_{it+1}-(1-\delta_T)\right]\left(\sum_{s\neq i}\rho_{si}\frac{l_{st+1}}{l_{it+1}}\right).$$

Derive the optimal subsidy (25) and (26) by equating (73) and (74). The second order conditions are trivially satisfied because the boundaries involve either no production or no R&D activity, and hence cannot be optimal. ■

Proof of Proposition 7. Consider two sectors with identical technology parameters except μ_i and ω_i. So (75) implies χ_{it}/χ_{it+1} are the same for the sectors. So

$$\begin{aligned} h_i - h_j &= \frac{\sum\rho_{si}\frac{l_s}{l_i}-\sum\rho_{sj}\frac{l_s}{l_j}}{\frac{\chi_{it}/\chi_{it+1}-(1-\delta_T)}{\gamma_i-(1-\delta_T)}-\kappa_i} = \frac{\left(\frac{1}{l_i}-\frac{1}{l_j}\right)\sum\rho_{si}l_s}{\frac{\chi_{it}/\chi_{it+1}-(1-\delta_T)}{\gamma_i-(1-\delta_T)}-\kappa_i} \\ h_i &> h_j \Leftrightarrow l_i < l_j \end{aligned}$$

and (74) implies

$$\begin{aligned} \frac{n_i}{l_i}-\frac{n_j}{l_j} &= \sum_s\left(\frac{\rho_{sj}}{\psi}\right)\frac{l_s}{l_i}-\sum_s\left(\frac{\rho_{si}}{\psi}\right)\frac{l_s}{l_j} = \left(\frac{1}{l_j}-\frac{1}{l_i}\right)\left(\sum_s\frac{\rho_{sj}l_s}{\psi}\right) \\ n_i/l_i &> n_j/l_j \Leftrightarrow l_i > l_j \end{aligned}$$

which is the same as $n_i/l_i > n_j/l_j \Leftrightarrow n_i > n_j$, which implies

$$h_i > h_j \Leftrightarrow l_i < l_j \Leftrightarrow n_i/l_i < n_j/l_j \Leftrightarrow n_i < n_j.$$

Using (72), (47) and (48), if either $i,j\in\{1,..m-1\}$ or $i,j\in\{m,...z\}$, then

$$\frac{n_i}{n_j} = \frac{\omega_i\left(1-1/\mu_i\right)\left(1-\tau_i\right)}{\omega_j\left(1-1/\mu_j\right)\left(1-\tau_j\right)} = \frac{\omega_i}{\omega_j}$$

where the equality follows from Proposition 7. ■

D Data

The NBER patent database, described in detail in Hall et al (2001), classifies patents according to their industry of origin and type of innovation. This involves tracking the industry of origin of each patent, and of the patents that each patent cites, for 16,522,438 citation entries. While data on patents begin in 1963, citations are only available for patents granted since 1975.

For most of the paper we place durables into 14 categories we could identify in the citation data.[25] The industry classification in Hall et al (2001) mostly coincides with that in Table 2. The exceptions were Aircraft, Ships and Boats, Autos and Trucks, and Structures, which we put together from their finer classification, including only rubrics that we could definitively associate with the industry in question. Autos and Trucks combines classes 180, 280, 293, 278, 296, 298, 305 and 301. Structures combines classes 14 and 52 (Bridges and Static Structures). Aircraft equals class 244 (Aeronautics), and Ships and Boats is class 114 (Ships). The full list of categories may be found at http://www.nber.org/patents/list_of_classes.txt

Patents from other categories were counted as non-durables. There is also an issue with the 15% of patent citations where the industry of origin of the cited patent was not available (i.e. the cited patent was older than 1963). When the industry of a citation was not known, we assumed that the industry distribution of citations was the same as for citations with a reported industry (whihc make up 85% of the database). Excluding these patents, or counting them as non-durables, did not affect results. Assuming no spillovers between durables and nondurables also had little impact on the matrix for durables.

TFP in the model is based on quality-adjusted relative prices of capital goods and the benchmark value of γ_c. See Cummins and Violante (2002) for details on the construction of the price indices.[26]

Our measure of R&D intensity is the median ratio of R&D expenditures to sales among firms in Compustat over the period 1950-2000. All firms in Compustat are assigned a 4-digit SIC industry code, which is used for industry assignments. Since firms in Compustat are arguably subject to weak if any financial constraints, this should reflect the "pure" technologically determined level of R&D intensity for the industry – see Ilyina and Samaniego (2007). We discard the top and bottom 1% of observations in the sample, to reduce the influence of outliers.

[25] The main limitation is the citation data. Hence Figure 1 displays results for all manufacturing indusitres, and Figures 2 and 3 report all the durables categories in the Cummins and Violante (2002) data.

[26] We are very grateful to Gianluca Violante for providing us with relative price data.

E Cross-industry spillover Matrix

Allowing for cross-industry knowledge spillovers, we can compute the cross-spillover matrix as follows. Letting $g_i = \log \gamma_i$, and letting R be the matrix of ρ_{ij}, with $\rho_{ii} \equiv \rho_i$, equation (63) can be written $-\log \left(\gamma_x^{\alpha/(1-\alpha)}\gamma_N\right)^{\psi} = \sum_j \rho_{ij}g_j - g_i$ or, in matrix form, $\mathbf{v} = \mathbf{Rg} - \mathbf{g}$.

Define C as the cross-citation matrix (as shown in Table 1), so c_{ij} is the number of times a patent in industry i cites one in industry j on average. How does C map into R? First, it is not the case that $R = C$: the mapping between citations and receptivity requires a scaling factor, reflecting the extent to which a cited patent aids the production of new knowledge. The scaling factor may differ by contributing and/or by recipient industry. The number of citations varies a lot by industry, reflecting differences across industries in the rate of ideas output, in the "ideas content" of patents, and possibly in the tendency to patent ideas (as opposed to opting for secrecy). An idea in Communications may not be the same as an idea in Mining and Oilfield Machinery, because these industries may differ in their tendency to patent, or because patents may represent different "increments" in knowledge in each industry. See Hall et al (2001) for a related discussion of industry fixed effects. Without any further data on the appropriate relative weights, we can still derive ρ_{ij} by assuming that the knowledge content of a given citation is constant regardless of the identity of the citing industry. Hence, $\rho_{ij} = c_{ij}w_j$ (c_{ij} is citations by patents in industry i of patents from industry j) or, in matrix notation, $\mathbf{R} = \mathbf{CW}$, where $\mathbf{W}$ is a diagonal matrix of the weights. Then, the relationship between citations and growth rates is $-\log \left(\gamma_x^{\alpha/(1-\alpha)}\gamma_N\right)^{\psi} = \sum_j c_{ij}g_jw_j - g_i$ or $\mathbf{v} = \mathbf{CGw} - \mathbf{g}$, where $\mathbf{w}$ is the vector of weights and $\mathbf{G}$ is the diagonal matrix of g_i. So, $\mathbf{w} = \mathbf{G}^{-1}\mathbf{C}^{-1}[\mathbf{v} + \mathbf{g}]$. The vectors $\mathbf{v}$ and $\mathbf{g}$ are given by the data, so given matrix $\mathbf{C}$ of citations, the vector of weighs is exactly identified. Given g_i, we can derive the unique vector of weights,[27] and compute the implied spillover matrix $\mathbf{R}$. Thus, for example, if i = Communications and j = Computers, ρ_{ij} equals the average number of citations of Computing patents by patents in Communications, weighted by the "ideas content" index of Computer patents $w_{COMPUTERS}$. Index $w_{COMPUTERS}$ is the value required to map between measured TFP growth rates and the citation matrix, using the structure of the model.

[27] In practice, we find that the weights do not differ very much. Among durable goods industries, the mean is 0.26 and, aside from two outliers, the weights lie in the range $0.14 - 0.38$. One outlier is Mining and Oilfield machinery (1.03), where patents tend to have very few references. The other is Electrical eq. n.e.c. (0.04).

In about 15% of citations, the industry of the cited patent is not known. In the reported results these citations were excluded, which is equivalent to assuming that their industry distribution is the same as that for reported citations. Including them all as Other (i.e. "Non-durables") affected the results negligibly. Table 7 reports the spillover matrix. Notably, the primary source of knowledge spillovers for each industry is the industry itself – as expected. Cross-industry spillovers do appear, but values of ρ_{ij} for $j \neq i$ tend to be small relative to ρ_i (reported along the diagonal). Thus, total receptivity $\sum_j \rho_{ij}$ (the sum of each row) is highly correlated with ρ_i (97%, or 86% excluding Other, and 93% without Electrical equipment n.e.c.). In addition, values of ρ_{ii} derived from the model using citation data are very close to the values of ρ_i in Table 2 (which assumes no cross-industry spillovers and does not use patent data). The correlation between g_i and ρ_{ii} is about 93% (or 63% without "Other", and 87% without Electrical equipment n.e.c.[28]). The relative importance of within-industry spillovers suggests that, while the 2-digit SIC codes used are based on product categories, and thus on product use, it turns out that, at least for these industries, this lines up with an alternative categorization based on similarities in the knowledge used in production.

		Spillover source															Total
Code	Spillover recipient	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Recep
1	Comp. & Office	0.87	0.09	0.00	0.04	0.01	0.01	0.05	0.03	0.00	0.01	0.03	0.00	0.00	0.00	-0.06	1.09
2	Commun.	0.08	0.71	0.00	0.04	0.00	0.00	0.06	0.01	0.00	0.01	0.01	0.00	0.00	0.00	-0.04	0.89
3	Aircraft	0.02	0.02	0.81	0.01	0.00	0.01	0.03	0.05	0.01	0.00	0.02	0.00	0.01	0.00	-0.09	0.90
4	Instr. & Photocop	0.02	0.03	0.00	0.74	0.01	0.00	0.03	0.01	0.00	0.01	0.03	0.00	0.00	0.00	-0.13	0.76
5	Fab. Met. Prod	0.01	0.00	0.00	0.01	0.65	0.00	0.05	0.03	0.00	0.00	0.05	0.00	0.00	0.00	-0.12	0.68
6	Autos and Trucks	0.01	0.01	0.00	0.00	0.00	0.62	0.02	0.04	0.01	0.00	0.05	0.00	0.02	0.00	-0.08	0.71
7	Electrical transm.	0.02	0.02	0.00	0.02	0.03	0.00	0.56	0.01	0.00	0.00	0.02	0.00	0.00	0.00	-0.07	0.62
8	Other Durables	0.02	0.01	0.00	0.01	0.02	0.01	0.01	0.57	0.00	0.00	0.06	0.00	0.01	0.00	-0.12	0.61
9	Ships and boats	0.00	0.01	0.01	0.00	0.00	0.01	0.01	0.02	0.63	0.00	0.03	0.00	0.01	0.00	-0.09	0.66
10	Electrical eq. n.e.c.	0.05	0.04	0.00	0.06	0.02	0.00	0.06	0.01	0.00	0.11	0.01	0.00	0.00	0.00	-0.08	0.29
11	Machinery	0.01	0.00	0.00	0.01	0.02	0.01	0.02	0.04	0.00	0.00	0.56	0.00	0.00	0.00	-0.14	0.55
12	Mining and oilfield	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.67	0.00	0.00	-0.06	0.65
13	Furniture and fixt	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.03	0.00	0.00	0.02	0.00	0.51	0.01	-0.15	0.48
14	Structures	0.00	0.00	0.00	0.00	0.02	0.00	0.01	0.05	0.00	0.00	0.02	0.00	0.02	0.45	-0.21	0.38
15	Other	0.01	0.01	0.00	0.03	0.02	0.00	0.02	0.03	0.00	0.00	0.05	0.01	0.01	0.00	-1.29	-1.09

Table 7 – Cross-spillover matrix ρ_{ij}. The matrix is derived from the NBER patent citation database and equation (22). A row corresponds to spillovers received by a given industry. Columns represent industries as *sources* of spillovers.

To assess the robustness of the matrix to omissions, lags, etc, we also computed the matrix based on data for citations within 5-year windows starting 1975-1979.

[28] An outlier is Electrical equipment n.e.c. This reflects the fact that it often cites the Computers, Communications and Electrical Transmission industries (which grow relatively fast) without this leading to high growth in Electrical equipment n.e.c.

These matrices vary very little over time. To get a sense of this, the correlation between ρ_i (the vector of diagonals) in each five year window vs. the values in the earliest window is 96% or higher. The correlation between the off-diagonal elements is 85% or higher. We conclude that our indicators of receptivity are all stable over time, which supports our assumptions and model structure.