

Black Hole Masses are Quantized

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Abstract

We give a simple argument showing that in any sensible quantum field theory the masses of black holes cannot assume continuous values and must be quantized. Our proof solely relies on Poincare-invariance of the asymptotic background, and is insensitive to geometric characteristics of black holes or other peculiarities of the short distance physics. Therefore, our results are equally-applicable to any other localized objects on asymptotically Poincare-invariant space, such as classicalons. By adding a requirement that in large mass limit the quantization must approximately account for classical results, we derive an universal quantization rule applicable to all classicalons (including black holes) in arbitrary number of dimensions. In particular, this implies, that black holes cannot emit/absorb arbitrarily soft quanta. The effect has phenomenological model-independent implications for black holes and other classicalons that may be created at LHC. We predict, that contrary to naive intuition, the black holes and/or classicalons, will be produced in form of fully-fledged quantum resonances of discrete masses, with the level-spacing controlled by the inverse square-root of cross-section.

1 Introduction and Summary

Theoretical realization of possibility of low quantum gravity scale [1,2], opened a prospect of potential experimental studies of quantum gravity at the particle accelerators. One of the model-independent signatures of strong gravity at LHC was predicted [2] to be formation of micro black holes. For subsequent studies of black hole formation in colliders see [3]. Of course, experimental searches are unimaginable without at least qualitative theoretical understanding of properties of micro black holes, to which we often mistakenly extrapolate idealized classical properties. In our previous paper [4] we have shown that micro black holes must be quantized, and have provided evidence that the quantization rule in high mass limit must reproduce the area quantization rule conjectured in pioneering work of [5–7]. In the present paper we shall give an exact prove of black hole mass quantization from the first principles of quantum physics, and derive universal quantization rule, that for high levels reproduces the particular result of [5–7].

The purpose of the present work is to unambiguously show that black hole masses (as masses of all the other bound-states in the quantum world) are quantized, and this fact determines the properties of micro black holes that may be accessed at LHC.

As it is well known, classically, the black holes are strongly-gravitating objects that can be produced as a result of a gravitational collapse of an arbitrary form of energy that can be localized within its own Schwarzschild radius. In quantum field theory framework, any form of energy can be described as some superposition of elementary particles that correspond to quantum degrees of freedom existing in the theory. When the energy is so large that the corresponding classical Schwarzschild radius exceeds all the relevant quantum length-scales in the problem, the configuration becomes effectively *classical*. In the particle language, classicality can be understood as the occupation number of particles involved in the configuration being large. The constituent particles that make up a given classical object, can be either on-shell or off-shell, but their number is always large, and this is what defines classicality. For example, for time-dependent configurations, such as gravity or electromagnetic waves, the constituent particles (gravitons or photons) are mostly on-shell. Contrary, in the static configurations, such as a black hole or an extended strong magnetic field, the constituents are off-shell. We can say, that such configurations are made out of longitudinal gravitons or photons of large occupation number. In both cases, classicality can be understood as the number of constituent quantum particles, being

large, $N \gg 1$. But, since underlying laws of physics are quantum, the configuration is always also quantum. For any finite N and finite size, the energy of any such configuration must be quantized. This is true about a hydrogen atom, a neutron star and a black hole.

Despite of the above obvious point, black holes are still often perceived as something exceptional from this rule. The reason for this misconception is perhaps our intuition based on the fact that in exact-classical limit ($\hbar \rightarrow 0$), black holes are described by solutions that are fully characterized by very few parameters, such as the mass, charge and an angular momentum. This fact is sometimes referred to as a black hole no-hair theorem [9], and it is a powerful tool for understanding certain approximate properties of real black holes. Nevertheless, it represents an idealized approximation, that can never be realized in real systems existing for the finite time. This idealized property, valid in an exact classical limit, we mistakenly often extend to real black holes.

The above misconception has most severe consequences for the experimental searches of micro black holes at LHC, because it suggest that in such searches we should look for semi-classical objects that evaporate *democratically* in all the light particle species with nearly a thermal spectrum. In reality, the bottom of a black hole tower that may be reached by LHC, will look nothing like this. The first resonances that we may be lucky to observe will be fully legitimate quantum resonances, with quantized mass spectrum and no obvious reason to decay democratically in all the species.

Quantization of their masses we shall prove unambiguously below. The issue of democracy was discussed in detail in [10]. As it was shown there, by unitarity and CPT the democracy must be a mass dependent property, with the lightest members of black hole tower exhibiting the lowest level of it. Further refinements of these results are beyond the scope of the present work.

In [4], we have argued that quantization of black hole masses follows from the fact that Planck length L_P is the shortest resolvable length scale in gravity, and is necessary for consistency of the transition between elementary particles and black holes required by self-completeness of gravity [8]. We showed, that assuming saturation of the holographic bound on information storage exactly reproduces the area quantization rule conjectured in [5,6]. We have further studied implications of mass quantization for LHC searches of black holes.

In the present paper, we shall prove inevitability of black hole mass quantization from the first principles of quantum theory and Poincare-invariance, with no other assumptions.

Because of this generality, our proof is not specific to black holes and holds for arbitrary localized states.

Other phenomenologically-interesting objects that automatically fall within our proof are recently-suggested *classicalons* [11]. They represent generalizations of black holes for non-gravitational theories that include bosonic fields (self)sourced by energy. Interest in these objects is particularly motivated by the idea that they can be an intrinsic part of the standard model unitarizing the scattering of longitudinal W -bosons via classicalization. In such a case, of course, proof of quantization is very important for the proper understanding their collider signatures. Our results show, that W -classicalons, just like micro black holes, must manifest themselves in form of a tower of quantum resonances of discrete masses.

Due to generality of our argument, under a *black hole* we shall mean an arbitrary configuration characterized by a mass m and a localization radius $r_*(m)$. At no point in our proof we shall use the geometric characteristics of a black hole. For example, we won't rely on any particular relation between r_* and m . For us, it will be sufficient that a characteristic size of a black hole can be defined for the cases of interest and is a finite quantity for finite m . Our reasoning will be performed from the point of view of an asymptotic observer for which a black hole is just another state of a finite mass, and for which its precise geometric structure is completely unimportant.

Of course, since we work in quantum field theory framework, at the end of the day, all the states are quantum (pure or mixed doesn't matter) and must be characterized by a corresponding wave-function ψ describing a state in the Hilbert space. The statement that a given state has a finite mass and a localization radius, implies that the corresponding wave-vector $|\psi\rangle$ has a finite norm. Thus, we are dealing with a localized wave-packet rather than a plane-wave. We wish to show, that the mass assumed by any such state, must be quantized.

We shall first consider a situation with a stable state, and then take into the account the effect of decay.

We shall consider two field theoretic arguments showing that allowing a continuum of masses leads us to a contradiction, and thus, the black hole mass must assume discrete values.

We shall summarize our argument is one line. The key point is Poincare invariance of the asymptotic background. On such a background, any $|in\rangle$ and $|out\rangle$ states in a scattering process can be labeled by quantum numbers describing irreducible representations

of the Poincare group. Consider a scattering process that takes an in-state $|in\rangle$ into a final states $|f_m\rangle$, that is characterized by a Poincare-invariant parameter m . Let the rate of this transition be $\Gamma_{in\rightarrow f_m}$. The total rate of the process is then obtained by summing up over m ,

$$\Gamma_{total} = \sum_m \Gamma_{in\rightarrow f_m}. \quad (1)$$

All the phase space integrations over the Poincare-non-invariant parameters (such as momenta of final particles) are already included in $\Gamma_{in\rightarrow f_m}$. So the remaining sum can only take place over Poincare-invariant parameters, such as masses, spins, and internal quantum numbers. Allowing these parameters to assume continuous values on any interval will inevitably result into an infinite transition rate, and would make no sense.

It is important to appreciate the power of the above simple argument, which only relies on Poincare invariance of an asymptotic background, and is therefore completely insensitive to the short-distance (or high energy) properties of the interaction that is responsible for the transition. This is why, the masses of localized objects must be quantized regardless what one is willing to imagine about short distance properties of quantum gravity.

From the first principles of quantum physics, we thus prove that masses of black holes or any other classicalons must be quantized. However, to uncover the precise quantization rule we need more input. We use as such an input the requirement that in $m \rightarrow \infty$ limit the quantization rule should reproduce the classical relation between r_* and m . We then obtain the following *universal* quantization rule that is accurate in this limit,

$$(mr_*) = N. \quad (2)$$

Notice, that for a four-dimensional Einsteinian black hole this exactly reproduces earlier conjectures [5,6], of quantization of black hole area in L_P^2 -units. However, the rule given by (2) is more generic and universally applies to a quantization of arbitrary classicalons (including black holes) in arbitrary number of dimensions. In the light of ref. [12], the above rule has a clear physical interpretation. It was shown there that certain properties of classicalons (and in particular black holes) can be understood by realizing that they represent configurations of large occupation number of the soft bosons of wavelength r_* , with the number given by (2). The quantization rule, then simply can be understood as trivial consequence of the quantization of the occupation number. For large N contribution coming from shorter wave-length particles is less and less important, and the

rule becomes more and more accurate. This result is also understandable in the light of an underlying holographic connection between different classicalizing theories explained in [12], and provides extra evidence for this connection.

Finally, we shall show that the mass-quantization of sharp black hole resonances is insensitive to the possible fine level splitting of underlying micro-states, and is given by the scale of the production cross section

$$\Delta m = 1/\sqrt{\sigma}, \quad (3)$$

even if the underlying "true" mass levels are extremely densely spaced. In phenomenological terms the above rule tells us the following. Let us imagine that in a scattering process we can resolve a sufficiently narrow black hole resonance of mass \bar{m} that is produced with a cross section σ . The rule then tells us that a neighboring resolvable resonance will be shifted approximately by (3). This rule does not preclude the existence of the substructure of levels, but at each resolution level the same rule should apply to the level spacing.

We shall now turn to more detailed discussion. Before doing so, let us briefly comment on notations. Throughout the paper we shall denote by \bar{m} the masses of resonances, whereas the masses of exact mass eigenstates will appear without bar, m . Also in some obvious places we shall drop coefficients of order one.

2 The Need for Mass-Quantization

2.1 Infinite Creation Rate for Black Holes with Non-Quantized Masses

First argument shows that creation rate of black holes with continuum masses would be infinite, and thus such theories cannot make physical sense.

To see this, let us assume, that the black hole spectrum in some interval of masses is continuous. Let us pick one member from this continuum of mass m . As said above, because a black hole is a localized state, and can be produced with finite probability in a collision process, it must have a finite norm in Hilbert space. We shall show that relaxing the condition of the finite norm will not change our conclusions, since only the finite-norm states can be created in scattering experiments.

Consider thus any scattering process that creates such an objects in a final state. For example, this can be a process in which some initial quantum particles scatter into a

black hole plus some other particles,

$$\text{initial particles} \rightarrow \text{BH}_m + \text{some final particles}, \quad (4)$$

or simply a process of a black hole pair-creation out of some initial state,

$$\text{initial particles} \rightarrow \text{BH}_m + \overline{\text{BH}}_m. \quad (5)$$

Here BH_m and $\overline{\text{BH}}_m$ stand for a BH of a definite mass m and its charge-conjugated state respectively. Basically, any process in which a black hole can appear in a final state would do the job.

In any sensible field theory an amplitude of such a process,

$$A \equiv \langle \text{initial particles} | \text{BH}_m + \text{something} \rangle, \quad (6)$$

must exist and be non-zero at least for some initial states.

Obviously, for many initial states, such a transition may be extremely suppressed, but this is no-problem for us. As long as the amplitude is finite, we can build our argument. The finiteness of such an amplitude follows from the fact that BH_m corresponds to a finite norm vector in Hilbert state. If norm were infinite, then it would mean that probability of creating BH_m is zero, and only a distribution (a resonance composed out of continuum of mass eigenstates) can be created. This is obvious from the fact that the expectation value of any Hermitian operator \mathcal{M} over such a state would vanish

$$\langle \text{BH}_m | \mathcal{M} | \text{BH}_m \rangle = 0. \quad (7)$$

Let for some fixed energy of an initial state, \sqrt{s} , the total rate of such a process in which the black hole of a particular mass appears, be $\Gamma(m)$. By analyticity of the scattering amplitudes, the function $\Gamma(m)$ must be non-singular at least in some finite interval of masses

$$m_0 < m < m_0 + \Delta m. \quad (8)$$

This must be true regardless of particularities of the dynamics that is responsible for black hole creation.

Since this is the most crucial step, let us elaborate on it. Continuity of the production rate in the processes (4) means, that if in the collision at center of mass energy $\sqrt{s} > m$, one can produce a black hole of mass m with four momentum p_μ and a particle of some four-momentum q_μ , the amplitude of a similar process with infinitesimally close value

of mass ($m' = m + \epsilon$) and some respective values of four-momenta (p'_μ, q'_μ) must be infinitesimally close.

After integrating over final four-momenta the resulting rate $\Gamma(m)$ must be a continuous function of m , meaning that $\Gamma(m + \epsilon) \rightarrow \Gamma(m)$ for $\epsilon \rightarrow 0$. The same is true about the process of black hole pair creation (5).

Because of this continuity, one cannot assume, for instance, that a black hole of mass m can only be created on a mass-threshold. This would inevitably violate unitarity and the asymptotic Poincare-invariance in such interactions.

In other words, once we make the assumption that black holes can be created in particle collision process in a theory in which particle interactions respect unitarity, we are forced to assume that black hole creation processes also respect the same consistency rules. In a consistent particle theory we cannot think of black holes as special objects that are not subject to common rules of the game, since any violation of unitarity rules by black hole processes would contaminate also the interactions of ordinary particles.

The only way to give black holes a special status, is to completely isolate them, and eliminate possibility of their production in ordinary particle processes. Such complete elimination is hard to understand in the view of *CTP*-invariance of the theory, according to which a pair-production of any finite energy/norm state at high enough energy must be allowed by all possible super-selection rules. For example, since a black hole and its charge conjugated state can annihilate into gravitons, the inverse transition must also be allowed. But even if one somehow can imagine a consistent loophole out of this argument, we are not interested in theories in which black holes cannot be produced in particle collisions. Speaking the least, such black holes will never be observed in collider experiments.

Let us thus proceed by accepting the fact that continuity of m implies continuity of $\Gamma(m)$ at least on some interval (8). Let the minimal value of Γ on this interval be Γ_{min} . In order to obtain the total rate of emission within this interval we have to sum up over all the members of the continuum, which obviously will give an infinite result,

$$\Gamma_{total} = \sum_m \Gamma(m) = \lim_{n \rightarrow \infty} \sum_{k=0}^n \Gamma(m_0 + k\Delta m/n) > \lim_{n \rightarrow \infty} n\Gamma_{min} = \infty. \quad (9)$$

Where, in order to make the divergence of the sum explicit, we have discretized the interval of masses by n steps and then have taken n to infinity.

Notice, that the above sum is not a simple integration over $\Gamma(m)dm$ with an infinitesimal measure, which of course would give a finite result. The reason is, that we are

summing up over an infinite number (continuum) of final states, each with *finite* norm. The above result summarizes our point. The equations (6), (7) are incompatible with masses being continuous on any interval (8), since such continuum leads to an infinite production rate, which makes no sense.

In a consistent *CPT*-invariant quantum field theory anything that has finite energy and can fit within a finite size box, can be pair produced with a non-zero probability, provided one pumps enough energy within that box.

Relaxing any of the two conditions (6) or (7) would mean eliminating the black hole states with finite mass and a finite norm from the Hilbert space of the theory.

As said above, we are not interested in such a situation. The only less radical step is to assume, that the black holes of definite mass do exist, but the possible values of the mass are discrete. This makes perfect sense, and is fully compatible with everything we know about quantum physics.

2.2 Infinitely Strong Couplings Mediated by Virtual Black Hole States with Continuous Masses

The second argument is based on processes mediated by virtual black hole states. Again, we can show that if the black hole masses are not quantized, integrating out such virtual states would lead to effective interactions of infinite strength among the low-energy fields.

Indeed, consider a black hole state of a definite mass $|BH_m\rangle$. If such a black hole state with a well-defined mass and a finite norm exists, then in any sensible quantum field theory it should also appear in form of an intermediate state in some transitional process. That is, an amplitude,

$$\langle \text{initial particles} | BH_m \rangle \langle BH_m | \text{final particles} \rangle \neq 0 \quad (10)$$

must be non-zero at least for some initial and final states. Then, by integrating out such virtual black hole states one should generate some effective operators describing interactions among the light fields at low energies. Let an operator generated by integrating out the above member of the black hole continuum of a given mass m be,

$$\Phi_1 \dots \Phi_K A(m), \quad (11)$$

where Φ_j , $j = 1 \dots K$ stands for some light field of different possible spins and other quantum numbers. The coefficient $A(m)$ controls the strength of the interaction. Now

again, in any sensible field theory with finite norm states of mass m , at least some low energy operators must be generated by integrating these states out. This means that $A(m)$ must be finite and non-singular at least in some interval (8). Moreover, we can always choose the interval in such a way that $A(m)$ has a definite sign on it, and a lowest value A_{min} . Then, by integrating over all possible virtual black holes on such an interval we would generate a low energy interaction of an infinite strength,

$$\Phi_1 \dots \Phi_K \sum_m A(m) = \Phi_1 \dots \Phi_K \lim_{n \rightarrow \infty} \sum_{k=0}^n A(m_0 + k\Delta m/n) = (\Phi_1 \dots \Phi_K) \infty. \quad (12)$$

Thus exchange of continuum of virtual black holes, no matter how suppressed for individual ones, would generate low energy interactions of an infinite strength.

Of course, one can assume that summing over all possible black holes may result in a destructive interference, but if we had to rely on such interference, this would be the end of low energy physics as we know it, since in order to compute any low energy process, one would need to rely on cancellations among the virtual superheavy objects that separately give infinite contributions. Of course, on the basis of first principles we cannot exclude such a miraculous cancellation of contributions in the low energy effective operators. But, even if this happens, this cannot save the blow-up of a scattering amplitude demonstrated in the previous section, since the black holes there appear in the final states, and all the contributions to the production rate are positive.

We thus conclude, that existence of stable black holes (or any other localized bound-states) with continuous values of masses is impossible in a consistent quantum framework that allows quantum production of these objects in the final states. An immediate consequence of this fact is that the masses of extremal black holes, e.g., RN ones, must be quantized. Of course, such quantization follows from quantization of an electric charge. But, we are arriving to the necessity of quantization from completely independent side, which makes mass quantization a necessity even if charge could take a continuous value.

2.3 Why the Infinite Norm Cannot Undo Quantization of Masses

Can a black hole state in the Hilbert space have an infinite norm? As we shall now show, this will not change anything in our conclusion, since even if one formally prescribes to the black hole states in the Hilbert space an infinite norm, the states that can be physically produced and observed will anyway have a quantized mass spectrum.

To see this, let us assume that black hole states of definite mass (call them $|BH_m\rangle$) have no finite norm, and thus are allowed to have a continuum of masses m . To form a physically-accessible Hilbert sub-space, they should be at least δ -function-normalizable,

$$\langle BH_{m'} || BH_m \rangle = \delta(m' - m), \quad (13)$$

and (if needed, together with some additional states) can be used to form an orthonormal complete set.

Obviously, in any local physical process, we can only produce a finite-norm superposition of the states $|BH_m\rangle$. Let us denote these observable finite-norm states by $|\bar{B}H_{\bar{m}}\rangle$. Of course, by default, these finite-norm observable states are *distributions*, and cannot have a definite mass. Each of them represents a superposition (wave-packet) of the definite mass (but infinite norm) states,

$$|\bar{B}H_{\bar{m}}\rangle = \int dm \psi_{\bar{m}}(m) |BH_m\rangle, \quad (14)$$

where $\psi_{\bar{m}}(m)$ define the relative weights by which an m -th black hole state enters in a given \bar{m} -th superposition. Because of orthogonality properties (13), we have

$$\langle \bar{B}H_{\bar{m}} || \bar{B}H_{\bar{m}} \rangle = \int dm \rho_{\bar{m}}(m), \quad (15)$$

where $\rho_{\bar{m}}(m) \equiv |\psi_{\bar{m}}(m)|^2$ is a spectral density function. Of course, all the states $\bar{B}H_{\bar{m}}$ are unstable, but a well-defined resonance exists only if the spectral density function is peaked about some value \bar{m} , with the width satisfying

$$\Delta(\bar{m}) \ll \bar{m}. \quad (16)$$

If these conditions are not met, the state is so broad, that not even approximately shares any properties with black holes. Obviously, this is not the case of our interest.

We thus focus on the case when (16) is satisfied.

Let us now ask, if such resonant states can ever come in form of a continuum? To see that this is not possible, we can repeat the infinite production rate argument but now applied to the $\bar{B}H_{\bar{m}}$ -states. These states are labeled by \bar{m} . Let us ask, if parameter \bar{m} can take a continuum of values.

Since the state $BH_{\bar{m}}$ is normalizable, the production rate $\bar{\Gamma}(\bar{m})$ must be finite and non-singular on some interval,

$$\bar{m}_0 < \bar{m} < \bar{m}_0 + \Delta\bar{m}. \quad (17)$$

Let the minimal value of $\bar{\Gamma}(\bar{m})$ on this interval be $\bar{\Gamma}_{min}$. Then again summing up over all the members of the continuum, will give an infinite result,

$$\bar{\Gamma}_{total} = \sum_{\bar{m}} \bar{\Gamma}(\bar{m}) = \lim_{n \rightarrow \infty} \sum_{k=0}^n \bar{\Gamma}(\bar{m}_0 + k\Delta\bar{m}/n) > \lim_{n \rightarrow \infty} n\bar{\Gamma}_{min} = \infty. \quad (18)$$

This result illustrates that the number of distinguished finite norm states, both the mass eigenvalues as well as sharp resonances, must be quantized.

This fact may come as a surprise, because naively one could argue in the following way. Let us start with a theory in which a number of finite-norm mass-eigenstates (or sharp resonances) is discrete. Now let us pick up two such distinct mass eigenstates denoted by $|m\rangle$ and $|m'\rangle$. Then one may expect that in a scattering process starting from some initial state, $\langle i|$, one should be able to produce a normalized superposition of the two states,

$$|\alpha\rangle \equiv \cos(\alpha)|m\rangle + \sin(\alpha)|m'\rangle, \quad (19)$$

plus some additional state, which we shall denote by $|f\rangle$. Of course, the state $|\alpha\rangle$ will evolve in time, because the two mass eigenstates will oscillate. But, one could argue, that the mass difference can be taken arbitrarily small, so that the oscillation period $\tau \sim (m - m')^{-1}$ can be made arbitrarily longer than m^{-1} . In this way, the state $|\alpha\rangle$ can be made into an arbitrarily-sharp resonance, with the lifetime exceeding all the other time scales in the problem. In such a case, the final state can be treated as direct product,

$$|\alpha, f\rangle \equiv |\alpha\rangle \otimes |f\rangle. \quad (20)$$

Can the parameter α take continuum values in such process? Our argument says that this is impossible. Indeed, if the transition amplitude $\langle in|\alpha, f\rangle$ where finite, and continuous on some interval

$$\alpha_0 < \alpha < \alpha_0 + \Delta\alpha_0, \quad (21)$$

The total rate that would be obtained by summation over this interval would be infinite.

But, what is the fundamental reason for the discreteness of α ? The reason, in fact, is Poincare-invariance. By Poincare-invariance, the parameter α can only depend on *Poincare-invariant* characteristics of the state $|f\rangle$. These are, eigenvalues of Poincare Casimir operators (masses and spin) and/or internal quantum numbers that are Poincare-singlets. Since number of particles in $|f\rangle$ is finite and discrete, all these characteristics are discrete. Thus, the summation over the label α is always discrete.

Impossibility of creating continuous superpositions of two states can be easily visualized on the following simple example. Consider a theory in which an initial state, let us say a heavy fermion χ , decays into a superposition of two nearly degenerate massive fermions, $\nu_\alpha \equiv \nu_m \cos(\alpha) + \sin(\alpha) \nu_{m'}$ and a light scalar ϕ ,

$$\chi \rightarrow \nu_\alpha + \phi. \quad (22)$$

Can one write down an effective vertex, that would allow in this process to produce continuum of different α -superpositions? The answer is no, because of Poincare-invariance. The corresponding vertex has a form

$$\chi \nu_\alpha \phi \quad (23)$$

and the only way to make different values of α possible in this process, is to make α sensitive to the Poincare-invariant continuous parameters that characterize final state of ϕ . But because of Poincare symmetry, such continuous parameters do not exist. In the absence of Poincare invariance, we could make α being explicitly dependent on four-momentum of ϕ , but Poincare-invariance forbids such dependence indicating that α can only depend on eigenvalues of Poincare Casimir operators, such as four-momentum square $p_\mu p^\mu$, or other operators commuting with the Poincare group. For any given ϕ , the set of such parameters is fixed and is discrete. So there is no way to vary this values as functions of four-momenta of the final states.

As a result, different $|\alpha\rangle$ -states can only be produced if we also make ϕ -state α -dependent. We can achieve this for example either by introducing different ϕ_α -fields and writing set of interactions

$$\chi \sum_\alpha \nu_\alpha \phi_\alpha, \quad (24)$$

or introducing operators that couple to different powers of ϕ , say,

$$\chi \sum_\alpha \nu_\alpha \phi^\alpha. \quad (25)$$

Obviously in both cases, the set of possible operators is discrete.

So the observable black hole states, must form a discretuum.

The above is a manifestation of a simple, but very deep, quantum-mechanical truth: *Resonances are normalizable states built out of continuum, because of this property they come in discrete numbers!*

In other words, the continuum of resonances makes no quantum-mechanical sense.

The above consideration can be straightforwardly applied for accounting other types of instabilities, such as the Hawking thermal instability for a black hole. Instability can be easily accounted by treating a black hole state as a distribution (a resonance), of spectral density $\rho_{\bar{m}}(m)$. Then we can automatically repeat the previous arguments, and conclude that \bar{m} must be quantized.

3 How Quantized?

We have concluded that masses of localized objects, such as black holes and other classicalons, must be quantized. Can we derive a quantization rule? To do this we need a more detailed information about the objects in question.

In order to derive an approximate quantization rule, we can proceed in the historic spirit of quantization, by demanding that the quantization rule for large m (meaning $m \gg L_*^{-1}$) should approximately reproduce the results of semi-classical computation obtained in the approximation of a continuum mass.

Interestingly, for classicalons (and in particular black holes) it turns out to be easier to guess an universal rule and then discuss its supporting evidence in concrete cases. In making such a guess, we shall rely on the results of ref [12], which showed that (seemingly mysterious) physical properties of classicalons can be well-understood if we realize that these objects represent superpositions of soft (wavelength $\sim r_*$) quanta with occupation number given by (2).

The quantization rule then follows from the following simple consideration. For large m ($N \gg 1$) the main contribution to the mass (and other properties) of the configuration comes predominantly from the wavelengths $\sim r_*$. Contribution from short wave-lengths is negligible. So the quantization rule then can be understood as the quantization of the occupation number of these cold bosons. This rule automatically accounts for all the existing classical limits.

In each concrete case, we can gether more supporting evidence for it, which we shall now do, separately for the black holes and other classicalons.

3.1 Black Hole Mass-Quantization

One of the well-known results obtained in the semi-classical approximation of continuous mass is , that black holes decay via emitting a thermal Hawking radiation of temperature $T = r_*^{-1}$. Thus, in this approximation a black hole of mass m emits predominantly the quanta of frequency T . We therefore require that the quantization rule should accommodate this result, meaning that for large mass (small T), the separation between the levels must decrease as T .

In order to accommodate the semi-classical limit for large masses, the quantization rule should allow a black hole of mass m to go to a lower level m' by emitting a quantum of some massless or massive (subject to Boltzmann-suppression) particle, γ of energy T ,

$$\text{BH}_m \rightarrow \text{BH}_{m'} + \gamma. \quad (26)$$

For large m (small T) the back reaction is higher order in T/m , and can be ignored and the level difference is approximately given by $m - m' = T$. Now, even without knowing a concrete dependence between m and T , but solely relying on the fact that in each elementary emission the back reaction is small, we can conclude that the number of quanta a black hole needs to emit before substantially reducing its mass is

$$N = m/T = mr_*. \quad (27)$$

So whatever the quantization rule is, for large N it has to reproduce this information.

In order to translate this in terms of mass quantization, we need a concrete relation between $r_*(T)$ and m , which is more model dependent, e.g., depends on number of extra dimensions. But, the rule (27) is unchanged.

For example, for Einsteinian gravity in four dimensions, we have $r_* = L_P^2/m$. If we make a further reasonable assumption that the transition probability is maximal between the neighboring levels, the resulting quantization rule, in the leading order in N , comes out to be

$$m = \sqrt{N}/L_P, \quad (28)$$

which exactly reproduces conjecture of [5–7]. Also note, that when applied to black holes in $4 + d$ dimensions, the rule (2) reproduces the quantization rule obtained in [4] of $2 + d$ -dimensional black hole area $A = r_*^{2+d}$ in units of $1 + d$ -dimensional fundamental Planck area, L_{4+d}^{d+2} . Or in terms of masses this quantization rule reads,

$$m = N^{\frac{1}{2+d}}/L_{4+d}. \quad (29)$$

An alternative estimate of the quantization rule, which gives the same result, comes from the requirement that a black hole creation cross-section in two-particle collisions at very high center of mass energies m , should approach a geometric cross section set by the area of a Schwarzschild black hole of mass m [13–18],

$$\sigma_m = r_*(m)^2. \quad (30)$$

This process, is accompanied by creation of additional soft quanta, so that schematically we can write down,

$$\gamma + \gamma \rightarrow BH_{m+\Delta m} + \gamma, \quad (31)$$

where γ stands for some initial and final particles. Since the scattering takes place at a macroscopic distance r_* , the characteristic momentum transfer is $\sim 1/r_*$, which limits the typical momentum of the final particle from above. Obviously, if we want the total cross section of such a process,

$$\sigma_{total} = \sum_{\delta m} r_*^2(m + \Delta m), \quad (32)$$

not to exceed (at least significantly) the geometric cross-section, we cannot allow the values of Δm much smaller than $1/r_*(m)$. Thus, again we have to admit that the level separation is $\Delta m \sim 1/r_*$, which again leads us to the quantization rule that approximately is given by relation (2).

3.2 Quantization of Classicalon Masses

Another category of localized finite mass configurations, that generalize notion of black holes, to a more general class of theories, are classicalons [11, 12]. In fact, as shown, black holes are simply a particular form of classicalons. The crucial unifying property of classicalons is that their size r_* grows with their mass. The mass (and thus size) appear at the level of the classical equations of motions as an integration constant that like a black hole mass, can assume a continuum of values. But this continuity, just like in a black hole case is an artifact of classical approximation. Just like black holes, classicalons can be viewed as bound-states of many bosons, with characteristic wave-length r_* . For total energy m , the occupation number is obviously given by (2). For a concrete case of black holes the role of r_* is played by a gravitational Schwarzschild radius, but relation (2) holds also for other classicalons.

From all these features, it is obvious that the classicalon masses must be quantized according to the same rule. To formulate a quantization rule only in terms of mass and N , we need to know dependence of r_* on energy. For large classicalons, $r_* \gg L_*$, the dependence can be parameterized by a power-law, $r_* \sim L_*(L_*m)^\gamma$, where γ is a parameter, that defines how efficiently the field is (self)sourced by the energy. The quantization rule, now can be found out from the requirement, that classicalon production cross section for high m , goes as a geometric cross section r_*^2 . Which, just like black holes implies the quantization of mass in units of r_* , according to (2). Translated for a mass we get,

$$m = N^{\frac{1}{1+\gamma}} L_*^{-1}. \quad (33)$$

Our results are fully understandable in the light of findings of ref [12], which makes close parallel between the generic classicalons and black holes in terms of field configurations composed out of N soft (wavelength $\sim r_*$) bosons. The quantization rule (2) is then just represents a requirement that the number of bosons is quantized.

4 Level-Splitting?

We have proven the necessity of the discreteness of the black hole states along the mass vertical. A level of a fixed mass m is expected to have an internal degeneracy, which we shall refer to as the *horizontal* degeneracy.

Such a degeneracy would mean that a black hole of mass m can be in a number of different internal states which we can further label by index a , with all of them having a common mass m_a . When we discuss a production rate of a black hole of a given mass $\Gamma(m)$, all such internal degeneracies are already summed up. That is,

$$\Gamma(m) = \sum_a \Gamma(m_a). \quad (34)$$

The existence of a horizontal degeneracy is suggested by the Bekenstein's entropy counting arguments, and is usually estimated to be $\sim e^{mr_*}$. Since we showed that for black holes the quantity mr_* is a discrete number N , our findings have some bearing on the above horizontal degeneracy and suggest that horizontal degeneracy is also discrete and given by e^N .

Can one use the latter statement against our proof, and suggest that discreteness of the degeneracy could result into the level-splitting which could effectively "wash-out" the discreteness of the spectrum? As we can easily see, this is not the case.

First, notice that there is absolutely no evidence that the levels can be split in a Poincare-invariant way. Having a background field that violates asymptotic Pioncare symmetry is beyond our interest. An explicit example of such unremovable degeneracy is given by string levels. So the first pint is that level-splitting doesn't follow from anything we know about black hole physics.

However, even if we assume that the degeneracy can be removed, this will not affect our proof. Indeed, such a level-splitting would populate the neighborhood of a any mass m , with exponentially large number of black hole states of distinct mass m_a . This changes nothing in our previous reasoning, since all these new states continue to be extremely sharp resonances. What is extremely important to realize is, that what count is the relation between the total width $\Delta(m_a)$ of an individual resonance to its mass m_a , and *not* a distance to a nearest level. As long as the two levels m_a and m_b satisfy the condition

$$\Delta(m_a) \ll m_a, \quad \Delta(m_b) \ll m_b, \quad (35)$$

the distance between the levels $m_a - m_b$ is irrelevant. This is obvious from the fact that when levels are not split $m_a = m_b$, we are back to the old situation, when each state counts.

Now the remaining question is whether a densely populated spectrum can imitate the effect of the continuum. To see that this cannot be the case, let us assume an extreme scenario in which the split levels are uniformly spread. In such a case the distance between the neighboring levels instead of $\Delta m = r_*^{-1}$, now becomes $\Delta m = r_* e^{-mr_*}$. It seems that such closely spaced levels in any scattering experiment will be indistinguishable from continuum. However, this is just an illusion, in reality the resonances that are produced by a geometric cross section r_*^2 , will continue to obey the quantization rule (2), and be spaced by r_*^{-1} .

To see that the resonances level splitting must be set by $\Delta \bar{m} = r_*^{-1}$ and not by $\Delta m = r_* e^{-mr_*}$, let us work in the approximation in which $\Delta m \rightarrow 0$. In this limit we recover exactly the situation discussed in section 2.3. The production rate of each state of a definite mass m now becomes infinitely suppressed, and in any scattering process instead we create a distribution, a resonance, characterized by a spectral function $\rho_{\bar{m}}(m)$. The point is that each of these distributions obeys the condition of a sharp resonance, and thus counts in the total rate. So the values of \bar{m} must be discrete, and moreover to reproduce the geometric cross-section the masses should obey the quantization rule (2) in

which m is replaced by \bar{m} .

4.1 Implications for LHC Black Hole Resonances

To finish this discussion, let us apply the above reasoning to a phenomenologically most relevant case of black hole production in the collision of Standard Model particles. Let us consider a creation of a black hole resonance of mass \bar{m} , in a collision process of some standard model particles and show that the mass splitting of the observable sharp resonances $\Delta\bar{m}$, will be set by their production cross-section scale, $1/\sqrt{\sigma}$, even if the "true" mass eigenstate levels are much closed spaced, $\Delta m \ll \Delta\bar{m}$.

Consider a collision process of some Standard Model particles in some in-state $|SM_{in}\rangle$, which results into a creation of a black hole resonance $|BH_{\bar{m}}\rangle$ and some Standard Model particles in a final state $|SM_{out}\rangle$,

$$SM_{in} \rightarrow BH_{\bar{m}} + SM_{out}. \quad (36)$$

Let us assume that at the center of mass energy \sqrt{s} the cross section of the above process is dominated by production of a black hole resonance of mass $\bar{m} \sim \sqrt{s}$ and is at least approximately given by a geometric area $\sigma \sim r_*^2(\bar{m})$ according to (32). A precise dependence of r_* on \bar{m} is unimportant, as long as r_* is a dominant length scale, growing with \bar{m} .

We should keep in mind that the state $|BH_{\bar{m}}\rangle$ we observe in this process is a distribution of the "true" mass levels, m ,

$$|\bar{B}H_{\bar{m}}\rangle = \sum_m \psi_{\bar{m}}(m) |BH_m\rangle, \quad (37)$$

where the coefficients $\psi_{\bar{m}}(m)$ define the relative weights by which an m -th black hole level enters in a given \bar{m} -th resonance, and $\rho_{\bar{m}}(m) \equiv |\psi_{\bar{m}}(m)|^2$ is a spectral density function.

It is clear that the problem we are dealing with is a discretized version of the one already discussed in section 2, and in particular (37) is just a discretized version of (14). However, since the level spacing is small, all the previous conclusions apply. The only new thing is to adopt this conclusion to the constraint that the cross section must be geometric, given by (32).

Let us now ask, how finely the resonance states \bar{m} can be spaced? Obviously, production of a nearest resonance \bar{m}' from the same initial state $|SM_{in}\rangle$ implies the change

of the coefficients $\psi_{\bar{m}}(m)$, but this is impossible without changing the Poincare-invariant characteristics of the final state $|SM_{out}\rangle$. In other words, $|SM'_{out}\rangle$ corresponding to a new resonance \bar{m}' must be a *different* Poincare state. For example, it has to include a different number and/or type of the Standard Model particle species. To be concrete, we can imagine that a creation of a lighter black hole of mass $\bar{m}' < \bar{m}$ in a same two particle collision in $|SM_{in}\rangle$ is accompanied by emission of more particle quanta in $|SM'_{out}\rangle$.

Obviously, such possible final states are discretized, and moreover their number is not nearly enough to accommodate almost equal opportunities for the appearance of black hole resonances if they are more finely spaced than $\Delta\bar{m} \sim r_*(\bar{m})^{-1}$, without being inconsistent with the starting cross section.

We thus are reaching a surprisingly general prediction, that the level spacing of observable sharp resonances is restricted by their cross section scale, according to (3), regardless of the underlying fine structure of mass eigenstates.

5 Conclusions

We are pointing out that there is a fundamental reason why the mass/energy spectrum of any localized bound-state cannot be continuous and must be quantized. Physical reason is that continuous spectrum of finite norm states (stable or resonances) would inevitably imply infinite production rate.

Mathematically, discreteness of masses follows from the discreteness of the finite-norm eigenfunctions of any hermitian operator.

And this constraint comes from first principles of quantum physics of any Poincare-invariant asymptotic background and is independent on particular short-distance properties of the state. Black holes, and classicalons in general, are no exception from this rule.

Mass-quantization of such states can be proven unambiguously from the first principles, but the precise rule of quantization requires more input. We have provided such an input, by linking the level spacing to the production cross-section. This gives the rule (3). Requiring that classicalon production cross section in high-energy scattering asymptotically approaches the geometric value given in terms of the area r_*^2 , for large m , gives the quantization rule (2), which for particular case of Schwarzschild black holes implies that area is quantized in the units of fundamental length L_* , in arbitrary number

of dimensions.

Most important phenomenological implication of our results is, that micro black holes and other classicalons that may be accessed by collider experiments will come in form of quantum resonances, and we can only distinguish them from ordinary particles by probing higher and higher states. Of course, for the lightest black holes, the quantization rule (2) can be modified by order one coefficients, but quantization will inevitably play a most important role there. To uncover the precise form of the quantization rule for lowest black hole resonances, we need more experimental input, and we are looking forward to it.

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