Contemporary models of elastic nucleon scattering and their predictions for LHC

Jan Kašpar[a,b], Vojtěch Kundrát*[b], Miloš Lokajíček[b]

[a] CERN, Geneva
[b] Institute of Physics of the AS CR, v.v.i., 182 21 Prague 8, Czech Republic

Abstract

The analyses of elastic collisions of charged nucleons have been based standardly on West and Yennie formula. However, this approach has been shown recently to be inadequate from experimental as well as theoretical points of view. The eikonal model seems to be more pertinent as it enables to determine physical characteristics in impact parameter space. The contemporary phenomenological models cannot give, of course, any definite answer as the elastic collisions may be interpreted differently, as central or peripheral processes. Nevertheless, the predictions for the planned LHC energy have been given on their basis and the possibility of exact determination of luminosity has been considered.

1. Introduction

The measurements of elastic scattering of charged nucleons at present high energies [1]–[3] have attained ample statistics enabling to perform very precise analyses of data measured in a broad interval of the four momentum transfer squared t. The region of t's where the differential cross section $\frac{d\sigma}{dt}$ can be determined covers not only the region where nearly the pure hadron (nuclear) scattering is dominant, i.e., $|t| \gtrsim 10^{-2}$ GeV^2, but also the region where the Coulomb scattering plays an important role, i.e., $|t| \lesssim 10^{-2}$ GeV^2 (the latter region being sometimes subdivided into Coulomb and interference parts). The complete scattering amplitude $F^{C+N}(s,t)$, fulfilling (in the normalization used by us) the relation

$$\frac{d\sigma(s,t)}{dt} = \frac{\pi}{sp^2}|F^{C+N}(s,t)|^2 \tag{1.1}$$

has been commonly decomposed according to Bethe [4] into the sum of the Coulomb scattering amplitude $F^C(s,t)$ known from QED and the hadronic amplitude $F^N(s,t)$ bound mutually by a relative phase $\alpha\Phi(s,t)$:

$$F^{C+N}(s,t) = e^{i\alpha\Phi(s,t)}F^C(s,t) + F^N(s,t); \tag{1.2}$$

s is the square of the center of mass energy, p is the momentum of an incident nucleon in the same system and $\alpha = 1/137.036$ is the fine structure constant. The influence of spins of all particles involved in the elastic scattering has been neglected at the highest energies.

The complete elastic scattering amplitude $F^{C+N}(s,t)$ used in the past has been established by West and Yennie [5] and equals in the first approximation to

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t}f_1(t)f_2(t)e^{i\alpha\Phi(s,t)} + \frac{\sigma_{tot}(s)}{4\pi}p\sqrt{s}\,(\rho(s) + i)e^{B(s)t/2}. \tag{1.3}$$

*Corresponding author
Email addresses: jan.kaspar@cern.ch (Jan Kašpar), kundrat@fzu.cz (Vojtěch Kundrát), lokaj@fzu.cz (Miloš Lokajíček)

The first term corresponds to the Coulomb scattering amplitude while the second term represents the elastic hadronic amplitude. The upper (lower) sign corresponds to the scattering of particles with the same (opposite) charges. The two form factors $f_1(t)$ and $f_2(t)$ in Eq. (1.3) describe the electromagnetic structure of each nucleon (commonly in a dipole form) as

$$f_j(t) = \left(1 + \frac{|t|}{0.71\,\mathrm{GeV}^2}\right)^{-2}. \tag{1.4}$$

Formula (1.3) is valid provided the hadronic elastic amplitude (the second term on its right hand side) has a constant diffractive slope B together with constant quantity ρ (the ratio of the real to imaginary parts of hadronic amplitude in forward direction). Similarly as the total cross section σ_{tot} they can depend only on the energy. The relative phase $\alpha\Phi(s,t)$ in Eq. (1.3) has been shown by West and Yennie [5] and independently by Locher [6] to be

$$\alpha\Phi(s,t) = \mp\alpha\,(\ln(-B(s)t/2) + \gamma) \tag{1.5}$$

where $\gamma = 0.577215$ is the Euler constant.

Formulas (1.3) and (1.5) have been used for fitting the experimental data of differential cross section for small $|t|$ values (in the Coulomb, interference and also in a small adjacent part of hadronic domain) and the three mentioned quantities σ_{tot}, B and ρ have been determined. At larger $|t|$ values (i.e., in the hadronic region) the influence of Coulomb scattering has been usually fully neglected and elastic scattering has been described with the help of phenomenological elastic hadronic amplitude $F^N(s,t)$ which usually has exhibited a much more complicated t dependence in this hadronic region than in Eq. (1.3). The different regions of differential cross section have been described by two different formulas (moreover based on incompatible assumptions) which has been recognized as important deficiency.

In the following section (Sec. 2) we will show in more detail which assumptions the formulas (1.3)–(1.5) are based on and what are the limits of their use in analyses of contemporary experimental data. In Sec. 3 we will then discuss the approach based on the eikonal model which not only removes the corresponding limitations but also which describes the common influence of both the Coulomb and hadronic interactions in the whole measured region of momentum transfers uniquely with only one formula for the complete elastic amplitude. The t dependence of the elastic hadronic scattering amplitude $F^N(s,t)$ derived from experimental data within the eikonal model enables to determine some physical characteristics in the framework of impact parameter space.

The aim of the presented paper is then contained mainly in the next three sections. The eikonal model approach will be used for analysis of four phenomenological models proposed for a description of the elastic pp scattering at the nominal LHC energy of 14 TeV; the model predictions will be given and discussed in Sec. 4. The problems connected with the estimation of luminosity on the basis of elastic nucleon scattering will be analyzed in Sec. 5. The calculated root-mean-square (RMS) values of total, elastic and inelastic impact parameters corresponding to individual analyzed models will be given and discussed in Sec. 6. And the results obtained on the basis of our approach will be summarized and discussed in Sec. 7.

2. The West and Yennie formula

The original function $\alpha\Phi(s,t)$ entering into Eq. (1.2) has been derived by West and Yennie [5] within the framework of Feynman diagram technique in the case of charged point-like particles and for $s \gg m^2$ (m stands for nucleon mass) as

$$\alpha\Phi_{WY}(s,t) = \mp\alpha\left[\ln\frac{-s}{t} - \int_{-4p^2}^{0}\frac{dt'}{|t'-t|}\left(1 - \frac{F^N(s,t')}{F^N(s,t)}\right)\right], \tag{2.1}$$

which has been further simplified and Eqs. (1.3)–(1.5) have been obtained. However, simplified formulas (1.3)–(1.5) could hardly be considered as a fully adequate tool for analyzing elastic nucleon scattering data already in time when they were derived. The issue is that formula (2.1) has contained the integration over all kinematically allowed values of t while experimental data has only covered a limited interval of t. Some assumptions defining and limiting the t dependence of the hadronic amplitude, i.e., its modulus and phase defined in our case as

$$F^N(s,t) = i|F^N(s,t)|e^{-i\zeta^N(s,t)}, \tag{2.2}$$

has had to be accepted to enable the integration. As nothing was known about the diffractive structure in $\frac{d\sigma}{dt}$ at that time, the two following crucial assumptions have been accepted:

- the t dependence of the modulus of the elastic hadronic amplitude is purely exponential for all kinematically allowed t values,
- both the real and imaginary parts of the elastic hadronic amplitude exhibit the same t dependence for all admitted t values.

In addition to these crucial assumptions, some high energy approximations has been added (see, e.g., Refs. [5]-[9]). Then the complete scattering amplitude has been written in the simplified form [5] (for details see Ref. [9]). Even if the standard fits obtained in the Coulomb and interference domains may seem to be good one cannot be sure about the actual meaning of fitted parameters since the data for higher $|t|$ values have not been taken into account quite correctly.

In some papers (see, e.g., Refs. [10, 11]) the complete scattering amplitude $F^{C+N}(s,t)$ has been, therefore, described with the help of Eq. (1.3) containing the standard West and Yennie phase (1.5) and the elastic hadronic amplitude $F^N(s,t)$ (substituting the second term in Eq. (1.3)) constructed on the basis of some phenomenological ideas deviating from the two assumptions under which Eqs. (1.3) and (1.5) were derived. Such an approach may be regarded, however, as very approximate.

It might seem that a correct way may be reverting back to integral formula (2.1) in combination with formulas (1.1)-(1.5). However, that is not possible, either, if the phase $\alpha\Phi_{WY}(s,t)$ should be real. The relative phase factor $\alpha\Phi_{WY}(s,t)$ can be real only provided the phase of the hadronic amplitude $\zeta^N(s,t)$ is t independent in the whole region of kinematically allowed t values [12]; i.e., the quantity $\rho(s,t)$ should be constant in the whole interval of t. The contemporary experimental data as well as the phenomenological models of high energy elastic nucleon scattering show, however, convincingly that the quantity ρ cannot be t independent. Therefore, one should conclude that also the integral formula (2.1) should be designated as inadequate for the description of elastic hadronic scattering. It is necessary to give decisive preference to a new and more suitable approach based on eikonal model. In the following we should like to demonstrate the possibilities and advantages of the eikonal model which is more general and more appropriate than that of West and Yennie.

3. Eikonal model approach and mean-squares of impact parameters

The complete elastic scattering amplitude $F^{C+N}(s,t)$ is related by Fourier-Bessel transformation to the complete elastic scattering eikonal $\delta^{C+N}(s,b)$ [13]

$$F^{C+N}(s,q^2=-t) = \frac{s}{4\pi i}\int_{\Omega_b} d^2b\, e^{i\vec{q}\vec{b}}\left[e^{2i\delta^{C+N}(s,b)}-1\right], \tag{3.1}$$

where Ω_b is the two-dimensional Euclidean space of the impact parameter $\vec{b}$.

When formula (3.1) is to be applied at finite energies some problems appear as the amplitude $F^{C+N}(s,t)$ is defined in a finite region of t only. Mathematically consistent use of Fourier-Bessel transformation requires, however, the existence of the reverse transformation. And it is necessary to take into account the values of elastic amplitude from unphysical region where the elastic hadronic amplitude is not defined; for details see Refs. [13]). This issue has been resolved in a unique way by

Islam [14,15] by analytically continuing the elastic hadronic amplitude $F^N(s,t)$ from the physical to the unphysical region of t; see also Ref. [16].

The individual eikonals may be defined as integrals of corresponding potentials [17]; and due to their additivity also the complete elastic eikonal $\delta^{C+N}(s,b)$ may be expressed as the sum of both the Coulomb $\delta^C(s,b)$ and hadronic $\delta^N(s,b)$ eikonals at the same value of impact parameter b [18]:

$$\delta^{C+N}(s,b) = \delta^C(s,b) + \delta^N(s,b). \tag{3.2}$$

The complete elastic scattering amplitude can be then written as [18]-[20]

$$F^{C+N}(s,t) = F^C(s,t) + F^N(s,t) + \frac{i}{\pi s}\int_{\Omega_{q'}} d^2q' F^C(s,q'^2) F^N(s,[q-q']^2), \tag{3.3}$$

where Ω_q is the two-dimensional set of kinematically allowed vectors q.

This equation containing the convolution integral differs substantially from Eq. (1.2). In the final form (valid at any s and t) it may be written [20] as

$$F^{C+N}(s,t) = \pm\frac{\alpha s}{t} f_1(t) f_2(t) + F^N(s,t)\,[1 \mp i\alpha G(s,t)], \tag{3.4}$$

where

$$G(s,t) = \int_{-4p^2}^{0} dt' \left\{ \ln\left(\frac{t'}{t}\right)\frac{d}{dt'}[f_1(t')f_2(t')] + \frac{1}{2\pi}\left[\frac{F^N(s,t')}{F^N(s,t)} - 1\right] I(t,t') \right\}, \tag{3.5}$$

and

$$I(t,t') = \int_0^{2\pi} d\Phi'' \frac{f_1(t'')f_2(t'')}{t''}, \quad t'' = t + t' + 2\sqrt{tt'}\cos\Phi''. \tag{3.6}$$

Instead of the t independent quantities B and ρ, it is now necessary to consider corresponding t dependent quantities defined as

$$B(s,t) = \frac{d}{dt}\left[\ln\frac{d\sigma^N}{dt}\right] = \frac{2}{|F^N(s,t)|}\frac{d}{dt}|F^N(s,t)| \tag{3.7}$$

and

$$\rho(s,t) = \frac{\Re F^N(s,t)}{\Im F^N(s,t)}. \tag{3.8}$$

The total cross section derived with the help of the optical theorem is then

$$\sigma_{tot}(s) = \frac{4\pi}{p\sqrt{s}}\Im F^N(s,t=0). \tag{3.9}$$

The form factors $f_1(t)$ and $f_2(t)$ reflect the electromagnetic structure of colliding nucleons and form a part of the Coulomb amplitude from the very beginning. But instead of using the dipole form factor (1.4) as it has been done in Eq. (1.3) it has been suggested to use more convenient formula from Ref. [21]:

$$f_j(t) = \sum_{k=1}^{4}\frac{g_k}{w_k - t}, \quad j = 1,2 \tag{3.10}$$

where the values of the parameters g_k and w_k are to be taken from the quoted paper.

As the Coulomb part in formula (3.4) is known the complete amplitude depends in principle on hadronic amplitude $F^N(s,t)$ only. Thus it can be used in two complementary ways:

- one can test the predictions of different models of high-energy elastic hadronic scattering that provide hadronic amplitudes $F^N(s,t)$. Then, with the help of formula (3.4) one can calculate complete amplitudes $F^{C+N}(s,t)$ that can be compared to experimental data by employing Eq. (1.1),

- one may resolve phenomenological t dependence of elastic hadronic amplitude $F^N(s,t)$ at a given s (and for all measured t values), by fitting experimental elastic differential cross section data with the help of Eq. (1.1) and (3.4). The crucial point here is then a suitable parameterization of the hadronic amplitude $F^N(s,t)$.

The eikonal approach brings the possibility of determining mean values of impact parameter for different kinds of scattering processes. These quantities characterize the ranges of forces responsible for the elastic, inelastic and total scattering. If the unitarity condition and the optical theorem are applied to the mean-squared values of impact parameter for different processes may be determined directly from the t dependence of elastic hadronic amplitude $F^N(s,t)$.

The elastic mean-square can be determined by means of the formula (see Refs. [16], [22]-[25])

$$
\begin{aligned}
< b^2(s) >_{el} \; = \; & 4\,\frac{\int_{t_{min}}^{0} dt\,|t|\left(\frac{d}{dt}|F^N(s,t)|\right)^2}{\int_{t_{min}}^{0} dt\,|F^N(s,t)|^2} + 4\,\frac{\int_{t_{min}}^{0} dt\,|t|\,|F^N(s,t)|^2\left(\frac{d}{dt}\zeta^N(s,t)\right)^2}{\int_{t_{min}}^{0} dt\,|F^N(s,t)|^2} \\
\equiv \; & < b^2(s) >_{mod} + < b^2(s) >_{ph}\,;
\end{aligned}
\tag{3.11}
$$

where the modulus of elastic hadronic amplitude itself determines the first term and the phase (its derivative) influences the second term only; note that both terms are positive.

The total mean-square can be determined with the help of the optical theorem by (see Ref. [24])

$$
\langle b^2(s)\rangle_{tot} = 2B(s,0)\,; \tag{3.12}
$$

the diffractive slope $B(s,t)$ being defined by Eq. (3.7).

According to the unitarity equation the averaged inelastic mean-square is related to the total and elastic mean-squares as [24]

$$
\langle b^2(s)\rangle_{inel} = \frac{\sigma_{tot}(s)}{\sigma_{inel}(s)}\langle b^2(s)\rangle_{tot} - \frac{\sigma_{el}(s)}{\sigma_{inel}(s)}\langle b^2(s)\rangle_{el}\,. \tag{3.13}
$$

4. Model predictions for pp elastic scattering at the nominal LHC energy

In connection with the TOTEM [26, 27] and the ATLAS ALFA [28] experiments where elastic pp scattering will be studied, the predictions of four models proposed by Islam et al. [29], Petrov, Predazzi and Prokhudin [30], Bourrely, Soffer and Wu [31] and Block, Gregores, Halzen and Pancheri [32] will be discussed. Two different alternatives for the model of Petrov et al. [30] with two pomerons (2P) and with three pomerons (3P) will be considered. The mentioned models contain some free parameters in the formulas describing their s and t dependences. Their values can be found in the quoted papers. The predictions for the nominal energy of 14 TeV are shown in Fig. 1 (small $|t|$ region) and Fig. 2 (large $|t|$ range).

The total cross section $\sigma_{tot}(s)$, the diffractive slope $B(s,t)$ and the quantity $\rho(s,t)$ have been determined with the help of formulas (3.9), (3.7) and (3.8) for each model. The integrated elastic hadronic cross sections have been determined by integration of modified Eq. (1.1) containing only $F^N(s,t)$. The values of all these quantities are given in Table 1; the corresponding graphs are shown in Figs. 3 – 4. It is evident that the predictions of divers models differ rather significantly;

the total cross section predictions range from 95 mb to 110 mb. Another value of 101.5 mb following from the formula

$$\sigma_{tot}(s) = 21.70 \left(\frac{s}{s_0}\right)^{0.0808} + 56.08 \left(\frac{s}{s_0}\right)^{-0.4525} \text{ mb}; \qquad s_0 = 1\text{ GeV}^2 \tag{4.1}$$

has been given by Donnachie and Landshoff [33] with the help of Regge pole model fit of pp total cross sections performed at lower energies. A higher value of σ_{tot} has been established by COMPETE collaboration [34] $\sigma_{tot} = 111.5 \pm 1.2^{+4.1}_{-2.1}$ mb which has been determined by extrapolation of the fitted lower energy data with the help of dispersion relations technique. Let us remark that there is no reliable theoretical prediction for this quantity: e.g., the latest prediction on the basis of QCD for this quantity has been 125 $\pm$ 25 mb [35]. The predictions of $\frac{d\sigma}{dt}$ values for higher values of $|t|$ are shown in Fig. 2; they differ significantly for different models. Let us point out especially the second diffractive dip being predicted by Bourrely, Soffer and Wu model [31]. The predictions for the t dependence of the diffractive slopes $B(t)$ are shown in Fig. 3. They differ significantly from the constant dependence required in the simplified West and Yennie formula (1.3). Fig. 4 displays the t dependence of the quantity $\rho(t)$ that is not constant, either, as it would be required by the second assumption needed for validity of formula (1.3). Figs. 3 and 4 represent,

model	σ_{tot} [mb]	σ_{el} [mb]	$B(0)$ [GeV^{-2}]	$\rho(0)$
Islam et al.	109.17	21.99	31.43	0.123
Petrov et al. (2P)	94.97	23.94	19.34	0.097
Petrov et al. (3P)	108.22	29.70	20.53	0.111
Bourrely et al.	103.64	28.51	20.19	0.121
Block et al.	106.74	30.66	19.35	0.114

Table 1: The values of basic parameters predicted by different models for pp elastic scattering at energy of 14 TeV.

therefore, further support for the use of the eikonal formula for the complete elastic scattering amplitude (3.4). Fig. 5 shows then the t dependence of the ratio of interference to hadronic contributions of the $\frac{d\sigma}{dt}$ for all of the given models, i.e., of the quantity

$$Z(t) = \frac{|F^{C+N}(s,t)|^2 - |F^C(s,t)|^2 - |F^N(s,t)|^2}{|F^N(s,t)|^2}. \tag{4.2}$$

The graphs show clearly that the influence of the Coulomb scattering may hardly be fully neglected also at higher values of $|t|$. It is interesting that at least for small $|t|$ the given characteristics are very similar.

5. Luminosity estimation on the basis of pp elastic scattering at the LHC

An accurate determination of the elastic amplitude is very important in the case when the luminosity of the collider is to be calibrated on the basis of elastic nucleon scattering. The luminosity L relates the experimental elastic differential counting rate $\frac{dN_{el}}{dt}(s,t)$ to the complete elastic amplitude $F^{C+N}(s,t)$ (see Eq. (1.1) and Refs. [36, 37]) by

$$\frac{1}{L}\frac{dN_{el}}{dt}(s,t) = \frac{\pi}{sp^2}|F^{C+N}(s,t)|^2. \tag{5.1}$$

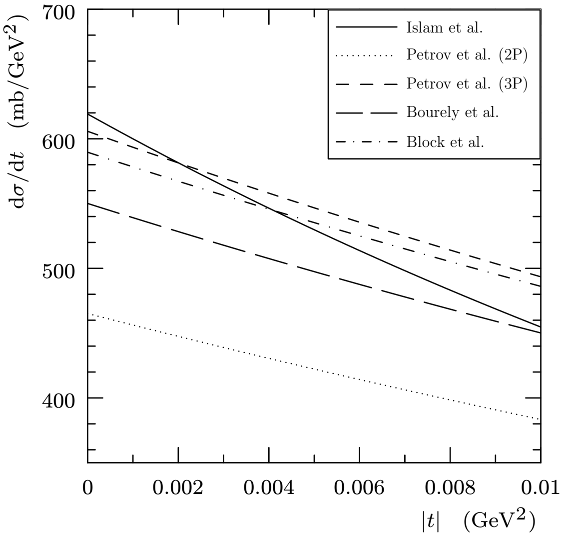

Figure 1: $\frac{d\sigma}{dt}$ predictions at low $|t|$ for pp scattering at 14 TeV according to different models (in very forward direction).

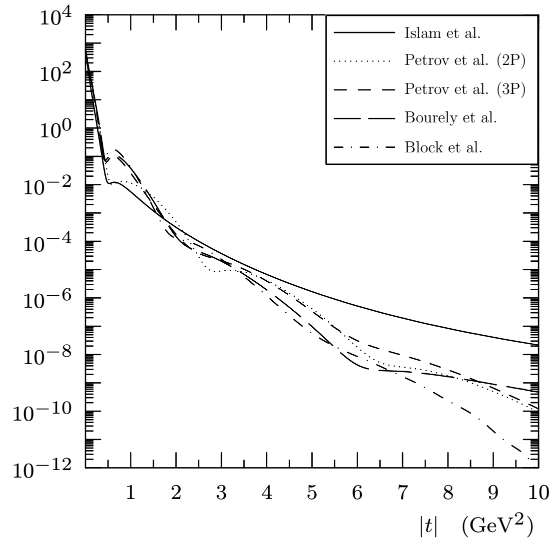

Figure 2: $\frac{d\sigma}{dt}$ predictions for pp scattering at 14 TeV according to different models (in a larger interval of t).

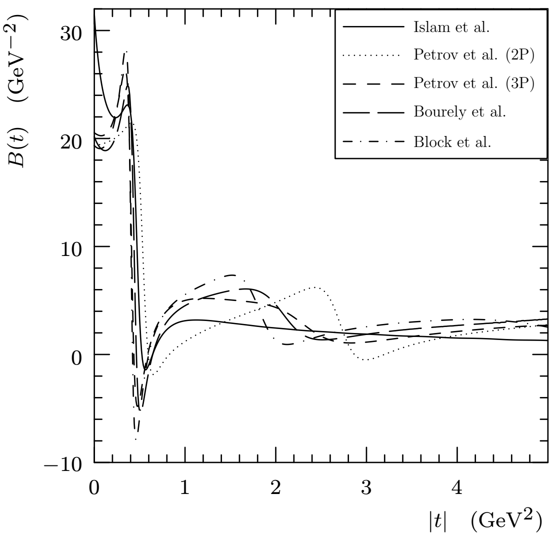

Figure 3: The diffractive slope predictions for pp scattering at 14 TeV according to different models.

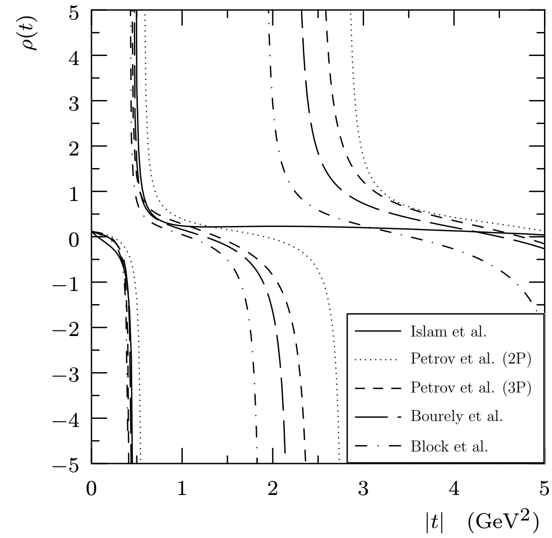

Figure 4: The $\rho(t)$ predictions for pp scattering at 14 TeV according to different models.

Eq. (5.1) is valid for any admissible value of t. The value L might be in principle calibrated by measuring the counting rate in the region of the smallest $|t|$ where the Coulomb amplitude is dominant. However, this region can hardly be reached at the nominal LHC energy due to technical limitations. A procedure allowing to avoid these difficulties may be based on Eq. (5.1), when the elastic counting rate may be, in principle, measured at any t which can be reached, and the complete elastic scattering amplitude $F^{C+N}(s,t)$ may be determined with required accuracy at any $|t|$, too. However, in this case it will be very important which formula for the complete elastic amplitude $F^{C+N}(s,t)$ will be used.

We have studied the differences between the West and Yennie simplified formula (see Eqs. (1.3) and (1.5)) and the eikonal model (Eqs. (3.4)–(3.6)). The differences can be well visualized by the

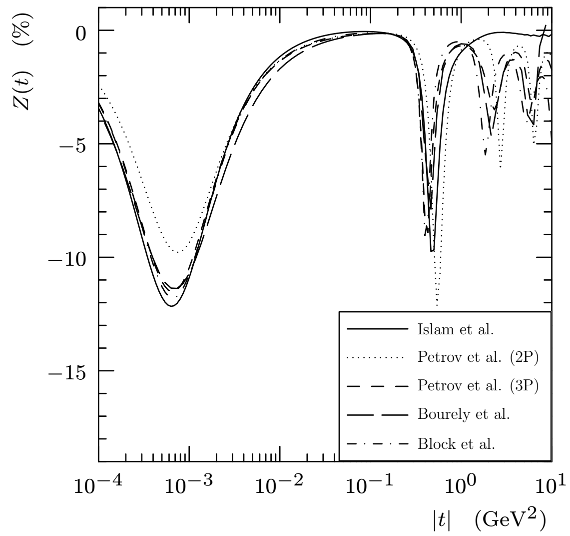

Figure 5: The t dependence of the ratio of the interference to the hadronic contributions to the $\frac{d\sigma}{dt}$ for pp elastic scattering at 14 TeV according to different models.

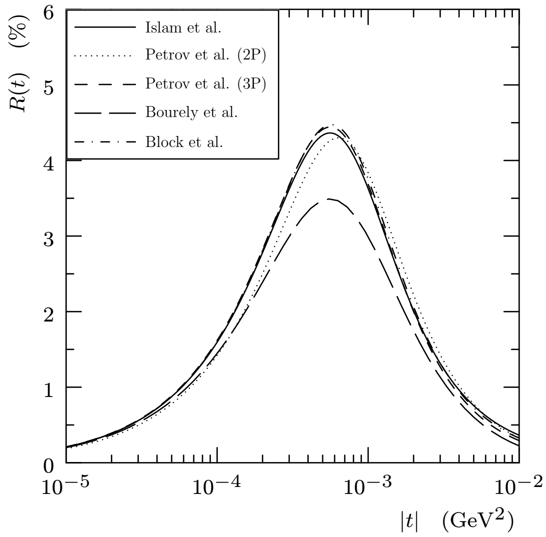

Figure 6: The $R(t)$ quantity predictions for pp scattering at 14 TeV according to different models.

quantity

$$R(t) = \frac{|F^{C+N}_{eik}(s,t)|^2 - |F^{C+N}_{WY}(s,t)|^2}{|F^{C+N}_{eik}(s,t)|^2}, \tag{5.2}$$

where $F^{C+N}_{eik}(s,t)$ is the complete elastic scattering eikonal model amplitude, while $F^{C+N}_{WY}(s,t)$ is the West and Yennie one. The quantity $R(t)$ is plotted in Fig. (6) for several models.

The maximum deviations lie approximately at [36]

$$|t_{int}| \approx \frac{8\pi\alpha}{\sigma_{tot}} \approx 0.00064\ \mathrm{GeV}^2, \tag{5.3}$$

where the Coulomb and the hadronic effects are expected to be practically equal. Let us emphasize that the differences between the physically consistent eikonal model and the West and Yennie formula may reach almost 5 %. It means that the luminosity derived on the basis of elastic pp scattering at the energy of 14 TeV might be burdened by a non-negligible systematic error, if determined only from a small t region around t_{int}.

6. Root-mean-squared values of impact parameters

The impact parameter representation of elastic hadronic amplitude $F^N(s,t)$ allows to establish different root-mean-squared (RMS) values of impact parameters that represent in principle the ranges of hadronic interactions. Their values calculated with the help of formulas (3.11)–(3.13) for each of the analyzed models and expected at LHC nominal energy are shown in Table 2. The values of elastic RMS are in all cases lower than the corresponding values of the inelastic ones. It means that the elastic pp collisions would be much more central then the inelastic ones; similarly as in the case of pp scattering at the ISR energies for all these models; this should be recognized as a puzzle, see Ref. [39]. It can be interpreted as a consequence of admitting only a weak (standard) t dependence of elastic hadronic phase in all models. The given puzzle can be removed if the used elastic hadronic phase $\zeta^N(s,t)$ is allowed to have a more general shape of t dependence (see Refs. [7,8,12,16,20,23]).

model	$\sqrt{\langle b^2_{tot} \rangle}$ [fm]	$\sqrt{\langle b^2_{el} \rangle}$ [fm]	$\sqrt{\langle b^2_{inel} \rangle}$ [fm]
Islam et al.	1.552	1.048	1.659
Petrov et al. (2P)	1.227	0.875	1.324
Petrov et al. (3P)	1.263	0.901	1.375
Bourrely et al.	1.249	0.876	1.399
Block et al.	1.223	0.883	1.336

Table 2: The values of root-mean-squares predicted by different models.

While the t dependence of modulus $|F^N(s,t)|$ can be determined from the measured elastic hadronic differential cross section the t dependence of phase remains rather arbitrary (as already mentioned). And it is possible to choose significantly different phase dependences [24].

It is, however, almost generally assumed that the imaginary part of elastic hadronic amplitude is dominant in a broad region of $|t|$ around the forward direction; it is taken as slowly decreasing with rising $|t|$ and vanishing at the diffractive minimum. The real part is assumed to start at small value at $|t| = 0$ and to decrease, too, having still non-zero value at the diffractive minimum. It means that the t dependence of the phase $\zeta^N(s,t)$ is very weak and becomes significant only in the region of diffractive minimum. However, the existence of diffractive minimum does not require zero value for its imaginary part at this point. It means only that the sum of both the squares of real and imaginary parts should be minimal at this point. The mentioned requirement of vanishing imaginary part represents much stronger and more limiting condition then the physics requires.

Regarding Eq. (3.11) it is evident that very different elastic RMS values may be obtained according to the chosen t dependence of the phase $\zeta^N(s,t)$. One should distinguish between the so called central picture (the first term dominates) and peripheral picture (decisive contribution comes from the second term when the phase increases quickly with rising t and reaches $\pi/2$ at $|t| \cong 0.1$ GeV^2). The value $\langle b^2 \rangle_{el}$ is lesser than $\langle b^2 \rangle_{inel}$ in the central case while $\langle b^2 \rangle_{el}$ is greater than $\langle b^2 \rangle_{inel}$ in the peripheral case. The proton in the central case has been regarded as relatively transparent object which still represents a puzzling question (see, e.g., Refs. [38] and [39]). And more detailed models of elastic hadronic scattering giving the peripheral distribution of elastic hadronic scattering should be considered and proposed. Only in such a case one may avoid the situation when the elastic hadronic scattering at high energies is more central than the inelastic ones as it follows immediately from the Fourier-Bessel transformation of elastic hadronic amplitude. Thus no a priori limitations of elastic hadronic amplitude should be introduced in the corresponding analysis of experimental data and different possibilities should be analyzed.

As to the profiles in the impact parameter space the peripheral behavior seems to be slightly preferred on the basis of analysis of pp experimental data at 53 GeV and p̄p at 541 GeV (see [20]). The peripheral picture is supported also by analysis of elastic scattering of α particles on various targets (^1H; ^2H; ^3He; ^4He) [40] performed with the help of Glauber model where the 'elementary' nucleon-nucleon elastic hadronic amplitude has exhibited similar t dependence of phase $\zeta^N(s,t)$ as in our peripheral case [20].

7. Conclusion

In the past the analyses of high energy elastic nucleon scattering data in the region of very small $|t|$ were performed with the help of the simplified interference formula proposed by West and Yennie and including the influence of both Coulomb and hadronic interactions. At higher values of momentum transfers the influence of Coulomb scattering was neglected and the elastic scattering of nucleons was described only with the help of a hadronic amplitude having dominant imaginary part in a broad region of t and vanishing only at the diffractive minimum. And it is evident that such a description of elastic nucleon scattering with the help of two different formulas for the complete amplitude represents significant deficiency.

A more general eikonal model has been proposed. It describes elastic charged nucleon collisions at high energies with only one formula for the complete elastic amplitude in the whole kinematical region of t. This model is adequate for any t dependence of the elastic hadronic amplitude and has been successfully used for the analysis of elastic pp and pp scattering data at lower energies – see, e.g., Ref. [20].

The attention of this paper has been devoted also to the LHC experiments that will measure proton-proton elastic scattering [27,28]. Several phenomenological model predictions for dynamical quantities of interest have been discussed. A certain problem may be seen, however, in the fact that practically all considered allow central behavior only.

Attention has been devoted also to the problem of luminosity determination as the values of all other quantities are affected by its value. The model predictions indicate that a systematic difference up to 5 % might occur between the eikonal and the West and Yennie formulas.

It is also necessary to call attention to the fact that the contribution of the Coulomb scattering cannot be fully neglected at rather high $|t|$ values, either. However, the main open question concerns the fact that the experimental data of the differential cross section give directly the t dependence of the modulus, while the t dependence of the phase is only little constrained and may depend on some other assumptions or degrees of freedom. Any analysis of experimental data should, therefore, always contain a statistical evaluation of two different alternatives: central and peripheral; peripheral behavior corresponding better to usual picture of collision processes. And the attention should be devoted to a construction of the model which would be able to represent a realistic picture of elastic hadronic scattering of charged nucleons.

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