# Towards small-$x$ resummed DIS phenomenology

Juan Rojo[1], Guido Altarelli[2], Richard D. Ball[3] and Stefano Forte[1],

[1]Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano,
Via Celoria 16, I-20133 Milan, Italy

[2]Dipartimento di Fisica "E.Amaldi", Università Roma Tre and INFN, Sezione di Roma Tre
Via della Vasca Navale 84, I–00146 Roma, Italy,
CERN, Department of Physics, Theory Division, CH-1211 Genève 23, Switzerland

[3]School of Physics, University of Edinburgh, Edinburgh EH9 3JZ, Scotland

We report on recent progress towards quantitative phenomenology of small $x$ resummation of deep–inelastic structure functions. We compute small $x$ resummed $K$–factors with realistic PDFs and estimate their impact in the HERA kinematical region. These $K$–factors, which match smoothly to the fixed order NLO results, approximately reproduce the effect of a small $x$ resummed PDF analysis. Typical corrections are found to be of the same order as the NNLO ones, that is, a few percent, but with opposite sign. These results imply that resummation corrections could be relevant for a global PDF analysis, especially with the very precise combined HERA dataset.

*Small x resummation in the LHC era*   The so-called small $x$ regime of QCD is the kinematical region in which hard scattering processes happen at a center-of-mass energy which is much larger than the characteristic hard scale of the process. An understanding of strong interactions in this region is therefore necessary to do physics at high–energy colliders. In this sense, HERA was the first small $x$ machine, and LHC is going to be even more of a small $x$ accelerator.

As is by now well known, perturbative corrections become large at small $x$. Due to the accidental vanishing of some coefficients, the leading large corrections cannot be seen in LO and NLO splitting functions; however, the first subleading correction can already be seen in the NNLO splitting functions which have been computed recently, as well as in NNLO coefficient functions: they are large enough to make recent NNLO parton fits unstable at small $x$ [1].

This suggests dramatic effects from yet higher orders, so the success of NLO perturbation theory at HERA, as demonstrated by the scaling laws it predicts, has been for a long time very hard to explain. In the last several years this situation has been clarified [2, 3, 4, 5, 6], showing that, once all relevant large terms are included, the effect of the resummation of terms which are enhanced at small $x$ is perceptible but moderate — comparable in size to typical NNLO fixed order GLAP corrections in the HERA region. A recent status report on small $x$ resummation, including a comparison of various approaches and a more complete list of references, can be found in Ref. [7].

Recently, in Ref. [4] a full small $x$ resummation including quarks and the resummation of deep-inelastic coefficient functions was presented, so that resummed expressions for deep-inelastic structure functions can be obtained. Furthermore, the resummation of hard partonic cross sections has been performed in several LHC processes such as heavy quark production [8], Higgs production [9, 10], Drell-Yan [11, 12] and prompt photon production [13]. This will enable fully resummed phenomenology. In this contribution we present a first step in this direction.
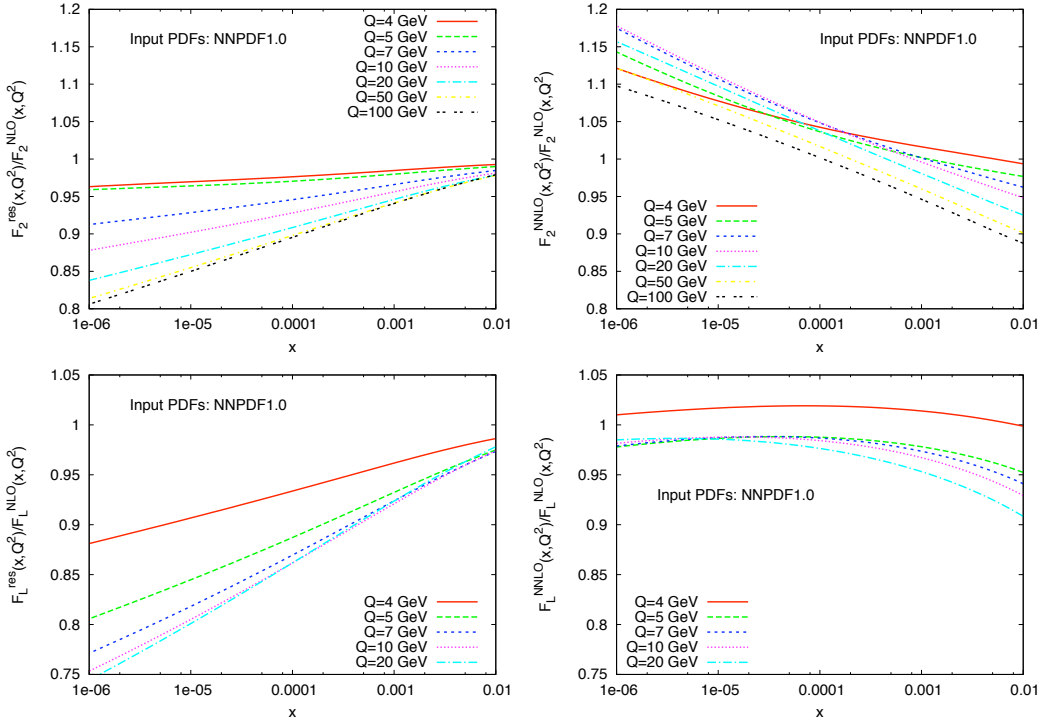
*DIS 2009*

Figure 1: Upper left: the small $x$ resummed $K$–factors for $F_2(x, Q^2)$ for various values of $Q$. Upper right: analogous $K$–factors for the NNLO case. Lower left: the small $x$ resummed $K$–factors for $F_L(x, Q^2)$ for various values of $Q$. lower right: analogous $K$–factors for the NNLO case. In all cases, the matching scale has been taken to be $Q_0 = 3$ GeV and the input PDF set is NNPDF1.0.

*Small $x$ resummed $K$–factors*   Using the results of Ref. [4], it is now possible to determine resummed predictions for deep–inelastic structure functions. Eventually, these should be combined with resummed expressions for various parton-level cross-sections in order to determine parton distributions at the resummed level, and use them for a fully resummed treatment of hard processes.

However, as a first step in this program, it is convenient to provide a qualitative estimate of the impact of small $x$ resummation on DIS phenomenology. Such an estimate can be obtained by first computing the structure functions $F_2$ and $F_L$ with a given set of NLO PDFs, and then assuming that the structure functions are kept fixed at some scale $Q_0$: this is then enough to determine the resummed singlet quark and gluon distributions at that scale. These quark and gluon distributions are close to those which would be obtained if PDFs were determined from a fit to DIS data mostly clustered around $Q_0$. They can thus be used to compute observables at any other scale.

We have used them to recompute the structure functions $F_2$ and $F_L$ in the resummed formalism and we have then determined $K$–factors as ratios of the results from the whole procedure at the resummed and at the NLO levels, respectively. In the following, we will show results obtained with the choice $Q_0 = 3$ GeV, which roughly corresponds to the typical

scale of the smallest $x$ HERA data used in a parton fit. Different choices of $Q_0$ lead to rather different results, so that this $K$–factor approach can only be used to get a first qualitative feeling for the phenomenological impact of resummation.

$K$–factors thus obtained for the $F_2(x,Q^2)$ and $F_L(x,Q^2)$ structure functions are shown in Fig. 1. In order to allow a more direct comparison, NNLO $K$–factors have been computed in the same way as the NLO small $x$ resummed ones. As one can observe, the impact of the resummation at the "HERA scale" $Q_0$ is comparable to that of NNLO corrections, but it goes in the opposite direction: it tends to suppress the starting PDFs while the NNLO tends to enhance them. Note that in the small $x$ resummed case asymptotic freedom is at work: for large values of $Q$, the $K$–factors become independent of its precise value.

The determination of $K$–factors requires the use of an input parton set. The $K$–factors shown in Fig. 1 have been computed using the NNPDF1.0 [14] parton set[a]. We have explicitly checked that different choices, for example using as input PDFs not the central NNPDF1.0 set but a set of PDFs which roughly sit on the associated $\pm 1$-$\sigma$ PDF uncertainty band, have a negligible impact on the procedure.

As already mentioned, from Fig. 1 the dominant qualitative feature of the $K$–factors is that resummation leads to a suppression of the structure functions $F_2$ and $F_L$ at small values of $x$. Note also the smooth matching with the GLAP NLO result, since the $K$–factor tends to 1 as expected at moderate and large-$x$.

It is important to understand the meaning of these $K$–factors, which increasingly deviate from one at larger $Q^2$, and not at low $Q^2$ as one might expect. Indeed, these $K$–factors provide the change in prediction, from NLO to the resummed level, for structure functions at large $Q^2$, assuming that the structure functions at the low scale $Q_0^2$ are given and fixed. As such, they provide a model for the change due to resummation in prediction at LHC scales, when current HERA data are used to determine parton distributions at small $x$.

*Small x resummed phenomenology*  Once the small $x$ resummed $K$–factors have been computed, they can be used to obtain resummed predictions for structure functions using any input NLO set of PDFs. In order to get an estimate of the quantitative impact of small $x$ resummation in the HERA region, in Fig. 2 we show the relative differences between the NLO computation and the small $x$ resummed one in the $(x,Q^2)$ kinematics of the published HERA data. As before, the matching scale has been taken to be $Q_0 = 3$ GeV and the input PDF set is NNPDF1.0. We observe that the typical effect is a negative difference (the resummation decreases $F_2$) of the order of a few percent. Since the upcoming combined HERA data set will have an accuracy of $\sim 1\%$, this implies that the effects of small $x$ resummation should be relevant in a global PDF analysis.

As another example of the applications of the resummed $K$–factors, we show in Fig. 3 a comparison between H1 data on the longitudinal structure function $F_L(x,Q^2)$ [18] and the NNPDF1.0 prediction both at NLO and with the small $x$ resummed $K$–factors. Interestingly, the suppression due to the resummation is generally larger than the one–sigma band due to PDF uncertainties, although the statistical accuracy of the measurements is still not enough to provide any discrimination power.

The impact of resummation on partonic cross sections for LHC signal, background or standard candle processes such as respectively Higgs [9], prompt photon [13] and Drell-

---

[a] The NNPDF approach to parton distributions [15, 14, 16, 17] provides sets of PDFs with faithful uncertainty estimation thanks to a combination of artificial neural networks as unbiased interpolations and Monte Carlo methods for robust error estimation and propagation.
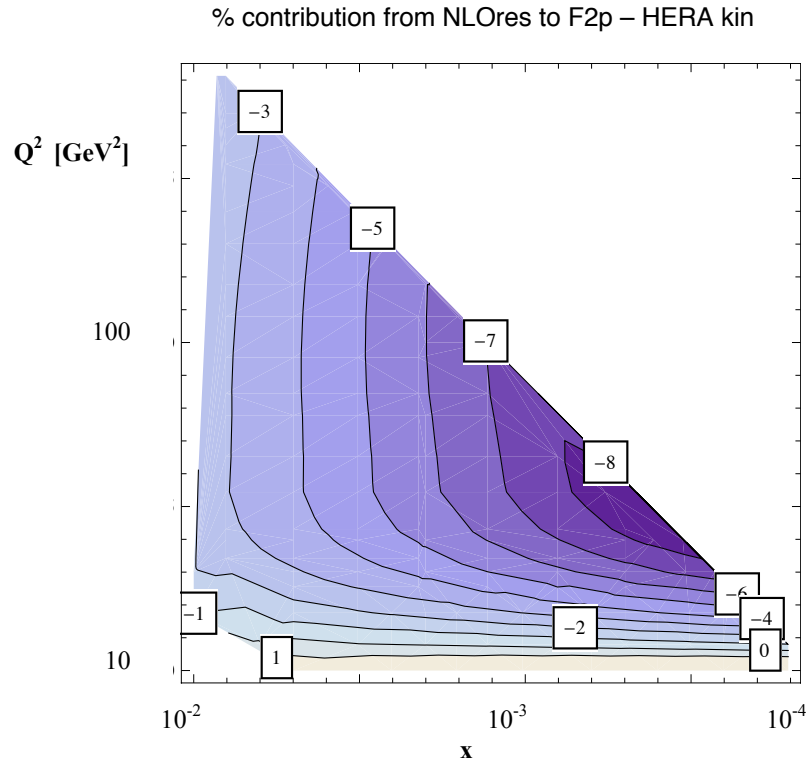
% contribution from NLOres to F2p – HERA kin

Figure 2: The impact of small $x$ resummation on the kinematics of published $F_2(x, Q^2)$ HERA data. The various contours show the relative percent difference between the NLO and the small $x$ resummed computations.

Yan [11] are now known to be qualitatively similar to the impact on deep-inelastic coefficient functions [19, 12]. Therefore, it is clear from the results presented above that resummation is necessary for LHC phenomenology at the percent level of accuracy, typical of NNLO computations.

On a more speculative level, resummation would also be very relevant for deep-inelastic scattering at a high energy electron-hadron collider based on the LHC, the so-called LHeC [20]. Indeed, in Ref. [21] it was show how the structure function $F_2$ in the LHeC kinematics varies on applying either resummed or NNLO $K$–factors to the NLO prediction from the NNPDF1.0 parton set. As can be seen in Fig. 4, the expected accuracy of such a machine would clearly discriminate between the small $x$ resummed and NNLO cases, since at such high energies the difference between the two predictions is larger than both the expected PDF uncertainty and the accuracy of experimental data.

*Outlook*   This contribution summarizes ongoing work towards small $x$ resummed phenomenology of deep-inelastic scattering. In summary, the impact of small $x$ resummation is typically as large as that of NNLO corrections in the HERA region, and even larger at higher collider
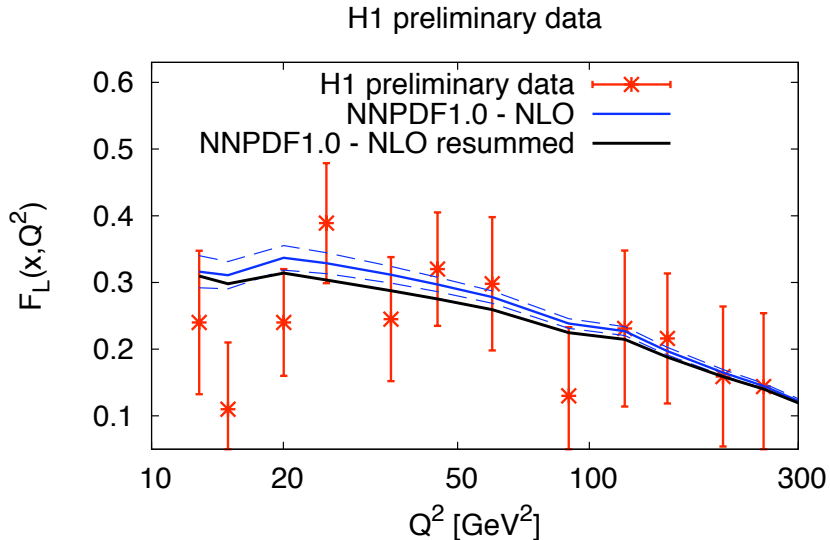
Figure 3: Comparison between the H1 measurent of $F_L(x, Q^2)$ [18] and the NLO and small $x$ resummed predictions computed with the NNPDF1.0 parton set.

energies, with the resummed results generally lying outside the PDF uncertainty bands.

Our results indicate the small $x$ resummed corrections could be disentangled with the ultimate precision of HERA data. However, a fully small $x$ resummed global PDF analysis is unavoidable in order to quantify the modifications in the PDFs due to small $x$ resummation, and to propagate these modifications into relevant observables at the LHC.

Such a programme will be necessary in order to achieve phenomenology at the percent level for many LHC signal, background and standard candle processes. It would be even more important for phenomenology at a future LHeC electron-proton collider, and mandatory for the treatment of extremely high-energy scattering processes, such as those induced by Ultra-High Energy cosmic neutrinos, which are currently under investigation.

# References

[1] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, "Parton distributions for the LHC," `arXiv:0901.0002 [hep-ph]`.

[2] G. Altarelli, R. D. Ball, and S. Forte, "An anomalous dimension for small x evolution," *Nucl. Phys.* **B674** (2003) 459–483, `arXiv:hep-ph/0306156`.

[3] G. Altarelli, R. D. Ball, and S. Forte, "Perturbatively stable resummed small x evolution kernels," *Nucl. Phys.* **B742** (2006) 1–40, `arXiv:hep-ph/0512237`.

[4] G. Altarelli, R. D. Ball, and S. Forte, "Small x Resummation with Quarks: Deep-Inelastic Scattering," *Nucl. Phys.* **B799** (2008) 199–240, `arXiv:0802.0032 [hep-ph]`.

[5] M. Ciafaloni, D. Colferai, G. P. Salam, and A. M. Stasto, "The gluon splitting function at moderately small x," *Phys. Lett.* **B587** (2004) 87–94, `arXiv:hep-ph/0311325`.

[6] M. Ciafaloni, D. Colferai, G. P. Salam, and A. M. Stasto, "A matrix formulation for small-x singlet evolution," *JHEP* **08** (2007) 046, `arXiv:0707.1453 [hep-ph]`.

[7] M. Dittmar *et al.*, "Parton Distributions," `arXiv:0901.2504 [hep-ph]`.

[8] R. D. Ball and R. K. Ellis, "Heavy quark production at high-energy," *JHEP* **05** (2001) 053, `arXiv:hep-ph/0101199`.

[9] S. Marzani, R. D. Ball, V. Del Duca, S. Forte, and A. Vicini, "Higgs production via gluon-gluon fusion with finite top mass beyond next-to-leading order," *Nucl. Phys.* **B800** (2008) 127–145, `arXiv:0801.2544 [hep-ph]`.

[10] S. Marzani, R. D. Ball, V. Del Duca, S. Forte, and A. Vicini, "Finite-top-mass effects in NNLO Higgs production," *Nucl. Phys. Proc. Suppl.* **186** (2009) 98–101, `arXiv:0809.4934 [hep-ph]`.

[11] S. Marzani and R. D. Ball, "High Energy Resummation of Drell-Yan Processes," *Nucl. Phys.* **B814** (2009) 246–264, `arXiv:0812.3602 [hep-ph]`.

[12] S. Marzani and R. D. Ball, "Drell-Yan processes in the high-energy limit," `arXiv:0906.4729 [hep-ph]`.

[13] G. Diana, "High-energy resummation in direct photon production," `arXiv:0906.4159 [hep-ph]`.

[14] **NNPDF** Collaboration, R. D. Ball *et al.*, "A determination of parton distributions with faithful uncertainty estimation," *Nucl. Phys.* **B809** (2009) 1–63, `arXiv:0808.1231 [hep-ph]`.

[15] **NNPDF** Collaboration, L. Del Debbio, S. Forte, J. I. Latorre, A. Piccione, and J. Rojo, "Neural network determination of parton distributions: The nonsinglet case," *JHEP* **03** (2007) 039, `arXiv:hep-ph/0701127`.

[16] **NNPDF** Collaboration, J. Rojo *et al.*, "Update on Neural Network Parton Distributions: NNPDF1.1," `arXiv:0811.2288 [hep-ph]`.

[17] **NNPDF** Collaboration, R. D. Ball *et al.*, "Precision determination of electroweak parameters and the strange content of the proton from neutrino deep-inelastic scattering," `arXiv:0906.1958 [hep-ph]`.

[18] A. Glazov, "These proceedings." 2009.

[19] R. D. Ball, "Resummation of Hadroproduction Cross-sections at High Energy," *Nucl. Phys.* **B796** (2008) 137–183, `arXiv:0708.1277 [hep-ph]`.

[20] J. B. Dainton, M. Klein, P. Newman, E. Perez, and F. Willeke, "Deep inelastic electron nucleon scattering at the LHC," *JINST* **1** (2006) P10001, `arXiv:hep-ex/0603016`.

[21] J. Rojo and F. Caola, "Parton distributions and small-x QCD at the Large Hadron Electron Collider," `arXiv:0906.2079 [hep-ph]`.
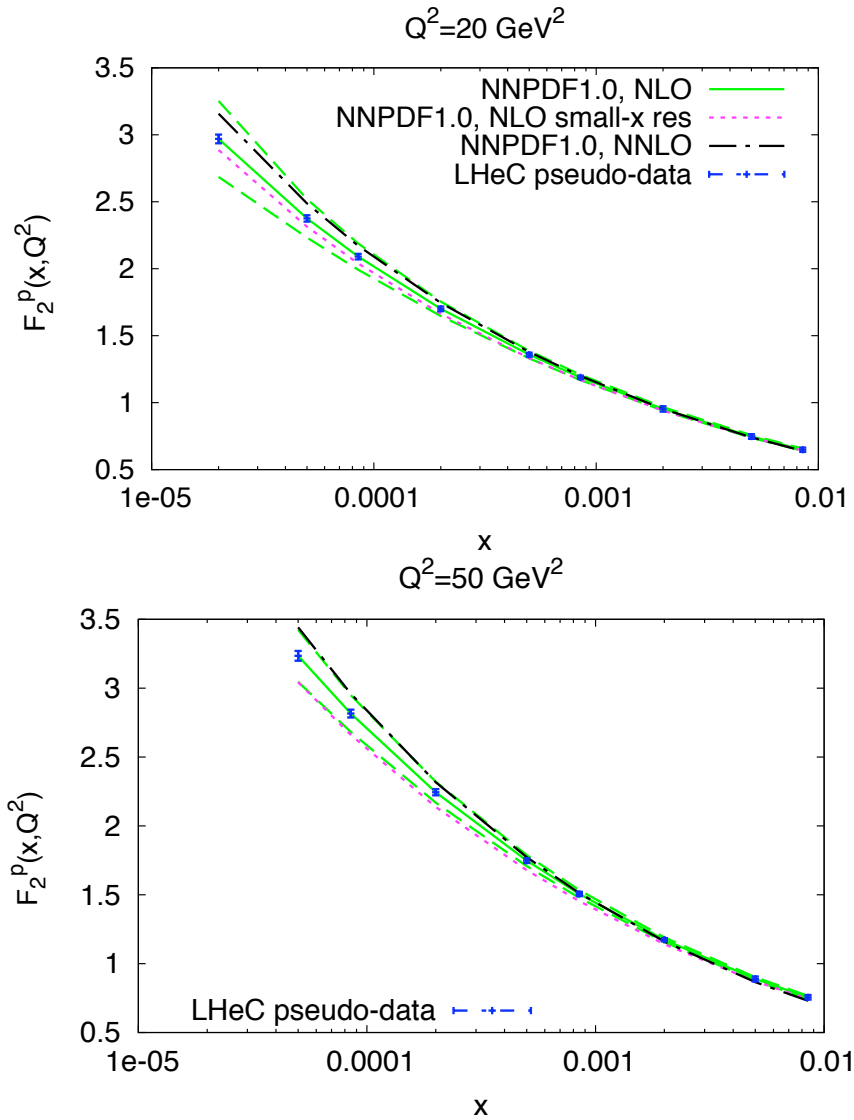
Figure 4: A comparison of various approximations to linear low-$x$ QCD for $F_2$ at the LHeC: the NNPDF1.0 prediction which includes PDF uncertainties and the NNPDF1.0 result corrected with the NNLO and small $x$ resummed $K$–factors. The expected experimental precision at the LHeC is also shown for illustration. The upper plot corresponds to $Q^2 = 20$ GeV$^2$, while the lower plot to $Q^2 = 50$ GeV$^2$.