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Heat transfer characteristics of a continuous stretching surface with variable temperature of two components fluid mixture

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Abstract : The non-linear partial differential equations describing the problem of heat transfer from a linearly stretching continuous surface with a power law temperature distribution of two-component fluids, are reduced by similarity transformation to non-linear ordinary differential equations. To obtain the numerical solution of this problem we used a modified Newton-Raphson shooting technique using Runge-Kutta Merson method with automatic error control as an initial value solver.

The heat transfer characteristics for this problem are found to be determined by the temperature λ and Prandtl number P_{r_1} and P_{r_2}. The magnitude of λ affects the direction and quantity of heat flow. For $\lambda = -1$, $P_{r_1} = 0.72$, $P_{r_2} = 3$, 10 and 100, the wall temperature gradient vanishes, (θ'_2 (o) = 0). In this case there is no heat transfer occurring between the continuous surface and the fluids. In general heat is transferred from the continuous surface to the fluids for $\lambda > -1$, $\lambda = -1$ in case of θ'_1 (o) and to the continuous surface for $\lambda < -1$ in case of θ'_1 (o). For $\lambda = -3$ and certain P_{r_2} values, unrealistic temperature distributions are encountered in case of θ'_2 (o). For the temperature profile θ_1 (η), thermal boundary layer thickness increases as λ decreases and no significant effect with different values of P_{r_2} is observed. The temperature profile θ_2 (η) is slightly affected by different values of λ and increases as P_{r_2} decreases. For a given λ and P_{r_1}, the smaller the P_{r_2}, the larger thermal boundary layer thickness. The velocity profiles F'_1, F'_2 and the shear stresses are not significantly affected by the variation of λ and P_{r_2}.

Keywords : Non-linear partial differential equations, Runge-Kutta Merson, Newton-Raphson shooting technique, Prandtl number.

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1. Introduction

The continuous surface heat transfer problems has many practical applications in industrial manufacturing processes, for example the extrusion of plastic sheets and the boundary layer along a liquid film in condensation processes. The boundary layer on a continuous surface moving steadily through stationary incompressible fluid was first studied theoretically by (Sakiadis 1961a). Most studies have been concerned with constant surface velocity and temperature (Tsou *et al* 1967) but for many practical applications the surface undergoes stretching and cooling or heating that cause surface velocity and temperature variations (Crane 1970, Vleggaar 1977 and Gupta and Gupta 1977) have analyzed the stretching problem with constant surface temperature while Soundalgekar and Ramana Murty (1980) investigated the constant surface case with power law temperature variation. However, Grubka and Bobba (1985) have analyzed the heat transfer from a linearly stretching continuous surface with a power law temperature distribution. The aim of this paper is to investigate the problem of the flow and heat transfer of a two-component fluid near a continuous linearly stretching surface. The effects of power law surface temperature variation and Prandtl number of one of the components are analyzed. Numerical results for local wall heat flux, temperature profiles for various values of temperature parameter and Prandtl number are given in tables and figures.

2. Analysis

The laminar velocity and thermal boundary layers of stationary incompressible mixture of fluids on a continuous stretching surface with velocity u_w and temperature T_w are considered when the physical properties are constant with the ambient temperature T_∞. Under the Boussinesq approximation and using the boundary layer approximation (Schlichting 1968), the fundamental equations for flow in the boundary layer are :

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \tag{1}$$

$$u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = f_1 \gamma_1 \frac{\partial^2 u_1}{\partial y^2} + (k/\rho_1)(u_2 - u_1), \tag{2}$$

$$u_1 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_1}{\partial y} = \alpha_1 \frac{\partial^2 T_1}{\partial y^2}, \tag{3}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0, \tag{4}$$

$$u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y} = f_2 \gamma_2 \frac{\partial^2 u_2}{\partial y^2} + (k/\rho_2)(u_1 - u_2), \tag{5}$$

$$u_2 \frac{\partial T_2}{\partial x} + v_2 \frac{\partial T_2}{\partial y} = \alpha_2 \frac{\partial^2 T_2}{\partial y^2}, \tag{6}$$

with the boundary conditions

$$\left.\begin{array}{ll} u_1 = cx, & u_2 = cx/a, \\ v_1 = 0, & v_2 = 0, \end{array}\right\} \text{at } y = 0 \tag{7}$$

$$T_1 = T_2 = T_w = T_\infty + A\,x^\lambda$$

$$u_1 = u_2 = 0,\ T_1 \longrightarrow T_{1\infty},\ T_2 \longrightarrow T_{2\infty}, \text{ at } y \longrightarrow \infty.$$

The x-axis runs along the continuous surface in the direction of motion, and the y-axis is perpendicular to it; u and v are the velocity components in the direction of x and y

respectively. Note that the viscous dissipation is neglected in the energy eq. (3) and (6). It should be remembered that from the definition of the volume fraction we have

$$f_1 + f_2 = 1. \tag{8}$$

The solution of eq. (1), (4) may be written in terms of the stream functions defined by the relations

$$u_1 = \frac{\partial \psi_1}{\partial y}, \qquad v_1 = -\frac{\partial \psi_1}{\partial x},$$

$$u_2 = \frac{\partial \psi_2}{\partial y}, \qquad v_2 = -\frac{\partial \psi_2}{\partial x}. \tag{9}$$

Introducing the usual similarity transformations and dimensionless temperature

$$\eta = y\,(c/\gamma_1)^{1/2},\ F_1(\eta) = \psi_1(x, y)\,/\,[x\,(\gamma_1 c)^{1/2}],\ F_2 = \psi_2\,/\,[x\,(\gamma_2 c)^{1/2}]$$

$$\theta_1 = (T_1 - T_\infty)\,/\,(T_w - T_\infty),\ \theta_2 = (T_2 - T_\infty)\,/\,(T_w - T_\infty). \tag{10}$$

The momentum eqs. (2), (5) and the energy eqs. (3), (6) can be written as

$$f_1 F_1''' + F_1 F_1'' - F_1'^2 + b\,(\frac{1}{a} F_2' - F_1') = 0, \tag{11}$$

$$f_2 F_2''' + a F_2 F_2'' - a F_2'^2 + (\rho_1/\rho_2)\, b a^2\,(a F_1' - F_2') = 0, \tag{12}$$

$$\theta_1'' + P_{r_1} F_1 \theta_1' - P_{r_1} \lambda F_1' \theta_1 = 0, \tag{13}$$

$$\theta_2'' + P_{r_2} a\,(F_2 \theta_2' - \lambda \theta_2 F_2') = 0. \tag{14}$$

at $\eta = 0$ $\quad F_1'(o) = F_2'(o) = 1$

$F_1(o) = F_2(o) = 0$

$\theta_1(o) = \theta_2(o) = 1,$ (15)

at $\eta \longrightarrow \infty$ $\quad F_1'(\infty) \longrightarrow 0 \qquad F_2'(\infty) \longrightarrow 0$

$\theta_1(\infty) \longrightarrow 0 \qquad \theta_2(\infty) \longrightarrow 0$

where primes denote order of differentiation with respect to η.

The local wall heat can be expressed as

$$q_{1w} = -k \left(\frac{\partial T_1}{\partial y}\right)_{y=0} = -kA\,(c/\gamma_2)^{1/2} x^{\lambda}\, \theta_1'(o),$$

$$q_{2w} = -k \left(\frac{\partial T_2}{\partial y}\right)_{y=0} = -kA\,(c/\gamma_1)^{1/2} x^{\lambda}\, \theta_2'(o).$$

3. Numerical treatment

Eqs. (11-15) are expressed in the following form

$$y_1' = y_2,\ y_2' = y_3,\ y_3' = (1/f_1)\,[-\,y_1 y_3 + y_2^2 - b\ ((1/a) y_5 - y_2)],\ y_4' = y_5,$$

$$y_5' = y_6,\ y_6' = (1/f_2)\,[-ay_4y_6 + ay_5^2 - 6a^2(\rho_1/\rho_2)(ay_2 - y_5)],\ y_7' = y_8, \tag{16}$$
$$y_8' = -P_{r_1}(y_1y_8 - \lambda y_2y_7),\ y_9' = y_{10},\ y_{10}' = -p_{r_2}a(y_4y_{10} - \lambda y_5y_9)$$

with the boundary conditions

$$y_1(o) = y_4(o) = 0,\ y_2(o) = y_5(o) = y_7(o) = y_9(o) = 1,\ y_2(\infty) = y_5(\infty) = y_7(\infty) = y_9(\infty) = 0 \tag{17}$$

where

$$y_1 = F_1,\ y_2 = F'_1,\ y_3 = F''_1,\ y_4 = F_2,\ y_5 = F'_2, \tag{18}$$
$$y_6 = F''_2,\ y_7 = \theta_1,\ y_8 = \theta'_1,\ y_9 = \theta_2 \text{ and } y_{10} = \theta'_2.$$

In order to solve the above system we apply a modified Newton-Raphson shooting technique (Hall and Watt 1976). The practical details are explained in the following steps :

(1) Set $\eta_f = 3$, $k = 0$ (where η_f is the terminal value of the independent variable η)

(2) Assume the missing initial conditions

$$y_3(o) = s_1^{(k)},\ y_6(o) = s_2^{(k)},\ y_8(o) = s_3^{(k)},\ y_{10}(o) = s_4^{(k)}. \tag{19}$$

(3) Integrate forward, the system (16) over an interval $[0, \eta_f]$ using Runge-Kutta Merson method with automatic error control where the local truncation error is bounded by the tolerance $E = 10^{-6}$; we get the solution $U(\eta, S)$ where $U = (u_1, u_2, \ldots\ldots, u_{10})$, $S = (s_1, s_2, s_3, s_4)$.

(4) Try to find S, such that the solution $U(\eta, S)$ satisfies the end conditions at $\eta = \eta_1$ i.e. We solve the system

$$U_r(\eta_f, s_1, s_2, s_3, s_4) = 0,\ r = 2, 5, 7, 9,\ \phi(s) = 0 \tag{20}$$

with $\phi_1 = u_2,\ \phi_2 = u_5,\ \phi_3 = u_7,\ \phi_4 = u_9$ (21)

by applying Newton-Raphson iteractive process

$$S^{(k+1)} = S^{(k)} - [J(S^{(k)})]^{-1}\,\phi(S^{(k)}) \tag{22}$$

where

$j(S^{(k)})$ is the 4×4 Jacobian matrix whose element in the i-th row and j-th column is

$$J_{i,j}(S^{(k)}) = \left(\frac{\partial \phi_1}{\partial s_j}\right)_s = s^{(k)}.$$

In order to get the Jacobian we use the approximation

$$\frac{\partial \phi_i}{\partial s_j} = (\phi_i(S_1, \ldots\ldots, S_j + \delta S_j, \ldots\ldots, S_4) - \phi_i(S_1, \ldots\ldots, S_4))\ /\ \delta S_j$$
$$i, j = 1, 2, 3, 4 \tag{23}$$

i.e. we have to solve the system (16) five times, once with the current values of the parameters δ_j and once with each of the four parameters perturbed in turn.

Note that the perturbations δS_j must be larger than the local truncation error allowed in the integration method used in solving the system; otherwise the truncation errors will dominate in (23).

In our case we take $\delta S_j = 10^{-3}$

(5) Set $k = k + 1$ and to step (2) and repeat the process until $\| S^{(k+1)} - S^{(k)} \| < \varepsilon$ (we take $\varepsilon = 10^{-6}$).

(6) In this case we integrate (16) forward with the full set of initial values and print the solution values at the required intermediate points.

(7) Repeat the process from (2) to (6) by increasing the value of η_f in small steps until we notice that no significant changes have occurred to the solution from one step to the next. Then we accept this value of η_f as our practical infinity.

In our case it is acceptable to take $\eta_f = 5$.

4. Results and discussion

Eqs. (11-14) with boundary conditions (15) are solved numerically using shooting method with Runge-Kutta Merson with automatic error control as an initial value solver. Numerical calculations are carried out for fluids having different Prandtl numbers of the second phase and constant Prandtl number of the first phase with various values of λ. Temperature profiles θ_1, θ_2 were obtained for $P_{r_1} = 0.72$, $P_{r_2} = 0.01$, 1, 3, 10 an 100 with λ ranging

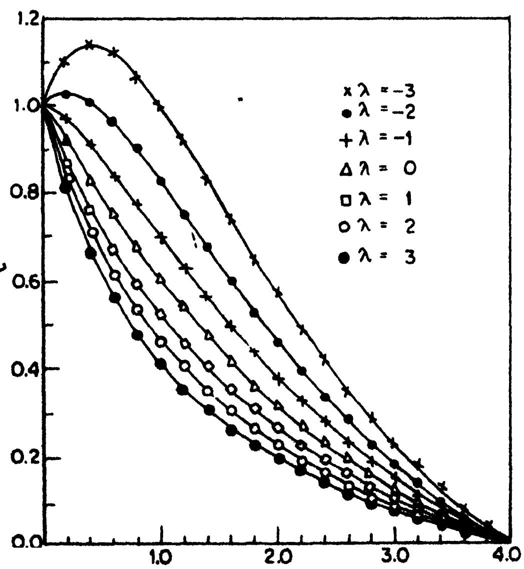

Figure 1. Temperature profiles for $P_{r_1} = 0.72$

between −3 and 3. Plots for the various parameter ombinations are shown in Figures 1 and 2. Both parameters are seen to have a significant effect on the temperature profiles θ_1 with λ

and θ_2 with P_{r_2}. The temperature θ_2 is slightly affected by the different values of λ, temperature θ_1 is not affected by the change of P_{r_2}. For given values of P_{r_1} and P_{r_2}, the

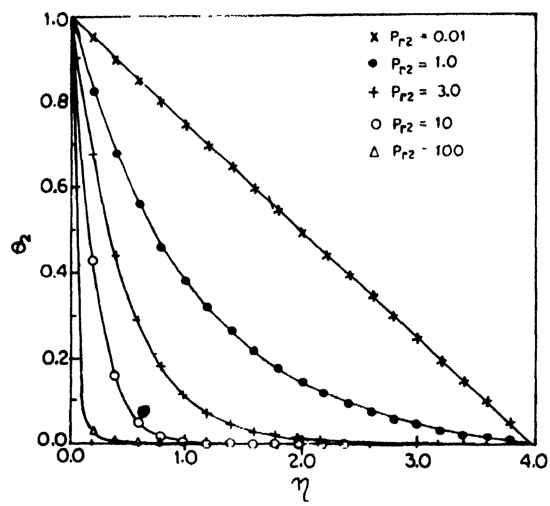

Figure 2. Temperature profiles for P_{r_2} values.

temperature increases as the temperature parameters decrease. Further thermal boundary layer thickness increases when λ decreases. For a given λ and P_{r_1}, the smaller the P_{r_2} the larger the thermal boundary layer thickness. To discuss the effect of λ, it is helpful to examine Tables 1, 2 which give a tabulation of the wall temperature gradient $\theta'_1(o)$, $\theta'_2(o)$. From

Table 1. Wall temperature gradient of the first phase θ'_1 (o) values for Prandtl number P_{r_1} = 0.72 and values of temperature parameter λ and Prandtl number of the second phase P_{r_2}.

λ/P_{r_2}	C.01	1	3	10	100
−3	0.7076	0.7076	0.7076	0.7076	0.7076
−2	0.2519	0.2519	0.2519	0.2519	.2519
−1	−.1185	−.1185	−.1185	−.1185	−.1185
0	−.4306	−.4306	−.4306	−.4306	−.4306
1	−.7009	−.7009	−.7009	−.7009	−.7009
2	−.94	−.94	−.94	−.94	−.94
3	−1.155	−1.155	−1.155	−1.155	−1.155

Table 1, for $\lambda > -1$ the wall temperature gradient is negative and heat flows from the continuous surface to the ambient. The magnitude of the wall temperature gradient increases with decreasing λ and no significant effect with different values of P_{r_2} is observed. For $\lambda < -1$, the sign of the temperature gradient changes and heat flows into the continuous surface from the ambient fluid. From Table 2, we find that the magnitude of the wall temperature gradient increases with decreasing λ. For $\lambda > -1$, the wall temperature gradient is negative and heat flows from the continuous surface to the ambient. When $\lambda = -1$, P_{r_2} = 3, 10 and 100, there is no heat transfer between the continuous surface and the ambient fluid.

Table 2. Wall temperature gradient of the second phase $\theta'(o)$ values for Prandtl number of the first phase (P_{r_1} = 0.72) and various values of temperature parameter λ and Prandtl number of the second phase P_{r_2}.

λ / P_{r_2}	0.01	1		10	100
−3	−.2354	1.967	69.86	−7.758	−5.364
−2	−.2413	0.696[illegible]	2.504	8.447	85.85
−1	−.2471	−.554[illegible]	$-.7263(10)^{-3}$	$-.1541(10)^{-6}$	$-1935(10)^{-6}$
0	−.2529	−.585[illegible]	−1.172	−2.378	−8.132
1	−.2587	−.997[illegible]	−1.961	−3.8600	−12.9
2	−.2645	−1.338[illegible]	−2.578	−4.9930	−16.5
3	−.2702	−1.631	−3.094	−5.9380	−19.49

For $\lambda = -2$, $P_{r_2} \geq 1$ and $\lambda = -3$, $P_{r_2} = 1, 3$ the sign of the temperature gradient changes and heat flows into the continuous surface from the ambient fluid. For $\lambda = -3$, $P_{r_2} = 0.01$, 10 and 100 the sign of the temperature gradient changes again and the heat is directed from the continuous surface to the free stream. Temperature distributions for the above λ and P_{r_2} values are found to have regions of temperature less than that of the ambient fluid. The velocity profiles F'_1, F'_2 and the shear stresses are not affected by the variation of λ and P_r.

Appendix

Nomenclature

C = Constant.
A = Constant.
F_1, F_2 = Dimensionless stream functions.
P_{r_1}, P_{r_2} = Prandtl number of the first and second fluids.
u_1, u_2 = Velocity components in the x-direction of the first and second fluids.
v_1, v_2 = Velocity components in the y-direction of the first and second fluids.
x = Coordinate measuring distance in the direction of surface motion.
y = Coordinate measuring distance normal to surface.
α_1, α_2 = Thermal diffusivities of the first and second fluids.
λ = Temperature parameter.
η = Dimensionless similarity variable
θ_1, θ_2 = Dimensionless temperatures of the first and second fluids.
γ_1, γ_2 = Kinematic viscosity of the first and second fluids.
ρ_1, ρ_2 = The densities of the first and second fluids
ψ_1, ψ_2 = Stream functions of the first and second fluids.
f_1, f_2 = The volume fractions of the first and second fluids.
K = The Rakhmatulin coefficient.
a = Constant.
h = Constant, interaction between two phases.

Subscripts

1, 2 = Correspond to the first and second fluids.
W = Continuous surface conditions.
∞ = Ambient conditions.

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