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Propagation and stability of ion cyclotron modes in a deuterium-hydrogen-oxygen fusion plasma

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Abstract : The stability of ion cyclotron modes, propagating nearly perpendicular to the ambient magnetic field, has been considered in a mirror confined fusion plasma that has deuterium as the majority species, hydrogen as the minority species and multiply ionised oxygen as the impurity constituent. Two modes can exist in the plasma ; one with a frequency slightly higher (the HF mode) than the deuterium gyro-frequency Ω_D and the other with a frequency slightly lower (the LF mode) than Ω_D. These modes have a wavelength larger than the ion gyro-radius r_{LD} (that is $k_\perp r_{LD} < 1$) ; the plasma itself being characterised by large ion plasma frequencies ($\omega_{pD}^2 \gg \Omega_D^2$). The necessary condition for an instability is that the loss-cone index $j > 3$. It is found that the growth rate is largest in a two-ion plasma ; the wavelength region of instability, however, decreases with increasing charge on the oxygen ion.

Keywords : Ion cyclotron mode, propagation and stability, fusion plasma dispersion relation, loss-cone velocity distribution.

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1. Introduction

The resonances between the natural motion of an ion and an electromagnetic (EM) wave at the same frequency provides an excellent alternative to the conventional method of ohmic heating of plasmas ; EM waves in the ion cyclotron range of frequencies (ICRF) has distinguished itself as a very successful one.

Fusion plasmas are often made up of deuterium and hydrogen (in a relatively low concentration). These plasmas, which are in the keV range, often contain impurities such as molybdenum (0.5 %), carbon (5 %) and multiply ionised oxygen (10 %) (Petrov 1977, Janzen 1981). These heavy impurity ions, even in small concentrations and low temperatures, can substantially effect the plasma confinement efficiency, the growth of instabilities and the propagation and amplification of EM waves. Even more interesting is the emergence of new resonances and cut-offs, the alteration of the wave group velocity and the strong coupling that may occur in the vicinity of the cross-over frequency.

Experimentally, as mentioned above, heating of fusion plasmas by waves in the ICRF has been very successful ; substantial increases in both ion and electron temperatures have been achieved in plasmas containing deuterium as the majority species and either hydrogen or helium as the minority constituent (Owens *et al* 1983 and Davies *et al* 1983). More recent relevant ICRF mirror experiments have used both the fast wave ($\omega \geqslant \Omega_i$; ω being the wave angular frequency and Ω_i the ion gyro-frequency) and the slow wave ($\omega \leqslant \Omega_i$) (Ichimaru *et al* 1988 and Golovato *et al* 1989).

This paper examines whether this successful method of heating could give rise to the reverse process in the form of instabilities and the influence of multiply ionised oxygen on them. This question of stability is very important since magnetic mirrors are subject to micro-instabilities such as the drift cyclotron loss-cone (DCLC) mode (Gary *et al* 1984) and the Alfven ion cyclotron (AIC) mode (Casper and Smith 1982) which produces fluctuations in the ICRF regime that is, with frequencies $\omega \approx \Omega_i$.

To pursue our objective we have derived a dispersion relation for the near perpendicular propagation of ion cyclotron (IC) modes with a frequency around the fundamental Ω_D of deuterium and a wavelength much larger than its ion Larmour radius ($k_\perp r_{LD} < 1$). The plasma is composed of deuterium (D), hydrogen (H) and multiply ionised oxygen (O) as the majority, minority and impurity constituent respectively. The solution of the dispersion relation yields two modes – a high frequency (HF) mode which starts at z_D $(=\omega/\Omega_D) \lessapprox 1$ and a low frequency (LF) mode which starts at a lower frequency and finally reaches a value of $z_D \lessapprox 1$. The evaluation of the growth rate reveals the instability to be restricted to certain wavelength regimes ; in a multi-ion component plasma the region of interaction (and hence the instability) is increased but extended to lower wavelengths. Also when the charge on the oxygen ion is increased, the region of growth decreases.

2. The dielectric tensor

We start with the wave equation

$$(c^2/\omega^2).[\underset{\sim}{k} \times (\underset{\sim}{k} \times \underset{\sim}{E})] + \underset{\approx}{K}.\underset{\sim}{E} = 0 \tag{1}$$

where c is the velocity of light and $\underset{\sim}{k}$ the wave vector which has components

$$\underset{\sim}{k} = (k_x = k_\perp,\ k_y = 0 \quad \text{and} \quad k_z = k_\parallel)$$

$\underset{\approx}{K}(\equiv K_{l,m}$; $l, m = x, y, z)$ is the hot plasma dielectric tensor while the symbols $\perp$ and $\parallel$ refer to the directions perpendicular and parallel to the magnetic field respectively.

We intend to study IC propagation in a mirror confined 3-ion plasma made up of deuterium (D), hydrogen (H) and multiply ionised oxygen (O). Mirror confined plasmas tend to have a loss-cone velocity distribution except when they are collisional or flow confined (Golovato et *al* 1989). We have therefore chosen this distribution to model all the four (3-ion and one electron) constituents of our plasma and it is given by

$$f_0 = \frac{1}{j!\pi^{3/2}W^{(2j+2)}U}(v_\perp)^{2j} \exp\left[-\left(\frac{v_\perp^2}{W^2}+\frac{v_\parallel^2}{U_2}\right)\right]. \tag{2}$$

In eq. (2) j is the loss-cone index while T and m denote the temperature and mass respectively. These are related to U and W as

$$U^2 = \frac{2T_\parallel}{m} \quad \text{and} \quad W^2 = \frac{2T_\perp}{m(j+1)}$$

The expressions for the elements of the dielectric tensor $\underset{\approx}{K}$, when the distribution (2) is used have been derived earlier (Chandu Venugopal 1983 ; here-in-after referred to as I) and is therefore given in an appendix, namely Appendix-A for completeness.

3. The approximation scheme

We use the dielectric tensor elements ($A-1$) to study the near perpendicular propagation ($k_\parallel \ll k_\perp$) of the EM-IC wave with a frequency around the fundamental harmonic of the ion gyro-frequency Ω_D of deuterium. The wave has a wavelength larger than the ion gyro-radius of deuterium ($k_\perp r_{LD} < 1$) and can have a small range of frequencies both above and below Ω_D in a plasma characterised by large ion plasma frequencies. We denote this deviation from the fundamental harmonic by a small parameter ϵ. In the MARS tandem mirror experiment, where the plasma was made up of deuterium and tritium ions, the ratio of the effective temperature T_i of the ions to T_e of the electrons was observed to be approximately 1.17 ; in their theoretical modelling of the observed loss-cone instabilities the effective temperatures of the two ion species were considered equal (Ho et *al* 1988). We thus make the simplifying assumption that the plasma is approximately temperature isotropic not only with respect to $T_\perp$ and $T_\parallel$ but also among the different species themselves. These assumptions regarding the wave and the plasma in which it propagates permits us the following ordering scheme (in terms of the parameter ϵ) :

$$\gamma_D = 1 - z_D^2 \sim \epsilon \;;\; \theta,\ I_{\perp D},\ I_{\parallel D} \quad \text{and} \quad \frac{1}{\omega_{pD}^2} \sim \epsilon\ T_{eD},\ T_{HD},\ T_{OD}$$

$$\text{and} \quad T_\perp/T_\parallel \sim 1 \quad \text{and} \quad m_e/m_D \sim \epsilon^2 \tag{3}$$

The definitions of the various parameters in eq. (3) are given in Appendix-A.

From the general definition of γ we can, on converting, the other gyro-frequencies to Ω_D, easily show that

$$\gamma_H = \frac{1}{4}\left[3+\gamma_D\right] \quad \text{and} \quad \gamma_O = \frac{c_0^2-64}{c_0^2}\left[1+\frac{64}{c_0^2-64}\gamma_D\right] \tag{4}$$

where c_0 is the charge on the oxygen ion. In the derivation of eq. (4) we have used the following simplifying relations namely,

$$m_D = 2m_H \text{ and } m_O = 8m_D. \tag{5}$$

Using the definition of $I_\perp$, namely eq. (A-8), and (5) we can easily show that

$$I_{\perp H} = \frac{1}{2} T_{\perp HD} I_{\perp D} \text{ and } I_{\perp O} = \frac{8}{c_0^2} T_{\perp OD} I_{\perp D} \text{ are of } \sim \epsilon. \tag{6}$$

4. The dispersion relation

The expressions for the various tensor elements $K_{l,m}$ in (A-1), using the ordering of eq. (3) and retaining terms of order ϵ^{-1}, 1 and ϵ have been derived earlier for a single ion (hydrogen) loss-cone plasma and can be adapted from them for the deuterium contribution. The hydrogen, oxygen and electron contributions can be derived in a similar manner and converted to the D terms using eqs. (4), (5) and (6).

In addition to extending our earlier work to a multi-ion component plasma we have, in this paper, also considered the contributions from the imaginary part of the function $E(t)$ defined in terms of the plasma dispersion function (A-9). The expression for this term is (Landau and Cuperman 1971 ; here-in-after referred to as II)

$$e_n = i \left[\frac{\pi}{8\theta^2 I_{\parallel}}\right]^{1/2} \exp\left[-\frac{(z-n)^2}{2\theta^2 I_{\parallel}}\right] \tag{7}$$

These contributions are, however, restricted only to the $n=1$ deuterium and $n=0$ electron terms since we are considering a resonant instability $(1-z_D^2 \sim \epsilon)$. They can be derived keeping in mind the ordering in eq. (3) and following the method used for an anisotropic Maxwellian plasma (II).

The expressions for the tensor elements for a single ion plasma considering only the real part of $E(t)$ were rather lengthy (I). The new ones are even longer since the real and imaginary parts of $E(t)$ (for deuterium and electrons) and the real part of $E(t)$ (for hydrogen and oxygen) now contribute. On substituting these tensor elements into the formula for the dispersion relation (I, II) we can, after a long simplification, arrive at the dispersion relation which we write as

$$Re\, D(\omega, k_{\parallel}, k_{\perp}) + i Im D(\omega, k_{\parallel}, k_{\perp}) = 0 \tag{8}$$

The expression for $Re\, D(\omega, k_{\parallel}, k_{\perp})$ is

$$A\gamma_D^2 - B\gamma_D + C - D = 0 \tag{9}$$

The coefficients A, B, C and D are given in Appendix-A where it is also shown that the D-term cannot contribute to eq. (9) giving our dispersion relation the final form

$$A\gamma_D^2 - B\gamma_D + C = 0 \tag{10}$$

The expression for $Im\,D(\omega, k_\perp, k_\parallel)$ is

$$ImD(\omega, k_\parallel, k_\perp) = -\gamma_D^2\left\{\left[B - \frac{\gamma_D}{2}\left(\frac{1}{2} + \frac{I_{\perp D}}{\beta_{\perp D}}\right) - \frac{2C}{\gamma_D}\right]e_{1D} + \frac{1}{\gamma_D}\left[2N_{eD}(1-A_e)I_{\perp e}\right]\tilde{e}_0\right\} \quad (11)$$

where $\tilde{e}_0 = -e_0$. This re-definition is necessary to make $\tilde{e}_0$ positive as it is intrinsically negative due to the factor $(I_{\parallel e})^{1/2}$ in (7) (II).

To solve eq. (8) (with the $Re\ D$ and $Im\ D$ being given by eqs. (10) and (11) respectively) we substitute

$z_D = 1 + a + ib$ with $a \ll 1$ and $b \ll a$ into it. Following the procedure for an anisotropic plasma (II) we get the final expression for the growth or damping rate b as

$$b = \frac{2a^2}{4Aa^2 - C}\left\{\left[(-4Aa^2 + C) + 2a^2\left(\frac{1}{2} + \frac{I_{\perp D}}{\beta_{\perp D}}\right)\right]\epsilon - \left[2N_{eD}\frac{T_{\perp e}}{T_{\parallel e}(j+1)}I_{\perp e}\right]\tilde{e}_0\right\} \quad (12)$$

As a check on relation (12) we note that it can be shown to reduce to the corresponding expression in (II) which was for an anisotropic Maxwellian plasma (this is a plasma for which the loss-cone index $j=0$).

5. Results

We plot relations (10) and (12) for typical fusion conditions namely $n = 4 \times 10^{12}$ cm^{-3}, $T_\perp = 1$ keV and $B_0 = 5$ kG. With these parameters $\beta_{\perp D} = 0.08046$. For simplicity we have set all temperature ratios equal to 1.0.

Figure 1 depicts the variation of z_D versus $k_\perp r_{LD}$. The dispersion diagrams consist of two modes—a low frequency (LF) mode which starts at $z_D < 1.0$ and finally reaches a value of $z_D \approx 1.0$ and a high frequency (HF) mode which starts at approximately this value and reaches higher frequencies. Plot 1(a) is for a plasma containing deuterium alone for $j=0$ (indicated by dotted lines and representing a Maxwellian plasma) and $j=4$. For $j=4$ the two modes coalesce at $k_\perp r_{LD} = 0.35$ resulting in a pair of complex conjugate roots and hence indicating an instability. On the other hand for $j=0$, the two modes are well separated and the plasma is stable. Plot 1(b) is for a plasma containing 10% of hydrogen. The two modes coalesce over a greater region of wavelength from $k_\perp r_{LD} = 0.35$ to 0.36. The same is true for plot 1(c) which has 5% of O^{8+}. However in this case the instability has moved to a higher region, from 0.37 to 0.38. And finally plot 1(d) is for a plasma containing deuterium, hydrogen (10%) and oxygen (5%). It resembles plot 1(c) in all aspects. We can thus conclude that the instability exists over a greater wavelength region in a multi-ion component plasma.

In order to explain physically the occurrence of the instability an expression for E^2/E^2_∞ was obtained and evaluated numerically. We found that, in general, the HF mode possesses a positive electrical energy and the LF mode a negative

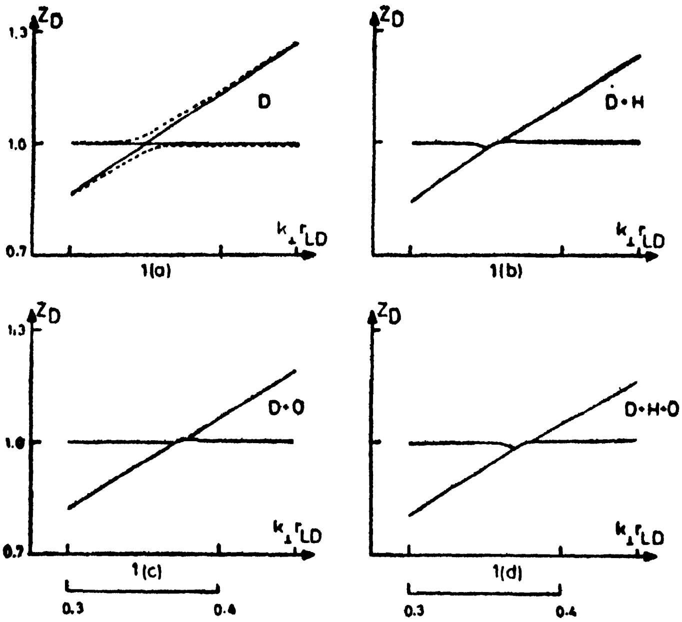

Figure 1. Plot of z_D versus $k_\perp r_{LD}$ for $\beta_{\perp D}$ =.08046. Plots 1(a) to 1(d) are for plasmas containing deuterium, deuterium+10% hydrogen, deuterium+5% O^{2+} and deuterium+10% hydrogen+5% O^{2+}. The dotted line in plot 1(a) is for $j=0$ and the solid lines in the plots for $j=4$.

one ; the instabilities can thus be interpreted as being due to an interaction of modes of opposite electrical energy (Hasegawa 1975).

Figure 2 is a plot of b, (the growth or damping rate) versus $k_\perp r_{LD}$ for $j=5$ and $\theta=0.013$. We have set $a=0.01$ ($z_D=1.0+a+ib$), so that the IC wave has a frequency slightly greater than Ω_D. We find the wave to be unstable over a small wavelength region ; the instability shifts towards higher $k_\perp r_{LD}$ in accordance with the dispersion diagrams of Figure 1. Also the maximum wave growth occurs in a plasma containing deuterium and doubly ionised oxygen.

A plot similar to Figure 2, but as a function of the charge on the oxygen ion for a given oxygen density, reveals that the growth rate decreases with increasing charge on the oxygen ion. This could be due to the increased Landau damping by the electrons which increases with $c_O(n_e=n_D+n_H+n_O\times c_O)$.

And finally, it should be noted that since eq. (12) contains only the second powers of "a" (that is, only a^2), our conclusions regarding the stability of the wave are also applicable to ion cyclotron modes with a frequency slightly less than $\Omega_D^-(z_D^-=1.0-a)$.

6. Discussion

We shall now discuss relations (10) and (12) to explain the results of the previous section. To simplify our discussion we consider a single ion plasma ; though our conclusions are valid for a multi-ion plasma as well. In a single ion plasma $A=1.0$ and the discriminant B^2-4AC can become negative leading to a pair of complex conjugate roots (and hence an instability) if and only if $j \geqslant 3$.

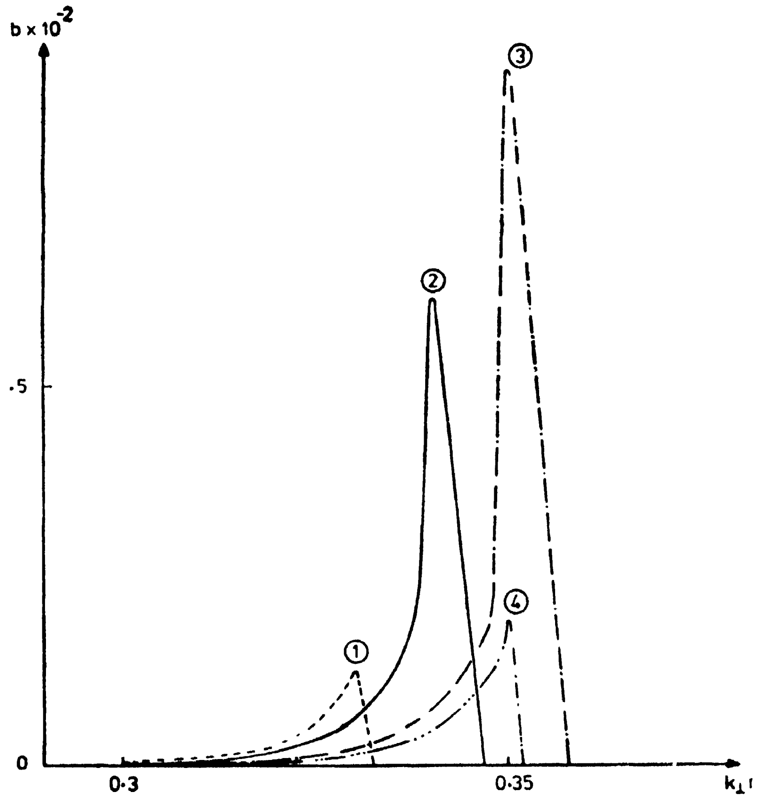

Figure 2. Plot of b versus $k_\perp r_{LD}$ for $j=5$, $\theta=.013$ and $\omega=1.01\ \Omega_D$. The numbers on the curves indicate the compositions of the plasmas in Figure 1.

As regards (12) let $j=0$ also. The term in front of the curly bracket is now positive definite. The expression within the curly bracket will be positive only if

$$2a^2\left(\frac{1}{2}+\frac{l_{\perp D}}{\beta_{\perp D}}\right)$$

dominates the other three negative terms. This is possible only if $(l_{\perp D}/\beta_{\perp D}) \gg 1.0$ a requirement which violates our assumption regarding its value (also a $\leqslant 1.0$), Thus the wave is damped for $j=0$ in agreement with the earlier conclusion for an anisotropic Maxwellian plasma (II). However, numerical computation is necessary to find out the exact region of instability. It may also be mentioned that for the case of extreme temperature anisotropy ($T_{\perp} \ll T_{\parallel}$) the degeneracy of the two modes disappear in a Maxwellian plasma and we have only one mode with $z \approx 1.0$ (II). The same should be true for the present case also.

7. Conclusions

We have, in this paper, studied the propagation and stability of ion cyclotron modes that has deuterium as the majority species, hydrogen as the minority species and multiply ionised oxygen as the impurity constituent. The necessary condition for an instability is that the loss-cone index $j \geqslant 3$. The wave growth is large in a two-ion plasma ; the region of growth however, decreases with increasing charge on the oxygen ion.

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Appendix A

The expressions for the elements of the dielectric tensor $\underset{\approx}{K}$ are

$$K_{xx} - 1 \qquad \frac{n^2}{\alpha^2} \quad I^j_p \qquad\qquad I^{j-1}_p$$

$$K_{xy} = \sum_s \frac{\omega_p^2}{z} C \sum_n \frac{in}{\alpha} \left\{ I^j_{\alpha p} \left[\frac{1-E}{n-z} - \frac{AE}{z} \right] - j \left[\frac{1-E}{n-z} - \frac{E}{z} \right] I^{j-1}_{\alpha p} \right\}$$

$$K_{yy} - 1 \qquad I^j_{\alpha\beta p} \qquad\qquad I^{j-1}_{\alpha\beta p}$$

$$K_{zx} \qquad \frac{n}{\theta\alpha^2} \quad I^j_p \qquad\qquad I^{j-1}_p$$

$$= \sum_s \frac{\omega_p^2}{z} C \sum_n \left\{ \left[E + \frac{A}{z}(n-z)E \right] - j \left[E + \frac{1}{z}(n-z)E \right] \right\}$$

$$K_{zy} \qquad \frac{i}{\theta\alpha} \quad I^j_{\alpha p} \qquad\qquad I^{j-1}_{\alpha p}$$

and

$$K_{zz} - 1 = \sum_s \frac{\omega_p^2}{z} C \sum_n \frac{1}{\theta^2\alpha^2} \left\{ I^j_p \left[\frac{nA}{z} + \frac{W^2}{U^2} \right] (z-n)E \right.$$

$$\left. - j \left[\frac{n}{z}(z-n)E \right] I^{j-1}_p \right\} \qquad \text{(A-1)}$$

In the above, '*s*' indicates a summation over the constituents ; the definitions of the other parameters, with the subscript '*s*' suppressed, are :

$$\Omega=\pm\frac{eB_0}{mc},\ \omega_p^2=\frac{4\pi ne^2}{m},\ \overline{\omega_p^2}=\frac{\omega_p^2}{\Omega^2},\ z=\frac{\omega}{\Omega},\ A=1-\frac{W^2}{U^2},$$

$$\theta=\frac{k_\parallel}{k_\perp},\ C=4[j!W^{2j+2}]^{-1} \text{ and } i=(-1)^{1/2} \tag{A-2}$$

Also

$$I_p^{(j)}=\frac{1}{W^2}L^jI(\alpha,\alpha),\ I_p^{(j-1)}=L^{j-1}I(\alpha,\alpha) \tag{A-3}$$

$$I_{\alpha p}^{(j)}=\frac{1}{W^2}L^j\left[\frac{d}{2d\alpha}I(\alpha,\alpha)\right],\ I_{\alpha p}^{(j-1)}=L^{j-1}\left[\frac{d}{2d\alpha}I(\alpha,\alpha)\right] \tag{A-4}$$

$$I_{\alpha\beta p}^{(j)}=\frac{1}{W^2}L^j\left[\frac{d^2}{d\alpha d\beta}I(\alpha,\beta)\right],\ I_{\alpha\beta p}^{(j-1)}=L^{j-1}\left[\frac{d^2}{d\alpha d\beta}I(\alpha,\beta)\right] \tag{A-5}$$

where

$$p=\frac{1}{W^2}$$

and

$$L^{j,(j-1)}=(-1)^{j,(j-1)}\frac{d^{j,(j-1)}}{dp^{j,(j-1)}} \tag{A-6}$$

The $dv_\perp$ integrations give rise to the functions I ; this in its most general form is given by

$$I=I(\alpha,\beta)=\frac{W^2}{2}\exp\left[-\frac{(\alpha^2+\beta^2)W^2}{4}\right]I_n\left(\frac{\alpha\beta W^2}{2}\right) \tag{A-7}$$

I_n is the modified Bessel function with an argument

$$l_\perp'=\alpha^2W^2=\frac{2}{(j+1)}\,l_\perp$$

where

$$l_\perp=\frac{k_\perp^2T_\perp}{\Omega^2m} \tag{A-8}$$

The derivatives used in (A-3), (A-4) and (A-5) can be obtained by appropriately manipulating relation (A-7) (*I*).

The function E(t) is from the $dv_\parallel$ integration and is defined by (Landau and Cuperman 1971)

$$E(t)=-\frac{1}{2}Z'(t/(2)^{1/2}) \tag{A-9}$$

where Z′ is the derivative of the plasma dispersion function. Its argument 't' is defined as

$$t=\frac{(z-n)}{\theta(l_{s})^{1/2}}$$

where

$$l_{s}=\frac{k_{\perp}^{2}T_{s}}{\Omega^{2}m} \qquad \text{(A-10)}$$

For ease of presentation, we also define the following density and temperature ratios, namely

$$N_{HD}=\frac{N_{H}}{N_{D}},\ N_{OD}=\frac{N_{O}}{N_{D}},\ N_{eD}=\frac{N_{e}}{N_{D}}$$

and

$$T_{\perp,HD}=\frac{T_{\perp H}}{T_{\perp D}},\ T_{\perp,OD}=\frac{T_{\perp O}}{T_{\perp D}},\ T_{\perp eD}=\frac{T_{\perp e}}{T_{\perp D}} \qquad \text{(A-11)}$$

The expressions for the coefficients A, B, C and D in our dispersion relation

$$A\gamma_{D}^{2}-B\gamma_{D}+C-D=0 \qquad \text{(A-12)}$$

are

$$A=\frac{2}{3}N_{HD}\left[\frac{4}{3}-4N_{eD}+2N_{HD}+4N_{OD}\frac{c_{0}^{2}(c_{0}-4)}{c_{0}^{2}-64}+\frac{l_{\perp D}}{\beta_{\perp D}}\right]$$

$$+N_{OD}\frac{c_{0}^{2}}{c_{0}^{2}-64}\left[N_{OD}c_{0}^{2}+8\frac{l_{\perp D}}{\beta_{\perp D}}+8-2N_{eD}c_{0}\right]$$

$$+8N_{OD}\frac{c_{0}^{2}}{(c_{0}+8)^{2}}+N_{eD}^{2}$$

$$B=1+\frac{2}{3}N_{HD}+16N_{OD}\frac{c_{0}}{(c_{0}+8)}-\frac{l_{\perp D}}{\beta_{\perp D}}-\delta,$$

with

$$\delta=\left[4N_{eD}+2\left(N_{eD}T_{\perp eD}+\frac{8}{15}N_{HD}T_{\perp HD}+N_{OD}T_{\perp OD}-\frac{2}{3}\right)\right.$$

$$-\frac{8}{3}N_{HD}-4N_{OD}\frac{c_{0}^{2}}{(c_{0}+8)}-\frac{24c_{0}+64}{(c_{0}+4)(c_{0}+8)}N_{OD}T_{\perp OD}$$

$$\left.-\frac{l_{\perp D}}{\beta_{\perp D}}\right]l_{\perp D}-\left(\frac{2}{\omega_{pD}^{2}}\right).$$

$$C=\frac{l_{\perp D}^{2}}{4}\frac{(j-2)}{(j+1)}$$

and

$$D = 4\frac{\theta^2 I_{\parallel D}}{\gamma_D}\left(1+\frac{2}{3}N_{HD}+16N_{OD}\frac{c_0}{(c_0+8)}-\frac{I_{\perp D}}{\beta_{\perp D}}\right)$$

In the above

$$\beta_{\perp D} = \frac{4\pi N_D T_{\perp D}}{B_0^2} \text{ with } \frac{I_{\perp D}}{\beta_{\perp D}} \sim 1 \cdot N_{HD} + N_0$$

As a check on our result we note that for $N_{HD}=N_{OD}=0$ $(N_{eD}=1.0)$, (A-12) reduces to our earlier dispersion relation (I). Examining it we find that for the first three terms to be of the order ϵ^2, we need to set

$$\left(1+\frac{2}{3}N_{HD}+16N_{OD}\frac{}{(c_0+8)}\ \frac{I_{\perp D}}{\beta_{\perp D}}\right) \ldots$$

Unfortunately the D-term is now of order ϵ^3 and thus does not contribute to (A-12), giving our dispersion relation the final form $A\gamma_D - B\gamma_D + C = 0$.

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