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Multiplicity distribution at high energy *pp* collision and semi inclusive scaling

S. R. Roy

Department of Physics, Jadavpur University, Calcutta 700032

D. P. Bhattacharyya and D. Basu

Indian Association for the Cultivation of Science, Calcutta 700032

AND

R. K. Roychoudhury

Department of Mathematics, Moulana Azad College, Calcutta 700013

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Recent experiments (Ammosov *et al* (1972), Chapman *et al* (1972), Charlton *et al* (1972) and Dao *et al* (1972)) on high energy *pp* collision reveals that the charged particle multiplicity distribution obeys the scaling law proposed by Koba Nielsen & Olesen (1972) at high energies. In the present work we have proposed a simple gamma distribution which incorporates the KNO scaling and explains adequately the data in the momentum range 50–303 GeV/c. Following Koba, Nielsen & Olesen (1972), we write

$$\sigma_{n_c}(s) = \frac{1}{<n>}\sigma_{in}(s)\psi\left(\frac{n}{<n>}\right), \qquad \dots \quad (1)$$

where $n = \frac{1}{2}n_c$, $\sigma_{in}(s) = \sum_n \sigma_{n_c}(s)$ and $\sigma_{n_c}(s)$ is the partial cross-section for n_c charged prongs at a centre of mass energy $\sqrt{s}$. The function ψ depends on s through the reduced multiplicity $z = n/<n>$. By using the following condition

$$\frac{1}{\sigma_{in}}\sum_n \sigma_{2n}(s) = \frac{1}{<n>}\frac{1}{\sigma_{in}}\sum_n n\sigma_{2n} = 1, \qquad \dots \quad (2)$$

we get the normalization condition on $\psi(n/<n>)$ as

$$\sum_n \frac{1}{<n>}\psi\left(\frac{n}{<n>}\right) = \sum \frac{n}{<n>^2}\psi\left(\frac{n}{< >}\right) = 1. \qquad \dots \quad (3)$$

If we replace the actual discrete spectrum of $z = n/<n>$ by a continuous spectrum (0 to ∞), we can write the conditions (3) as

$$\int_0^\infty \psi(z)dz = \int_0^\infty z\psi(z)dz = 1. \qquad \dots \quad (4)$$

Our proposed gamma distribution which satisfies the conditions is of the following form

$$\psi(z) = \frac{m^m}{\Gamma(m)} z^{m-1} e^{-mz}, \qquad \dots (5)$$

where m is a free parameter. Eq. (5) for $\psi(z)$ can also be obtained starting from a discrete distribution function as given below,

$$\psi\left(\frac{n}{<n>}\right) = An^{m-1}e^{-\alpha n}. \qquad \dots (6)$$

Using the normalization condition (3) we can write A and α in terms of m and $<n>$. For large $<n>$ i.e., at large s we get

$$A \sim \frac{m^m}{<n>^{m-1}\Gamma(m)} \quad \text{and} \quad \alpha \sim m/<n>,$$

which reproduces eq. (5).

The normalized moments of order k is defined by the relation

$$C_k = \frac{<n_c^k>}{<n_c>^k} = \sum_{n=1}^{n=\infty} \frac{\sigma_{2n}}{\sigma_{in}} \left(\frac{n}{<n>}\right)^k. \qquad \dots (7)$$

Our distribution (5) gives

$$C_k = \frac{\Gamma(m+k)}{m^k\Gamma(m)}. \qquad \dots (8)$$

Experimentally the ratio $<n_c>/(<n_c^2>-<n_c>^2)^{\frac{1}{2}}$ is found to be constant and approximately equals to 2 at high energies. From eq. (8) this ratio comes out to be $\sqrt{m}$. So we put $m = 4$ in eq. (8) and get

$$C_k = \frac{\Gamma(4+k)}{4^k\Gamma(4)}. \qquad \dots (9)$$

The values of C_k calculated from eq. (9) agree very well with the experimental values for $k \leqslant 3$. For $k \geqslant 4$ there are some discrepancies. This is because the experimentally measured moments C_k is actually given by

$$C_k = \sum_{n=1}^{\frac{1}{2}N_{max}} \frac{\sigma_{2n}}{\sigma_{in}} \left(\frac{n}{<n>}\right)^k, \qquad \dots (10)$$

where N_{max} is the highest prong number for which the partial cross section is measured. If we use eq. (10) instead of eq. (8), we get

$$C_k = \frac{\Gamma(4+k, x)}{4^k\Gamma(4)} \qquad \dots (11)$$

where $\Gamma(a, x)$ is the incomplete gamma function. The normalized moments calculated from eq. (11) agrees remarkably well with experimental values of Slattery (1972) and Dao *et al* (1973). The Mueller (1971) correlation parameter f_2 is defined by

$$f_2 = < n_c^2 > - < n_c >^2 - < n_c >.$$

Our distribution yields

$$f_2 = (1/4) < n_c > (< n_c > -4). \qquad \dots (12)$$

In figure 1 we have plotted

$$\tfrac{1}{2} < n_c > \frac{\sigma_{n_c}(s)}{\sigma_{in}} = \psi(z)$$

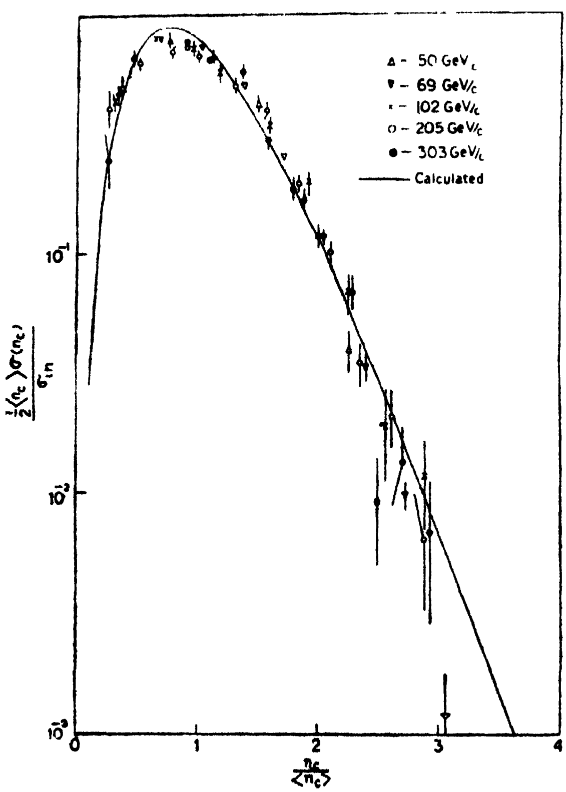

Fig. 1. $\psi(z)$ plotted as a function of z. Experimental results : Δ, ∇, Ammosov *et al* (1972), × Chapman *et al* (1972), O Charlton *et al* (1972), Dao *et al* (1972).

against z, for different moments from 50 to 303 GeV/c. The multiplicity scaling is found to work very well and is evident by the fact that the data corresponding to different momentum fall on the same curve given by eq. (5). Figure 2 represents the plot of f_2 against s along with the experimental results. Increase of f_2 with energy clearly reveals the broadening of the distribution curve at higher

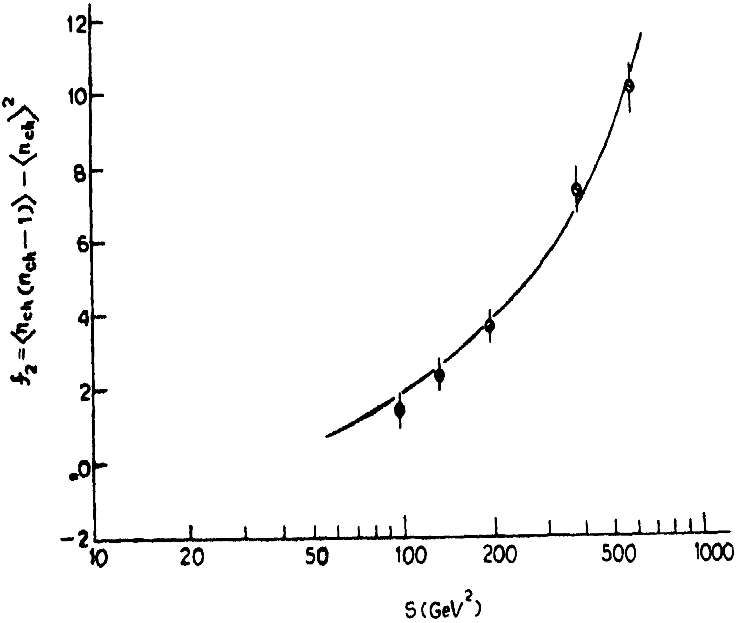

Fig. 2. The solid curve represents f_2 as a function of s. Data taken from Slattery (1973).

energies, a property which cannot be explained by a single Poisson distribution (Chapman *et al* 1972). The merit of our proposed distribution function is that it is simple and at the same time capable of explaining the detailed nature of the experimental multiplicity distribution.

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