Indian J. Phys. **48**, 921–924 (1974)

Triton binding energy for local square well potential

G. PURKAYASTHA AND N. C. SIL

Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Calcutta 700032

(*Received* 17 *April* 1974)

The applicability of a new method of approximating the two-body t-matrix in separable form for general local short-range interaction, as suggested earlier, has been tested in the calculation of triton binding energy with Faddeev formalism. The square well potential is considered since convergent theoretical binding energy values by Sturmian expansion method is available for this potential. Using a two term and three term expansion, the binding energy values of triton have been evaluated taking the triplet plus singlet states of deuteron. It is observed that the binding energy values for the second case is very close to the result obtained by Kharchenko and Storozhenko for the same potential using the Sturmian expansion method. The present method has the advantage over the Sturmian method that it is applicable to more general type short-range potentials and to higher partial waves.

1. INTRODUCTION

The formal theory of scattering and reactions for a system of three particles, in the case of two particle forces with a finite radius of action, has been developed by Faddeev (1961). In this theory it is possible to obtain a set of two-dimensional coupled integral equations admitting of a unique solution. These two-dimensional integral equations are to be solved numerically. The two-dimensional integral equations of the three nucleon problem can be reduced to one-dimensional equations by using a separable representation for the two-particle scattering amplitude and this reduction simplifies the calculation enormously. Several methods have been proposed for approximating the two-particle scattering amplitude by separable terms. Some authors (Mitra & Vasin 1963, Fairlie 1960, Weinberg 1963) have used non-local separable potentials. Kharchenko & Storozhenko (1969) have investigated neutron-deuteron problem with local square well and Hulthen interactions using a separable expansion of the two particle scattering amplitude in Sturmian function representation. A separable representation for the two-body scattering amplitude for a local Hulthen potential has been used by the present authors (Purkayastha *et al* 1971) in their calculatons of the binding energy of the triton and the neutron deuteron doublet and quartet scattering lengths, in which the physical deuteron states have been chosen as the expansion bases. The authors have also calculated triton binding energy for local Yukawa potentials, where, for the separable representation of the two-particle amplitudes, the Sturmian eigenfunctions for a suitable s-wave Hulthen potential have been used as expansion bases (Purkayastha *et al* 1972), the authors have proposed another separable representation of the partial wave two-body t-matrix for short-range central local potentials (paper I) (Purkayastha *et al* 1971) based on the separable approximation of the integral representation of the interaction potential in momentum space by suitable quadrature formula. In the present paper, we have considered an application of the separable approximation discussed in paper I, to a short-range square well potential. Our method has the advantage over the Sturmian method that it is applicable to more general short-range potentials and higher partial waves. Using a two term and three term separable expansion for the s-wave two-body potential, we have calculated the binding energy values of triton taking the triplet plus singlet states of deuteron, with the help of Faddeev formalism.

2. Theory

For identical particles and binding energy problems the Faddeev equations, for s-wave part of the two-body interactions are of the form

$$\psi(p,q,\beta;s) = -\frac{4\pi}{q\sqrt{3}} \sum_{\beta'} F(\beta|\beta') \int_0^\infty dq'^2 \int_L^U dp'^2 \frac{t_\beta(p,\bar{p};s-q^2)}{p'^2+q'^2-s} \psi(p',q',\beta';s), \tag{1}$$

where

$$\bar{p}^2 = p'^2+q'^2-q^2$$
$$U(q,q') = (2q+q')^2/3.$$
$$L(q,q') = (2q-q')^2/3.$$

$F(\beta|\beta')$ is the spin-isospin recoupling coefficient. The abbreviated notation β stands for the spin-isospin factor, s is the energy of three body system p and q describe the magnitudes of the relative momenta; p stands for relative momentum of two particles whereas q stands for the momentum of the third particle relative to the other two particles. Now the integral transform $V(p,p')$ of the potential in momentum space is represented by a series, separable in p and p' (paper I)

$$V_\beta(p,p') = \sum_n g_n^\beta(p) g_n^\beta(p'). \quad \ldots \quad (2)$$

Utilizing the expression (2) for $V_\beta(p,p')$ in two-body Lippmann-Schwinger equation,

$$t_\beta(p,p';s) = V_\beta(p,p') - 4\pi \int_0^\infty \frac{V_\beta(p,p'')t_\beta(p'',p';s)}{p''^2-s} dp',$$

we have t_β of the following form

$$t_\beta(p,\bar{p};s-q^2) = \sum_n g_n^\beta(p) \chi_n^\beta(\bar{p},s-q^2). \quad \ldots \quad (3)$$

χ_n can be obtained easily because of the degenerate form of the kernel. From eqs. (1) and (3) it is evident that the p-dependence of $\psi(p, q, \beta; s)$ can be written explicitly as

$$\psi(p, q, \beta; s) = \sum_m g_m{}^\beta(p) h_m{}^\beta(q, \beta; s). \tag{4}$$

Substituting expressions (3) and (4) respectively for $t_\beta(p, \bar{p}; s-q^2)$ and $\psi(p, q, \beta; s)$ in eq. (1) and equating the coefficients of $g_i{}^\beta$ we get a set of coupled integral equation for $h_i{}^\beta$.

$$h_i{}^\beta(q, \beta; s) = -\frac{4\pi}{q\sqrt{3}} \sum_{\beta'} \sum_j F(\beta|\beta') \int_0^\infty dq'^2 \int_L^U dp'^2 \frac{\chi_i{}^\beta(p; s-q^2)}{p'^2+q'^2-s}$$

$$\times g_j{}^{\beta'}(p') h_j{}^{\beta'}(q', \beta'; s). \tag{5}$$

3. Results and Discussions

We have carried out our calculations for the triton binding energy with two sets of parameters of the square well potential. These parameters—depth and range, taken from the paper of Kharchenko and Storozhenko (1969), are as follows

Set I			
	Triplet	Depth	$= 35{\cdot}2765$ Mev
		Range	$= 2{\cdot}043 \times 10^{-13}$ cm
	Singlet	Depth	$= 14{\cdot}0746$ Mev
		Range	$= 2{\cdot}586 \times 10^{-13}$ cm
Set II			
	Triplet	Depth	$= 33{\cdot}8882$ Mev
		Range	$= 2{\cdot}093 \times 10^{-13}$ cm
	Singlet	Depth	$= 15{\cdot}6222$ Mev
		Range	$= 2{\cdot}457 \times 10^{-13}$ cm

The g^β's and χ^β's have been computed utilizing expression (2) and (3) in the two-body Lippmann-Schwinger equation. In our calculation we have used expansions with two terms and three terms for $V_\beta(p, p')$ in the expression (3). For triplet and singlet states of deuteron, β takes two values and thus we get two sets of coupled integral equations containing four and six equations for the above two cases. The g^β's and χ^β's are stored as sub-routine to be used as input in the Faddeev equations.

In course of our calculations we come across integrals of the type

$$\int_r^U \frac{\sin \bar{p}a^\beta \sin p'a^\beta}{p^2+q'^2-s}\, dp'$$

Such integrations have been carried out with a Gaussian quadrature formula with suitable point distributions. We have used a mesh point distribution which depends on the variable lower and upper limits of integration. As the function present in the integrand are fluctuating sine functions, special care has been taken for large values of p, p' and the convergence of the integrals has been tested by increasing the number of quadrature points. In our case, 40 Gaussian points have been found to be adequate for convergent results. The resulting one-dimensional integral equations have been recast into matrix equation with the help of Gauss-Legendre quadrature method and the triton binding energy (E_T) has been evaluated by searching for the pole of the corresponding inverted matrix. Here also the convergence has been tested by increasing the gaussian quadrature points. In table 1, we have enlisted our results for the triton binding energy with triplet plus singlet states of deuteron together with the corresponding theoretical findings of Kharchenko & Storozhenko (1969) for the same. From the table it can be inferred that our result for three terms in the expression (2) is very near the convergent result of Kharchenko and Storozhenko. So our approximation for the two-body t-matrix can be applied to three body binding energy problem with good results. We have worked with square well potential because convergent results are available for this potential, our method has the advantage that it is applicable to more general type of short-range potentials, some of which are discussed in paper I, and to higher partial wave.

Table 1. Values of the triton binding energy

	Our calculation		Kharchenko and Storozhenkos' calculation
	with two terms	with three terms	
Set I	10.64	9.66	9.20
Set II	10.92	9.97	9.53

References

Faddeev L. D. 1961 *Sov. Phys. JETP* **12**, 1914
Fairlie D. S. 1960 *Proc. Cambridge Phil. Soc.* **56**, 182.
Kharchenko V. F. & Storozhenko S. A. 1969 *Nucl. Phys.* **A137**, 437.
Mitra A. N. & Vasin V. S. 1963 *Phys. Rev.* **131**, 1265.
Purkayastha G., Banerjee S. N. & Sil N. C. 1971 *Nucl. Phys.* **A167**, 376.
Purkayastha G., Banerjee S. N. & Sil N. C. 1972 *Nucl. Phys.* **A182**, 443.
Purkayastha G., Banerjee S. N. & Sil N. C. 1972 *J. Phys.* **A5**, 1495.
Weinberg S. 1963 *Phys. Rev.* **130**, 776.