Indian J. Phys. 48, 509-519 (1974)

Second quantization and interaction of electromagnetic fields for non-zero mass system in angular momentum basis : II

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(*Received* 14 *June* 1973, *revised* 24 *September* 1973)

The study of interaction and second quantization of electromagnetic fields for non-zero mass system has been undertaken in angular momentum basis and the selection rules for emission and absorption of a massive particle are derived. Reducton of real transverse electromagnetic vector potential for non-zero mass system has been derived in terms of irreducible representations of proper, ortho-chronous, inhomogeneous Lorentz group in angular momentum basis

1. INTRODUCTION

In the present paper we have undertaken the study of interaction of electromagnetic fields for non-zero mass system in angular momentum basis. For this purpose the transverse reduced expansion of real electromagnetic vector potential for non-zero mass system in linear momentum basis derived by Parkash et al (1974) (henceforth to be referred to as I) has been transformed to that in angular momentum basis in terms of vector spherical harmonics. Replacing the amplitude corresponding to particle wave-function and their complex conjugate in this reduced expansion in angular momentum basis by annihilation and creation operators respectively, the second quantization of real electromagnetic vector potential has been carried out in angular momentum basis and the commutation rules for these operators have been derived. Using the second quantized reduced expansion of real electromagnetic vector potential, the expression for field-Hamiltonian is derived in terms of particle number operators quanta in angular momentum basis, which correspond to circularly polarized field states.

Second quantized relativistic reduced expansions of purely transverse real electromagnetic vector potential operator in angular momentum basis has been used to derive the transition probability for emission and absorption of a massive particle in the interaction of massive electromagnetic fields with an atomic system and it has been shown that the probability of spontaneous emission in this case also, like that in linear momentum basis for non-zero mass system given in I and that in linear and angular momentum basis for zero mass system (Rajput 1970, 1971) is non-vanishing. Moreover it is shown that the probability of transition in both the cases (emission and absorption) is proportional to the magnitude of linear momentum of the field. The selection rules for the emission and absorption of a particle in this interaction are similar to those derived in the interaction of electromagnetic field for zero-mass system with an atom (Rajput 1971).

2. Reduced Expansion of Electromagnetic Vector Potential in Linear Momentum Basis

Reduced expansion of real transverse electromagnetic vector potential derived in I for non-zero mass system in linear momentum basis may also be written as follows for the general case

$$\boldsymbol{A}(x) = \boldsymbol{A}^{+}(x) + \boldsymbol{A}^{+*}(x), \quad \ldots (1)$$

where

$$\boldsymbol{A}^{+}(x) = \frac{1}{4\pi^{3/2}} \int \frac{d\boldsymbol{p}}{\omega(p)} f(\boldsymbol{p}) \exp [i\{\boldsymbol{p}.\boldsymbol{x} - \omega(p)t\}]$$

$$= \frac{1}{4\pi^{3/2}} \int \frac{d\mathrm{p}}{\omega(p)} [\boldsymbol{\epsilon}_1 f(\boldsymbol{p}, 1) + \boldsymbol{\epsilon}_2 f(\boldsymbol{p}, 2) + \boldsymbol{\epsilon}_3 f(\boldsymbol{p}, 3)] \exp [i\{\boldsymbol{p}.\boldsymbol{x} - \omega(p)t\}], \quad \ldots (2)$$

where $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$, $\boldsymbol{\epsilon}_3$ are the unit vectors along first, second, and third axes respectively. Let us define now the functions

$$f(\boldsymbol{p}, \gamma) = \frac{1}{\sqrt{2}} [f(\boldsymbol{p}, 1) + i\gamma f(\boldsymbol{p}, 2)] \quad \ldots (3)$$

where $\gamma = \pm 1$

and

$$f(\boldsymbol{p}, 3) = f(\boldsymbol{p}, 0), \quad \ldots (4)$$

and the corresponding unit vectors $\boldsymbol{\epsilon}_{+1}$, $\boldsymbol{\epsilon}_{-1}$ and $\boldsymbol{\epsilon}_0$ as

$$\boldsymbol{\epsilon}_{+1} = \frac{\boldsymbol{\epsilon}_1 - i\boldsymbol{\epsilon}_2}{\sqrt{2}}$$

$$\boldsymbol{\epsilon}_{-1} = \frac{\boldsymbol{\epsilon}_1 + i\boldsymbol{\epsilon}_2}{\sqrt{2}}$$

$$\boldsymbol{\epsilon}_0 = \boldsymbol{\epsilon}_3 . \quad \ldots (5)$$

Substituting eqs. (3), (4) and (5) in eq. (2) we get

$$\boldsymbol{A}^{+}(x) = \frac{1}{4\pi^{3/2}} \int \frac{d\boldsymbol{p}}{\omega(p)} [\boldsymbol{\epsilon}_{+1} f(\boldsymbol{p}, +1) + \boldsymbol{\epsilon}_{-1} f(\boldsymbol{p}, -1) + \boldsymbol{\epsilon}_0 f(\boldsymbol{p}, 0)]$$

$$\times \exp[i\{\boldsymbol{p}.\boldsymbol{x} - \omega(p)t\}]. \quad \ldots (6)$$

3. Reduction in Angular Momentum Basis

In angular momentum basis a wave function for non-zero mass system depends on the energy E, total angular momentum quantum number j, quantum number m of j (z—component of angular momentum operator), and on the variable α (the absolute value α of which is identified with the spin of the particle). Thus the wave-function in this case is represented as $F(E, j, m, \alpha)$ which is related to the components of corresponding wave-function in linear momentum basis by the following eq. (Moses 1967b)

$$f(p, \gamma) = \frac{1}{p}\left(\frac{4\pi}{3}\right)^{\frac{1}{2}} \sum_{\alpha=-1}^{1} \sum_{j=1}^{\infty} \sum_{m=-j}^{j} (-i)^{m-\gamma}(-1)^{1-\alpha}$$

$$X\, Y_j^{m,\alpha}(\theta, \phi) Y^{\gamma}{}_1{}^{,\alpha *}(\theta, \phi) F(E, j, m, \alpha). \qquad \ldots \quad (7)$$

where $\gamma = +1,\ 0,\ -1$, designates the components of the wavefunction, and $Y_j^{m,\alpha}(\theta, \phi)$ and $Y_1^{\gamma,\alpha *}(\phi, \theta)$ are the generalized spherical harmonics of θ and ϕ which are the polar angles of vector p and vary from 0 to π and from 0 to 2π respectively, $\boldsymbol{p} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$, $p = (E^2 - \mu^2)^{\frac{1}{2}}$. The well known expansion for $\exp(i\boldsymbol{p}.\boldsymbol{x})$ in this basis is given by

$$\exp(i\boldsymbol{p}.\boldsymbol{x}) = 4\pi \sum_{k=1}^{\infty} \sum_{m'=-k}^{k} (i)^k J_k[(E^2-\mu^2)^{\frac{1}{2}} r]$$

$$\times\, Y_k^{m',o}(\hat{\theta}, \hat{\phi})\, Y_k^{m',o*}(\hat{\theta}, \hat{\phi}), \qquad \ldots \quad (8)$$

where $r = |\boldsymbol{x}|$, $J_k[(E^2-\mu^2)^{\frac{1}{2}} r]$ is the spherical Bessel's function of order k, and $\hat{\theta}$ and $\hat{\phi}$ are the polar angles which describe the direction of $\boldsymbol{x}$.

Substituting eqs. (8) and (7) in eq. (6), we get

$$A^{\prime}(x, \gamma) = \frac{2}{3} \sum_{\alpha=-1}^{1} \sum_{j=\alpha}^{\infty} \sum_{m=-j}^{j} \sum_{k=1}^{\infty} \sum_{m'=-k}^{k} (i)^{k-m+\gamma+2-2\alpha}$$

$$\times \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \int dE\ J_k[(E^2-\mu^2)^{\frac{1}{2}} r] Y_k^{m',o}(\hat{\theta}, \hat{\phi})$$

$$\times\ Y_j^{m,\alpha}(\theta, \phi)\ Y_1^{\gamma,\alpha *}(\theta, \phi)\ Y_k^{m',o*}(\theta, \phi)$$

$$\times\ F(E, j, m, \alpha) \exp(-iEt). \qquad \ldots \quad (9)$$

This reduced expansion in angular momentum basis is used for calculating the field Hamiltonian in the next section. However, for the compact form of reduced expansion which may be conveniently used for the study of interaction, the following properties of generalized spherical harmonics are used (Moses 1967);

$$Y_1^{\gamma,\alpha *}(\theta, \phi) Y_k^{m',o*}(\theta, \phi) = \sum_{J=|k-1|}^{|k+1|} \left(\frac{3}{4\pi}\frac{2k+1}{2J+1}\right)^{\frac{1}{2}}$$

$$\times (k, m', 1, \gamma \,|\, k, 1, J, m'+\gamma)(k, o, 1, \alpha \,|\, k, 1, J, \alpha)$$

$$\times\, Y_J^{m'+\gamma,\alpha *}(\theta, \phi), \qquad \ldots \quad (10)$$

which may be readily derived from the well-known relation

$$\frac{1}{V}\int_{n} ei(\boldsymbol{p}-\boldsymbol{p}').\mathrm{x})d\boldsymbol{x} = \delta(\boldsymbol{p}-\boldsymbol{p}'),$$

and

$$\frac{1}{(2\pi)^3}\int\int\int (E^2-\mu^2)^{\frac{1}{2}} dE\, E \sin\theta\, d\theta d\phi \rightarrow \lim_{V\rightarrow\infty}\frac{1}{V}\sum_{E} \qquad \ldots \quad (22)$$

which is angular momentum analogue of the similar approximation

$$\frac{1}{(2\pi)^3}\int d\boldsymbol{p} = \lim_{V\rightarrow\infty}\sum_{p} \text{ in linear momentum basis.}$$

Similarly the volume integral of second term of $H^{+}(\boldsymbol{x})$ given by eq. (19) is obtained as

$$\hat{H}_2{}^{+} = \frac{1}{4}\sum_{k}\sum_{m}\sum_{\alpha}\int\frac{dE}{E}(E^2-\mu^2)^{\frac{1}{2}}$$

$$\times F^{*}(E,k,m,\alpha)\; F(E,k,m,\alpha), \qquad \ldots \quad (23)$$

where in addition to relations (21) and (22) we have used the following well known relations also (Edmonds 1957)

$$\nabla J_k[(E^2-\mu^2)^{\frac{1}{2}}r].Y_k{}^{m-\gamma,0}(\hat{\theta},\hat{\phi}) = \Big[-\left(\frac{k+1}{2k+1}\right)^{\frac{1}{2}}\left(\frac{d}{dr}-\frac{k}{r}\right)$$

$$\times J_k[(E^2-\mu^2)^{\frac{1}{2}}r]Y_{k,k+1,m-\gamma}(\hat{\theta},\hat{\phi})+\left(\frac{k}{2k+1}\right)^{\frac{1}{2}}\left(\frac{d}{dr}+\frac{k+1}{r}\right)J_k[E^2-\mu^2)^{\frac{1}{2}}r]$$

$$\times Y_{k,\,k-1,m-\gamma}(\hat{\theta},\hat{\phi})\Big], \qquad (24)$$

$$\frac{d}{dr}J_k\,[(E^2-\mu^2)^{\frac{1}{2}}r] = \frac{k}{r}\,J_k[(E^2-\mu^2)^{\frac{1}{2}}r]-(E^2-\mu^2)^{\frac{1}{2}}J_{k+1}[(E^2-\mu^2)^{\frac{1}{2}}r]$$

$$\frac{k+1}{r}J_k[(E^2-\mu^2)^{\frac{1}{2}}r]+(E^2-\mu^2)^{\frac{1}{2}}J_{k-1}[(E^2-\mu^2)^{\frac{1}{2}}r] \qquad (25)$$

and

$$\int_0^{\pi} d\hat{\varphi}\int_0^{\pi} d\hat{\theta}\sin\hat{\theta}\; Y_{k,k',m}(\hat{\theta},\hat{\phi})Y^{*}_{k,k',m'}(\hat{\theta},\hat{\phi}) = \delta_{k,k'}\delta_{m,n} \qquad (26)$$

$Y_{k,k',m}(\hat{\theta},\hat{\phi})$ are the usual vector spherical harmonics. The volume integration of third term in $H^{+}(\boldsymbol{x})$ is

$$\hat{H}_3{}^{+} = \frac{1}{4}\sum_{k}\sum_{m}\sum_{\alpha}\int\frac{dE}{E}\frac{\mu^2}{(E^2-\mu^2)^{\frac{1}{2}}}\;F^{*}(E,k,m,\alpha)F(E,k,m,\alpha). \qquad \ldots \quad (27)$$

Combining relations (20), (23) and (27) we get

$$\hat{H}^{+}=\frac{1}{2}\sum_{k}\sum_{m}\sum_{\alpha}\int\frac{E}{(E^{2}-\mu^{2})^{\frac{1}{2}}}dE\,F^{*}(E,k,m,\alpha)F(E,k,m,\alpha). \quad \ldots \quad (28)$$

In a similar manner one can calculate the contribution of quantized operator $\hat{A}^{+*}(x,\gamma)$ to the field Hamiltonian.

Thus

$$\hat{H}=\frac{1}{2}\sum_{k}\sum_{m}\sum_{\alpha}\frac{E}{(E^{2}-\mu^{2})^{\frac{1}{2}}}dE[F^{*}(E,k,m,\alpha)F(E,k,m,\alpha)$$

$$+F(E,k,m,\alpha)F^{*}(E,k,m,\alpha)]. \quad \ldots \quad (29)$$

Using the commutation relation given by eq. (16) and dropping the zero-point energy, we get

$$\hat{H}=\sum_{k}\sum_{m}\sum_{\alpha}\int\frac{E}{(E^{2}-\mu^{2})^{\frac{1}{2}}}dE\,F^{*}(E,k,m,\alpha)F(E,k,m,\alpha). \quad \ldots \quad (30)$$

Let us now define new annihilation and creation operators as

$$b(s,\alpha)=[1/(E^{2}-\mu^{2})^{1/4}]F(s,\alpha)$$

$$b^{*}(s,\alpha)=\frac{1}{(E^{2}-\mu^{2})^{1/4}}F^{*}(s,\alpha), \quad \ldots \quad (31)$$

where s collectively denotes the variables E, k, m. These operators satisfy the following commutation relations

$$[b(s,\alpha),b(s',\alpha')]=[b^{*}(s,\alpha),b^{*}(s',\alpha')]=0, \quad \ldots \quad (32)$$

$$[b(s,\alpha),b^{*}(s',\alpha')]=\delta(s-s')\delta_{\alpha\alpha'},$$

where

$$\delta(s-s')=\delta(E-E')\delta_{k,k'}\delta_{m,m'}.$$

In terms of these operators the field Hamiltonian becomes

$$\hat{H}=\sum_{k}\sum_{m}\sum_{\alpha}\int dE\,Eb^{*}(s,\alpha)b(s,\alpha)$$

$$=\sum_{k}\sum_{m}\int dE\,E[\hat{N}(s,+1)+\hat{N}(s,-1)+\hat{N}(s,0)], \quad \ldots \quad (33)$$

where

$$\hat{N}(s,+1),\ \hat{N}(s,-1)\ \text{and}\ \hat{N}(s,0)$$

are three different particle number operators given by

$$\hat{N}(s,\alpha)=b^{*}(s,\alpha)\,b(s,\alpha)$$

$$=\frac{1}{(E^{2}-\mu^{2})^{\frac{1}{2}}}F^{*}(s,\alpha)F(s,\alpha), \quad \ldots \quad (34)$$

which commute with each other and have the independent positive integer eigen values $n(s, \alpha)$. Thus the energy of the field is

$$E = \sum_{k} \sum_{m} \int EdE[n(s, +1)+n(s, -1)+n(s, 0)]. \tag{35}$$

Let us assume that all the quanta of this real field are aligned and move along the third axes (i.e., z-direction). $\boldsymbol{p} = (0, 0, p)$. Then $f(\boldsymbol{p}, 0) = 0$ due to condition* (2) and hence $\hat{N}(s, 0) = 0$

Then

$$\hat{\mathrm{H}} = \sum_{k} \sum_{m} \int dEE[\hat{N}(s, -1)+\hat{N}(s, +1)]$$

$$= \sum_{k} \sum_{m} \sum_{\alpha=\pm 1} \int dE\, Eb^{*}(s, \alpha)\, b(s, \alpha)$$

$$= \sum_{k} \sum_{m} \sum_{\alpha=\pm 1} \int dE\; E\hat{N}(s, \alpha), \qquad \ldots \tag{36}$$

where $\hat{N}(s, +1)$ and $\hat{N}(s, -1)$ are the number operators for the particles with helicity $+1$ and -1 respectively. The quanta of particle number operators $\hat{N}(s, \alpha)$ correspond to two circularly polarized field states (Part I).

The base vector which spans the Hilbert space can be chosen to contain these circularly polarized particles of mass μ and spin 1. Let us define the state which contains n-particles (each of mass μ) with variables $s_1, s_2, \ldots, s_n$ and helicities $\alpha_1, \alpha_2, \ldots, \alpha_n$, as

$$|n> = \prod_{i=1}^{n} b^{*}(s_i, \alpha_i)\,|0> \qquad \ldots \tag{37}$$

where $|0>$ designates the vacuum state. If all these n- particles correspond to a well defined quantum state i.e.,

$$s_1 = s_2 = \cdots = s_n = s, \;\; \alpha_1 = \alpha_2 = \quad = \alpha_n = \alpha$$

then

$$|n> = |n(s, 1)> = \frac{1}{\sqrt{n(s, \alpha)}}\; b^{*n}(s, \alpha)\,|0>. \qquad \ldots \tag{38}$$

Annihilation and creation operators act upon these basis vectors in the following manner

$$b^{*}(s, \alpha)\,|\,n(s. \alpha)> = [n(s, \alpha)+1]^{\frac{1}{2}}\,|\,n(s, \alpha)+1> \qquad \ldots \tag{39}$$

$$b(s, \alpha)\,|\,n(s, \alpha)> = [n(s, \alpha)]^{\frac{1}{2}}\,|\,n(s, \alpha)-1> \qquad \ldots \tag{40}$$

* Using reduced expansion (12) one can prove that div **A** is zero in angular momentum basis also.

In terms of operators $b(s, \alpha)$ and $b^*(s, \alpha)$, the reduced expansion for the operator $\hat{A}^+(x)$, transforms to.

$$\hat{A}^+(x) = \sum_{\gamma=\pm 1} e_\gamma \, A^+(x, \gamma)$$

$$= \frac{1}{\pi^{\frac{1}{2}}} \sum_{\alpha=\pm 1} \sum_{k=1}^{\infty} \sum_{m=-k}^{k} \sum_{\gamma=\pm 1} (i)^{k-m+\gamma} \int dE (E^2-\mu^2)^{1/4}$$

$$\times \Big[\exp(-iEt)\, b(s, \alpha)\, e_\gamma \Big\{ i \Big[\frac{k}{2(2k+1)} \Big]^{\frac{1}{2}}$$

$$\times (k+1, m-\gamma, 1, \gamma \,|\, k+1, 1, k, m)\, J_{k+1}[(E^2-\mu^2)^{\frac{1}{2}} r]\, Y_{k+1}^{m-\gamma, 0} (\hat{\theta}, \hat{\phi})$$

$$- i \Big[\frac{k+1}{2(2k+1)} \Big]^{\frac{1}{2}} (k-1, m-\gamma, 1, \gamma \,|\, k-1, 1, k, m) J_{k-1}[(E^2-\mu^2)^{\frac{1}{2}} r]$$

$$\times Y_{k-1}^{m-\gamma, 0} (\hat{\theta}, \hat{\phi}) - \frac{1}{\sqrt{2}} \alpha (k, m-\gamma, 1, \gamma \,|\, k, 1, k, m) J_k[(E^2-\mu^2)^{\frac{1}{2}} r]$$

$$\times Y_k^{m-\gamma, 0} (\hat{\theta}, \hat{\phi}). \Big\} \Big]. \quad \ldots (41)$$

Substituting the vector spherical harmonics

$$Y_{k, k', m}(\theta, \phi) = \sum_{\gamma} (i)^\gamma e_\gamma Y_{k'}^{m-\gamma, 0} (\hat{\theta}, \hat{\phi}) (k', m-\gamma, 1, \gamma \,|\, k', 1, k, m) \quad \ldots (42)$$

where

$$k' = k-1, k, k+1,$$

we get

$$\hat{A}^*(x) = \frac{1}{\pi^{\frac{1}{2}}} \sum_{\alpha=\pm 1} \sum_{k=1}^{\infty} \sum_{m=-k}^{k} (i)^{k-m} \int dE (E^2-\mu^2)^{1/4} b(s, \alpha)$$

$$\times \Big\{ i \Big[\frac{k}{2(2k+1)} \Big]^{\frac{1}{2}} J_{k+1}[(E^2-\mu^2)^{\frac{1}{2}} r]\, Y_{k, k+1, m}(\theta, \phi)$$

$$- i \Big[\frac{k+1}{2(2k+1)} \Big]^{\frac{1}{2}} J_{k-1}[(E^2-\mu^2)^{\frac{1}{2}} r]\, Y_{k, k-1, m}(\theta, \phi)$$

$$- \frac{1}{\sqrt{2}} \alpha \, J_k[(E^2-\mu^2)^{\frac{1}{2}} r]\, Y_{k, k-m}(\theta, \phi) \Big\}. \quad \ldots (43)$$

Reduced expansion of operator $A(x)$ may be readily derived by using the relation

$$\hat{A}(x) = \hat{A}^+(x) + \hat{A}^{+*}(x). \quad \ldots (44)$$

5. Interaction

In the angular momentum base, the transition probabilities of emission and absorption in the interaction of electromagnetic fields for non-zero mass case with an atomic system may be calculated in the manner similar to that for linear momentum basis given in I. For emission, the initial state before interaction consists of field state containing n (s, α) particles and atomic state with quantum numbers k, m and π for angular momentum, z-component of angular momentum and parity. The final state in emission consists of field state with $(n(s, \alpha)+1]$ particles and atomic state with quantum numbers k_f, m_f, π_f. The matrix element of interaction Hamiltonian for emission is thus given below

$$<\psi_F | \boldsymbol{A}(\boldsymbol{x}, 0) \cdot \boldsymbol{v} | \psi_I> = <\psi_f| <S | A^{+*}(\boldsymbol{x}, 0) \cdot \boldsymbol{v} | V> | \psi_i>.$$

$$-\frac{1}{(2\pi)^{\frac{1}{2}}} (-i)^{k-m}(E^2-\mu^2)^{1/4}(n(s, \alpha)+1]^{\frac{1}{2}}$$

$$\times \Big[-i\Big(\frac{k}{2k+1}\Big)^{\frac{1}{2}} J_{k+1}[(E^2-\mu^2)^{\frac{1}{2}}r]\Big\}$$

$$\times < \psi_f | Y^*_{k,k+1},m (\theta, \phi) \cdot \boldsymbol{v} | \psi_i > +i \Big(\frac{k+1}{2k+1} \Big) J_{k-1}[(E^2-\mu^2)^{\frac{1}{2}}r]$$

$$\times < \psi_f | Y^*_{k,k+1,m}(\theta, \phi) \cdot | \boldsymbol{v} | \psi_i > -\alpha J_k(E^2 \quad \mu^2)^{\frac{1}{2}} r$$

$$\times < \psi_f | Y^*_{k,k,m}(\theta, \phi) \cdot \boldsymbol{v} | \psi_i >], \tag{45}$$

where we have used eqs. (39) and (44) from which it is clear that only $\boldsymbol{A}^{+*}(\boldsymbol{x}, 0)$ part of $\boldsymbol{A}(\boldsymbol{x}, 0)$ contributes to the interaction Hamiltonian for emission, while the other part i.e., $A^{+}(\boldsymbol{x}, 0)$ contributes to the Hamiltonian for absorption. The matrix element given by eq. (48) consists of the terms like

$$< \psi_f | Y^*_{k,k'},m(\theta, \phi) \cdot \boldsymbol{v} | \psi >, \quad (k' = k+1, k)$$

which can be written in terms of quantum numbers of the initial and final states as follows

$$< k_f m_f \pi_f | Y^*_{k,k'}m(\theta, \phi) \cdot \boldsymbol{v} | k_i m_i \pi_i >, \tag{46}$$

where $Y^*_{k,k'},m(\theta, \phi) \cdot \boldsymbol{v}$ is an irreducible tensor of rank k. Applying Winger-Echart theorem, it follows that only those matrix element like (49) are non-vanishing for which, the following selection rules are satisfied.

$$k_i = k_f+k, \; k_f+k-1, \ldots | k_f-k |, \tag{47}$$

$$m_i = m_f+m. \tag{48}$$

The probability for the emission per unit time in transition from ψ_I to ψ_F is proportional to the square of matrix element (48). Hence it is proportional to $[(E^2 -\mu^2)^{\frac{1}{2}} n(s, \alpha)+1]$, which is non-vanishing even for $n(s, \alpha) = 0$.

In the absorption the initial state is the same as that of emission while the final field state consists of $(n(s,\alpha)-1]$ particle. The matrix element of interest in this case is $<\psi_{f'}A^{+}(x,0).\, v\psi_f>$

$$= \frac{1}{(2\pi)^{\frac{1}{2}}}[n(s,\alpha)]^{\frac{1}{2}}(E^2-\mu^2)^{1/4}(i)^{k-m}$$

$$\times \left[i\left(\frac{k}{2k+1}\right)^{\frac{1}{2}} J_{k+1}[(E^2-\mu^2)^{\frac{1}{2}}r] < \psi_f |\, Y_{k,k+1,m}(\theta,\phi)\,.\,\boldsymbol{v}\,|\,\psi_i > \right.$$

$$-i\left(\frac{k+1}{2k+1}\right)^{\frac{1}{2}} J_{k-1}[(E^2-\mu^2)^{\frac{1}{2}}r]$$

$$\times < \psi_f |\, Y_{k,k-1,m}(\theta,\phi)\,.\,\boldsymbol{v}\,|\,\psi_f > -\alpha J_k[(E^2-\mu^2)^{\frac{1}{2}}r]$$

$$\left. \times < \psi_f |\, Y^*_{k,k,m}(\theta,\phi)\,.\,\boldsymbol{v}\,|\,\psi_i > \right].$$

In the similar manner as discussed for emission we get the following selection rules for absorption,

$$k_f = k_i+k,\, k_i+k-1 \,\ldots\, |k_i-k| \qquad \ldots (49)$$

$$m_f = m_i+m. \qquad \ldots (50)$$

The probability for abosrption is proportional to the number of particles of a given kind in the initial state i.e., $[(E^2-\mu^2)^{\frac{1}{2}}n(s,\alpha)]$. The ratio of the probability of emission to that of absorption is proportional to $[n(s,\alpha)+1]/n(s,\alpha)$.

The selection rules for emission and absorption of a particle in this case are similar to those obtained in the interaction of electromagnetic fields for zero mass system with an atom. Transitions can take place between those atomic states quantum number of which satisfy the conditions (50) and (51) for emission and (52) and (53) for absorption. The interaction consists of transition of the atomic system from one quantum state to another. This transition results into an emission or absorption of a particle (which is assumed to have the same mass μ, energy E, angular momentum number k, z-component of quantum number m helicity α).

Acknowledgment

One of the authors (O.P.) is thankful to the C.S.I.R. for providing the financial assistance in the form of research fellowship.

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