Indian J. Phys. **48**, 161–172 (1974)

Problem of the Λnp system with the Hulthen Potential and the Faddeev equations: I

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(*Received* 23 *February* 1973)

Making use of the Faddeev equations, the problem of one lambda hyperon and two nucleon three body system is worked out with the interactions between each pair of particles determined by the local Hulthen Potential. The potentials between the pairs of particles are taken to be *s*-wave charge symmetric. The Hilbert-Schmidt separable expansion of the two body *t*-matrix projects the Faddeev equations in a set of one dimensional coupled integral equations. With this approach we have calculated the values of the binding energy of the Λ hyperon in $_{\Lambda}H^3$ for different sets of the low-energy Λ–*N* parameters.

1. Introduction

The non-relativistic three-body problem is widely investigated with the application of Faddeev's (1961a, 1961b, 1963) set of integral equations, which have unique mathematical solution. Lovelace (1964) reformulated these equations in such a way that they lead more directly to the *t*-matrix of the three body system. A practical solution is usually obtained for separable two particle potentials only.

In this paper we have made a study of the Λnp three body iso-spin zero system in doublet and quartet spin states when the two-particle forces are determined from the local Hulthen potential. A separable form of the two particle *t*-matrix and consequently of the interactions is most desirable in the three-body theory, because it reduces two continuous integration variables to one. The problem is thus to have some sort of an optimal separable representation for the local potential under consideration and to use it in the Faddeev equations. Recently some of the methods have been suggested by Gillespie (1967) and Efimov (1964) in which the two-body *t*-matrix is expanded in separable form. The method based on the Hilbert-Schmidt theorem (1931) wherein the expansion of the kernel of the Lippmann-Schwinger equation in terms of its eigen functions is used was developed by Weinberg (1963, 1964) to eliminate the divergence of the Born series of the two-body *t*-matrix.

The problem of the Λnp system was investigated in the frame work of the Faddeev formalism and the binding energy calculations of $_\Lambda H^3$ were carried out by Hetherington *et al* (1965), Schick *et al* (1967), Roy Choudhury *et al* (1973), but the two-body potentials used were the non-local separable potentials of Yamaguchi form. Our results for the binding energy of $_\Lambda H^3$ with two body local Hulthen potential are compared in Section 4 with the results of Hetherington *et al* (1965) and Roy Choudhury *et al* (1973) who made use of the non-local separable potential of Yamaguchi type.

Each of the three particles in the hypertriton is a spin $\frac{1}{2}$ fermion and the two-body potentials between the pairs of the particles are taken to be s-wave spin dependent. So the spin of the three particle Λnp system is either $\frac{1}{2}$ or 3/2. We neglect the neutron-proton mass difference and treat the N–N and Λ–N potentials to be charge symmetric. In the calculations only three two-body states have been considered. They are the N–N in triplet spin state and the Λ-N in singlet and triplet spin states. The nucleons are labelled 1 and 3, the lambda particle is numbered 2, the nucleons from the two dimensional representation of SU(2) and Λ belongs to the singlet representation.

2. Two Body t-Matrix with Hulthen Potentials

The two-body potentials in all the three channels are chosen to be of Hulthen type which in the configuration space are of the form

$$V(r) = -V_0 \exp(r/R-1)^{-1}, \qquad \ldots (1)$$

where the parameters V_0 and R are different for N–N triplet, Λ–N singlet and Λ–N triplet interactions. In the momentum space the Fourier transform of $V(r)$ reads as

$$V(\boldsymbol{k}-\boldsymbol{k}') = \int \exp(i(\boldsymbol{k}-\boldsymbol{k}').\boldsymbol{r})\,V(r)\mathrm{d}^3r, \qquad \ldots (2)$$

Its lth partial wave projection is

$$V_l(\boldsymbol{k},\boldsymbol{k}') = \int_{-1}^{1} \mathrm{d}x\, V[(k^2+k'^2-2kk'x)^{\frac{1}{2}}]P_l(x). \qquad \ldots (3)$$

This appears in the kernal of the general Lippmann-Schwinger equation for the two body t-matrix off the energy shell

$$t_l\left(k, k'; -\frac{Q^2}{2\mu}\right) = V_l(k,k') - \frac{\mu}{\pi^2}\int_0^\infty \frac{t_l\left(k, q; -\frac{Q^2}{2\mu}\right)}{(q^2+Q^2)} V_l(q,k')q^2\mathrm{d}q. \qquad \ldots (4)$$

where μ is the reduced mass of the two interacting particles and as the two-body energy is always negative in our analysis we have defined it to be $-(Q^2/2\mu)$.

The relation of the partial on shell t-matrix to the corresponding phase shifts is the following one

$$-\frac{\mu}{2\pi} t_l \left(k, k'; \frac{k^2}{2\mu}\right) = \frac{1}{k} \sin \delta_l(k) \exp (i\delta_l(k)). \qquad \ldots \quad (5)$$

As we are investigating the bound state problem of the three particles, the total energy E in the c.m.s. of the three particles is negative. Thus the two-body sub-system can have only negative energy. Further, suffix l will be suppressed because we have taken into account only s-wave two body interaction. As done by Kharchenko & Petrov (1969) following the Hilbert-Schmidt method (1931) the two-body t-matrix can be expanded in the following series in separable form

$$t\left(k,k'; -\frac{Q^2}{2\mu}\right) = -\sum_{n=1}^{\infty} \frac{1}{\eta_n^{-1}(Q)^{-1}} g_n(k, Q) g_n(k', Q). \qquad \ldots \quad (6)$$

The separable expansion of the two-body t-matrix is very useful for the three-body calculations in Faddeev's formalism. Taking more number of terms in the series (eqn. 6), the number of coupled integral equations for three-body t-matrix gets increased which makes the numerical computations more cumbersome. Details related to eq. (6) appear in the Appendix. The convergence of expansion (6) is discussed in detail by Kharchenko *et al* (1969).

The Hulthen potential parameters v and R for Λ–N singlet and triplet channels are evaluated from the respective scattering length and effective range by making use of the following relations of Kharchenko *et al.* (1969)

$$\left.\begin{aligned} a &= -2XR \sum_{n=1}^{\infty} [n(n^2-X)]^{-1} \\ r &= \frac{2}{3}a + \frac{16}{3}\frac{XR^3}{a^2} \sum_{n=1}^{\infty} \frac{3n^4-3n^2X-X^2}{n^3(n^2-X)^3} \end{aligned}\right\} \qquad \ldots \quad (7)$$

For lambda-nucleon scattering low-energy parameters a and r in singlet and triplet spin states, a number of sets are available in the literature on the basis of theoretical and experimental investigations. For our analysis of hypertriton problem, the sets of Hotherington *et al* (1965), Alexander *et al* (1966, 1967, 1968), Herndon *et al* (1967), Beck *et al* (1965), Fast *et al* (1969), Dietrich *et al* (1964), Sechi-Zorn *et al* (1968), de Swart *et al* (1962) we have picked up, and the corresponding Hulthen potential parameters obtained by us are given in table 3. For six sets of lambda-nucleon scattering lengths and effective ranges the convergence of series (7) is shown in table 1 and table 2. Table 1 illustrate the convergence of the series R/a and table 2 that of r/R upto six terms. How the series for r/a converges with n and how it depends on the variation of $X(vR^2)$ can be seen in figure 1.

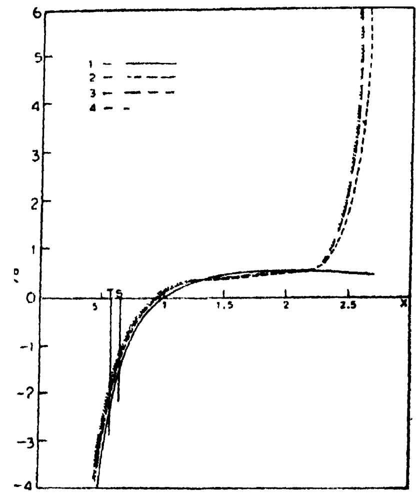

Fig. 1. r/a is plotted against $X(vR^2)$ for the Hulthen potential. The numbers marked on the corresponding curves indicate the number of terms taken in eq. (7) to evaluate r/a. Two barriers T and S shown in the figure are for Herndon and Tang's Triplet and Singlet Λ–N low energy parameters for Set 1.

3. Three Body Equations and Method for Calculating B_Λ

In the set of three-body coupled integral equations we discuss in this section, the off-shell two body t-matrix appears in the kernel. Even if we put the two-body t-matrix in one term separable form we face three coupled integral equations. The number of coupled integral equations multiply further if we take into account more number of terms in the separable expansion (6) of the two body t-matrix. Presently, we are doing the three-body calculations with only the first term of eq. (6). The s-wave set of integral equations for the Λnp three particle scattering matrix obtained from the Faddeev type multiple scattering analysis by Schick *et al* (1967) is as follows

$$R_{\alpha\beta}(q,q') = K_{\alpha\beta}(q,q') + \sum_{\gamma\lambda}\int_0^\infty K_{\alpha\gamma}(q,k)\tau^{\gamma\lambda}(k)R_{\lambda\beta}(k,q')(2\pi)^{-2}k^2\mathrm{d}k, \quad \ldots (8)$$

with

$$K_{\alpha\beta}(q,q') = W_{\alpha\beta}N_\alpha N_\beta \Bigg[\frac{a_{\alpha\beta}}{(a_{\alpha\beta}B_{\alpha\beta}-b_{\alpha\beta}A_{\alpha\beta})(a_{\alpha\beta}C_{\alpha\beta}+c_{\alpha\beta}A_{\alpha\beta})}\log\frac{A_{\alpha\beta}+a_{\alpha\beta}}{A_{\alpha\beta}-a_{\alpha\beta}}$$

$$+\frac{b_{\alpha\beta}}{(b_{\alpha\beta}A_{\alpha\beta}-a_{\alpha\beta}B_{\alpha\beta})(b_{\alpha\beta}C_{\alpha\beta}+c_{\alpha\beta}B_{\alpha\beta})}\log\frac{B_{\alpha\beta}+b_{\alpha\beta}}{B_{\alpha\beta}-b_{\alpha\beta}}$$

$$+\frac{c_{\alpha\beta}}{(a_{\alpha\beta}C_{\alpha\beta}+c_{\alpha\beta}A_{\alpha\beta})(b_{\alpha\beta}C_{\alpha\beta}+c_{\alpha\beta}B_{\alpha\beta})}\Bigg]\log\frac{C_{\alpha\beta}+c_{\alpha\beta}}{C_{\alpha\beta}-c_{\alpha\beta}}\Bigg] \quad \ldots (9)$$

where,

$$\left.\begin{aligned} A_{\alpha\beta} &= q'^2+\left(\frac{m_\alpha}{M_\alpha}\right)^2 q^2+\beta^2 \quad &; \quad a_{\alpha\beta} &= 2qq'\frac{m_\beta}{M_\alpha} \\ B_{\alpha\beta} &= q^2+\left(\frac{m_\alpha}{M_\alpha}\right)^2 q'^2+\beta_\beta^2 \quad &; \quad b_{\alpha\beta} &= 2qq'\frac{m_\alpha}{M_\beta} \\ C_{\alpha\beta} &= E-\frac{M_\beta}{2m_\alpha m_{\alpha\beta}}q^2-\frac{M_\alpha}{2m_\beta m_{\alpha\beta}}q'^2 \quad &; \quad c_{\alpha\beta} &= qq'\frac{1}{m_{\alpha\beta}} \end{aligned}\right\} \quad \ldots (10)$$

and

$$\left.\begin{aligned} N_\alpha &= \frac{2}{\sqrt{\pi R^3}}(1+QR)^{\frac{1}{2}}(1+2QR), \\ \beta &= \frac{1+QR}{R}. \end{aligned}\right\} \quad \ldots (11)$$

where Q is connected with two body energy and Q for the two particles in αth channel is given by

$$Q=\left[2\mu_\alpha\left(-E+\tfrac{1}{2}q'^2\frac{M}{m_\alpha M_\alpha}\right)\right]^{\frac{1}{2}} \quad \ldots (12)$$

where q' is the total momentum of the two particles in αth channel in c.m.s. of the three particles.

m_α is the mass of the spectator particle in the αth two particle channel, M is the total mass of all three particles, $M_\alpha = M-m_\alpha$ and $m_{\alpha\beta} = M-m_\alpha-m_\beta$. The Greek indices in this paper run from 1 to 3 for Λ–N singlet, N–N triplet and Λ–N triplet interactions respectively.

In eq. (8) the matrix $\tau^{\alpha\beta}$ is given by

$$\tau^{\alpha\beta} = \begin{bmatrix} \tau_1 & 0 & 0 \\ 0 & \tau_2 & 0 \\ 0 & 0 & \tau_3 \end{bmatrix}. \quad \ldots (13)$$

where again subscripts 1, 2, 3 refer to the respective two particle channels as indicated above. τ's are related to the respective two-body t-matrices eq. (6) when it is truncated after first term. The t-matrix is written after dropping the suffix $n = 1$ for the first term as

$$t\left(k, k'; -\frac{Q^2}{2\mu}\right) = g(k, Q)\tau g(k', Q), \quad \ldots (14)$$

where

$$\tau = -\frac{\pi^2}{\mu} \frac{1}{\eta^{-1}-1} \quad \ldots (15)$$

and

$$g(k, Q) = N(Q)/(k^2+\beta^2). \quad \ldots (16)$$

The matrix $[W_{\alpha\beta}]$ for Λnp doublet spin state is

$$[W_{\alpha\beta}] = \begin{bmatrix} \frac{1}{2} & \sqrt{3}/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 0 & -1/\sqrt{2} \\ -\sqrt{3}/2 & -1/\sqrt{2} & -\frac{1}{2} \end{bmatrix} \quad \ldots (17)$$

and that for quartet spin state is

$$[W_{\alpha\beta}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 1 \end{bmatrix} \quad \ldots (18)$$

To find out the binding energy B_Λ we evaluate the Fredholm determinant of the s-wave set of integral eq. (8) for a given value of E and repeat the calculations till we get the value of E say E_0 for which the determinant becomes zero. For each set of low energy lambda-nucleon parameters the same procedure is followed and E_0 is found out. Then to get B_Λ, the deuteron binding energy is subtracted out of E_0. The N-N triplet spin state Hulthen parameters, we have worked with are fitted to the deuteron binding energy, 2·225 MeV and the two nucleon triplet spin state scattering length, 5·378 F and the values of those parameters are $v = 1{\cdot}8509\ F^{-2}$ and $R = 0{\cdot}8708\ F$.

4. Results and Discussion

Our results for the three body binding energy of the Λnp system are recorded in table 3. This table contains twelve sets of low energy lambda-nucleon scattering parameters *i.e.*, the scattering length and effective range. The Hulthen potential parameters v and R obtained by us are also appearing there. While reporting the values of B_Λ for $_\Lambda H^3$ $(J = \frac{1}{2})$ from the present calculations with two-body local Hulthen potential in Table 3, we have also put the B_Λ values obtained when the two-body interaction is determined by the non-local separable potential of Yamaguchi form by Roy Choudhury *et al* (1973). As is evident from table 3 the general tendency of the local Hulthen potential calculations is to suppress the values of B_Λ as compared to their counterparts obtained from NLS potential calculations by Roy Choudhury *et al* (1973). The possibility of the Λnp $(J = \frac{1}{2})$ bound state disappeared for the present calculation for the sets 7 to 12. Three values of B_Λ obtained in increasing order are ·009, ·096 and

Table 1. The values of R/a are given for $n = 1, \ldots, 6$. The convergence of the series R/a for the six sets of singlet and triplet lambda-nucleon scattering lengths is shown by this table.

Set	State	a (F)	r (F)	vR^2	R/a $n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$
1[a]	Singlet	−2.76	3.05	0.6649	−0.2520	−0.2399	−0.2369	−0.2357	−0.2351	−0.2348
	Triplet	−1.96	3.50	0.5806	−0.3613	−0.3404	−0.3351	−0.3330	−0.3320	−0.3314
2[b]	Singlet	−3.30	1.83	0.7754	−0.1448	−0.1400	−0.1387	−0.1382	−0.1380	−0.1379
	Triplet	−0.64	3.70	0.3829	−0.8058	−0.7425	−0.7265	−0.7201	−0.7169	−0.7151
3[c]	Singlet	−4.60	1.70	0.8299	−0.1025	−0.0998	−0.0991	−0.0989	−0.0987	−0.0987
	Triplet	−0.53	3.88	0.3482	−0.9360	−0.8593	−0.8399	−0.8321	−0.8282	−0.8260
4[d]	Singlet	−3.60	2.00	0.7751	−0.1451	−0.1402	−0.1390	−0.1385	−0.1382	−0.1381
	Triplet	−0.53	5.00	0.3134	−1.0954	−1.0021	−0.9785	−0.9690	−0.9643	−0.9617
5[e]	Singlet	−2.89	1.94	0.7467	−0.1696	−0.1633	−0.1617	−0.1610	−0.1607	−0.1605
	Triplet	−0.71	3.75	0.3967	−0.7604	−0.7017	−0.6868	−0.6808	−0.6780	−0.6762
6[d]	Singlet	−2.40	2.00	0.7123	−0.2020	−0.1935	−0.1934	−0.1905	−0.1901	−0.1898
	Triplet	−0.52	4.00	0.3412	−0.9654	−0.8857	−0.8655	−0.8574	−0.8534	−0.8512

a—Herndon *et al* (1967). *b*—Beck *et al* (1965). *c*—Dietrich *et al* (1964). *d*—de Swart *et al* (1962). *e*—Herndon *et al* (1965).

Table 2. For $n = 1, \ldots, 6$, the values of r/R are given here. The convergence of the series r/R for the six sets of singlet and triplet lambda-nucleon effective ranges is shown here.

Set	State	a (F)	r (F)	vR^2	r/R $n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 4$	$n = 6$
1[a]	Singlet	−2.76	3.05	0.6649	6.0160	5.1021	4.8722	4.7800	4.7339	4.7075
	Triplet	−1.96	3.50	0.5806	6.8900	5.8428	5.5785	5.4725	5.4194	5.3890
2[b]	Singlet	−3.30	1.83	0.7754	5.1586	4.3642	4.1650	4.0852	4.0453	4.0225
	Triplet	−0.64	3.70	0.3829	10.4466	8.8030	8.3847	8.2162	8.1318	8.0834
3[c]	Singlet	−4.60	1.70	0.8299	4.8198	4.0680	3.8800	3.8043	3.7666	3.7450
	Triplet	−0.53	3.88	0.3482	11.4876	9.6615	9.1960	9.0085	8.9144	8.8605
4[d]	Singlet	−3.60	2.00	0.7751	5.1606	4.3659	4.1667	4.0869	4.0469	4.0241
	Triplet	−0.53	5.00	0.3134	12.7632	10.7110	10.1870	9.9757	9.8697	9.8090
5[e]	Singlet	−2.89	1.94	0.7467	5.3569	4.5362	4.3302	4.2477	4.2065	4.1829
	Triplet	−0.71	3.75	0.3967	10.0832	8.5020	8.1008	7.9389	7.8578	7.8113
6[d]	Singlet	−2.40	2.00	0.7123	5.6156	4.7593	4.5442	4.4580	4.4480	4.3902
	Triplet	−0.52	4.00	0.3412	11.7233	9.8556	9.3793	9.1874	9.0911	9.0360

a—Herndon *et al* (1967). *b*—Beck *et al* (1965). *c*—Dietrich *et al* (1964). *d*—de Swart *et al* (1962). *e*—Herndon *et al* (1965).

Table 3. Binding energy B_Λ of the Λ hyperon in ($J = \frac{1}{2}$) Λnp system for several sets of the low-energy Λ – N Hulthen Potential parameters. The N–N triplet spin state Hulthen Potential parameters used here are $R = 0{\cdot}8708\ F$ and $v = 1{\cdot}8509\ F^{-2}$.

Set	Ref.	Λ–N^1S_0				Λ–N^3S_1				B_Λ(MeV)		Gound state spin of $_\Lambda H^3$
		a (F)	r (F)	v (F^{-2})	R (F)	a (F)	r (F)	v (F^{-2})	R (F)	Present Calculations	NLS[k]	
1	*a*	−2.76	3.05	1.5839	0.6479	−1.96	3.50	1.3763	0.6495	0.096	0.625	1/2
2	*b*	−3.30	1.83	3.7466	0.4549	−0.64	3.70	1.8278	0.4577	0.310	1.195	1/2
3	*c*	−4.60	1.70	4.1836	0.4454	−0.53	3.88	1.8161	0.4378	0.738	1.941	1/2
4	*d*	−3.60	2.00	3.1377	0.4970	−0.53	5.00	1.2064	0.5097	0.236	1.010	1/2
5	*e*	−2.89	1.94	3.4713	0.4638	−0.71	3.75	1.7213	0.4801	0.163	0.900	1/2
6	*d*	−2.40	2.00	3.4320	0.4556	−0.52	4.00	1.7412	0.4426	0.009	0.404	1/2
7	*e*	−2.46	3.87	1.0846	0.7457	−2.07	4.50	0.8842	0.7856	< 0	0.203	1/2
8	*f*	−1.80	2.80	2.0650	0.5413	−1.60	3.30	1.6173	0.5858	< 0	0.188	1/2
9	*g*	−1.70	2.50	2.5490	0.4912	−1.50	2.00	3.8749	0.4039	< 0	0.302	3/2
10	*h*	−2.00	5.00	0.7485	0.8344	−2.20	3.50	1.3315	0.6719	< 0	0.042	3/2
11	*i*	−1.36	3.06	1.9327	0.5284	−1.62	2.93	1.9715	0.5416	< 0	0.016	3/2
12	*j*	−1.80	2.06	3.5042	0.4336	−0.40	4.00	1.9284	0.3982	< 0	0.050	1/2

a—Herndon *et al* (1967). *b*—Beck *et al* (1965). *c*—Dietrich *et al* (1964). *d*—de Swart *et al* (1962). *e*—Herndon *et al* (1965). *f*—Alexander *et al* (1968). *g*—Fast *et al* (1969). *h*—Sechi-Zorn *et al* (1968). *i*—Alexander *et al* (1967). *j*—Hetherington *et al* (1965). *l*—Roy Choudhury *et al* (1973). *k*—See Roy Choudhury *et al* (1973) for sets 1 to 3 and 6 to 11 and *j* for sets 4, 5 and 12.

·163 MeV for the low-energy Λ–N parameters of de Swart and Dullemond (1962), Herndon and Tang (1967), and Herndon, Tang and Schmid (1965) respectively. The values of B corresponding to the sets 1 and 6 are within the limits of experimental value (·06 ± ·06) MeV of Bhom *et al* (1968).

For Λnp system in $J = 3/2$ state we searched for the bound states corresponding to all the twelve sets but no evidence is found in favour of it. The bound state formation in $J = 3/2$ state is feasible when the three-body calculations are done with two terms in expansion (6) of the two-body t-matrix. Our results in case of $J = 3/2$ bound state match with the remarks of Herndon & Tang (1968) and Keyes *et a* (1968).

Sets 9, 10, and 11 are to be discarded because they favour the Λnp ground state in $J = 3/2$ state in contradiction to the experimental findings which predict $J = \frac{1}{2}$ as the spin of the $_{\Lambda}H^3$ ground state (Rayet *et al* 1966). For these three sets, the attractive force due to NLS potential was of sufficient strength to form the bound state of the Λnp system in $J = 3/2$ as well as in $J = \frac{1}{2}$ state. It is interesting to note that the force due to the Hulthen potential is weaker to the extent that for these sets it is not sufficient to form a bound state even in $J = 3/2$ spin state.

ACKNOWLEDGEMENTS

We are grateful to Professor D. Basu, Director, Indian Association for the Cultivation of Science, and Professor N. C. Sil for their kind interest in this work. All numerical work were carried out on CDC 3600 at the T.I.F.R., Bombay.

APPENDIX

Similar to the separable expansion of $t\left(k, k'; -\frac{Q^2}{2\mu}\right)$ the $V_l(k, k')$ of eq. (3) for $l = 0$ can be expanded in the separable form

$$V(k, k') = -\sum_{n=1}^{\infty} \eta_n(Q) g_n(k, Q) g_n(k', Q), \qquad \text{... (A1)}$$

where $g_n(k, Q)$ and $\eta_n(Q)$ are the eigenfunctions and eigenvalues as appearing in the equation

$$-\frac{1}{2\pi^2}\int_0^{\infty} V(k, q)(q^2+Q^2)^{-1} g_n(q, Q) q^2 \mathrm{d}q = \eta_n(Q) g_n(k, Q). \qquad \text{... (A2)}$$

The orthonormal property of the ligen functions is given by equation

$$\frac{1}{2\pi^2}\int_0^{\infty} g_n(k, Q) g_n(k, Q)(k^2+Q^2)^{-1} k^2 \mathrm{d}k = \delta_{n'n}. \qquad \text{... (A3)}$$

The series for t-matrix (6) and for kernel (A1) converges absolutely and uniformly with respect to both the variables k and k', (Kharchenko *et al* 1969).

The explicit expressions for η_n and g_n are as follows

$$\eta_n(Q) = X[n(n+2QR)]^{-1}, \quad X = vR^2 \text{ and } v = \frac{2\mu V_0}{\hbar^2}$$

$$g_n(k, Q) = C_n(Q) \sum_{\nu=1}^{n} A_{n\nu}(Q) v \eta_\nu^{-1}(Q) [k^2 + Q^2 + v\eta_\nu^{-1}(Q)]^{-1},$$

$$A_{n\nu}(Q) = (-1)^{\nu+1} \prod_{\sigma=1}^{\nu} \frac{n-\sigma+1}{n+\sigma-1} \frac{\eta_\sigma(Q)}{\eta_{\sigma+n-1}(Q)}, \qquad \ldots \text{(A4)}$$

$$C_n^2(Q) = \frac{\pi n X^{2n-1} R}{\mu(n!)^4} \left[\eta_{2n}(Q) \prod_{\nu=1}^{n-\nu} \eta_\nu^2(Q)\right]^{-1},$$

$$C_1^2(Q) = \frac{\pi}{\mu} \frac{XR}{\eta_2(Q)}.$$

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