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Heat transfer by fluctuating flow of a non-Newtonian fluid past a porous flat plate with time-varying suction

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The heat energy equation has been solved when a non-Newtonian liquid flows past a porous infinite wall. The suction velocity normal to the plate oscillates in magnitude but not in direction about a non-zero mean and the free stream velocity oscillates in time about a constant mean but not in direction. The liquid is taken to be slightly non-Newtonian. The heat flow phenomenon has been characterized by the parameters such as non-Newtonian parameter R, frequency parameter ω, variable suction parameter A, Eckert number E, and Prandtl number σ and the effects of these parameters on the temperature distribution and its fluctuating parts have been studied and the results have been presented by several graphs.

1. INTRODUCTION

Lighthill (1954) initiated an important class of two dimensional time dependent flow problems dealing with the response of the boundary layer to external unsteady fluctuations of the free stream velocity about a mean value. Stuart (1955) studied the oscillating flow over an infinite flat plate with constant suction. Messiha (1966) examined Stuart's problem for the case of variable suction at the plate. Kaloni (1967) and Soundalgekar & Puri (1969) studied the problems of Stuart and Messiha respectively replacing a viscous liquid by an elastico-viscous liquid. In a subsequent paper Soundalgekar (1972) solved the heat transfer part by the fluctuating flow of an elastico-viscous liquid. But in his work the energy dissipation term seems to be incorrect. So the same problem has been solved by Mishra & Acharya (1973) and they have made elaborate discussions of the heat flow characteristics.

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The aim of this paper is to solve the heat energy equation when a non-Newtonian liquid flows past a porous infinite wall. In an earlier paper the present authors (1974) have obtained the expression for the velocity distribution due to the fluctuating flow of a non-Newtonian fluid past a porous flat plate with time-varying suction and the same expression for the velocity field has been used in the heat energy equation of this paper. The non-Newtonian liquid model has been described in the same paper in detail and therefore, we do not repeat the same here.

2. Mathematical Analysis

In this problem X'-axis is chosen along the two-dimensional infinite wall and Y'-axis perpendicular to it. Under these conditions, the flow is independent of x'. Hence the equations of heat energy can be written as

$$\rho' c'\left[\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}\right] = k'\frac{\partial^2 T'}{\partial y'^2}+\phi', \qquad \dots (1)$$

where T' is the temperature, c' and k' are respectively the specific heat and thermal conductivity. ϕ' is the dissipation function given by

$$\phi' = \mu\left(\frac{\partial u'}{\partial y'}\right)^2. \qquad \dots (2)$$

But from continuity condition

$$v' = -v_0'(1+\epsilon A\exp(i\omega' t')), \qquad \dots (3)$$

In the liquid considered in this paper

$$\mu = \mu_0\left[1-\alpha\frac{\partial u'}{\partial y'}\right]. \qquad \dots (4)$$

With the transformations

$$\left.\begin{aligned} &\eta = \frac{y' v_0'}{\nu}, \quad t = \frac{v_0'^2 t'}{4\nu}, \quad \omega = \frac{4\nu\omega}{v_0'^2}, \quad u = \frac{u'}{U_0'}, \\ &R = \frac{2\alpha U_0' v_0'}{\nu}, \quad T = \frac{T'-T'_\infty}{T'_\infty}, \quad \lambda = \frac{k'}{\rho' c'}, \\ &\sigma = \frac{\nu}{\lambda}, \quad E = \frac{U_0'^2}{c' T^1}. \end{aligned}\right\} \qquad \dots (5)$$

We get from eq. (1).

$$\frac{\partial^2 T}{\partial \eta^2}+\sigma[1+\epsilon A\exp(i\omega t)]\frac{\partial T}{\partial \eta}-\frac{1}{4}\sigma\frac{\partial T}{\partial t}$$

$$= -\sigma E\left[\left(\frac{\partial u}{\partial \eta}\right)^2-\tfrac{1}{2}R\left(\frac{\partial u}{\partial \eta}\right)^3\right], \qquad \dots (6)$$

where

$$u = f_1(\eta)+\epsilon \exp(i\omega t)f_2(\eta), \quad \ldots (7)$$

$$f_1(\eta) = (1-\exp(\eta))\{1+\tfrac{1}{2}R \exp(-\eta)\}+0(R^2), \quad \ldots (8)$$

$$f_2(\eta) = 1-S\exp(-h\eta)-(1-S)\exp(-\eta)$$
$$+R[\tfrac{1}{2}(1-S)(\exp(-\eta)-\exp(-h\eta))+P(\exp(-h\eta)-\exp(-2\eta))$$
$$+Q(\exp(-h\eta)-\exp(-(1+h)\eta))]+0(R^2), \quad \ldots (9)$$

$$\left.\begin{aligned} S &= 1-\frac{4iA}{\omega} \quad h = \tfrac{1}{2}[1+(1+i\omega)^{\frac{1}{2}}], \\ P &= \frac{A+2(1-S)}{2-\frac{i\omega}{4}}, \quad Q = \frac{hS(1+h)}{h^2+h-\frac{1}{4}i\omega} \end{aligned}\right\} \quad \ldots (10)$$

and σ, E are resepctively the Prandtl number and Eckert number. It is assumed that there is no heat transfer between the fluid and the wall, which lead to the following boundary conditions

$$\frac{\partial T}{\partial \eta} = 0 \text{ at } \eta = 0 \text{ and } T = 0 \text{ at } \eta = \infty. \quad \ldots (11)$$

We take

$$T(\eta, t) = T_0(\eta)+\epsilon \exp(i\omega t)T_1(\eta). \quad \ldots (12)$$

For the heat transfer problem we take the terms up to first power of R in eq.(7). Substituting eqs. (7) and (12) into eq. (6) and comparing harmonic terms and neglecting coefficients of ϵ^2, we obtain

$$T_0''+\sigma T_0' = -E(f_1'^2-\tfrac{1}{2}Rf_1'^3) \quad \ldots (13)$$

$$T_1''+\sigma(T_1'+AT_0')-\frac{i\omega}{4}\sigma T_1 = -\sigma E(2f_1'f_2'-\tfrac{3}{2}Rf_1'^2f_2'). \quad \ldots (14)$$

Boundary conditions (11) now reduce to

$$\left.\begin{aligned} T_n' &= 0 \quad \text{at} \quad \eta = 0 \\ T_n &= 0 \quad \text{as} \quad \eta = \infty \end{aligned}\right\} \quad n \geqslant 0 \quad \ldots (15)$$

Solutions of eqs. (13) and (14) subject to boundary conditions (15) are

$$T_0 = -\frac{(2\sigma+\sigma R-\sigma)E}{2(\sigma-2)(\sigma-3)}\exp(-\sigma\eta)+\frac{\sigma E}{2(\sigma-2)(\sigma-3)}[(\sigma-3)(1-R\exp(-2\eta)$$
$$+(\sigma-2)R\exp(-3\eta)]. \quad \ldots (16)$$

$$T_1 = \frac{2Ai\sigma(2\sigma+\sigma R-\sigma)E}{m_1\omega(\sigma-2)(\sigma-3)}\,[\sigma\exp(-m_1\eta)-m_1\exp(-\sigma\eta)]-\frac{A\sigma^2E}{2m_1(\sigma-2)(\sigma-3)}\times$$

$$\left[\frac{2(\sigma-3)(1-R)}{\xi}(2\exp(-m_1\eta)-m_1\exp(-2\eta))+\frac{3(\sigma-2)R}{\zeta}(3\exp(-m_1\eta)\right.$$

$$\left.-m_1\exp(-3\eta))\right]$$

$$+\frac{\sigma E}{m_1}\left[\frac{2(1-S)(1-R)}{\xi}(2\exp(-m_1\eta)-m_1\exp(-2\eta))+\frac{R}{\zeta}\left(4P+\frac{1-S}{2}\right)\right.$$

$$\left.(3\exp(-m_1\eta)-m_1\exp(-3\eta))\right]$$

$$+\frac{h}{\Lambda}\{2S(1-R)+R(1-2P-2Q)\}\{(1+h)\exp(-m_1\eta)-m_1\exp(-1+h)\eta\}$$

$$+\frac{R}{\psi}\left\{2(1+h)Q+\frac{Sh}{2}\right\}\{(2+h)\exp(-m_1\eta)-m_1\exp(-(2+h)\eta)\}], \qquad \ldots (17)$$

where

$$\xi = 4-2\sigma-\frac{i\omega\sigma}{4}\,; \quad \zeta = 9-3\sigma-\frac{i\omega\sigma}{4},$$

$$\Lambda = (1+h)^2-\sigma(1+h)-\frac{i\omega\sigma}{4},$$

$$\psi = (2+h)^2-\sigma(2+h)-\frac{i\omega\sigma}{4},$$

$$m_1 = \tfrac{1}{2}\sigma\left\{1+\left(1+\frac{i\omega}{\sigma}\right)^{\frac{1}{2}}\right\}\,; \quad n_1 = \tfrac{1}{2}\left\{1+\left(1+\frac{2i\omega}{2}\right)^{\frac{1}{2}}\right\}.$$

We can now write

$$T = T_0+\epsilon\exp(i\omega t)(T_{1r}+iT_{1i}), \qquad \ldots (18)$$

where T_{1r} and T_{1i} are the real and imaginary parts of T_1. The numerical computation has been made to draw the graphs for T_{1r}, T_{1i}.

3. Discussion

Figure 1 shows that the non-Newtonian parameter R decreases the temperature at any point. In a thin liquid layer near the plate the temperature increases very rapidly and then asymptotically falls.

Figure 2 shows that for low values of ω, the temperature becomes negative near the plate and very sharply increases in a very thin liquid layer and beyond this layer the temperature also sharply falls. As ω gradually increases, the temperature attains the maximum value at the plate and then decreases to zero asymptotically. But it is interesting to note that for all values of ω, the curves coincide and then asymptotically falls to zero.

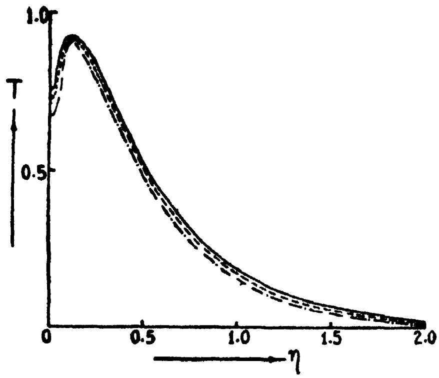

Fig. 1. Temperature distribution for different values of the non-Newtonian parameter
$A = 0.5, \quad \sigma = 10, \quad \omega = 10$
$E = 2, \quad \omega t = \pi/2, \quad \epsilon = 0.2$
$R = 0.0$——, $R = 0.05$— — — — — — — —
$R = 0.10$—.—.—.—.—.—.—.—

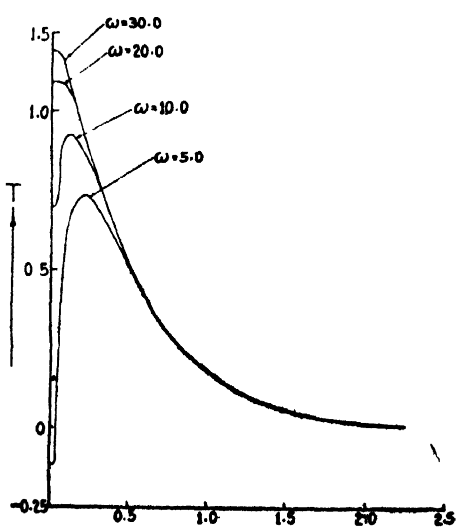

Fig. 2. Temperature distribution for different values of the frequency parameter
$\omega t = \pi/2, \quad R = 0.05, \quad \epsilon = 0.2$
$\sigma = 10, \quad E = 2, \quad A = 0.5$

In figure 3 it is noted that beyond a thin liquid layer, the curves for temperature profile for different values of the suction parameter A coincide and then tend to zero asymptotically. But within this layer, the temperature at any point decreases as A increases. For large values of A, the temperature at the plate becomes negative and then very sharply increases.

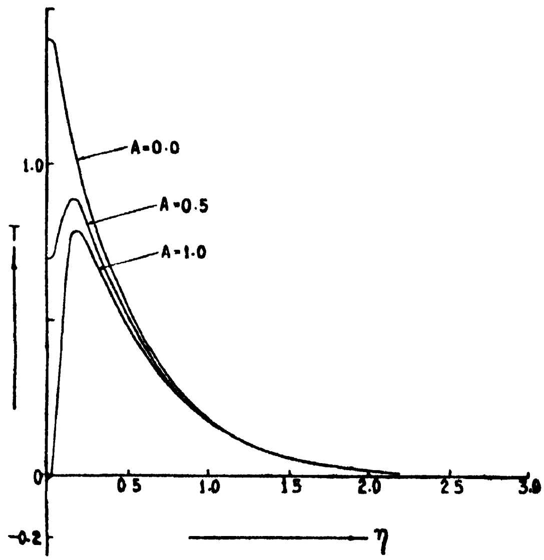

Fig. 3. Temperature distribution for different values of the variable suction parameter
$\epsilon = 0.2, \quad \omega = 10, \quad \omega t = \pi/2$
$R = 0.05\ E = 2, \sigma = 10$

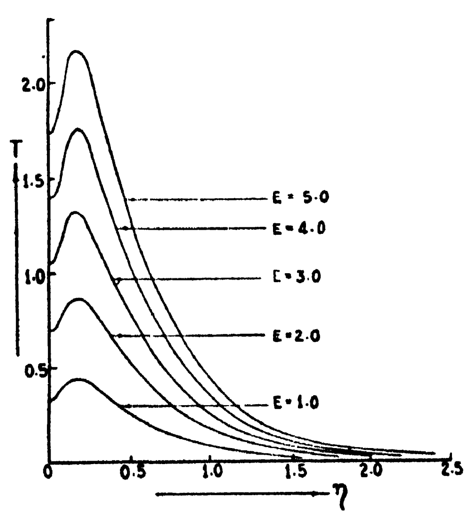

Fig. 4. **Temperature distribution for Different values of Eckert numbers**
$R = 0.05, \quad A = 0.5, \quad \omega t = \pi/2$
$\sigma = 10 \quad \omega = 10, \quad \epsilon = 0.2$

Figure 4 shows that the temperature at any point in the liquid increases as E increases. For large values of E, the increase of the temperature in a thin liquid layer at the plate very sharply increases and then asymptotically falls.

Figure 5 shows that for higher values of the Prandtl number, in a thin liquid layer near the plate, the temperature is negative and then very steeply increases to a maximum value whereafter, slowly falls to zero. The temperature at any point decreases with the increase in the value of σ.

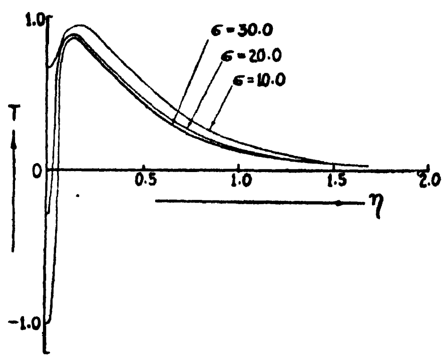

Fig. 5. Temperature distribution for different values of Prandtl number
$R = 0.05$, $A = 0.5$, $\epsilon = 0.2$, $\omega = 10$
$\omega t = \pi/2$, $E = 2$

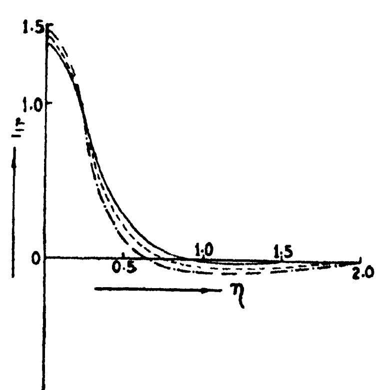

Fig. 6. Fluctuating part of the temperature profile
$E = 2.0$, $\sigma = 10.0$
$\omega = 10.0$, $A = 0.5$
$R = 0.00$———— $R = 0.05$- - - - - - - -
$R = 0.10$ —.—.—.—.—.—.—.—.—

Figures 6 and 7 are the fluctuating parts T_{1r}, T_{1i} of the temperature. Near the plate T_{1r} increases as the non-Newtonian parameter increases, but beyond a certain liquid layer the non-Newtonian parameter decreases the value of T_{1r}

at any point. It becomes negative at certain stage and then becomes zero. The behaviour of T_{1i} is almost the same as T_{1r} excepting the fact that it falls very steeply in a liquid layer near the plate.

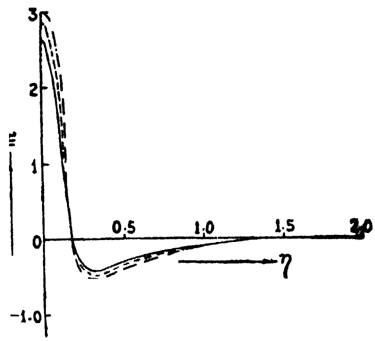

Fig. Fluctuating part of the temperature profile
$E = 2.0$, $\sigma = 10.0$ $\omega = 10$, $A = 0.5$
$R = 0.00$ ———————— $R = 0.05$ — — — — —
$R = 0.10$ —.—.—.—.—.—.—.—

4. Conclusions

The conclusions in the problem can be summarised as follows

(i) Non-Newtonian nature of the fluid decreases the temperature at any point in the fluid.

(ii) Temperature at a point near the wall increases with frequency of fluctuation of the liquid.

(iii) The variable suction parameter decreases the temperature at any point.

(iv) The temperature at any point increases with the Eckert number.

(v) The effect of the Prandtl number is to decrease the temperature at any point.

(vi) The non-Newtonian parameter increases the values of the fluctuating parts in a thin liquid layer at the plate and then an opposite effect takes place.

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