Indian J. Phys. **47**, 287–294 (1973)

Radial vibration of an aeolotropic cylindrical shell of varying density in a magnetic field

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(*Received* 4 *December* 1971)

In this paper, we have discussed the problem of vibration of cylindrical shell of aelotropic material of variable density for two different cases—first, when the density varies linearly and second, when it varies inversely as the radius vector.

1. INTRODUCTION

Yadava (1968) obtained the solution of the problem of vibration of a cylindrical shell in a magnetic field, the material of the shell being aeolotropic and density uniform. In this paper, the discussion has been extended to the problem of vibration of a cylindrical shell of aelotropic material of variable density. Two cases have been considered. The first, when the density varies linearly and the second, when it varies inversely as the radius vector. Such problems of magneto-elastic vibrations are of much importance in view of increasing investigations on radiation of electromagnetic energy into the vacuum adjacent to magneto-elastic bodies.

2. THE PROBLEM, FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

We consider an aeolotropic, perfectly conducting cylindrical shell of inner and outer radii r_1 and r_2 respectively and the space outside the shell to be vacuum. We consider the boundary of the shell to be mechanically stress free. Initially there exists an axial magnetic field of intensity H in the shell. Then the constitutive relations for aeolotropic bodies in cylindrical coordinates (r, θ, z) as given by Love (1944) are,

$$\begin{aligned}\sigma_{rr} &= c_{11}e_{rr}+c_{12}e_{\theta\theta}+c_{13}e_{zz}\\ \sigma_{\theta\theta} &= c_{21}e_{rr}+c_{22}e_{\theta\theta}+c_{23}e_{zz}\\ \sigma_{zz} &= c_{31}e_{rr}+c_{32}e_{\theta\theta}+c_{33}e_{zz}\end{aligned} \quad \dots \quad (2.1)$$

where σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} and e_{rr}, $e_{\theta\theta}$, e_{zz} are the components of stress and strain respectively. The equations of magneto-elasticity for a perfect conductor with unit permeability as deduced by Kailiski (1963) are,

$$\frac{\partial}{\partial r}(\sigma_{rr})+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}+\frac{1}{4\pi}\,[\mathrm{rot\,rot}(\boldsymbol{u}\times\boldsymbol{H})]\times\boldsymbol{H} = \rho\frac{\partial^2 u_r}{\partial t^2} \quad (2.2)$$

$$E = \frac{1}{c}\left(\frac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}\right), \quad \boldsymbol{h} = \text{rot}(\boldsymbol{u} \times \boldsymbol{H}) \qquad \ldots \quad (2.3)$$

where $\boldsymbol{u}$ is the mechanical displacement vector, $\boldsymbol{E}$ the electric intensity vector and $\boldsymbol{h}$ is the perturbation in the magnetic intensity vector.

The equations of electromagnetic field in vacuum are,

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E}^* = 0 \qquad (2.4)$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{h}^* = 0 \qquad (2.5)$$

$$\text{rot}\,\boldsymbol{E}^* = -\frac{1}{c}\frac{\partial \boldsymbol{h}^*}{\partial t} \qquad (2.6)$$

$$\text{rot}\,\boldsymbol{h}^* = \frac{1}{c}\frac{\partial \boldsymbol{E}^*}{\partial t}, \quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \qquad (2.7)$$

where $\boldsymbol{E}^*$, $\boldsymbol{h}^*$ denote the values of quantities $\boldsymbol{E}$ and $\boldsymbol{h}$, respectively, in vacuum. For radial vibration, we have,

$$u_\theta = u_z = 0, \qquad u_r = Ue^{i\omega t} \qquad (2.8)$$

$$e_{rr} = \frac{\partial U}{\partial r}e^{i\omega t}, \quad e_{\theta\theta} = \frac{U}{r}e^{i\omega t}, \quad e_{zz} = 0. \qquad (2.9)$$

Also the other corresponding quantities are,

$$\begin{aligned} h_r^* &= h_\theta^* = 0, & h_z^* &= h^* = Ve^{i\omega t} \\ H_r &= H_\theta = 0, & H_z &= H_1 \\ E_r^* &= E_\theta^* = 0, & E_z^* &= E^* = We^{i\omega t} \end{aligned} \qquad (2.10)$$

where U, V, W are functions of r alone. The equation (2.3) gives,

$$E = \frac{H_1}{c}\frac{\partial U}{\partial t} = \frac{i\omega}{c}H_1 U e^{i\omega t}$$

$$h = -\frac{H_1}{r}\frac{\partial}{\partial r}(r\dot{U}) = -H_1\left(\frac{\partial U}{\partial r} + \frac{U}{r}\right)e^{i\omega t}.$$

From (2.5) and (2.7) we get,

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\omega^2}{c^2}V = 0 \qquad .. \quad (2.12)$$

$$W = \frac{ic}{\omega}\frac{\partial V}{\partial r}. \qquad .. \quad (2.13)$$

The boundary conditions on the surface are,

$$\begin{aligned} \sigma_{rr}+T_{rr} &= T_{rr}^* && \text{on } r = r_1 \\ \sigma_{rr}+T_{rr} &= T_{rr}^* && \text{on } r = r_2 \\ E &= E^* && \text{on } r = r_1 \\ E &= E^* && \text{on } r = r_2 \end{aligned} \qquad \ldots \quad (2.14)$$

where T_{rr}, T_{rr}^* are Maxwell tensors in the shell and vacuum respectively and can be expressed as

$$T_{rr} = -\frac{H_1}{4\pi} h = \frac{H_1^2}{4\pi}\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right) e^{i\omega t}$$

$$T_{rr}^* = -\frac{H_1}{4\pi} h^* = -\frac{H_1}{4\pi} V e^{i\omega t} \qquad \ldots \quad (2.15)$$

while the elastic stress tensor σ_{rr} is expressible as,

$$\sigma_{rr} = \left(c_{11}\frac{\partial U}{\partial r} + c_{12}\frac{U}{r}\right) e^{i\omega t} \qquad \ldots \quad (2.16)$$

3. Method of Solution

Case 1. Let us assume,

$$\rho = \rho_0 r \qquad \ldots \quad (3.1)$$

where ρ_0 is a constant. The equation (2.2) with the help of (2.1), (2.8), (2.9) and (3.1) becomes,

$$\frac{\partial^2 U}{\partial r^2} + \left(1+\frac{c_{12}-c_{21}}{c_{11}+H_1^2/4\pi}\right)\frac{1}{r}\frac{\partial U}{\partial r} - \frac{c_{22}}{c_{11}+H_1^2/4\pi}\frac{U}{r^2} + \frac{\rho_0\omega^2 U r}{c_{11}+H_1^2/4\pi} = 0 \qquad \ldots \quad (3.2)$$

Putting $U = r^{-\frac{1}{2}}\xi$ we get,

$$\frac{\partial^2 \xi}{\partial r^2} + \left(\frac{c_{12}-c_{21}}{c_{11}+H_1^2/4\pi}\right)\frac{1}{r}\frac{\partial \xi}{\partial r} - \frac{m^2}{r^2}\xi + k^2\xi r = 0 \qquad \ldots \quad (3.3)$$

where

$$K^2 = \frac{\rho_0\omega^2}{c_{11}+H_1^2/4\pi}, \quad m^2 = \frac{1}{4}\left[\frac{4c_{22}+2c_{12}-2c_{21}}{c_{11}+H_1^2/4\pi} - 1\right.$$

Putting $z = 2/3Kr^{3/2}$ we get,

$$\frac{\partial^2 \xi}{\partial z^2} + \frac{1-\alpha}{z}\frac{\partial \xi}{\partial z} + \left(1-\frac{\lambda^2}{z^2}\right)\xi = 0, \qquad (3.4)$$

where

$$\alpha = \tfrac{2}{3}\left(1-\frac{c_{12}-c_{21}}{c_{11}+H_1{}^2/4\pi}\right)$$

and

$$\lambda^2 = 4m^2$$

Then the solution of (3.4) is

$$\xi = z^{\alpha/2}[AJ_n(z)+BY_n(z)] \qquad \dots \quad (3.5)$$

where $n^2 = \dfrac{4\lambda^2-\alpha^2}{4}$, A, B are constants and J_n, Y_n are Bessel functions of first and second kind. Consequently, the solution of (3.2) is

$$U = r^{3\alpha-2/4}[A_1J_n(\tfrac{2}{3}Kr^{3/2})+B_1Y_n(\tfrac{2}{3}Kr^{3/2})] \qquad (3.6)$$

where A_1, B_1 are constants given by

$$A_1 = (\tfrac{2}{3}K)^{\alpha/2}.A, \qquad B_1 = (\tfrac{2}{3}K)^{\alpha/2}.B.$$

Making use of recurrence formula,

$$J_n'(z) = J_{n-1}(z)-\frac{n}{z}\,J_n(z)$$

$$Y_n'(z) = Y_{n-1}(z)-\frac{n}{z}\,Y_n(z)$$

σ_{rr}, T_{rr} may be calculated with the help of (3 6). We have,

$$\sigma_{rr} = r^{3/4(\alpha-2)}[A_1\{Kr^{3/2}c_{11}J_{n-1}(\tfrac{2}{3}Kr^{3/2})+\nu J_n(\tfrac{2}{3}Kr^{3/2})\}$$
$$+B_1\{Kr^{3/2}c_{11}Y_{n-1}(\tfrac{2}{3}Kr^{3/2})+\nu Y_n(\tfrac{2}{3}Kr^{3/2})\}]e^{i\omega t} \qquad (3.7)$$

where

$$\nu = \frac{4c_{12}-(6n-3\alpha+2)c_{11}}{4}$$

and

$$T_{rr} = \frac{H_1{}^2}{4\pi}\,r^{3/4(\alpha-2)}\,[A_1\{Kr^{3/2}J_{n-1}(\tfrac{2}{3}Kr^{3/2})+\frac{2-6n+3\alpha}{4}\,J_n(\tfrac{2}{3}Kr^{3/2})]$$

$$B_1\{Kr^{3/2}Y_{n-1}(\tfrac{2}{3}Kr^{3/2})+\frac{2-6n+3\alpha}{4}\,Y_n(\tfrac{2}{3}Kr^{3/2})\}]\,e^{i\omega t}.$$

$$\dots \quad (3.8)$$

From (2.11) and (3.6) we get,

$$E = \frac{i\omega}{c}H_1r^{(3\alpha-2)/4}[A_1J_n(\tfrac{2}{3}Kr^{3/2})+B_1Y_n(\tfrac{2}{3}Kr^{3/2})]\,e^{i\omega t}. \qquad \dots \quad (3\cdot9)$$

The solution of (2.12) with conditions appropriate to the problem is,

$$V = CY_0\left(\frac{\omega r}{c}\right) \quad \text{for } r \geqslant r_2,$$
$$= DI_0\left(\frac{\omega r}{c}\right) \quad \text{for } r \leqslant r_1, \qquad \dots (3.10)$$

where Y_0 and I_0 are Bessel functions of order zero. From (2.15) we get,

$$T_{rr}^* = -\frac{H_1}{4\pi}CY_0\left(\frac{\omega r}{c}\right)e^{i\omega t} \quad \text{on } r \geqslant r_2$$
$$T_{rr}^* = -\frac{H_1}{4\pi}DI_0\left(\frac{\omega r}{c}\right)e^{i\omega t} \quad \text{on } r \leqslant r_1. \qquad \dots (3.11)$$

From (3.10), (2.13) and (2.10) we have,

$$E^* = iCY_1\left(\frac{\omega r}{c}\right)e^{i\omega t} \quad \text{on } r \geqslant r_2$$
$$E^* = iDI_1\left(\frac{\omega r}{c}\right)e^{i\omega t} \quad \text{on } r \leqslant r_1.$$

The boundary conditions (2.14) yields

$$A_1\{\theta_1 r^{3/2}J_{n-1}(\tfrac{2}{3}Kr_1^{3/2})+\theta_2 J_n(\tfrac{2}{3}Kr_1^{3/2})\}+B_1\{\theta_1 r_1^{3/2}Y_{n-1}(\tfrac{2}{3}Kr_1^{3/2})+$$
$$\theta_2 Y_n(\tfrac{2}{3}Kr_1^{3/2})\}+\frac{H_1}{4\pi r_1^{3/4(\alpha-2)}}DI_0\left(\frac{\omega r_r}{c}\right)=0$$

where (3.13)

$$\theta_1 + K\left(c_{11} = \frac{H_1^2}{4\pi}\right), \quad \theta_2 = \nu + \frac{H_1^2}{4\pi}\left(\frac{2-6n+3\alpha}{4}\right),$$

$$A_1\{\theta_1 r_2^{3/2}J_{n-1}(\tfrac{2}{3}Kr_2^{3/2})+\theta_2 J_n(\tfrac{2}{3}Kr_2^{3/2})\}+B_1\{\theta_1 r_2^{3/2}Y_{n-1}(\tfrac{2}{3}Kr_2^{3/2}$$
$$+\theta_2 Y_n(\tfrac{2}{3}Kr_2^{3/2})\}+\frac{H_1}{4\pi r_2^{3/4(\alpha-2)}}CY_0\left(\frac{\omega r_2}{c}\right)=0 \qquad (3.14)$$

$$A_1 J_n(\tfrac{2}{3}Kr_1^{3/2})+B_1 Y_n(\tfrac{2}{3}Kr_1^{3/2})-\frac{C}{\omega H_1 r_1^{(3\alpha-2)/4}}DI_1\left(\frac{\omega r_1}{c}\right)=0 \qquad (3.15)$$

$$A_1 J_n(\tfrac{2}{3}Kr_2^{3/2})+B_1 Y_n(\tfrac{2}{3}Kr_2^{3/2})-\frac{c}{\omega H_1 r_2^{(3\alpha-2)/4}}CY_1\left(\frac{\omega r_2}{c}\right)=0. \qquad \dots (3.16)$$

Eliminating A_1, B_1, C, D from (3.13) to (3.16) we obtain the frequency equation as,

$$\frac{\left[\frac{H_1 r_1^{3/4(2-\alpha)}}{4\pi} I_0\left(\frac{\omega r_1}{c}\right)\right]\left[Y_n(\tfrac{2}{3}Kr_1^{3/2})\right]+\left[\frac{c r_1^{(2-3\alpha)/4}}{\omega H_1} I_1\left(\frac{\omega r_1}{c}\right)\right]\left[\theta_1 r_1^{3/2} Y_{n-1}(\tfrac{2}{3}Kr_1^{3/2})+\theta_2 Y_n(\tfrac{2}{3}Kr_1^{3/2})\right]}{[\theta_1 r_1^{3/2} J_{n-1}(\tfrac{2}{3}Kr_1^{3/2})+\theta_2 J_n(\tfrac{2}{3}Kr_1^{3/2})]\left[\frac{c}{\omega H_1 r_1^{(3\alpha-2)/4}} I_1\left(\frac{\omega r_1}{c}\right)\right]-\left[\frac{H_1 r_1^{3/4(2-\alpha)}}{4\pi} I\left(\frac{\omega r_1}{c}\right)\right] J_n(\tfrac{2}{3}Kr_1^{5/2})}$$

$$=\frac{\left[\frac{H_1}{4\pi} r_2^{3/4(2-\alpha)} Y_0\left(\frac{\omega r_2}{c}\right)\right]\left[Y_n(\tfrac{2}{3}Kr_2^{3/2})\right]+\left[\theta_1 r_2^{3/2} Y_{n-1}(\tfrac{2}{3}Kr_2^{3/2})+\theta_2 Y_n(\tfrac{2}{3}Kr_2^{3/2})\right]\frac{c}{\omega H_1 r_2^{(3\alpha-2)/4}} Y_n\left(\frac{\omega r_2}{c}\right)}{\left[\frac{c}{\omega H_1 r_2^{(3\alpha-2)/4}} Y_1\left(\frac{\omega r_2}{c}\right)\right]\left[\theta_1 r_2^{3/2} J_{n-1}(\tfrac{2}{3}Kr_2^{3/2})+\theta_2 J_n(\tfrac{2}{3}Kr_2^{3/2})\right]+\left[J_n(\tfrac{2}{3}Kr_2^{3/2})\right]\frac{H_1}{4\pi r_2^{3/4(\alpha-2)}} Y_0\left(\frac{\omega r_2}{c}\right)} \quad \ldots \quad (3.17)$$

Case 2. Let us assume,

$$\rho_0 = \frac{\rho_0}{r} \quad \ldots \quad (3.11)$$

where ρ_0 is a constant. The equation (2.2) with the help of (2.1), (2.8), (2.9) and (3.18) becomes,

$$\frac{\partial^2 U}{\partial r^2}+\left(1+\frac{c_{12}-c_{21}}{c_{11}+\frac{H_1^2}{4\pi}}\right)\frac{1}{r}\frac{\partial U}{\partial r}-\frac{c_{22}}{c_{11}+\frac{H_1^2}{4\pi}}\frac{U}{r_2}+\frac{\rho_0\omega^2}{c_{11}+\frac{H_1^2}{4\pi}}\frac{U}{r}=0 \quad \ldots \quad (3.19)$$

Putting $U = r^{-\frac{1}{2}}\eta$ we get as in the previous case,

$$\frac{\partial^2\eta}{\partial r^2}+\frac{(c_{12}-c_{21})}{c_{11}+\frac{H_1^2}{4\pi}}\frac{1}{r}\frac{\partial\eta}{\partial r}-\frac{m^2}{r^2}\eta+K^2\frac{\eta}{r}=0 \quad \ldots \quad (3.20)$$

Putting $z = 2Kr^{\frac{1}{2}}$ we get,

$$\frac{\partial^2\eta}{\partial r^2}+\frac{1-\beta}{z}\frac{\partial\eta}{\partial z}+\left(1-\frac{\nu^2}{z^2}\right)\eta=0 \quad \ldots \quad (3.21)$$

where,

$$\beta=\frac{3}{2}-\frac{c_{12}-c_{21}}{c_{11}+\frac{H_1^2}{4\pi}}, \quad \nu^2=4m^2.$$

The solution of (3.21) is,

$$\eta = z^{\beta/2}[AJ_n(z)+BY_n(z)] \qquad \dots \quad (3.22)$$

where A and B are constants and J_n, Y_n are Bessel functions of order n. Consequently the solution of (3.19) becomes,

$$U = r^{(3\beta-2)/4}[A_1J_n(2Kr^{\frac{1}{2}})+B_1Y_n(2Kr^{\frac{1}{2}})] \qquad \dots \quad (3.23)$$

where A_1, B_1 are constants given by

$$A_1 = (\tfrac{1}{2}K)^{\beta/2}A, \qquad B_1 = (\tfrac{1}{2}K)^{\beta/2}.B.$$

Then σ_{rr} and T_{rr} in this case may be calculated with the help of (3.23) as,

$$\sigma_{rr} = r^{3/4(\beta-6)}[A_1\{c_{11}Kr^{\frac{1}{2}}J_{n-1}(2Kr^{\frac{1}{2}})+\delta J_n(2Kr^{\frac{1}{2}})\} + B_1\{c_{11}Kr^{\frac{1}{2}}Y_{n-1}(2Kr^{\frac{1}{2}})+\delta Y_n(2Kr^{\frac{1}{2}})] \, e^{i\omega t} \qquad \dots \quad (3.24)$$

where,
$$\delta = \frac{\beta-2n-2}{4}\, c_{11}+c_{12}$$

and

$$T_{rr} = \frac{H_1^2}{4\pi}\, r^{3(\beta-6)/4}\left[A_1\left\{Kr^{\frac{1}{2}}J_{n-1}(2Kr)^{\frac{1}{2}}+\frac{\beta-2n+2}{4}\,J_n(2Kr^{\frac{1}{2}})\right\} + B_1\left\{Kr^{\frac{1}{2}}Y_{n-1}(2Kr^{\frac{1}{2}})+\frac{\beta-2n+2}{4}\,Y_n(2Kr^{\frac{1}{2}})\right\}\right]e^{i\omega t} \qquad \dots \quad (3.25)$$

$$E = \frac{i\omega}{c}H_1r^{(\beta-2)/4}[A_1J_n(2Kr^{\frac{1}{2}})+B_1Y_n(2Kr^{\frac{1}{2}})] \, e^{i\omega t}. \qquad \dots \quad (3.26)$$

$T_{rr}{}^*$ and E^* however, remain the gme as in case 1. The boundary concdition (2 14) yields,

$$A_1\{\phi_1r_1^{\frac{1}{2}}J_{n-1}(2Kr_1^{\frac{1}{2}})+\phi_2J_n(2Kr_1^{\frac{1}{2}})\}+B_1\{\phi_1r_1^{\frac{1}{2}}Y_{n-1}(2Kr_1^{\frac{1}{2}})+\phi_2Y_n(2Kr_1^{\frac{1}{2}})\} + \frac{H_1}{4\pi}\, r_1^{3(6-\beta)/4}\, DI_0\left(\frac{\omega r_1}{c}\right) = 0 \qquad \dots \quad (3.27)$$

where,

$$\phi_1 = \left(c_{11}+\frac{H_1^2}{4\pi}\right)K, \qquad \phi_2 = \delta+\frac{\beta-2n+2}{4}$$

$$A_1\{\phi_1r_2^{\frac{1}{2}}J_{n-1}(2Kr_2^{\frac{1}{2}})+\phi_2J_n(2Kr_2^{\frac{1}{2}})\}+B_1\{\phi r_2^{\frac{1}{2}}Y_{n-1}(2Kr_2^{\frac{1}{2}})+\phi_2Y_n(2Kr_2^{\frac{1}{2}}) - \frac{H_1}{4\pi}\, r_2^{3(6-\beta)/4}CY_0\left(\frac{\omega r_2}{c}\right) = 0 \qquad \dots \quad (3.28)$$

$$A_1J_n(2Kr_1^{\frac{1}{2}})+B_1Y_n(2Kr_1^{\frac{1}{2}})-\frac{c}{\omega H_1}\, r_1^{(2-\beta)/4}DI_1\left(\frac{\omega r_1}{c}\right) = 0 \qquad \dots \quad (3.29)$$

$$A_1J_n(2Kr_2^{\frac{1}{2}})+B_1Y_n(2Kr_2^{\frac{1}{2}})-\frac{c}{\omega H_1r_2^{(\beta-2)/4}}\, CY_1\left(\frac{\omega r_2}{c}\right) = 0. \qquad \dots \quad (4.30$$

Eliminating A_1, B_1, C and D from (3.27), (3.28), (3.29) and (3.30) we get the frequency equation as,

$$\left[\frac{H_1}{4\pi} r_1^{(6-\beta)/4} I_0\left(\frac{\omega r_1}{c}\right)\right]\left[Y_n(2k_1 r^{\frac{1}{2}})\right]+\left[\frac{c}{\omega H_1} r_1^{(2-\beta)/4} I_1\left(\frac{\omega r_1}{c}\right)\right]$$

$$\left[\phi_1 r_1^{\frac{1}{2}} Y_{n-1}(2Kr_1^{\frac{1}{2}})+\phi_2 Y_n(2Kr_1^{\frac{1}{2}})\right]$$

$$\left[\phi_1 r_1^{\frac{1}{2}} J_{n-1}(2Kr_1^{\frac{1}{2}})+\phi_2 J_n(2Kr_1^{\frac{1}{2}})\right]\left[\frac{c}{\omega H_1} r_1^{(2-\beta)/4} I_1\left(\frac{\omega r_1}{c}\right)\right]$$

$$+\left[\frac{H_1}{4\pi} r_1^{(6-\beta)/4} I_0\left(\frac{\omega r_1}{c}\right)\right] J_n(2Kr_1^{\frac{1}{2}})$$

$$\left[\frac{H_1}{4\pi} r_2^{(6-\beta)/4} Y_0\left(\frac{\omega r_2}{c}\right)\right]\left[Y_n(2kr_2^{\frac{1}{2}})\right]-\left[\phi_1 r_2^{\frac{1}{2}} Y_{n-1}(2Kr_2^{\frac{1}{2}})+\phi_2 Y_n(2K_2 r^{\frac{1}{2}})\right]$$

$$\left[\frac{c}{\omega H_1} r_2^{(2-3\beta)/4} Y_1\left(\frac{\omega r_2}{c}\right)\right]$$

$$\left[\frac{c}{\omega H_1} r_2^{(2-3\beta)/4} Y_1\left(\frac{\omega r_2}{c}\right)\right]\left[\phi_1 r_2^{\frac{1}{2}} J_{n-1}(2Kr_2^{\frac{1}{2}})+\phi_2 J_n(2Kr_2^{\frac{1}{2}})\right]$$

$$-\left[J_n(2Kr_2^{\frac{1}{2}})\right]\frac{H_1}{4\pi} r_2^{(6-\beta)/4} Y_0\left(\frac{\omega r_2}{c}\right). \qquad \ldots (3.31)$$

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