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On the performance of slider bearing under fluctuating speed using a non-Newtonian visco-elastic fluid as lubricant

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In the present paper effect of fluctuations on the performance of a slider bearing has been considered using a non-Newtonian visco-elastic fluid as lubricant It has been obtained that load-carrying capacity of the bearing always exceeds its value in steady case except for a brief initial period. The effect of visco-elasticity has been found to be advantageous for the increase of load-carrying capacity.

1 INTRODUCTION

The problem of hydrodynamic lubrication of certain types of non-Newtonian fluids have been investigated in its theoretical and experimental aspects by many workers (Saibel 1962, Slibar & Paslay 1956, Shukla 1964 and Srivastava 1964) In the study of bearing lubrication of both Newtonian and non-Newtonian fluids, most of the problems have been investigated under the conditions of steady load and steady speed and literature on the performance of such bearings is almost exhaustive. Comparatively less attention has been paid towards the problems of bearings which have been subjected to the variable conditions of load and speed. The theoretical analysis of the problem concerning the variable load has been carried out by Harrison (1920) and Swift (1937) Ladanyi (1948) has given approximate solution for the effect of bearing acceleration during unsteady state of speed while Burwell (1947) has developed an analytical method for computing the behaviour of any dynamically loaded bearings.

The present problem aims to investigate the effect of slider fluctuations on the performance of bearing using a non-Newtonian visco-elastic fluid as lubricant Verma (1969) has investigated the same problem and has only obtained a modified Reynolds equation which includes the terms on account of slider oscillations. But no attempt has been made to discuss the pressure distribution, load-carrying capacity etc Moreover a term has been missing in his equation (2 7). The present problem has been solved using a technique different to that of Verma and taking into consideration the term dropped out by him. It is interesting to note that except for a brief initial period, the load-carrying capacity always exceeds its value in steady case Effect of visco-elasticity has been found to be advantageous in increase of load capacity.

2. Problem Formulation

The classical theory of lubrication has been developed by Fuller (1961), Pinkus & Sternlicht (1961) and Tipei (1962) to explain the fluid mechanical behaviour of lubricants acting as thin load-bearing films between almost—mating smooth metal surfaces moving relative to one another. The assumptions of the original theory are :

(1) the fluid is Newtonian, and of constant viscosity;

(2) the flow is laminar, and is dominated by viscous and pressure forces;

(3) the fluid is incompressible;

(4) the flow is steady;

(5) the pressure in the film is a function of the coordinates only measuring position on either surface, but not of the coordinate measuring position between opposing surfaces

The justification for these five assumptions is usually that the films are very thin and so the length scales in the plane of the surfaces are large compared with the film thickness. As the assumptions are independent, we have relaxed the assumptions (1) and (4) to include—

(a) non-Newtonian behaviour due to visco-elasticity;

(b) unsteadiness due to oscillating fluctuations.

The constitutive equations of incompressible second order visco-elastic fluid following Colueman & Noll (1960) are

$$\tau_{ij} = \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)ik} A_{(1)kj}, \qquad \ldots \quad (2.1)$$

where

$$A_{(1)ij} = v_{i,j} + v_{j,i},$$

$$A_{(2)ij} = a_{i,j} + a_{j,i} + 2v_{m,i} v_{m,j},$$

and

$$S_{ij} = \tau_{ij} - p g_{ij}, \qquad \ldots \quad (2.2)$$

such that S_{ij} is the stress tensor, g_{ij} the metric tensor, v_i and a_i the velocity and acceleration vectors, respectively, and p pressure, μ_1, μ_2 and μ_3 material constants.

The flow takes place between two almost parallel surfaces sliding past one another and spaced apart by a variable distance $h(x, y)$ and the bounding surfaces are $z = 0$ and $z = h(x, y)$.

3 Equations of Motion

With the usual assumptions of the lubrication flow, the equations of motion of a incompressible second-order fluid in cartesian coordinates, making use of

(2.1), (2.2) and equation of continuity, can be written as :

$$\rho\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\left[-p+(2\mu_2+\mu_3)\left(\frac{\partial u}{\partial z}\right)^2\right]+\mu_1\frac{\partial^2 u}{\partial z^2}+\mu_2\frac{\partial^3 u}{\partial t\partial z^2} \quad \dots \quad (3.1)$$

and

$$0=\frac{\partial}{\partial x}\left(\mu_1\frac{\partial u}{\partial z}+\frac{\partial}{\partial z}\right)-p+(2\mu_2+\mu_3)\left(\frac{\partial u}{\partial z}\right)^2\Big]. \quad \dots \quad (3.2)$$

Here, the first equation of (2.7) of Verma (1969) is corrected as (3.1) in which additional term $\mu_2\frac{\partial^3 u}{\partial t\partial z^2}$ occurs in the right hand side.

The above equations may be written as

$$\left.\begin{aligned}\rho\frac{\partial u}{\partial t}&=-\frac{\partial P}{\partial x}+\mu_1\frac{\partial^2 u}{\partial z^2}+\mu_2\frac{\partial^3 u}{\partial t\partial z^2},\\ 0&=-\frac{\partial P}{\partial z}+\frac{\partial}{\partial x}\left(\mu_1\frac{\partial u}{\partial z}\right),\end{aligned}\right\} \quad \dots \quad (3.3)$$

where P is the modified pressure (Bhatnagar 1961).

As the characteristic length in the x-direction is much greater than the characteristic length in the z-direction, hence $\frac{\partial}{\partial x}$ is small in comparison to $\frac{\partial}{\partial z}$ and thus second equation of the (3.3) reveals that $P=P(x)$ and (3.3) reduces to

$$\frac{\partial u}{\partial t}=-\lambda+\nu_1\frac{\partial^2 u}{\partial z^2}+\nu_2\frac{\partial^3 u}{\partial t\partial z^2}, \quad \dots \quad (3.4)$$

where $\lambda=-\frac{1}{\rho}\frac{\partial P}{\partial x}$, $\nu_1=\frac{\mu_1}{\rho}$ and $\nu_2=\frac{\mu_2}{\rho}$.

4. Solutions

We shall solve the equation (3.4) by the technique due to Lighthill (1949), i.e., we assume the expression for velocity as

$$u=u_0(z)+\epsilon u_1(z,t). \quad \dots \quad (4.1)$$

Substituting (4.1) in (3.4), we have

$$\epsilon\frac{\partial u_1}{\partial t}=-\lambda+\nu_1\left[\frac{\partial^2 u_0}{\partial z^2}+\epsilon\frac{\partial^2 u_1}{\partial z^2}\right]+\nu_2\left[\epsilon\frac{\partial^3 u_1}{\partial t\partial z^2}\right]. \quad \dots \quad (4.2)$$

On equating the coefficients of ϵ on both the sides of (4.2), we have following equations:

$$\frac{\partial^2 u_0}{\partial z^2}-\frac{\lambda}{\nu_1}=0, \qquad \dots (4.3)$$

and

$$\frac{\partial u_1}{\partial t}=\nu_1\frac{\partial^2 u_1}{\partial z^2}+\nu_2\frac{\partial^3 u_1}{\partial t\partial z^2}. \qquad \dots (4.4)$$

Equation (4.3) is to be solved subject to the following boundary conditions:

$$u_0=U; \qquad \text{when } z=0$$
$$u_0=0; \qquad \text{when } z=h.$$

Its solution is

$$u_0=\frac{1}{2\mu_1}\frac{\partial P}{\partial x}z(z-h)+\frac{U}{h}(h-z). \qquad \dots (4.5)$$

Now in order to solve the equation (4.4), we define the Laplace transform

$$\bar{u}_1(z,p)=\int_0^\infty \exp(-pt)u_1(z,t)dt. \qquad \dots (4.6)$$

Equation (4.4) is to be solved subject to the following boundary conditions

$$\left.\begin{array}{lll}\text{(a)} & u_1=U\exp(int) & \text{on } z=0;\ t>0\\ \text{(b)} & u_1=0 & \text{on } z=h,\ t>0\\ \text{(c)} & u_1=0 & \text{on } t=0.\end{array}\right\} \qquad \dots (4.7)$$

Multiplying both the sides of (4.4) by $\exp(-pt)$ and integrating between the limits 0 to ∞, we have the Laplace transform of (4.4) as

$$\frac{\partial^2\bar{u}_1}{\partial z^2}-\frac{p}{\nu_1+\nu_2 p}\bar{u}_1=0, \qquad \dots (4.8)$$

and transformed boundary conditions are

$$\left.\begin{array}{lll}\text{(a)} & \bar{u}_1=\dfrac{U}{p-in} & \text{on } z=0,\\ \text{(b)} & \bar{u}_1=0 & \text{on } z=h.\end{array}\right\} \qquad \dots (4.9)$$

Now the solution of (4.8) subject to (4.9) is

$$\bar{u}_1=\frac{U\sinh\left[\left(\frac{p}{(\nu_1+\nu_2 p)}\right)^{\frac{1}{2}}(h-z)\right]}{(p-in)\sinh\left[\left(\frac{p}{\nu_1+\nu_2 p)}\right)^{\frac{1}{2}}h\right]} \qquad \dots (4.10)$$

The inverse Laplace integral of (4.10) is evaluated by transforming the path of integration into a closed contour and applying the calculus of residues (Carslaw & Jaeger 1941). We obtain

$$u_1 = U \exp(int) \frac{\sinh[\lambda_0(h-z)]}{\sinh(\lambda_0 h)}$$

$$+\sum_{K=1}^{\infty} \frac{U(-1)\ 2\pi k\nu_1 \sin\left[k\pi\left(1-\frac{z}{h}\right)\right]\exp\left[-t\left(\frac{\nu_1 k^2\pi^2}{h^2+\nu_2 k^2\pi^2}\right)\right]}{\left(1+\frac{\nu_2 k^2\pi^2}{h^2}\right)[\nu_1 k^2\pi^2+in(h^2+\nu_2 k^2\pi^2)]} \quad \ldots \quad (4.11)$$

where

$$\lambda_0 = \left(\frac{n(\nu_2 n+i\nu_1)}{\nu_1^2+\nu_2^2 n^2}\right)^{1/2}.$$

Thus we have from equations (4 1), (4.5) and (4.11)

$$u = -\frac{1}{2\mu_1}\frac{\partial P}{\partial x} z(h-z)+\frac{U}{h}(h-z)+\epsilon\Big[U \exp(int)\frac{\sinh(\lambda_0(h-z))}{\sinh(\lambda_0 h)}$$

$$+\sum_{K=1}^{\infty} \sin\left(k\pi\left(1-\frac{z}{h}\right)\right)F(k,h,t)\Big] \quad \ldots \quad (4.12)$$

where

$$F(k,h,t) = \frac{(-1)kU2\pi k\nu_1 \exp\left[-t\left(\frac{\nu_1 k^2\pi^2}{h^2+\nu_2 k^2\pi^2}\right)\right]}{\left(1+\frac{\nu_2 k^2\pi^2}{h^2}\right)[\nu_1 k^2\pi^2+in(h^2+\nu_2\pi^2 k^2)]}.$$

5. Rate of Flow and Pressure Distribution

The flow rate Q is given by

$$Q = \int_0^h u\,dz \quad \ldots \quad (5.1)$$

Substituting (4.12) in (5.1) and integrating, we obtain

$$Q = \frac{Uh}{2}-\frac{h^3}{12\mu_1}\frac{\partial P}{\partial x}+\epsilon\left(-\frac{U\exp(int)}{\lambda_0}\left\{\frac{1}{\sinh(\lambda_0 h)}-\coth(\lambda_0 h)\right\}+\sum_{K=1}^{\infty}\frac{2k}{k\pi}Fk,h,t)\right), \quad \ldots \quad (5.2)$$

where k is an odd integer.

Applying the condition that there is no side leakage, we have

$$\frac{\partial Q}{\partial x} = \frac{U}{2}\frac{dh}{dx}-\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu_1}\frac{\partial P}{\partial x}-\epsilon\left\{-\frac{U\exp(int)}{\lambda_0}\left(\frac{1}{\sinh(\lambda_0 h)}-\coth(\lambda_0 h)\right)\right.\right.$$

$$\left.\left.+\sum_{K=1}^{\infty}\frac{2h}{k\pi}F(k,h,t)\right\}\right)=0, \quad \ldots \quad (5.3)$$

$$\frac{U}{2}\frac{dh}{dx}=\frac{\partial}{\partial x}\left(\frac{h^3}{12\mu_1}\frac{\partial P}{\partial x}-\epsilon\left\{-\frac{U\exp(int)}{\lambda_0}\left(\frac{1}{\sinh(\lambda_0 h)}-\coth(\lambda_0 h)\right)+\sum_{K=1}^{\infty}\frac{2h}{k\pi}F(k,h,t)\right\}\right). \quad \dots (5.4)$$

Equation (5.4) on integration yields

$$\frac{U}{2}(h-c_1)=\frac{h^3}{12\mu_1}\frac{\partial P}{\partial x}-\epsilon\left\{-\frac{U\exp(int)}{\lambda_0}\left(\frac{1}{\sinh(\lambda_0 h)}-\coth(\lambda_0 h)\right)+\sum_{K=1}^{\infty}\frac{2h}{K\pi}F(K,h,t)\right\},$$

where c_1 is constant of integration, or

$$\frac{\partial P}{\partial x}=6U\mu_1\left(\frac{1}{h^2}-\frac{c_1}{h^3}\right)+\frac{12\mu_1\epsilon}{h^3}\left(-\frac{U\exp(int)}{\lambda_0}\left\{\frac{1}{\sinh(\lambda_0 h)}-\coth(\lambda_0 h)\right\}+\sum_{K=1}^{\infty}\frac{2h}{k\pi}F(k,h,t)\right). \quad \dots (5.5)$$

Equation (5.5), which is similar to Reynold's (1886) equation for steady speed except that effects of slider fluctuations are also included, is the governing equation and all the operating characteristics of the flow may be derived from it.

6. Pressure Distribution and Load Capacity for Small Time

In order to obtain the pressure distribution, we collect the real part of equation (5.5) and integrate the equation so obtained. Further integration will yield us the expression for the load capacity.

On collecting the real part of (5.5) and approximating,

$$\exp\left(-t\left(\frac{\nu_1 k^2\pi^2}{h^2+\nu_2 k^2\pi^2}\right)\right)$$

for small values of t, we have

$$\frac{\partial P}{\partial x}=6\mu_1 U\left(\frac{1}{h^2}-\frac{c_1}{h^3}\right)-\frac{12\mu_1\epsilon U}{h^3}\Bigg[\frac{1}{l(b^2+c^2)\{\cosh(2bhl)-\cos(2chl)\}}$$
$$\times\{(b\cos nt+c\sin nt)[2\sinh(bhl)\cos(chl)-\sinh(2bhl)]$$
$$+(b\sin nt-c\cos nt)[2\cosh(bhl)\sin(chl)-\sin(2chl)]\}$$
$$+\sum_{K=1}^{\infty}\frac{4\pi^2\nu_1^2k^2h^3\left\{1-t\left(\frac{\nu_1k^2\pi^2}{h^2+\nu_2k^2\pi^2}\right)\right\}}{(h^2+\nu_2k^2\pi^2)\{\nu_1^2k^4\pi^4+n^2(h^2+\nu_2k^2\pi^2)^2\}}\Bigg] \quad \dots (6.1)$$

where

$$b = [\tfrac{1}{2}\{(\nu_1^2+\nu_2^2n^2)^{\frac{1}{2}}+n\}]^{\frac{1}{2}},$$

$$c = [\tfrac{1}{2}\{(\nu_1^2+\nu_2^2n^2)^{\frac{1}{2}}-n\}]^{\frac{1}{2}},$$

and

$$\frac{1}{t} = \left(\frac{n}{\nu_1^2+\nu_2^2n^2}\right)^{1/2}.$$

Now as h is small, equation (6.1) can further be simplified by neglecting the terms of order h and higher. We thus obtain

$$\frac{\partial P}{\partial x} = 6\mu_1 UB\left(\frac{1}{h^2}-\frac{c_1}{h^3}\right)+12\mu_1 UB\epsilon\left(\frac{\cos nt}{2h^2}+\right.$$

$$\left.\left\{\frac{1^2}{24}[2bc\sin nt+3\cos nt(b^2-c^2)]-\frac{4\nu_1^2\left(1-t\frac{\nu_1}{\nu_2}\right)}{\nu_2\pi^4(\nu_1^2+\nu_2^2n^2)}\right\}\right) \qquad (6.2)$$

where $\bar{x} = x/B$.

In the case of slider bearing with a film given by the equation

$$h = h_2(a-a\bar{x}+\bar{x}), \quad a = \frac{h_1}{h_2}, \qquad (6.3)$$

we have

$$\frac{\partial P}{\partial x} = \frac{6\mu_1 UB}{h_2^2}\left(\frac{1}{(a-a\bar{x}+\bar{x})^2}-\frac{c_1}{h_2(a-a\bar{x}+\bar{x})^3}\right)+\frac{12\mu_1 UB\epsilon}{h_2^2}\left(\frac{1}{2}\frac{\cos nt}{(a-ax+x)^2}\right.$$

$$\left.+h_2^2\left\{\frac{1^2}{24}[2bc\sin nt+3\cos nt(b^2-c^2)]-\frac{4\nu_1^2\left(1-t.\frac{\nu_1}{\nu_2}\right)}{\nu_2\pi^4(\nu_1^2+\nu_2^2n^2)}\right\}\right). \qquad \dots \quad (6.4)$$

By using the boundary conditions $P(0) = P(1) = 0$, the integrated pressure is

$$P = 6\mu_1 UB \quad (a-1)\bar{x}(1-\bar{x}) \quad +12\mu_1 UB\epsilon\left(\cos nt\times\right.$$

$$\frac{(a-1)\bar{x}(1-\bar{x})}{(a+1)(a-a\bar{x}+\bar{x})^2}-\frac{h_2^2}{(a^2-1)}\left\{\frac{a^2}{(a-a\bar{x}+\bar{x})^2}-\bar{x}(a^2-1)-1\right\}$$

$$\left.\times\left\{\frac{1^2}{24}[2bc\sin nt+3\cos nt(b^2-c^2)]-\frac{4\nu_1^2\left(1-t.\frac{\nu_1}{\nu_2}\right)}{\nu_2\pi^4(\nu_1^2+\nu_2^2n^2)}\right\}\right) \qquad \dots \quad (6.5)$$

Having obtained pressure distribution, Load capacity W can be obtained from the following:

$$W = LB\int_0^1 Pd\bar{x}. \qquad \dots \quad (6.6)$$

From (4.5) and (4.6), we obtain

$$W = \frac{6\mu_1 UB^2L}{h_2^2(a-1)^2}\left(\log a - \frac{2(a-1)}{(a+1)}\right)$$

$$-\frac{12\mu_1 UB^2L\epsilon}{h_2^2}\left(\frac{1}{2}\frac{(a-1)h_2^2}{(a+1)}\left\{\frac{4\nu_1^2\left(1-t.\frac{\nu_1}{\nu_2}\right)}{\nu_2\pi^4(\nu_1^2+\nu_2^2n^2)}-\right.\right.$$

$$\left.\left.-\frac{1^2}{24}[2bc\sin nt+3(b^2-c^2)\cos nt]\right\}-\frac{\cos nt}{2(a-1)^2}\left\{\log a-\frac{2(a-1)}{(a+1)}\right\}\right). \tag{6.7}$$

Equation (6.7) can also be written as

$$\frac{W-W_s}{W_s} = -\frac{2\epsilon(a-1)^2}{\log a-\frac{2(a-1)}{(a+1)}}\left(\frac{(a-1)h_2^2}{2(a+1)}\left\{\frac{4\nu_1^2\left(1-t.\frac{\nu_1}{\nu_2}\right)}{\nu_2\pi^4(\nu_1^2+\nu_2^2n^2)}-\right.\right.$$

$$\left.\left.-\frac{1^2}{24}[2bc\sin nt+3(b^2-c^2)\cos nt]\right\}-\frac{\cos nt}{2(a-1)^2}\left\{\log a-\frac{2(a-1)}{(a+1)}\right\}\right) \tag{6.8}$$

where

$$W_s = \frac{6\mu_1 UB^2L}{h_2^2(a-1)^2}\left(\log a-\frac{2(a-1)}{(a+1)}\right).$$

Expression for the centre of pressure has been obtained as

$$x = \frac{1}{W}\left(\frac{6\mu_1 UB^3L}{h_2^2}\left\{\frac{-5(a^2-1)+2a(a+2)\log a}{2(a^2-1)(a-1)}\right\}\right.$$

$$+\frac{12\mu_1 UB^3L\epsilon}{h_2^2}\left\{\frac{\cos nt}{2}\left(\frac{-5(a^2-1)+2a(a+2)\log a}{2(a^2-1)(a-1)}\right)\right.$$

$$+h_2^2\left(\frac{(2a^3-8a^2+a-1)}{6(a^2-1)(a-1)}+\frac{a^2\log a}{(a^2-1)(a-1)^2}\right)$$

$$\times\frac{1^2}{24}\{2bc\sin nt+3\cos nt(b^2-c^2)\}-\frac{4\nu_1^2\left(1-t\frac{\nu_1}{\nu_2}\right)}{\nu_2\pi^4(\nu_1^2+\nu_2^2n^2)} \tag{6.9}$$

7. Discussion

The equations (6.7) and (6.8) for load capacity and relative reduction of load are examined numerically and the graphs are shown in figures 1 and 2. The relative reduction of load capacity in (6.8) as shown in figure 2 is to be understood only as an instantaneous effect. As the acceleration persists, the velocity

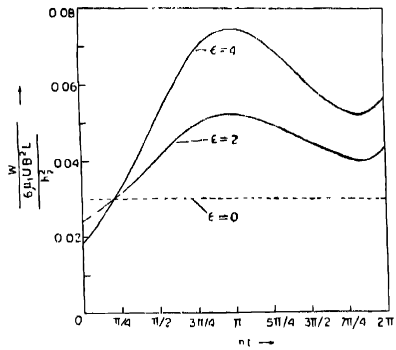

Figure 1.

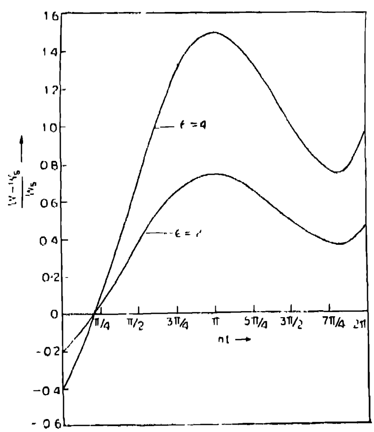

Figure 2.

will increase and the load capacity will rise with time. The slider bearing is given a pulsating motion hence, the load capacity is maximum at $nt = \pi$ and then it will decrease with time but always remain positive In figure 1, the load capacity is plotted against nt and it will always remain greater than the load capacity in case of steady motion, except for very small time in the beginning, which can be considered as the instantaneous effect.

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