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On the attenuation of longitudinal wave propagated along a magneto-strictive material

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Attenuation calculation in a ferromagneto material placed in a magmetic field has been worked out in the present paper following operational method. Expression for attenuation/wavelength obtained in the present method is exactly the same as that derived by Leslie (1950). The expression has been successfully used for computing attenuation in a nickel tube at no biasing and the agreement with the result of Wegel & Walther (1935) in the same material is found to be excellent.

Introduction

Measurement of attenuation in solids has been carried out by many workers, namely Siegel (1944) Hillier (1949), Lesli (1950). The measurement of attenuation in a ferromagnetic material has a special significance because of the extensive use of magnetic materials in industry The attenuation in magnetic material is a drawback owing to the incremental permeability adding to the disturbing factor, in the use of ferro-magnetic materials for design purposes From theoretical point of view, propagation of acoustic wave through a magnetostrictive material involves losses both of micro and macro order due to internal friction, electromechanical coupling and eddy current losses. As no attempt has yet been made to include all these, we shall solve the problem by setting up steady waves in a ferro-magnetic rod or tube and finding the attenuation losses due to internal friction and heat arising from magnetostriction. Since we consider low amplitude vibrations, the alternating flux produced, owing to Villari effect, is necessarily small. Therefore eddy loss can be neglected particularly if we consider a thin rod or a tube.

Solution of the Problem

For mathematical convenience, it is common in problems on vibrations to consider the relation between the stress, p, and the strain, e, in a solid to be of the form :

$$p = E.e+\eta \frac{de}{dt} \tag{1}$$

where E and η are the associated elastic and viscous moduli.

In the present problem the strain in the magneto-striotive bar consists of two parts. Firstly that due to mechanical stressing and secondly due to magnetic biasing.

So equation (1) may be written in the form

$$p = E\,(e_{\text{mech}}+e_{\text{mag}})+\eta\frac{\mathrm{d}}{\mathrm{d}t}(e_{\text{mech}}+e_{\text{mag}}) \quad \ldots \quad (2)$$

The production of strain due to magnetic biasing (the biasing field being steady) has been considered by Kar (1967) as a function of the flux density B to be

$$e_{\text{mag}} = \chi B^2 \quad \ldots \quad (3)$$

for values of B much less than its saturation value.

χ is the proportionality constant, being positive and negative for magneto-striotive expansion and contraction, respectively. For nickel χ is negative.

Thus the stress-strain relation (2) comes out as

$$p = E(e_{\text{mech}}+\chi B^2)+\eta\frac{\mathrm{d}}{\mathrm{d}t}\,(e_{\text{mech}}) \quad \ldots \quad (4)$$

$$= E\frac{\mathrm{d}\omega}{\mathrm{d}s}+\chi EB^2+\eta\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\omega}{\mathrm{d}s}\right). \quad \ldots \quad (5)$$

where s = longitudinal co-ordinate,

ω = longitudinal displacement.

The equation of longitudinal motion in a thin bar or tube exhibiting strain-rate effect is given by

$$\rho\ \frac{\mathrm{d}^2\omega}{\mathrm{d}t^2} = \frac{\mathrm{d}p}{\mathrm{d}s} \quad \ldots \quad (6)$$

where ρ is the density of the bar.

Substituting the value of p from (5) in equation (6) we get

$$\rho.\frac{\mathrm{d}^2\omega}{\mathrm{d}t^2} = E\left(\frac{\mathrm{d}^2\omega}{\mathrm{d}s^2}\right)+\lambda_m\left(\frac{\mathrm{d}B}{\mathrm{d}s}\right)+\eta\ \frac{\mathrm{d}^3\omega}{\mathrm{d}t,\mathrm{d}s^2} \quad \ldots \quad (7)$$

where λ_m is the magnetostriction constant and is given by

$$\lambda_m = 2\chi EB. \quad \ldots \quad (7a)$$

Here B is taken as the mean flux density over ds and the total changes in the flux-density may be taken to be due to the change in external magnetic field (H) and due to the change in mechanical strain.

It follows that the relation between these quantities may be given by (Kar 1967)

$$\frac{\mathrm{d}B}{\mathrm{d}s} = \mu\left[\frac{\mathrm{d}H}{\mathrm{d}s} + 4\pi\lambda_m \frac{\mathrm{d}^2\omega}{\mathrm{d}s^2}\right] \quad \ldots \quad (8)$$

where μ = the permeability at the point B. As the external magnetic field may be taken constant over the small distance ds, we may take $\frac{\mathrm{d}H}{\mathrm{d}s} = 0$

Thus equation (8) comes out as

$$\frac{\mathrm{d}B}{\mathrm{d}s} = 4\pi\mu\lambda_m \cdot \frac{\mathrm{d}^2\omega}{\mathrm{d}s^2} \quad \ldots \quad (9)$$

and equation (7) becomes

$$\rho \cdot \frac{\mathrm{d}^2\omega}{\mathrm{d}t^2} = E_1 \cdot \frac{\mathrm{d}^2\omega}{\mathrm{d}s^2} + \eta \frac{\mathrm{d}^3\omega}{\mathrm{d}t.\mathrm{d}s^2} \quad \ldots \quad (10)$$

where $$E_1 = (E + 4\pi\mu\lambda^2{}_m) \quad \ldots \quad (10a)$$

Equation (10) is equivaent to

$$\frac{\mathrm{d}^2\omega}{\mathrm{d}s^2} = \frac{D_1{}^2\omega}{C_1{}^2} \quad \ldots \quad (11)$$

where $$\mathrm{D}_1 = \mathrm{D}(1+\eta_1\mathrm{D})^{-\frac{1}{2}}$$

$$C_1 = E_1/\rho \quad \ldots \quad (11a)$$

$$\eta_1 = \eta/E_1 \quad \ldots \quad (11b)$$

and $$\mathrm{D} = \text{the operator } \frac{\mathrm{d}}{\mathrm{d}t}.$$

The operational solution of the equation (11) gives (Ghosh 1953)

$$\omega_s = \psi(s)_{s=0}\left[\cosh^2\frac{\beta s}{C_1} - \sin^2\frac{ns}{C_1}(4-3\alpha)\right]^{\frac{1}{2}}.\exp(i(nt+\epsilon) \quad \ldots \quad (12)$$

when a periodic stress F.exp (int), $(i = \sqrt{-1})$ of peak value F is applied at one end $(s = l)$ of the bar.

Here $\psi(s)_{s=0}$ is the amplitude of vibration at the free end $(s = 0)$ and give by,

$$\psi(s)_{s=0} = \frac{F}{\frac{\beta l}{C_1}(\alpha^2n^2+E_1)[\cosh^2\frac{\beta^2}{C_1} - \cos^2\frac{nl}{C_1}(4-3\alpha)]^{\frac{1}{2}}}$$

where $$\alpha = 1+\tfrac{1}{8}\eta_1{}^2n^2, \qquad \beta = \tfrac{1}{2}\eta_1 n^2. \qquad \ldots \quad (12a)$$

n = angular frequency of vibration, and

$$\tan\,\epsilon = \tan h\frac{\beta s}{C_1}\ \tan\ \frac{ns}{C_1}\ (4-3\alpha) \qquad \ldots \quad (12b)$$

In the present problem we are not interested about the phase and so we may ignore the phase term in equation (12). For most metals the value of $\eta_1{}^2.n^2$ is very small. So α may be taken to be equal to unity.

Equation (12) thus reduces to

$$\omega_s = \psi(s)_{s=0}\left[\cosh^2\frac{\beta s}{C_1}-\sin^2\frac{ns}{C_1}\right]^{\frac{1}{2}}.\ \ \exp\,(\text{int}) \qquad \ldots \quad (13)$$

Due to reverse magnetostriction or Villari effect (Smith 1930) it follows that there will be an induced voltage along the rod and this voltage at any point is proportional to the rate of change of strain at that point.

Therefore differentiating equation (13) first with respect to s and then with respect to t we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}\omega_s}{\mathrm{d}s}\right) = \frac{\psi(s)_{s=0}\,in\left(\frac{\beta}{C_1}\sinh\frac{2\beta s}{C_1}-\frac{n}{C_1}\sin\frac{2ns}{C_1}\right)}{2\left(\cosh^2\frac{\beta s}{C_1}-\sin^2\frac{ns}{C_1}\right)^{\frac{1}{2}}}\ .\ \exp\,(\text{int}) \qquad \ldots \quad (14)$$

The induced voltage at s is obtained as

$$V_s = P\,.\frac{\mathrm{d}}{\mathrm{d}t}\ \left(\frac{\mathrm{d}\omega_s}{\mathrm{d}s}\right) \qquad \ldots \quad (14a)$$

where P is the proportionality constant.

Equation (14a) with the help of equation (14) gives

$$V_s = P\ \frac{\psi(s)_{s=0}\ \ in}{}\ .\frac{\left(\frac{\beta}{C_1}\sinh\frac{2\beta s}{C_1}-\frac{n}{C_1}\sin\frac{2ns}{C_1}\right)}{2\left(\cosh^2\frac{\beta s}{C_1}-\sin^2\frac{ns}{C_1}\right)^{\frac{1}{2}}}\cdot\exp\,(\text{int}) \qquad \ldots \quad (15)$$

As the value of $\beta s/C_1$ is very small we make the following reasonable assumption

$$\cosh\frac{\beta s}{C_1}\simeq 1, \quad \sinh\frac{2\beta s}{C_1}\simeq\frac{2\beta s}{C_1}\,.$$

So

$$V_s = \frac{P\psi(s)_{s=0}}{2}\ \text{in}\ \frac{\left(\frac{2\beta^2 s}{C_1{}^2}-\frac{n}{C_1}\sin\frac{2ns}{C_1}\right)}{\cos\frac{ns}{C_1}}\ .\ \exp\,(\text{int}).$$

Neglecting $2\beta s/C_1^2$ we have

$$V_s = -\frac{P\psi(s)_{s=0}\, in^2}{C_1} \sin\frac{ns}{C_1} \exp(int). \quad \ldots \quad (15a)$$

Thus it is seen that the induced voltage will be maximum or minimum according as $\sin(ns/C_1)$ is maximum or minimum.

Now, $\sin\frac{ns}{C_1}$ is maximum when $\frac{ns}{C_1} = (2N+1)\frac{\pi}{2}$, *i.e*,

when $$s = \frac{C_1}{2n}\cdot(2N+1)\pi \quad \text{where} \quad N = 0, 1, 2, \ldots; \quad \ldots \quad (16a)$$

Again $\sin\frac{ns}{C_1}$ is minimum when $\frac{ns}{C_1} = N\pi$, *i.e*, when $s = \frac{C_1 N\pi}{n}$ ... (16b)

where $N = 0, 1, 2, \ldots$.

Thus the maximum and minimum values of induced voltage can be obtained by using the conditions (16a) and (16b) in equation (15) as

$$V_{\max} = \frac{P\psi(S)_{s=0} \text{ in } \beta}{C_1} \cdot \cosh\frac{\beta}{n}(2N+1))\frac{\pi}{2} \cdot \exp(\text{int}) \quad \ldots \quad (17a)$$

and

$$V_{\min} = \frac{P\psi(s)_{s=0} \text{ in } \beta}{C_1} \cdot \sinh\frac{\beta N\pi}{n} \exp(\text{int}). \quad (17b)$$

Therefore from equations (17a) and (17b) we get

$$\frac{(V_{\min})_{s=2\pi C_1/n} - (V_{\min})_{s=\pi C_1/n}}{(V_{\max})_{s=3\pi C_1/2n}} = \frac{\sinh\frac{\beta}{n}(2\pi) - \sinh\frac{\beta}{n}(\pi)}{\cosh\frac{\beta}{n}\left(\frac{3\pi}{2}\right)}$$

$$= 2\sinh\left(\frac{\beta\pi}{2n}\right) \sim \frac{\beta\pi}{n} = \eta_1 n\pi \quad \ldots \quad (18)$$

An inspection of equation (13) shows that minimum values of displacement (nodes) occur at positions given by

$$\frac{ns}{C_1} = N' \cdot \frac{\pi}{2}$$

where $N' = 1, 3, 5, \ldots$ and the amplitude in the simplest mode at reasonance at the point s is

$$|\omega| = \psi(s)_{s=0} \sinh\frac{\beta_r s}{C_1} = \psi(s)_{s=0} \sinh\frac{\eta_1 n_r^2 s}{2C_1}$$

where n_r is the resonance frequency and $\beta_r = \frac{1}{2}\eta_1 n_r^2$.

Now let the frequency n_r be changed in equation (13) to a value $(n_r+\Delta)$ to get amplitude at the same point s changed to $\sqrt{2}\,|\omega_s|$ then,

$$\sqrt{2}\,|\omega_s| = \sqrt{2}\psi(s)_{s=0}\sinh\frac{\eta_1 n_r^2 s}{2C_1}$$

$$= \psi(s)_{s=0}\left[\cosh^2\frac{\eta_1 n_r^2 s}{2C_1} - \cos^2\frac{\Delta s}{C_1}\right]^{\frac{1}{2}}$$

(neglecting Δ/n_r compared to unity)

or, $$\cos^2\frac{\Delta s}{C_1} = 1-\sinh^2\frac{\eta_1 n^2 s}{2C_1}$$

Expanding, $$\Delta^2 = \frac{\eta_1^2 n_r^4}{4}$$

or, $$\Delta = \pm\frac{\eta_1^2 n_r^2}{2}$$

Therefore the frequency spread between the values for which the amplitude is increased by 3db from that at resonance peak (minima) is

$$2\Delta = \eta_1 n_r^2 \qquad \ldots \quad (19b)$$

For a constantly driven system, the quality (Q), resonance frequency (n_r) and band-width (Δ) at 3db point is related by (Mason 1964)

$$Q = \frac{n_r}{2\Delta} \quad \text{or,} \quad Q = \frac{1}{\eta_1 n_r} \qquad \ldots \quad (19c)$$

Again the quality and attenuation in a material is simply related by (Mason, 1964)

$$A\lambda = \frac{\pi}{Q} \quad \text{nepers} \qquad \ldots \quad (19d)$$

where A is the attenuation coefficient and λ is the wave length of sound wave in the material. Comparing equations (19c) and (19d) we get

$$A\lambda = \pi\eta_1 n_r^2 \qquad \ldots \quad (19)$$

So, if the experiment be made such that the specimen rod be excited at simplest mode of resonance ($i.e$, $n = n_r$) we get from (18) and (19)

$$\frac{(V_{\min})_{s=2\pi C_1/n} - (V_{\min})_{s=\pi C_1/n}}{(V_{\max})_{s=3\pi C_1/2n}} = \frac{A\lambda}{2} \qquad \ldots \quad (20)$$

Equation (20) is the same as that obtained by Leslie (1950).

From equations (19), (11b) and (10a)

$$A\lambda = \frac{\eta\pi n}{E+4\pi\mu\lambda_m^2}$$

$$= \frac{\eta\pi n}{(E+16\pi\mu\chi^2E^2B^2)} \quad (21)$$

Equation (21) represents the dependence of the attenuation per wave length with μ and B consequent upon change of magnetic biasing at a fixed periodic force of excitation.

When there is no biasing field, *i.e.*, with no polarizing flux, $\lambda_m = 0$ and equation (21) becomes

$$[A\lambda]_{\text{no-bias}} = \frac{\eta\pi n}{E} \quad \ldots \quad (21a)$$

With a nickel rod if the periodic stressing is of frequency

21×10^3 cs/sec. *i.e* $n = 21\times10^3\times2\pi$,

taking $\eta = 2\times10^4$ poise,

$E = 20\,2\times10^{11}$ dynes/cm^2,

we get attenuation/wavelength at no biasing to be equal to 0041 nepers/wavelength. The value of attenuation/wavelength at no biasing field as obtained by Wegel & Walther (1935) at the same excitation frequency was .0048 nepers/wavelength for a nickel rod specimen of 1 cm diameter. Thus the agreement is excellent

Discussion

Expression for attenuation per wave-length in a nickel rod at different biasing field can be easily obtained by writing equation (4) as (Kar 1967)

$$p = E(e_{\text{mech}}-\chi B^2)+\eta\frac{d}{dt}(e_{\text{mech}}). \quad (4a)$$

Using this value of stress we get equation (21) transformed as

$$A\lambda = \frac{\eta\pi n}{(E-16\pi\mu\chi^2E^2B^2)}; \quad (22)$$

The equations (21) and (22) show that as $\mu\chi^2E^2B^2$ is a positive quantity the presence of biasing field increases the attenuation in a nickel-like material while it decreases the attenuation in others. The increase of attenuation with biasing field on, in nickel, is supported by the experimental work of Leslie (1950).

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