The author is thankful to Dr. G. P. Bhattacharjee and Dr. S. K. Dutta Roy for their interest and help in the work. He also thanks the authorities of the Indian Institute of Technology, Kharagpur, India, for providing facilities for using the electronic computer.

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Indian J. Phys. **44**, 63–,65 (1970)

Hydrogen excitation in H-α collision by second Born approximation

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(*Received* 31 *March* 1970)

Previous investigations on the excitation of atomic hydrogen by alpha particle have been made by Bates (1959). Using the impact parameter treatment he has shown that the introduction of allowance for distortion leads to much smaller cross-sections than those obtained by the first Born approximation at low and moderate energies. The purpose of our present work is to investigate the same problem in the second Born approximation and to compare our findings with the previous results.

We consider α-particle B to be moving with a constant velocity v in a straight line and the target nucleus to be at rest at A. The Hamiltonian H corresponding to the motion of the electron is given by

$$H = -\tfrac{1}{2}\nabla^2 - \frac{1}{r_A} - \frac{2}{r_B} \quad \text{(in atomic units)}$$

The total electronic wave function may be represented by the expansion

$$\Psi(r, t) = \sum_n a_n(t)\psi_n$$

where $$\psi_n = \phi_n(r)\exp(-i\epsilon_n t),$$

$\phi_n(r)$ and ϵ_n being the eigenfunctions and eigenenergies of the hydrogen atom.

The collision is then described by the time-dependent Schrödinger equation $H\Psi(t) = i\frac{\partial}{\partial t}\Psi(t)$. Proceeding in the customary manner it can be shown that

$$\frac{d}{ds} a_m(s) = i\Sigma F_{mn}a_n(s) \tag{1}$$

where $s = vt$

$$F_{mn} = -\int \bar{\psi}_m V_i \psi_n dV$$

and $V_i = -2/r_B$, is the perturbing potential

The cross section for excitation from ground state to m-th state is

$$Q_m = 2\pi \int_0^\infty |a_m|^2 p\,dp, \quad p \text{ being the impact parameter.}$$

From equation (1) together with the initial conditions

$$a_1(-\infty) = 1$$

and

$$a_n(-\infty) = 0, \; n \neq 1$$

we have,

$$a_m(s) = \delta_{m1} + \frac{i}{v} \Sigma \int^s F_{mn}a_n ds \tag{2}$$

The 1st Born approximation for a_m is given by

$$a_m(s) = \delta_{m1} + \frac{i}{v} \int_{-\infty}^{s} F_{m1}\, ds \tag{3}$$

From (2) and (3) the second Born approximation is obtained. Neglecting the effect of all states other than the ground state and the final state m, we obtain

$$\underset{m\neq 1}{a_m(\infty)} = \frac{i}{v}\int_{-\infty}^{\infty} F_{m1}\, ds$$

$$+\frac{1}{v^2}\int_{-\infty}^{\infty}\left\{\left(\int_{-\infty}^{s} F_{11}ds_1\right)F_{m1} + \left(\int_{-\infty}^{s} F_{m1}ds_1\right)F_{mm}\right\}ds$$

The total expression for cross section in the Born approximation is

$$Q = \int_0^\infty \left[\frac{1}{v^2}\left(\int_{-\infty}^{\infty} F_{m1R}\, ds\right)^2 - \frac{2}{v^3}\left(\int_{-\infty}^{\infty} F_{m1R}\, ds\right)\right.$$

$$\left.\left(\int_{-\infty}^{\infty}\left\{\left(\int_{-\infty}^{s} F_{11}ds_1\right)F_{m1I} + \left(\int_{-\infty}^{s} F_{m1I}\, ds_1\right)F_{mm}\right\}ds\right)\right] pdp \tag{4}$$

In equation (4) we have neglected the two terms of the fourth order in interaction energy. Since other terms of the same order would also come from higher Born

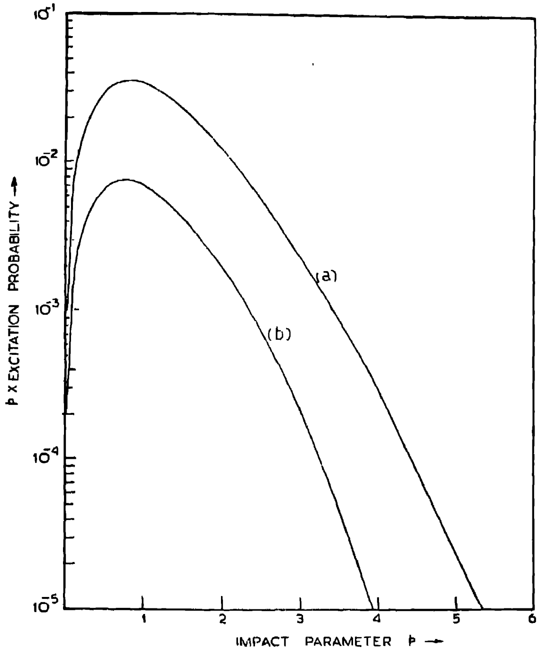

Figure 1. The product of excitation probability and the impact parameter p against p
(a) For 1s-2s excitation at 1412 *keV*
(b) For 1s-3s excitation at 1412 *keV*.

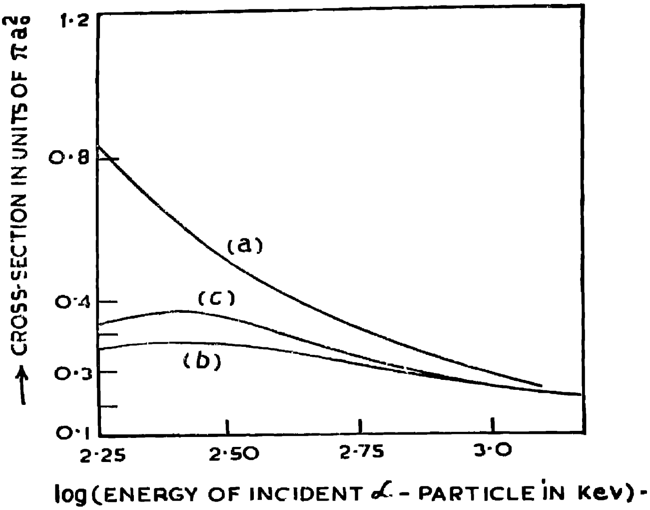

Figure 2. Excitation cross sections for $1s$-$2s$ transition by
(a) 1st Born, (b) distortion, and (c) 2nd Born approximations.

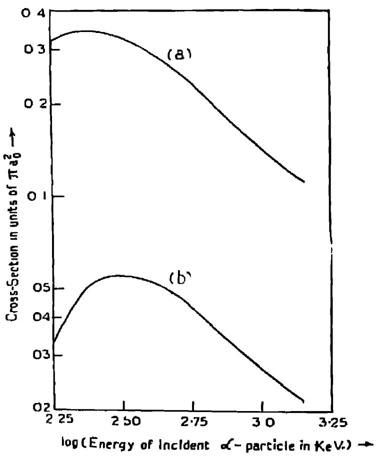

Figure 3. Second Born excitation cross sections for
(a) $1s$-$2s$ transition
(b) $1s$-$3s$ transition.

approximation and such terms are not known, hence for consistency those two terms are not included.

In figure 1 we have have plotted the product of the excitation probabilities and the impact parameter p against p at the incident energy 1412 keV.

In figure 2 we have compared the cross-sections obtained from first Born distortion and second Born approximation for $1s$–$2s$ transition. It is seen that at about 1000 keV incident energy, the cross-section due to 1st Born, 2nd Born and the distortion approximation are almost the same. As the energy decreases, the discrepancy between the results of the 1st Born and the distortion increases, where the second Born results are comparatively close to the distortion results.

Figure 3 shows our results of the cross-section of $1s$–$2s$ and $1s$–$3s$ excitations. It is found that $1s$–$3s$ excitation cross-section is much lower than that of $1s$–$2s$ excitation.

This work is preliminary to a series of investigations of Alpha-Hydrogen Collision where we propose to take into account the capture state of He^{+} as well.

The authors are thankful to Dr. N. C. Sil and Prof. D Basu for helpful discussions.

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Indian J. Phys. **44**, 65–67, (1970)

On parameter estimation of Gibbs' canonical distribution

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(*Received* 26 *March—Revised* 20 *July* 1970)

In this note first considering Gibbs' canonical distribution as a simple statistical distribution, the fluctuation of temperature has been obtained using the asymptotic properties of maximum-likelihood estimate. Secondly, Einstein's formula for the fluctuation probability has been derived purely from statistical consideration.

After Gibbs, we consider a system in equilibrium as a small part of a large system exchanging its energy with its environment (heat bath). In this case the