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Laminar source flow of second order fluid between two parallel coaxial stationary infinite porous disks

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The laminar source flow of second order fluid between two stationary infinite porous disks has been studied The solution has been determined in the form of a double series expansion. The effect of non-Newtonian parameters on velocity profile, pressure and shear stress has been discussed.

1. Introduction

Livesey (1962) investigated the incompressible radial flow between two parallel stationary disks using the integral approach and the asumption of a parabolic velocity profile Savage (1964) obtained the solution by expanding velocity components and pressure in terms of the downstream coordinate. But he has omitted the no-slip condition on the disk $z = 0$. Similar problems have also been studied by Peube (1963), Chen & Peube (1964), Geiger *et al* (1964).

Flow between porous boundaries is of practical as well as theoretical interest. The problems of gaseous diffusion, boundary cooling and lubrication of porous bearings are a few examples. Elkouh (1969) obtained a solution for source flow between parallel stationary porous disks He has given the effect of either equal suction or equal injection The solution is in the form of a perturbation from the creeping flow solution, valid for small values of the wall Reynolds number (Rw), and of the reduced source Reynolds number (Re^*). (Rw) is based on equal suction or injection velocities at the disks. In a subsequent paper Elkouh (1971) has investigated laminar source flow between parallel porous disks with equal suction and injection. The solution is in the form of an infinite series expansion about the solution at infinite radius and is valid for all suction and injection rates. Khan (1968) has investigated the laminar source flow between two parallel coaxial porous disks rotating at the same speed taking into account suction at one disk and an equal injection at the other.

In the present paper, the source flow of the second order fluid between two parallel coaxial infinite porous disks with equal permeability has been investigated.

The constitutive equations of an incompressible second order fluid as suggested by Rivilin & Ericksen (1955) are :

$$\tau_{ij} = -pg_{ij}+\phi_1 A_{ij}+\phi_2 B_{ij}+\phi_3 A_{ik}A_{kj}, \qquad \ldots (1)$$

$$A_{ij} = v_{i,j}+v_{j,i}, \qquad \ldots (2)$$

and

$$B_{ij} = a_{i,j}+a_{j,i}+2v_{m,i}v_{m,j}. \qquad \ldots (3)$$

where τ_{ij} is the stress sensor, g_{ij} is the metric tensor, v_i is the velocity vector, a_i is the acceleration vector, p is the pressure, ϕ_1, ϕ_2, ϕ_3 are the fluid parameters and comma in the suffix denotes covariant differentiation.

The series expansion method used by Elkouh (1969) has been adopted to obtain the solution. The effect of non-Newtonian parameters on radial velocity perturbations and pressure drop has been exhibited graphically. It is noted that the visco-elasticity and cross viscosity increase the magnitude of the radial velocity perturbations. The results obtained in this paper are valid at a distance $r > \sqrt{(Re)}$ from the source, where (Re) denotes source Reynolds number.

2. Equations of Motion

The momentum equation for the incompressible flow are

$$\rho v_j v_{i,j} = \tau_{ij,j} \qquad \ldots (4)$$

and the equation of continuity is

$$v_{i,i} = 0. \qquad \ldots (5)$$

We shall use cylindrical polar coordinates $(\bar{r}, \theta, \bar{z})$ and consider the axially symmetric steady flow between two infinite stationary porous disks $\bar{z} = -a$ and $\bar{z} = +a$. The line source is situated on the z-axis. Let the volumetric flow rate of the source and magnitude of the constant injection velocity at the disks be Q and W, respectively The boundary conditions are .

$$\left.\begin{aligned} &\bar{u}(\bar{r}, \pm a) = 0, \\ &\bar{w}(\bar{r}, \pm a) = \mp W, \\ \text{and} \quad &\int_{-a}^{+a} 2\pi\bar{r}\bar{u}\,d\bar{z} - 2\pi\bar{r}^2 W = Q, \end{aligned}\right\} \qquad \ldots (6)$$

which is the overall conservation of mass equation

We introduce the following non-dimensional quantities :

$$\left.\begin{aligned} &r = \bar{r}/a, \quad z = \bar{z}/a, \quad u = \rho\bar{u}a/\phi_1, \quad w = \rho\bar{w}a/\phi_1 \\ &p = \rho\bar{p}a^2/\phi_1, \quad K = \phi_2/\rho a^2, \quad S = \phi_3/\rho a^2. \end{aligned}\right\} \qquad \ldots (7)$$

Equations (6) and (7) give the boundary conditions in the following form

$$\left.\begin{aligned} & u(r, \pm 1) = 0, \\ & w(r, \pm 1) = \mp (Rw), \\ & \int_{-1}^{+1} u\,dz - (Rw)r = \frac{2(Re)}{r} \\ & (Rw)(\text{wall Reynolds number}) = \frac{Wa\rho}{\phi_1}, \\ & (Re)(\text{source flow Reynolds number}) = \frac{Q\rho}{4\pi a\phi_1}, \end{aligned}\right\} \quad \ldots (8)$$

where (Rw) is taken positive for injection and negative for suction. The radial and axial velocity components are related to a stream function ψ in the following form

$$u = \frac{1}{r}\frac{\partial\psi}{\partial z}, \; w = -\frac{1}{r}\frac{\partial\psi}{\partial r} \quad \ldots (9)$$

For small values of $(Re)/r^2$, let us expand ψ and p as follows :

$$\psi = \tfrac{1}{2} r^2(Rw)f_{-1}(z) + (Re)\Big[f_0(z) + \Big(\frac{(Re)}{r^2}\Big) f_1(z) + \Big(\frac{(Re)}{r^2}\Big)^2 f_2(z) + \ldots \Big], \quad (10)$$

and

$$p = \tfrac{1}{4} r^2(Rw)h_{-1}(z) + h(z) + (Re)\Big[h_0(z)\ln r + \Big(\frac{(Re)}{r^2}\Big) h_1(z) + \Big(\frac{(Re)}{r^2}\Big)^2 h_2(z) + \ldots \Big] \quad (11)$$

From (10) and (11) we infer that the solution so obtained will be valid for $r > \sqrt{(Re)}$ We transform equations (1) to (5) in cylindrical polar coordinate system, and use (7), (9), (10), (11) and equate the coefficients of like powers of r. This gives us an infinite set of simultaneous ordinary differential equations. For the sake of brevity we record only two systems below. However solution in following section has been given upto system IV.

System I .

$$\begin{aligned} & f'''_{-1} + (Rw)(f_{-1}f''_{-1} - \tfrac{1}{2}f'^2_{-1}) \\ & \quad = h_{-1} + K(Rw)[f_{-1}f^{iv}_{-1} - f''^2_{-1}] + S(Rw)[-\tfrac{1}{2}f''^2_{-1}], \end{aligned} \quad (12a)$$

$$h'_{-1} = 2(Rw)(2K+S)f''_{-1}f'''_{-1}, \quad (12b)$$

and

$$\begin{aligned} h' = & -(Rw)(f''_{-1} + (Rw)f'_{-1}f_{-1}) + K(Rw)[(Rw)(-f_{-1}f'''_{-1} + 5f'_{-1}f''_{-1} \\ & + 8f'''_{-1}f'_{-1} + 6f''^2_{-1} + 2f^{iv}_{-1}f_{-1}) + 2(Re)(f'''_{-1}f''_0 + f''_{-1}f'''_0)] + \\ & S(Rw)[7(Rw)f'_{-1}f''_{-1} + (Re)(f'''_{-1}f''_0 + f''_{-1}f'''_0)]. \end{aligned} \quad (13)$$

System II :

$$f'''_0+(Rw)f_{-1}f''_0 = h_0+K(Rw)[f'_{-1}f'''_0+f_{-1}f^{iv}_0+f''_0f''_{-1}+f'_0f'''_{-1}]+ +S(Rw)[f'''_{-1}f'_0+f''_{-1}f''_0+f''_{-1}f'''_0]. \quad \dots (14a)$$

and $$h'_0 = 0. \quad \dots (14b)$$

In the above equations the primes denote the differential coefficients with respect to z. The equation of continuity is identically satisfied by equation (9) Equations (8), (9) and (10) give the modified conditions as

$$\left.\begin{aligned} &f'_n(\pm 1) = 0, \quad n = -1, 0, 1, 2, \dots, \\ &f_n(\pm 1) = 0, \quad n = 1, 2, \dots, \\ &f_{-1}(\pm 1) = \pm 1, \\ \text{and} \quad &f_0(1)-f_0(-1) = 2. \end{aligned}\right\} \quad \dots (15)$$

We choose $f_0(-1) = -1$, so that $f_0(1) = 1$

We note that the differential equations in above systems and the governing boundary conditions are independent of (Re), hence the solutions of the above systems will be independent of (Re). However, (Re) appears as a parameter in the solution of system IV.

3. Solutions for Small (Rw)

The sets of differential equations in the above systems have been solved by expanding the different functions in powers of Rw (for small values of the suction or injection Reynolds number) Thus

$$f_n = \sum_{\alpha=0}^{\infty} (Rw)^\alpha f_{n,\alpha}. \quad (16)$$

in which $f_{n,\alpha}$ are independent of (Rw).

The corresponding boundary conditions for $f_{n,\alpha}$ are

$$\begin{aligned} &f'_{n,\alpha}(\pm 1) = 0, \quad \text{for} \quad n = -1, 0, 1, 2, \text{ and all } \alpha \\ &f_{n,\alpha}(\pm 1) = 0, \quad n = 1, 2, \text{ and all } \alpha, \\ &f_{n,0}(\pm 1) = \pm 1, \quad n = -1, 0, \\ &f_{n,\alpha}(\pm 1) = 0, \quad n = -1, 0 \text{ and } \alpha \geqslant 1. \end{aligned} \quad (17)$$

Solution of System I :

Integration of equation (12b) gives

$$h_{-1} = C_{-1}+(2K+S)(Rw)(f''^2_{-1}), \quad (18)$$

where C_{-1} is a constant Equations (12a) and (18) give

$$f'''_{-1}+(Rw)(f_{-1}f''_{-1}-\tfrac{1}{2}f''^2_{-1}) = C_{-1}+K(Rw)[f_{-1}f^{iv}_{-1}+f''^2_{-1}] +S(Rw)[\tfrac{1}{2}f''^2_{-1}] \quad \ldots \quad (19)$$

Let us further assume

$$C_{-1} = \sum_{\alpha=0}^{\infty} (Rw)^{\alpha} C_{-1\cdot\alpha}. \quad \ldots \quad (20)$$

We substitute equations (16) and (20) in (19) and equate the coefficients of like powers of (Rw). We get a set of ordinary linear differential equations. The solution of these differential equations subjected to the modified boundary conditions (17) is

$$f_{-1\cdot 0} = \tfrac{1}{2}z(3-z^2), \quad \ldots \quad (21)$$

$$f_{-1\cdot 1} = -\frac{1}{560}z(z^6-21z^4+39z^2-19)+\frac{3}{40}z(2K+S)(z^4-2z^2+1), \quad \ldots \quad (22)$$

$$f_{-1\cdot 2} = -\frac{1}{258700}z(63z^{10}-1540z^8+11682z^6-15708z^4+2215z^2+3288)$$
$$+\frac{1}{11200}Kz(45z^8-684z^6+2022z^4-2172z^2+789)+\frac{1}{2240}Sz\left(\frac{255}{3}z^8-1332z^6\right.$$
$$\left.+2286z^4-916z^2-123\right)-\frac{9}{700}K^2z(10z^6-49z^4+68z^2-29)$$
$$-\frac{3}{1400}S^2z(10z^6-21z^4+12z^2-1)-\frac{1}{1400}KSz(150z^6-567z^4+684z^2-267), \quad \ldots \quad (23)$$

and

$$h_{-1} = -3+(Rw)\left[-\frac{54}{35}+\frac{9}{10}(2K+S)(10z^2-1)\right]+(Rw)^2\left[-\frac{151}{2695}\right.$$
$$+\frac{9}{350}K(35z^6-350z^4+195z^2-54)+\frac{1}{700}S(315z^6-3150z^4+1755z^2-136)$$
$$-\frac{18}{175}K^2(350z^4-210z^2+51)-\frac{9}{175}S^2(175z^4-105z^2+3)$$
$$\left.-\frac{9}{175}KS(700z^4-420z^2+57)\right]+0(Rw)^3 \quad \ldots \quad (24)$$

It is observed that this solution is affected by non-Newtonian parameters Elkouh (1969) pointed out that second order perturbation solution of system I in case of Newtonian fluid is reasonably accurate for $|(Rw)| \leqslant 3\,0$ approximately.

Solution of System II :

Proceeding in a manner similar to that adopted in obtaining the solution of system I, we get

$$f_{0,0}=\tfrac{1}{2}z(3-z^2), \qquad \ldots \quad (25)$$

$$f_{0,1}=-\frac{1}{280}z(2z^6-21z^4+36z^2-17)+\frac{3}{10}z(K+S)(z^4-2z^2+1), \qquad \ldots \quad (26)$$

$$f_{0,2}=-\frac{1}{5174400}z(812z^{10}-13475z^8+70092z^6-108570z^4+56704z^2-5563)+$$
$$+\frac{1}{11200}Kz(235z^8-2484z^6+6834z^4-7156z^2+2571)+$$
$$+\frac{1}{16800}Sz(245z^8-2610z^6+6408z^4-5966z^2+1923)-\frac{3}{1400}K^2z$$
$$\times(205z^6-987z^4+1359z^2-577)-\frac{3}{560}S^2z(55z^6-189z^4+213z^2-79)$$
$$-\frac{3}{2800}KSz(685z^6-2919z^4+3783z^2-1549)\Big), \qquad \ldots \quad (27)$$

and

$$h_0=-3-\frac{27}{35}(Rw)[1-7(K+S)]-(Rw)^2\Big[\frac{886}{13475}+\frac{1}{175}(309K+11S$$
$$+1404K^2-280S^2+1224KS)\Big]+0(Rw)^3. \qquad \ldots \quad (28)$$

Solution of System III :

$$f_{1,0}=-\frac{3}{280}z(z^6-7z^4+12z^2-5)+\frac{3}{10}z(K+S)(z^4-2z^2+1), \qquad \ldots \quad (29)$$

$$f_{1,1}=-\frac{1}{184800}z(85z^{10}-1155z^8+4950z^6-8778z^4+6901z^2-2003)$$
$$+\frac{1}{5600}Kz(335z^8-2796z^6+7602z^4-8156z^2+3015)+\frac{1}{4200}Sz$$
$$\times(170z^8-1377z^6+3339z^4-3227z^2+1045)-\frac{3}{350}K^2z(115z^6-546z^4+747z^2-316)$$
$$-\frac{3}{700}S^2z(165z^6-567z^4+639z^2-237)-\frac{3}{700}KSz(395z^7-4977z^4+6399z^2-2607),$$
$$\ldots \quad (30)$$

and

$$h_1=-\frac{27}{35}+\frac{18}{5}K(5z^2-2)+\frac{9}{10}S(10z^2-3)+(Rw)\Big[-\frac{37}{1925}+\frac{1}{700}K(4410z^6$$
$$-18900z^4+9400z^2+507)+\frac{1}{140}S(441z^6-189z^4+945z^2+59)$$
$$-\frac{1}{700}K^2(75600z^4-45360z^2-1377)-\frac{1}{280}S^2(15120z^4-9072z^2+27)$$
$$-\frac{1}{1400}KS(226800z^4-130480z^2-2619)\Big]+0(Rw)^2. \qquad (31)$$

Solution of System IV :

$$f_{20} = -\frac{1}{431200}z(168z^{10}-1925z^8-7524z^6-15246z^4+14780z^2-5301)$$

$$+\frac{1}{2800}Kz(145z^8-996z^6+2706z^4-3004z^2+1140)$$

$$+\frac{1}{2800}Sz(95z^8-636z^6+1446z^4-1364z^2+459)-\frac{1}{350}K^2z(255z^6-1197z^4+1629z^2$$

$$-687)-\frac{1}{350}S^2z(165z^6-567z^4+639z^2-237)-\frac{1}{25}KSz(30z^6-126z^4+162z^2-66), \quad \ldots \quad (32)$$

and

$$h_2 = -\frac{78}{2695}+\frac{3}{175}K(315z^6-1050z^4+445z^2-13)$$

$$+\frac{3}{350}S(315z^6-1050z^4+495z^2+42)-\frac{72}{175}K^2(175z^4-105z^2+6)$$

$$-\frac{36}{175}S^2(175z^4-105z^2+2)-\frac{36}{25}KS(75z^4-45z^2+2)+0(Rw) \quad \ldots \quad (33)$$

Chen Che-Pen (1966) in his experimental investigation for flow between non-porous disks, assumed $(Re) > 1000$ On the assumption that (Re) is of the same order as used by Che-Pen, the terms containing $\frac{1}{(Re)}$ in (32) and (33) have been neglected

On integrating equation (13) we have

$$h = -(Rw)(f'_{-1}+\tfrac{1}{2}f_{-1}^2(Rw))+K(Rw)[(Rw)(2f_{-1}f'''_{-1}-f_{-1}f''_{-1}+6f'_{-1}f''_{-1}$$

$$+3f''^2_{-1})+2(Re)f''_{-1}f''_0]+S(Rw)[\tfrac{7}{2}(Rw)f'^2_{-1}+(Re)f''_0f''_{-1}]+\text{const} \quad \ldots \quad (34)$$

where the constant is determined from a known pressure at a point in the flow

Above expressions are in agreement with Elkouh (1969) on taking $K = S = 0$. Equations (21), (25), (29) and (32) represent the solution of laminar source flow between non-porous disks, while equations (22), (23), (26), (27) and (30) are due to interaction between the flow of system I and the source flow

4. Velocity Distribution

The component of radial velocity, in terms of the average radial velocity is given by

$$u^* = \frac{u}{<u>} = \frac{\tfrac{1}{2}(Rw)f'_{-1}+\frac{(Re)}{r^2}\left[f'_0+\left(\frac{(Re)}{r^2}\right)f'_1+\left(\frac{(Re)}{r^2}\right)^2 f_2'\right]}{\left(\tfrac{1}{2}(Rw)+\frac{(Re)}{r^2}\right)} \quad \ldots \quad (35)$$

where
$$\langle u \rangle = \int_0^1 u\,dz = \tfrac{1}{2}\, r(Rw) + \frac{(Re)}{\ldots}$$

As we have taken (Re) to be very large, that is, f'_2 is independent of (Re), the radial velocity distribution is only a function of (Rw) and (Re^*) $(= (Re)/r^2)$. Equation (35) will be valid at large distance from the source. The series representation for u^* can converge only for $r > \sqrt{(Re)}$. For $r < \sqrt{(Re)}$, that is, in the neighbourhood of the source, the solution given by (35) will not be valid In the absence of porosity the flow in the neighbourhood of source will be entirely radial. The effect of non-Newtonian parameters has been shown graphically on various

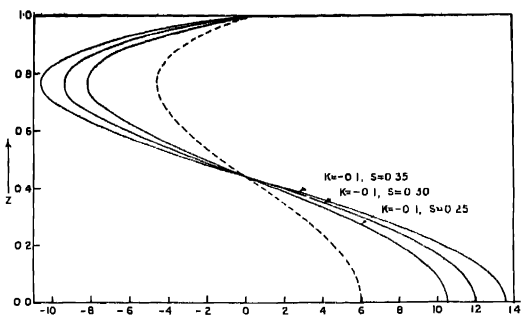

Fig I $f'_{0,1} \times 10^2$ AGAINST Z —— NON-NEWTONIAN, ------ NEWTONIAN

Figure 1.

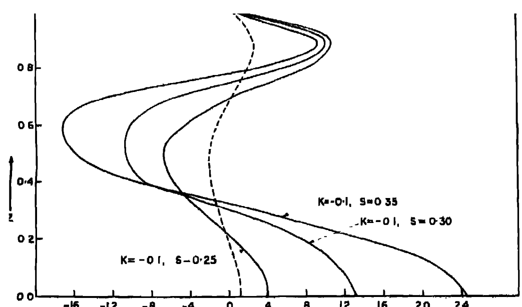

FIG 2 $f'_{0,2} \times 10^3$ AGAINST Z —— NON-NEWTONIAN, -----NEWTONIAN

Figure 2.

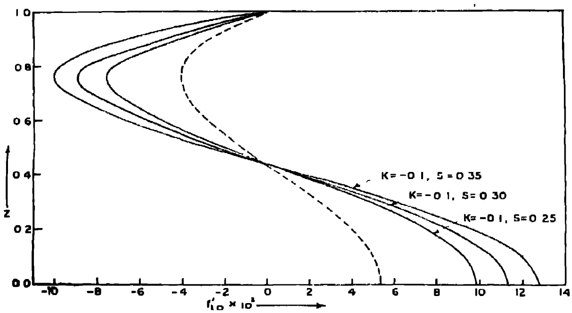

Fig 3 $f'_{1,0} \times 10^3$ AGAINST Z ——— NON-NEWTONIAN, - - - - - NEWTONIAN

Figure 3.

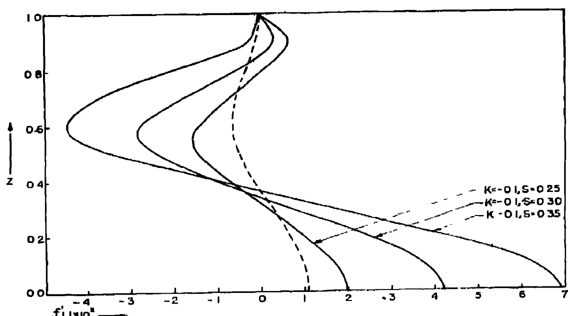

FIG 4 $f'_{1,1} \times 10^2$ AGAINST Z ——— NON-NEWTONIAN - - - - NEWTONIAN

Figure 4.

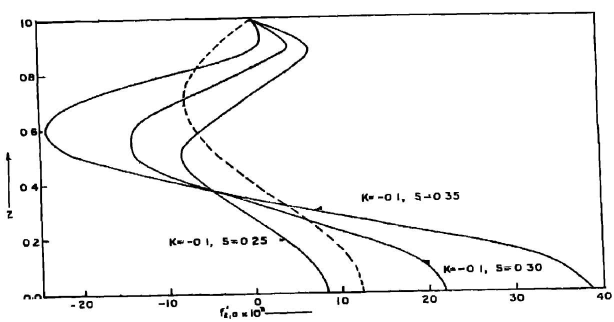

Fig. 5 $f'_{2,0} \times 10^3$ AGAINST Z ——— NON-NEWTONIAN, - - - - - - NEWTONIAN

Figure 5.

perturbation terms. For numerical work (Rw) has been taken to be 1 Figure 1 shows the effect of visco-elasticity and cross-viscosity on $f'_{0,1}$ and compared with Newtonian profile The magnitude of $f'_{0,1}$ in second order fluid is always greater than that in Newtonian viscous fluids This difference goes on decreasing as we move from the central region towards the upper disk, till it becomes zero, and then it again shows a variation and the profiles coincide again at the upper disk The effects of non-Newtonian parameters on other radial velocity perturbations $f'_{0,2}, f'_{1,0}, f'_{1,1}$ and $f''_{2,0}$ have been depicted in figures 2,3,4 and 5 respectively.

5. Streamlines

The stream function ψ is given by equation (10) This equation is rewritten in the following form .

$$\bar{\psi} = \frac{1}{2} R^2(Rw)f_{-1}(z) + f_0(z) + \frac{1}{R^2}f_1(z) + \dots , \qquad \dots (36)$$

where $\bar{\psi} = \psi/(Re)$ and $R = r/\sqrt{(Re)}$.

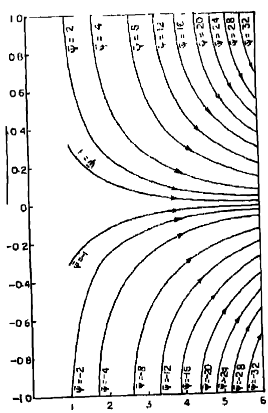

Figure 6.

The streamlines have been drawn in figure 6 for $(Rw) = 0.5$, $K = -0.1$ and $S = 0.3$ from $R = 1$ to $R = 6$. It is noted that the injected fluid particles will first move towards the mid-plane and then will be moving parallel to the mid-plane.

6. Pressure Distribution

Equations (11), (24), (28), (31), (33) and (34) give the pressure distribution $p(r,z)$. The constant of integration has been determined by the assumption that the pressure at a point $(R, +1)$ in the flow is known The pressure drop in the radial direction is defined as

$$p^* = p(r,z) - p(R,z). \tag{37}$$

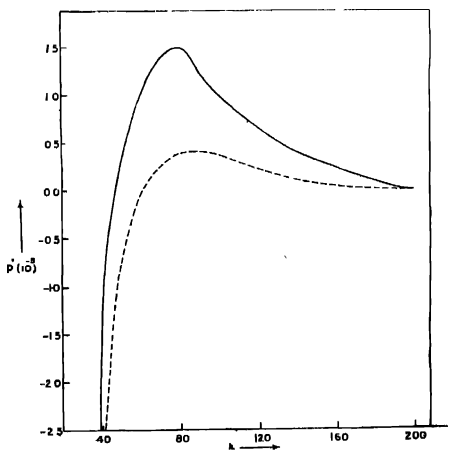

Fig 7 PRESSURE DROP IN RADIAL DIRECTION FOR Rw=-0.50, Re=10^4 ——— NON-NEWTONIAN (K=-0.1, S=.25), ------ NEWTONIAN

Figure 7.

Here we note a significant difference between Newtonian and non-Newtonian case In Newtonian case the pressure drop in radial direction is independent of axial distance, that is, remain constant on all planes perpendicular to z-axis, as shown by Elkouh (1969). In the present case it is dependent on axial distance and therefore will be different on different planes perpendicular to z-axis The difference of pressure drop in Newtonian and non-Newtonian case has been shown figure 7 For numerical work we have taken $(Re) = 10^4$, $R = 200$ and $(Rw) = -0.50$, $K = -0.1$, $S = 0.25$ The magnitude of pressure drop in the present case is more than that in the Newtonian fluids.

7. Skin Friction

The non-dimensional radial shear stress is given by

$$T_{rz} = \left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)+K\left[\frac{\partial}{\partial z}\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)+\frac{\partial}{\partial r}\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)\right.$$

$$\left.+2\frac{\partial u}{\partial r}\frac{\partial u}{\partial z}+2\frac{\partial w}{\partial r}\frac{\partial w}{\partial z}\right]+S\left[2\frac{\partial u}{\partial r}\left(\frac{\partial w}{\partial z}+\frac{\partial w}{\partial r}\right)+2\frac{\partial w}{\partial z}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right], \qquad (38)$$

where $$\tau = \left(\frac{\psi_1}{a^2\rho}\right)T_{rz}. \qquad (38)$$

From equations (9) and (38) T_{rz} at upper disk is given by

$$(T_{rz})_{z=1} = \tfrac{1}{2}\, r(Rw)\left[-3-(Rw)\left\{-\frac{9}{35}-\frac{3}{5}(2K+S)\right\}-\frac{1}{175}(Rw)^2\left\{\frac{447}{77}-\right.\right.$$

$$\left.\left.-30K+25S-342K^2+27S^2-117KS\right\}\right]+\frac{(Re)}{r}\left[-3-(Rw)\left\{-\frac{3}{7}-\frac{12}{5}(K+S)\right\}\right.$$

$$-\frac{1}{175}(Rw)^2\left\{\frac{1811}{231}-99K-4S-1116K^2-180S^2-1296KS\right\}+\frac{(Re)}{r^2}\left\{\frac{12}{35}\right.$$

$$\left.+\frac{12}{5}(K+S)-\frac{1}{175}(Rw)\left(-\frac{2}{33}-306K-76S-2412K^2-432S^2-2844KS\right)\right\}$$

$$\left.+\frac{1}{175}\left(\frac{(Re)}{r^2}\right)^2\left\{\frac{636}{77}+294K+54S+1728K^2+288S^2+2016KS\right\}\right]-$$

$$-K(Rw)\left[\tfrac{1}{2}r(Rw)\left\{-3+(Rw)\left\{\frac{51}{35}+\frac{18}{5}(2K+S)\right\}\right\}+\frac{(Re)}{r}\left\{-3+(Rw)\left\{-\frac{183}{70}\right.\right.\right.$$

$$\left.+\frac{72}{5}(K+S)\right\}+\frac{1}{175}(Rw)^2\left\{-\frac{6661}{77}-564K-791S+3006K^2-1395S^2\right.$$

$$\left.\left.\left.+1611KS\right\}\right\}\right] \qquad \ldots \quad (39)$$

The skin friction τ at the upper disk is defined as :

$$\tau = -(T_{rz})_{z=1} \qquad \ldots \quad (40)$$

The expression for skin friction agrees with Elkouh (1969) if $K = S = 0$. The following table shows the effect of non-Newtonian parameters on $\tau^*(=\tau/r)$ for different values of $r^*(=r/\sqrt{(Re)})$. For numerical work we have taken $(Rw) = 0.25$ and $K = S = 0$, $K = -0.01$, $S = 0.025$.

Table 1

	$(K - S\ 0)$	$(K - -0.01, S - 0\ 025)$
1 5	1.5822	1 5789
2 0	1.0690	1 0671
2 5	0 8216	0 8206
3 0	0 6816	0 6841
3 5	0 6013	0 6009
4 0	0 5169	0 5157

Here we find that the skin friction in the case of non-Newtonian is less then that of Newtonian case

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