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Heat transfer from two parallel coaxial disks rotating at different speeds with a source on the axis of rotation

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Heat transfer from two parallel coaxial disks rotating at different speeds in the presence of a source on the axis of rotation has been investigated The solution has been obtained in the form of double series expansion The effect of rotation on temperature profile and Nusselt's number has been discussed

Introduction

Source flow between two parallel disks rotating with the same velocity has been studied by Breitner & Pohlhausen (1962) and Kreith & Peube (1965, 1966). Kreith & Viviand (1967) considered the axisymmetric flow between two disks, rotating at different angular velocities with a line source at the centre The equations of motion are solved by double series expansion about a known solution at a large radius. The results are valid for small rotational Taylor numbers of the disks and at a distance $r \gg (\mathrm{Re})^{\frac{1}{2}}$.

In the present paper the nature of heat transfer has been investigated between two parallel coaxial disks rotating at different speeds. A line source has been assumed to be present on the axis of rotation The surfaces of the two disks are taken to be at constant temperatures. The fluid is incompressible, so that the momentum equations are independent of the heat transfer phenomena. Temperature distribution has been obtained as a double series expansion The energy equation is simplified by expanding temperature in powers of downstream coordinate. The resulting equations have been solved for small Prandtl number The effect of rotation on temperature distribution has been shown graphically. Nusselt's numbers for both the disks have also been calculated.

Statement of Problem

Consider the flow of a viscous fluid between two parallel rotating disks with a source on the axis of rotation. We shall work with cylindrical polar coordinates $(\bar{r}, \theta, \bar{z})$. Let the middle point of the axis of rotation be the origin. The surfaces ' ' ' ,ve shall ks are defined by $\bar{z} = +a$ and $\bar{z} = -a$, respectively. The upper propagation. constant angular velocity ω_2 and the lower one with ω_1. The flow rate of the source is Q. The boundary conditions on the velocity profile are

$$\left.\begin{array}{l} \bar{u} = \bar{w} = 0, \quad \text{at } \bar{z} = \pm a, \\ \bar{v} = \bar{r}\omega_1, \text{ at } \bar{z} = -a, \; \bar{v} = \bar{r}\omega_2, \text{ at } \bar{z} = +a, \\ \int_{-a}^{a} 2\pi r u \, \mathrm{d}\bar{z} = Q, \end{array}\right\} \qquad \dots \quad (1)$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are the components of velocity along $\bar{r}$, θ, $\bar{z}$ directions.

The axisymmetric form of energy equation in cylindrical polar coordinates is

$$\rho c_p\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}\right) = k\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) + \bar{\Phi}, \qquad \dots \quad (2)$$

where c_p, k, ρ and $\bar{T}$ are specific heat, thermal conductivity, density and temperature, respectively. Viscous dissipation ($\bar{\Phi}$) of the fluid in axisymmetric case is given by

$$\bar{\Phi} = 2\mu\left[\left(\frac{\partial \bar{u}}{\partial \bar{r}}\right)^2 + \left(\frac{\bar{u}}{\bar{r}}\right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2 + \frac{1}{2}\left(\frac{\partial \bar{v}}{\partial \bar{z}}\right)^2 + \right.$$

$$\left. + \frac{1}{2}\left(\frac{\partial \bar{w}}{\partial \bar{r}} + \frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 + \frac{1}{2}\left(\frac{\partial \bar{v}}{\partial \bar{r}} - \frac{\bar{v}}{\bar{r}}\right)^2\right]. \qquad \dots \quad (3)$$

where μ is the coefficient of viscosity. The boundary conditions for temperature are

and

$$\left.\begin{array}{l} \bar{T} = \bar{T}_1 \text{ at } \bar{z} = -a, \\ \bar{T} = \bar{T}_2 \text{ at } \bar{z} = +a. \end{array}\right\}. \qquad \dots \quad (4)$$

Appropriate dimensionless variables are defined by the following relations

$$\left.\begin{array}{l} \bar{r} = ar, \; \bar{z} = az, \; \bar{u} = \dfrac{u\nu}{a}, \; \bar{v} = \dfrac{v\nu}{a}, \; \bar{w} = \dfrac{w\nu}{a}, \\ \bar{T} = \dfrac{\nu^2}{a^2 c_p} T, \quad \bar{\Phi} = \dfrac{\rho\nu^3}{a^4}\phi, \end{array}\right\} \qquad \dots \quad (5)$$

where $\frac{\mu}{\rho}$.

ns (2), (3) and (5) give

$$+w\frac{\partial T}{\partial z} = \frac{1}{P}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right.$$

$$\left.\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{u}{r}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2, \quad (6)$$

(Prandtl number) $= \frac{\mu c_p}{k}$.

lified boundary conditions on the fluid and temperature are

$$u = w = 0, \text{ at } z = \pm 1,$$

$$\int_{-1}^{1} u \, dz = 2\frac{(\text{Re})}{r}, \quad (7)$$

$$v = \alpha_1 r, \text{ at } z = -1, \; v = \alpha_2 r, \text{ at } z = +1,$$

$$T = s_1 \text{ at } z = -1, \; T = s_2 \text{ at } z = +1, \quad (8)$$

$$\alpha_1 = \frac{\omega_1 a^2}{\nu}, \quad \alpha_2 = \frac{\omega_2 a^2}{\nu}, \quad (\text{Re}) = \frac{Q}{4\pi a\nu},$$

$\bar{T}_1$, and $s_2 = \frac{a^2 c_p}{\,^2}\bar{T}_2$

Solution of Equations

ing Kreith & Viviand (1967) the forms of u, v and w are taken as the

$$rf'_{-1}(z) + (\text{Re})^{\frac{1}{2}}\left[\frac{(\text{Re}^{\frac{1}{2}})}{r}f_1'(z) + \frac{(\text{Re})^{3/2}}{r^3}f_3'(z) + \ldots\ldots\right],$$

$$-2f_{-1}(z) + 2\frac{(\text{Re})^2}{r^4}f_3(z) + \ldots\ldots, \quad (9)$$

$$rg_{-1}(z) + (\text{Re})^{\frac{1}{2}}\left[\frac{(\text{Re})^{\frac{1}{2}}}{r}g_1(z) + \frac{(\text{Re})^{3/2}}{r^3}g_3(z) + \ldots\ldots\right],$$

ime denotes differential coefficient with respect to z. $f_n(z)$ and $g_n(z)$ are nless functions to be determined from momentum equations These have been determined by Kreith & Viviand (1967) for small values of α_1 and α_2 Casal (1950) states that the expansions in the case $\alpha_1 = 0$, are convergent for $|\alpha_2| < 0.17$. Equations (6) and (9) give

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}-\frac{1}{\mathrm{P}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=r^2(g'^2_{-1}+f''^2_{-1})$$

$$+2(6f'^2_{-1}+(\mathrm{Re})g'_{-1}g'_1+(\mathrm{Re})f''_{-1}f''_1)+\frac{(\mathrm{Re})^2}{r^2}(g'^2_1+4g'_{-1}g'_3+2f''_{-1}f''_3 \quad \ldots \quad (10)$$

where terms upto $(1/r^2)$ have been retained on right hand side. Equation (10) readily suggests that the form of T should be

$$T(r,z) = r^2 T_{-2}(z) + T_0(z) + \frac{1}{r^2}T_2(z) + \ldots \qquad (11)$$

We substitute (11) into (10) and equate the coefficients of equal powers of r on both sides. This gives us a set of ordinary differential equations. The first two differential equations are

$$T''_{-2} = \mathrm{P}(2f'_{-1}T_{-2} - 2f_{-1}T'_{-2} - g'^2_{-1} - f''^2_{-1}), \qquad \ldots \quad (12)$$

$$T''_0 = -T_{-2} + \mathrm{P}(2(\mathrm{Re})f'_1 T_{-2} - 2f_{-1}T'_0 - 12f'^2_{-1} - 2(\mathrm{Re})g'_{-1}g'_1 - 2(\mathrm{Re})f''_{-1}f''_1) \quad \ldots (13)$$

The modified boundary conditions for temperature are

$$\left.\begin{array}{l} T_{-2}(\pm 1) = 0, \\ T_0(-1) = s_1, \quad T_0(+1) = s_2 \end{array}\right\} \qquad \ldots \quad (14)$$

Equation (12) is a non-linear ordinary differential equation. Its solution is obtained by a perturbation method in powers of Prandtl number P in the form

$$T_{-2} = T_{-2,0} + \mathrm{P}\, T_{-2,1} + \mathrm{P}^2 T_{-2,2} + \ldots \qquad \ldots \quad (15)$$

We substitute (15) in (12) and equate like powers of P on both sides. This gives a set of linear ordinary differential equations. These equations together with modified boundary conditions give

$$T_{-2} = \frac{1}{8}\mathrm{P}(\alpha_1+\alpha_2)^2(1-z^2) + 0(\mathrm{P}^3). \qquad \ldots \quad (16)$$

Proceeding in a similar manner, the solution of equation (13) is

$$T_0 = \tfrac{1}{2}[(s_2-s_1)z + (s_2+s_1)] + \mathrm{P}[(\alpha_1+\alpha_2)^2\phi_1 + (s_2-s_1)\phi_2$$
$$+ (\mathrm{Re})(\alpha_1^2\phi_3 + \alpha_1\alpha_2\phi_4 + \alpha_2^2\phi_5)] + \mathrm{P}^2[(\mathrm{Re})(\alpha_1+\alpha_2)^2\phi_6] + 0(\mathrm{P}^3) \qquad \ldots \quad (17)$$

where

$$\phi_1 = \frac{1}{96}(z^4-6z^2+5),$$

$$\phi_2 = \frac{1}{50400}\alpha_1^2(5z^7-35z^6-21z^5+175z^4+35z^3-525z^2-19z+385)$$

$$+\frac{1}{50400}\alpha_2^2(5z^7+35z^6-21z^5-175z^4+35z^3+525z^2-19z-385)$$

$$-\frac{1}{25200}\alpha_1\alpha_2(5z^7-21z^5+35z^3-19z),$$

$$\phi_3 = -\frac{1}{240}(z^6-12z^5+9z^4-40z^3-21z^2+52z+11),$$

$$\phi_4 = \frac{1}{120}(z^6+9z^4-21z^2+11),$$

$$\phi_5 = -\frac{1}{240}(z^6+12z^5+9z^4+40z^3-21z^2-52z+11),$$

$$\phi_6 = \frac{1}{80}(z^6-5z^4+15z^2-11).$$

In obtaining the results given in (15), (16) the values of $f_{-1}(z)$, $g_{-1}(z)$, $f_1(z)$ and $g_1(z)$ have been taken from Kreith & Viviand (1967).

DISCUSSIONS

We introduce another dimensionless temperature in the form

$$T^* = \frac{T-s_1}{s_2-s_1}. \qquad \dots (19)$$

Equations (11), (16), (17) and (19) give

$$T^* = \frac{1}{8}E_2 Pr^2(\alpha+1)^2(1-z^2)+\frac{1}{2}(1+z)+P[E_2(\alpha+1)^2\phi_1+\phi_2+$$

$$+E_2(Re)(\alpha^2\phi_3+\alpha\phi_4+\phi_5)]+P^2E_2(Re)(\alpha+1)^2\phi_6 \qquad \dots (20)$$

where E_2(Eckert number) $= \dfrac{\alpha_2^2}{s_2-s_1} = \dfrac{\omega_2^2a^2}{c_p(T_2-T_1)}$,

and $\alpha = \dfrac{\alpha_1}{\alpha_2}$

The dimensionless temperature in absence of source ($\bar{T}^*$) can be obtained by taking (Re) = 0 in equation (20) Hence we have

$$T^*-\bar{T}^* = \mathrm{PE_2(Re)}(\alpha^2\phi_3+\alpha\phi_4+\phi_5)+\mathrm{P^2E_2(Re)}(\alpha+1)^2\phi_6. \quad \ldots \quad (21)$$

The variation of $(T^*-\bar{T}^*)/\mathrm{E_2(Re)}$ against z has been shown in figure 1, for different values of α. For numerical work we have taken P = 1 We note from the

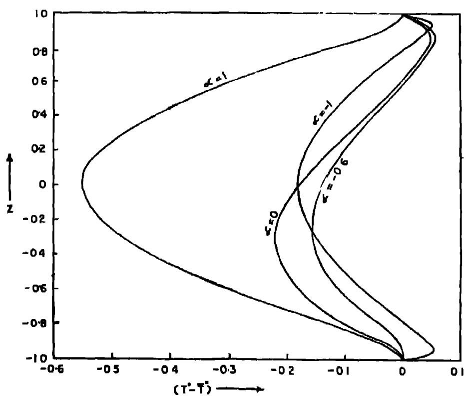

Figure 1

figure that $T^*-\bar{T}^*$ is symmetrical, when the two disks rotate with equal angular velocity in the same or in opposite directions. For $\alpha = 1$, $T^* < \bar{T}^*$ at every point of the region. For $\alpha = -1$, $T^* > \bar{T}^*$ near both the disks, and at other points of the region $T^* < \bar{T}^*$ For $\alpha = 0, -0.6$, $T^* > \bar{T}^*$ near the upper disk only and at other points of the region $T^* < \bar{T}^*$ This indicates that if the two disks rotate with different angular velocities, then $T^*-\bar{T}^*$ is positive only near the disk which rotates with greater angular velocity.

The Nusselt's number of the lower disk is given by

$$(\mathrm{Nu})_{\bar{z}=-a} = 2a(Q^*)_{\bar{z}=-a}/k(\bar{T}_2-\bar{T}_1), \quad \ldots \quad (22)$$

where $(Q^*)_{\bar{z}=-} \quad \dfrac{1}{\pi(\bar{r}^2-\bar{r}_0^2)} \displaystyle\int_{\bar{r}_0}^{\bar{r}} 2\pi\bar{r}(q)_{\bar{z}=-a}\,\mathrm{d}\bar{r}$,

and $(q)_{\bar{z}=-a} = -k\left(\dfrac{\partial T}{\partial \bar{z}}\right)_{\bar{z}=-a}$

such that $\bar{r}_0$ is the distance of a given point on the disk from the axis of rotation, From equations (5), (11), (16) (17) and (22) we have :

$$(Nu)_{\bar{z}=-a} = -1 - \frac{1}{4}PE_2(\alpha+1)^2(r^2+r_0^2) - P\Big[\frac{1}{6}E_2(\alpha+1)^2 + \frac{4}{175}\alpha_1^2 - \frac{34}{1575}\alpha_2^2 - \frac{2}{1575}\alpha_1\alpha_2 - \frac{16}{15}E_2(Re)(1-x^2)\Big] + \frac{2}{5}(Re)P^2E_2(\alpha+1)^2. \quad (23)$$

$$(Nu)_{\bar{z}=+a} = -1 + \frac{1}{4}PE_2(\alpha+1)^2(r^2+r_0^2) + P\Big[\frac{1}{6}E_2(\alpha+1)^2 + \frac{34}{175}\alpha_1^2 - \frac{4}{175}\alpha_2^2 + \frac{2}{175}\alpha_1\alpha_2 + \frac{16}{15}E_2(Re)(1-\alpha^2)\Big] - \frac{2}{5}(Re)P^2E_2(\alpha+1)^2. \quad \ldots (24)$$

The effect of different speeds of rotation on the Nusselt's number of the disk $\bar{z} = -1$ has been shown in figures 2 and 3.

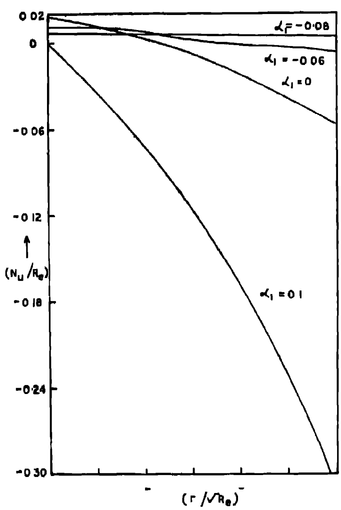

Figure 2

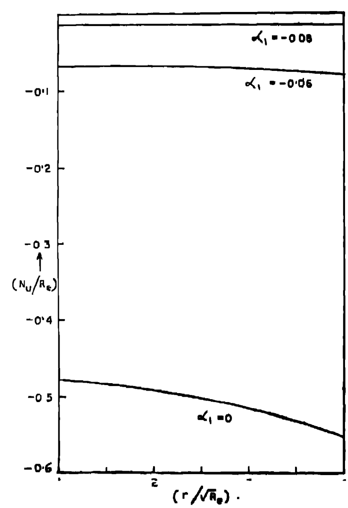

Figure 3

For numerical work we have assumed (Re) $= 1000$, $P = 1, E_2 = 0.02$, $\alpha_2 = 0.1$ The graphs have been drawn for $(r_0/(Re)^{\frac{1}{2}}) = 1$ and 10. We note from figures 2 and 3 that with decrease in the value of α_1 the slope of the curve decreases At $\alpha_1 = -0.1$, it will be almost parallel to the axis representing $(r/(Re)^{\frac{1}{2}})$.

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