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Response of a moving-coil galvanometer in a vacuum tube circuit

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The nature of response of a moving-coil galvanometer to an exponentially decreasing input voltage pulse in a VTVM circuit has been theoretically analysed. The output pulse, as revealed by the galvanometer deflection consists of the following two components :

(i) first component decreases exponentially with time;

(ii) second component varies sinusoidally with time, having constant amplitude, provided the effect of galvanometer damping is negligible. However, when the damping is effective, the amplitude of the oscillatory component decreases exponentially with time.

These findings have been experimentally verified as far as practicable.

Introduction

It is customary to use a high-impedance device for measuring voltage and a low impedance device for measuring current. However, for measurement of very small voltages and currents, just the reverse is true, since power is required to operate the instrument. For example, if the number of turns per unit cross-section is changed in a galvanometer coil, the quantity that is held constant for a given deflection is the power (*i.e.* I^2R or E^2/R). Hence high current sensitivity requires a high-resistance coil, and high voltage sensitivity requires a low-resistance coil.

For the most sensitive galvanometer made the power per unit deflection is approximately $10^{-19}W$. Thus the best voltage sensitivity is about $10^{-9}V$ (for $R \approx 10$ ohms) and the best current sensitivity is about 10^{-11} amp (for $R \approx 1000$ ohms).

For measurements of currents smaller than about 10^{-10} or 10^{-11} amp, a galvanometer becomes impractical, and an electrometer, which is essentially a voltage measuring instrument with a sensitivity of the general order of 1 mv per scale deflection, can be used in conjunction with a shunt resistor of 10^{12} ohms to obtain a sensitivity of 10^{-15} amp. Alternatively, a simple nonfeedback type electronic amplifier consisting of an electrometer type tube with a sensitive galvanometer as the plate load may be used. The grid current for large bias voltages is made very small for proper tube design and may be about 10^{-16} amp for the best tubes. The fundamental circuit, is a single-tube dc amplifier, in which the ionisation current or photo-electric current passes through a grid-leak resistance ($\sim 10^{12}$ ohms) and the resulting voltage alters the grid potential of the electrometer tube The consequent change in plate current is read on the galvanometer. Montgomery & Montgomery (1940) have discussed the circuit diagram of a vacuum tube electrometer when used in conjunction with an Ionization Chamber. Swann (1946) has published the replica of a cosmic ray nuclear burst as recorded by such an arrangement. Weisz & Ramsey (1942) have used a more elaborate arrangement to study the ionizing capacity of individual particles of cosmic ray ionization. The charge produced by each discharge in a proportional counter is converted into a voltage pulse, which is amplified by a three stage linear amplifier and fed into a vacuum-tube voltmeter consisting of a type-38 tube. Finally the pulse is recorded photographically by the amount of deflection of a galvanometer, whenever a ray passes through a path in the proportional counter tube which is defined by three trays of GM counters in a telescopic arrangement.

Detailed analysis of the influence of the time-constants of various coupling stages of a linear amplifier has been worked out by Wilson (1941) and Chatterjee (1944) Lewis (1942) has also indicated the distorted form of an ionization chamber pulse, after three-stage amplification by a R-C coupled linear amplifier.

It may be noted, however, that in almost all the cases cited above, the galvanometer has been used merely as an indicating instrument, whose deflection has been assumed to be proportional to the amplitude of the input pulse. No account has been taken of the influence of the galvanometer constants in influencing the size and the wave-form of the output pulse. These factors have now been taken into consideration in determining the nature of response of a moving coil galvanometer in a vacuum-tube circuit whose input is a transient pulse simulating an Ionization Chamber pulse The relevant circuit is actually that of a valve tube voltmeter (VTVM). Since a voltage sensitive galvanometer is essentially a low resistance instrument,it cannot be used in a high impedance circuit. And so a single-stage dc electronic amplifier is used to amplify the signal and to match a high input impedance to that of a sensitive d'Arsonval type galvanometer. In our present set up, the galvanometer has been transformer coupled to the plate load of the vacuum tube in order to avoid the balancing of plate-current for zero adjustment.

Theoretical considerations

A galvanometer has been transformer coupled to the plate load of the vacuum tube (triode) as shown in figure 1. A transient voltage pulse,

$$\epsilon = \epsilon_0 \, e^{-t/\tau}$$

where t = time and τ = time-constant of the grid-system, has been applied between the grid and the filament of the triode,

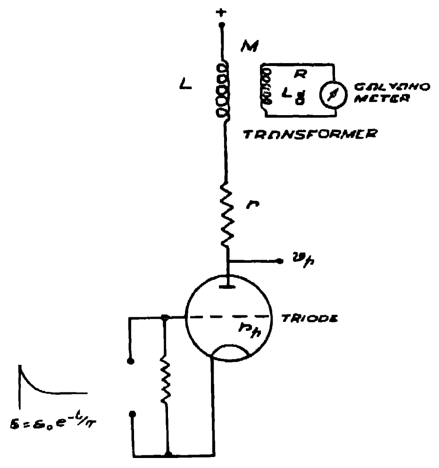

Figure 1. Basic vacuum tube circuit.

Assuming that plate current i_p and plate voltage v_p represent changes from steady current conditions,

$$i_p = \frac{v_p}{r_p} + \frac{\mu e}{r_p} \qquad \ldots \quad (1)$$

where r_p = plate resistance of the tube and μ = amplification factor.

The plate of the triode consists of a protective resistance r in series with the primary of an air- core transformer, while the secondary is coupled to a moving coil galvanometer.

(*a*) *Basic equations*

Now, if L be the co-efficient of self-inductance of the primary and L_g that of the galvanometer circuit and M the mutual inductnce, then the governing equations are the following :

$$L\frac{di_p}{dt} + M\frac{di_g}{dt} + ri_p + v_p = 0 \qquad \ldots \quad (2)$$

and

$$L_g\frac{di_g}{dt} + M\frac{di_p}{dt} + Ri_g = 0 \qquad \ldots \quad (3)$$

where i_g = current in the glvanometer circuit, R = total resistance in the galvanometer circuit.

Eliminating di_p/dt between the equations (2) and (3) and then substituting v_p by the value as obtained from equation (1)

$$(LL_g-M^2)\frac{di_g}{dt}+LR\,i_g-M(r+r_p)i_p=-M\mu\epsilon_0 e^{-t/\tau} \qquad \dots \quad (4)$$

Differentiating equation (4) and then eliminating di_p/dt with the help of equation (3),

$$A\,.\frac{d^2i_g}{dt^2}+B\,.\,\frac{di_g}{dt}+C\,i_g=\frac{M\mu\epsilon_0}{\tau}\;e^{-t/\tau} \qquad \dots \quad (5)$$

where

$$A=LL_g-M^2$$
$$B=LR+(r+r_p)\,.\,L_g.$$
$$C=(r+r_p)\,.\,R$$

(*b*) *Solution for i_g with large plate resistance r_p*

It will, at this stage, be desirable to make such approximations as will apply to the present case. The quantity r_p is of the order of 10^5 ohms. There is no need for r to have more than a protective significance. In view of this large value of r_p, the auxiliary equation of the equation (5) will have the roots

$$-\frac{R}{L_g}\quad\text{and}\quad-\frac{r_p}{L-\dfrac{M^2}{L_g}}$$

while the dominant term in the denominator of the particular integral will be

$$r_p(R\tau^2-L_g\tau)$$

Thus, determination of complementary function and particular integral under the above specification leads the solution of equation (5) into the form :

$$i_g=\frac{M\mu\epsilon_0}{r_p(R\tau-L_g)}\left[e^{-t/\tau}+\alpha\,.\,e^{-\frac{Rt}{L_g}}+\beta\;e^{-\frac{r_pt}{L-\frac{M^2}{L_g}}}\right] \qquad \dots \quad (6)$$

and

$$\frac{di_g}{dt}=\frac{M\mu\epsilon_0}{r_p(R\tau-L_g)}\,.\left[-\frac{e^{-t/\tau}}{\tau}-\alpha\frac{R}{L_g}\;e^{-\frac{Rt}{L_g}}-\beta\frac{r_p}{L-\dfrac{M^2}{L_g}}\,.\,e^{-\frac{r_pt}{L-\frac{M^2}{L_g}}}\right] \quad \dots \quad (7)$$

At $t=0$, $\qquad i_g=0$ and $i_p=0$

Hence from equation (6)

$$1+\alpha+\beta=0. \qquad \dots \quad (8)$$

and from the equations (4) and (7),

$$\left(\frac{di_g}{dt}\right)_{t=0} = -\frac{M\mu\epsilon_0}{LL_g-M^2} = -\frac{M\mu\epsilon_0}{r_p(R\tau-L_g)}\left[\frac{1}{\tau}+\frac{\alpha R}{L_g}+\frac{\beta r_p}{1-\frac{M^2}{L_g}}\right]$$

$$\approx -\frac{M\mu\epsilon_0}{R\tau-L_g}\cdot\frac{\beta}{L-\frac{M^2}{L_g}} \text{ (approximately)}$$

since r_p is large.

Therefore,

$$\beta = \frac{R\tau}{L_g}-1$$

and

$$\alpha = -(\beta+1) = -\frac{R\tau}{L_g} \quad \text{by equation (8).}$$

Substituting these values of α and β in equation (6)

$$i_g = \frac{M\mu\epsilon_0}{r_p(R\tau-L_g)}\left[e^{-t/\tau}-\frac{R\tau}{L_g}\cdot e^{-Rt/L_g}+\left(\frac{R\tau}{L_g}-1\right)e^{-L_g r_p t/(LL_g-M^2)}\right] \quad \dots (9)$$

(c) *Galvanometer deflection with negligible damping*

Due to the current i_g, the galvanometer coil would be deflected through an angle θ. If R be large enough so that the damping in the galvanometer circuit is negligible, then the differential equation giving θ, is

$$K\frac{d^2\theta}{dt^2}+C\theta = \frac{JG^{\frac{1}{2}}.M\mu\epsilon_0}{r_p(R\tau-L_g)}\left[e^{-t/\tau}-\frac{R\tau}{L_g}\,e^{-Rt/L_g}+\left(\frac{R\tau}{L_g}-1\right)e^{-r_pL_gt/(LL_g-M^2)}\right] \quad \dots (10)$$

where K = moment of inertia of the suspended system and $JG^{\frac{1}{2}}$ galvanometer constant.

Solving this equation (10) under the initial conditions *viz*,

at $t = 0$, $\theta = 0$ and $\frac{d\theta}{dt} = 0$

and introducing the quantity τ_g defined by $\tau_g = \frac{L_g}{R}$, the deflection θ is,

$$\theta = \frac{W}{4\pi^2}\left[\frac{e^{-t/\tau}}{1+\frac{T^2}{4\pi^2\tau^2}} - \frac{\frac{\tau}{\tau_g}.e^{-t/\tau_g}}{1+\frac{T^2}{4\pi^2\tau_g^2}} + \frac{\sin\left(\frac{2\pi t}{T}-\phi_1\right)}{\left(1+\frac{T^2}{4\pi^2\tau^2}\right)^{\frac{1}{2}}} - \frac{\frac{\tau}{\tau_g}\sin\left(\frac{2\pi t}{T}-\phi_2\right)}{\left(1+\frac{T^2}{4\pi^2\tau^2_g}\right)^{\frac{1}{2}}}\right] \quad \ldots \quad (11)$$

where T = free period of the galvanometer,

$$\tan\phi_1 = \frac{2\pi\tau}{T}, \qquad \tan\theta_2 = \frac{2\pi\tau_g}{T}$$

and $$W = \frac{4\pi^2 \,.\, JG^{\frac{1}{2}} \,.\, M\mu\epsilon_0}{Cr_p(R\tau - L_g)}.$$

The solution (11) has been written on neglecting the term which involves the large quantity r_p^2 in the denominator. The equation (11) gives θ as a function of time t; its maximum occurs when

$$\frac{d\theta}{dt} = 0,$$

i.e. when

$$-\frac{\frac{1}{\tau}.e^{-t/\tau}}{1+\frac{T^2}{4\pi^2\tau^2}} + \frac{\frac{1}{\tau_g}\left(\frac{T}{\tau_g}\right).e^{-t/\tau_g}}{1+\frac{T^2}{4\pi^2\tau_g^2}}$$

$$+\frac{\frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}-\phi_1\right)}{\left(1+\frac{T^2}{4\pi^2\tau^2}\right)^{\frac{1}{2}}} - \frac{\frac{2\pi\tau}{T\tau_g}\cos\left(\frac{2\pi t}{T}-\phi_2\right)}{\left(1+\frac{T^2}{4\pi^2\tau_g^2}\right)^{\frac{1}{2}}} = 0.$$

In view of the complexity of the expression, it is probably better to obtain the maximum by plotting θ against t.

Now

$$\frac{W}{4\pi^2} = \frac{JG^{\frac{1}{2}} \,.\, M\mu\epsilon_0}{Cr_p(R\tau - L_g)}.$$

Since G represents the resistance of the galvanometer coil and R the resistance of the secondary of the transformer,

$$R = R_c + G$$

and so

$$\frac{W}{4\pi^2} = \frac{J.\mu\epsilon_0}{cr_p\tau}.\left(\frac{G^{\frac{1}{2}}.M}{R_c+G-\frac{L_g}{\tau}}\right)$$

Also for a given coil space in the secondary and a fixed primary, L_g is proportional to the square of the number of turns and so to R_c; hence L_g/τ becomes equal to gR_c, where g is the constant of proportionality.

Also M is proportional to $R_c^{\frac{1}{2}}$. Thus for a given resistance R_1 of the primary coil, $G = R_c(1-g)$ for a maximum and the maximum is proportional to $R_c^{\frac{1}{2}}/R_c^{\frac{1}{2}}$. and so, is independent of R_c Of course, θ involves G and R_c other than through W. However, the importance of G and R relationship lies in W.

Now, $JG^{\frac{1}{2}}$ is the coefficient of current in the expression for couple per unit angle of twist. Hence for a deflection $\delta\theta$ due to a current δi_g,

$$c\delta\theta = JG^{\frac{1}{2}}\,.\,\delta i_g.$$

If σ be the current sensitivity of the galvanometer, then

$$\sigma = \operatorname*{Lt}_{\delta i_g \to 0} \frac{\delta\theta}{\delta i_g} = \frac{JG^{\frac{1}{2}}}{c},$$

and so

$$\frac{W}{4\pi^2} = \frac{\sigma M\mu\epsilon_0}{r_pR\tau\left(1-\frac{\tau_g}{\tau}\right)}$$

This looks as though θ tends to become infinite as τ_g approaches τ; but it may be noted also that as τ_g approaches τ, the quantity inside the square bracket in equation (11) tends to become zero. It is of interest to inspect the order of

$$\frac{\sigma M\mu\epsilon_0}{r_pR\tau}.$$

If σ be of the order of 10^9, $\mu \approx 10^3$ and $\epsilon_0 \approx 10^{-5}$, thus remembering that $r_p = 10^5$ and $\tau = 10^{-1}$, it may be seen that

$$\frac{\sigma M\mu\epsilon_0}{r_nR\tau} \approx \frac{10^9\times10^3\times10^{-5}}{10^5\times10^{-1}}.\frac{M}{R} = 1000\,\frac{M}{R}.$$

(*d*) *Galvanometer deflection after a long interval of time*

It is interesting to note the amplitude of oscillation long after the incident pulse had been applied to the input terminals *i.e.* at $t = \infty$.

At $t = \infty$, as follows from equation (11),

$$\theta_\infty = \frac{\sigma M \mu \epsilon_0}{r_p R\tau\left(1-\frac{\tau_g}{\tau}\right)}\left[\left(\frac{x}{1+x^2}-\frac{y^2/x}{1+y^2}\right)^2+\left(\frac{1}{1+x^2}-\frac{y/x}{1+y^2}\right)\right] \sin(\zeta-\phi)$$

where
$$\tan\phi = \frac{\frac{1}{1+x^2}-\frac{y/x}{1+y^2}}{\frac{x}{1+x^2}-\frac{y^2/x}{1+y^2}}$$

$$x = \frac{T}{2\pi\tau},\ y = \frac{I}{2\pi\tau_g},\ \zeta = \frac{2\pi t}{T}$$

and
$$\frac{y}{x} = \frac{\tau}{\tau_g}$$

(x and y are constants but ζ varies with time t).

Hence the amplitude of θ_∞ is

$$|\theta_\infty| = \frac{\sigma M \mu \epsilon_0}{r_p R\tau\left(1-\frac{\tau_g}{\tau}\right)}\left[\frac{1}{1+x^2}+\frac{y^2/x^2}{1+y^2}-2\frac{(y^2+y/x)}{(1+x^2)(1+y^2)}\right]^{\frac{1}{2}} \qquad \ldots (12)$$

It is of interest to investigate $|\theta_\infty|$ for different values of y and x.

Case I

Let $y/x = 1+\eta$, such that η^2 is negligible. If $y = 1$, the result is indeterminate. Substituting this in equation (12) and remembering that η is small,

$$|\theta_\infty| = -\frac{\sigma M \mu \epsilon_0}{r_p R\tau_g\eta}\left[\frac{2\eta\left(1-\frac{x^2}{1+x^2}-\frac{1}{1+x^2}\right)}{1+x^2}\right]^{\frac{1}{2}}$$

The radical vanishes, as it should, to the first order.

Case II

Let $y/x = 10$, $x = 2$, $y = 20$.

Substituting these in equation (12)

$$|\theta_\infty| = \frac{\sigma M \mu \epsilon_0}{r_p R\tau(0.9)}\left[\frac{1}{5}+\frac{100}{(1+400)}-\frac{2(400+10)}{5(1+400)}\right]^{\frac{1}{2}}$$

$$= \frac{\sigma M \mu \epsilon_0}{r_p R\tau(0.9)}\cdot[0.2+0.25-0.4]^{\frac{1}{2}} \text{ approximately.}$$

Therefore

$$\frac{|\theta_\infty|}{\dfrac{\sigma M\mu\epsilon_0}{r_p R\tau}} = \frac{2.23}{9}$$

Furthermore, let $y/x = 10$, $x = 4$, $y = 40$

In this case, by substitution in equation (12)

$$|\theta_\infty| = \frac{\sigma M\mu\epsilon_0}{r_p R\tau(0.9)}\left[\frac{1}{17}+\frac{1600}{16(1+1600)}-\frac{2(1600+40/4)}{(1+16)(1+1600)}\right]$$

$$= \frac{\sigma M\mu\epsilon_0}{r_p R\tau(0.9)}\left(\frac{1}{16^2}\right)^{\frac{1}{2}} \quad \text{approximately}$$

Hence

$$\frac{|\theta|}{\dfrac{\sigma M\mu\epsilon_0}{r_p R\tau}} = \frac{0.625}{9}$$

Hence it is important to have x not so large. Remembering that where T = free period of the galvanometer and τ, the time-constant of the grid system, it is advantageous to have the free period of the galvanometer to be small and the time constant of the grid system correspondingly large.

Note : By making y/x very small, the quantity inside the square bracket in equation (12), may be approximated to $[1(1+a^2)]^{\frac{1}{2}}$ This is reasonable because it is equivalent to lengthening τ to some extent although the external factor $1-\tau_g/\tau$ is admittedly affected

(e) *Conclusion*

The expression for θ in equation (11) suggests that the galvanometer deflection is a consequence of two components. (*i*) One of these decreases exponentially with time, and (*ii*) the other varies sinusoidally with time, provided the galvanometer damping factor is negligibly small.

The exponential component dominates so long as the time t is comparable with the time constant of the grid system and also with the time constant of the galvanometer circuit.

After a long interval, only the sinusoidal component persists. The amplitude of this component depends upon the following :

(*i*) time constant τ of the grid system

(*ii*) free period T of the galvanometer

(*iii*) time constant τ_g of the galvanometer-circuit.

The amplitude may also be increased by decreasing the quantities $T/2\pi\tau$ and τ/τ_g Since T is fixed for a given galvanometer, it is advantageous to increase both τ and τ_g, to get a larger amplitude. Furthermore, it may be seen that the quantity inside the square bracket in equation (12) is maximum when $T' = 2\pi\sqrt{\tau\tau_g}$ i.e. when the free-period of the galvanometer is 2π times the geometric mean of the time constants of the grid system and the galvanometer circuit.

It has been mentioned earlier that an increase in τ_g is an advantage. This can be done by decreasing the resistance R in the galvanometer circuit. However, if R be diminished beyond a certain limit, the damping factor in the galvanometer will prevail which will nullify the equation (10) governing the galvanometer motion

(*f*) *Solution without approximations*

In the foregoing derivation, the solution for the galvanometer deflection θ has been obtained under the following assumptions :

(*i*) the plate resistance r_p of the vacuum tube is large in comparison with similar elements in the circuit;

and (*ii*) the damping factor in the galvanometer circuit is negligibly small.

Those approximations have been removed in the following steps For the sake of simplicity it will be convenient to adopt the following notations :

$$g_0 = 1 - \frac{M^2}{LL_g}$$

$$\frac{1}{\lambda_1} = \frac{B}{2A} - \left(\frac{B^2}{4A^2} - \frac{C}{A}\right)^{\frac{1}{2}}$$

$$\frac{1}{\lambda_2} = \frac{B}{2A} + \left(\frac{B^2}{4A^2} - \frac{C}{A}\right)^{\frac{1}{2}}$$

$$\frac{1}{\lambda_0} = \frac{1}{\lambda_2} - \frac{1}{\lambda_1} = 2\left(\frac{B^2}{4A^2} - \frac{C}{A}\right)^{\frac{1}{2}}.$$

τ = time constant of grid system, $\tau_g = L_g/R$, $\tau_p = L/(r+r_p)$.

T_1 = time constant of galvanometer circuit with L_g coupled to L as in use.

T' = damped galvanometer period divided by 2π.

T = undamped galvanometer period divided by 2π.

In terms of these notations, the solution of the equation (5), under the initial conditions, *viz.*;

$$\text{at} \quad t = 0, \ i_g = 0, \ i_p = 0, \ \text{and} \left(\frac{di_g}{dt}\right)_{t=0} = -\frac{M\mu e_0}{g_0 LL_g}$$

$$\text{is} \quad i_g = \frac{M\mu e_0}{h_0}\left[e^{-t/\tau} + \alpha . e^{-t/\lambda_1} + \beta . e^{-t/\lambda_2}\right] \qquad \dots (13)$$

where $h_0 = R(r+r_p)\tau\left[1-\frac{\tau_g}{\tau}-\frac{\tau_p}{\tau}+\frac{\tau_p\tau_g}{\tau^2}\cdot g_0\right]$,

$$\alpha = \lambda_0\left(\frac{1}{\tau}-\frac{1}{\lambda_2}-h\right),$$

$$\beta = \lambda_0\left(h+\frac{1}{\lambda_2}-\frac{1}{\tau}\right)-1.$$

$$h = \frac{h_0}{g_0 L L_g}.$$

Taking into consideration the damping of the galvanometer, the deflection θ arising due to i_g, satisfies the differential equation

$$K\frac{d^2\theta}{dt^2}+b\frac{d\theta}{dt}+c\theta = \frac{JG^{\frac{1}{2}}\,.\,M\mu\epsilon_0}{h_0}\left[e^{-t/\tau}+\alpha\,.\,e^{-t/\lambda_1}+\beta\,.\,e^{-t/\lambda_2}\right]$$

where $JG^{\frac{1}{2}}$ is the co-efficient of i_g in the expression of couple and b, a constant. In view of the relations

$$T'^2 = K/c,\text{ and } b = \frac{2K}{T_1} = 2c\frac{T'^2}{T_1},$$

the above equation of motion becomes

$$\frac{d^2\theta}{dt^2}+\frac{2}{T_1}\frac{d\theta}{dt}+\frac{1}{T'^2}\theta = \frac{JG^{\frac{1}{2}}\,.\,M\mu\epsilon_0}{h_0}\left[e^{-t/\tau}+\alpha e^{-t/\lambda_1}+\beta e^{-t/\lambda_2}\right]$$

This equation when solved under the initial conditions *viz.*,

at $\quad t = 0,\quad \theta = 0\quad$ and $\quad\frac{d\theta}{dt} = 0,$

gives the solution

$$\theta = \frac{\sigma M\mu\epsilon_0}{h_0}\left[e^{-t/\tau}+\left(\frac{T'}{\tau}-\frac{T'}{T_1}\right)\,.\,e^{-t/T_1}\sin\frac{t}{T'}-e^{-t/T_1}\cos\frac{t}{T'}\right.$$

$$+\alpha\,\frac{e^{-t/\lambda_1}+\left(\frac{T'}{\lambda_1}-\frac{T'}{T_1}\right)\cdot e^{-t/T_1}\sin\frac{t}{T'}-e^{-t/T_1}\cos\frac{t}{T'}}{1-\frac{2T'^2}{\lambda_1 T_1}+\frac{T'^2}{\lambda_1^2}}$$

$$\left.+\beta\,\frac{e^{-t/\lambda_2}+\left(\frac{T'}{\lambda_2}-\frac{T'}{T_1}\right)e^{-t/T_1}\sin\frac{t}{T'}-e^{-t/T_1}\cos\frac{t}{T'}}{1-\frac{2T'^2}{\lambda_1 T_2}+\frac{T'^2}{\lambda_2^2}}\right] \qquad \ldots \quad (14)$$

provided $T_1 > T'$,

where $\sigma = \frac{JG^{\frac{1}{2}}}{c} = \mathrm{Lt}_{\delta i_g \to 0} \frac{\delta\theta}{\delta i_g}$ = galvanometer sensitivity,

$$\frac{1}{\lambda_1} = \frac{1}{2g_0}\left[\frac{1}{\tau_g}+\frac{1}{\tau_p}-\left\{\frac{1}{\tau_g^2}+\frac{1}{\tau_p^2}+\frac{2-4g_0}{\tau_p\tau_g}\right\}^{\frac{1}{2}}\right],$$

$$\frac{1}{\lambda_2} = \frac{1}{2g_0}\left[\frac{1}{\tau_g}+\frac{1}{\tau_p}+\left\{\frac{1}{\tau_g^2}+\frac{1}{\tau_p^2}+\frac{2-4g_0}{\tau_p\tau_g}\right\}^{\frac{1}{2}}\right]$$

It may be observed that the quantities λ_0, λ_1, λ_2 are practically always concerned with the ratios such as T/λ_1, etc., and so should be calculated in these terms. Thus,

$$\frac{T}{\lambda_1} = \frac{1}{2g_0}\left[\frac{T}{\tau_g}+\frac{T}{\tau_p}-\left\{\frac{T^2}{\tau_g^2}+\frac{T^2}{\tau_p^2}+(2-4g_0)\frac{T^2}{\tau_p\tau_g}\right\}^{\frac{1}{2}}\right]$$

and so on

The expression for θ in equation (14) shows that the galvanometer deflection consists of two components :

(*i*) one decreases exponentially with time,

and (*ii*) the other is damped during an oscillatory motion.

In the earlier analysis where the damping has been neglected, the oscillatory component persists and exhibits a steady amplitude. But in the present case, where damping effect dominates, the amplitude of the oscillatory component decreases exponentially with time. As the damping of the galvanometer is decreased by increasing the total resistance R of the galvanometer circuit, the rate of decrease of the amplitude of deflection of the coil diminishes.

Experimental arrangement

Figure 2, is a diagrammatic representation of the experimental arrangement. Initially the key K is closed and a negative voltage $(-v)$, equal to PD across the potentiometer wire, is applied to the grid terminal of the triode vacuum tube. The condenser G_1 is also charged to the same potential $(-v)$. When the key K is opened, the condenser C_1 discharges through the resistance R_1 and the grid voltage rises exponentially towards zero with a time constant $\tau = R_1C_1 = 10$ milliseconds approximately. The shape of the voltage pulse, recorded with a Tektronix CRO is shown in figure 3A.

Due to the exponentially varying input voltage applied to the grid, the primary current in the transformer varies with time. Consequently an emf is induced in the transformer secondary and a current flows in the galvanometer circuit. The shape of the current pulse as revealed by the voltage drop across the resistance R in the secondary circuit is shown by the oscillogram pattern in figure 3B. It may be noted that the output current pulse does not correspond with the sharp exponential incident voltage pulse pattern. The complexity of the nature of the galvanometer current pulse has already been discussed in an earlier section.

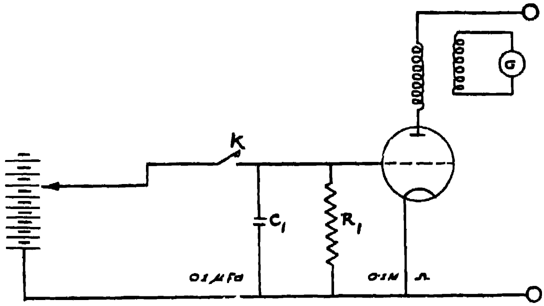

Figure 2. Schematic experimental arrangement.

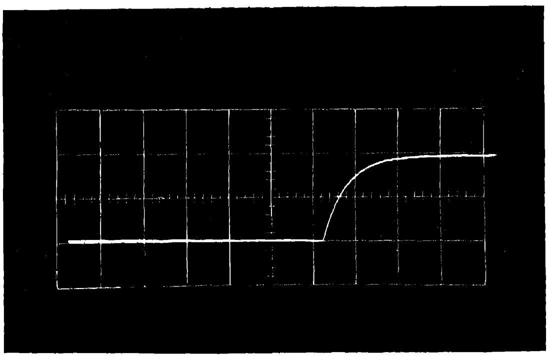

Figure 3A Oscillogram of input voltage pulse.

The actual experimental set up for recording the galvanometer deflections is schematically represented in figure 4. Light from a straight-filament lamp is focussed by means of an adjustable system of lenses so that after reflection at the galvanometer mirror, a luminous vertical line is formed on the cylindrical lens. The latter focusses this light on a narrow horizontal slit behind which a sensitive photographic paper is smoothly drawn at constant speed.

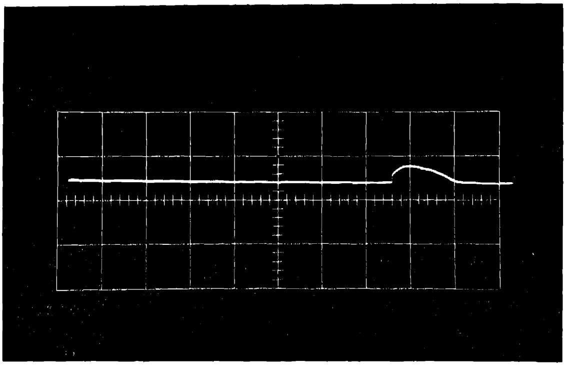

Figure 3B. Oscillogram of current pulse in the galvanometer.

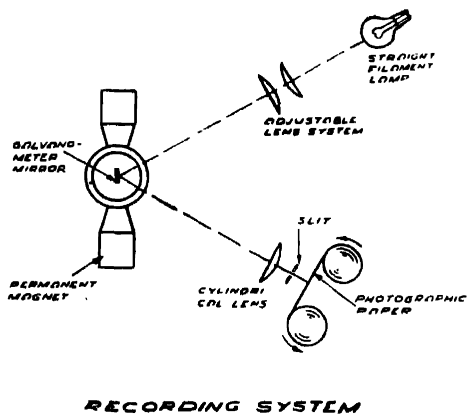

Figure 4. Galvanometer deflection recording arrangement.

Figure 5A, is a photographic record of the resultant deflection when the galvanometer is slightly underdamped, while figure 5B represents the case when the damping factor is reduced by increasing the total resistance in the galvanometer circuit. The oscillatory motion of the galvanometer spot of light associated with its logarithmic decrement is recognizable in figure 5B.

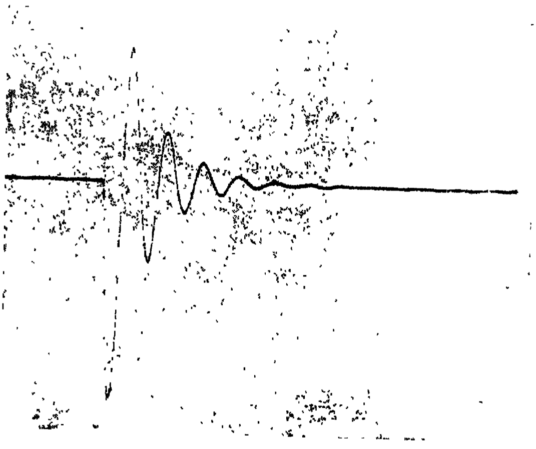

Figure 5A. Photographic record of 'slightly underdamped' galvanometer deflection.

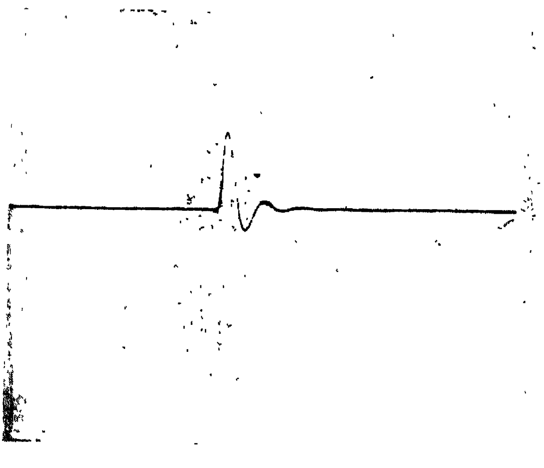

Figure 5B. Photographic record 'undamped' galvanometer deflection

Finally, the total resistance of the galvanometer circuit is made equal to its critical damping resistance. The galvanometer deflections can now be easily recorded. Figure 6 represents the calibration curve for transient pulses measured in our experimental set up. It may be noted that the calibration curve is no longer a straight line as the input pulse is increased in magnitude. This may be due to the distortion introduced by transformer coupling and the consequence of attenuating factors enunciated in earlier equations.

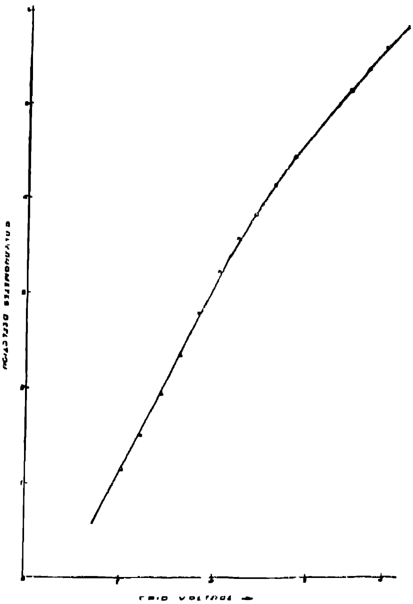

Figure 6. Calibration curve for transient input voltage pulses.

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