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Interaction of electromagnetic field with matter (angular momentum basis)

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Reduced expansions of electromagnetic fields are derived in terms of irreducible representations of proper, orthochronous, inhomogeneous Lorentz group in angular momentum basis. The second quantized expansions, derived by replacing photon wavefunctions and their complex conjugates by annihilation and creation operators in the reduced expansion, are given in terms of vector spherical harmonics with annihilation and creation operators as amplitudes For calculating the interaction Hamiltonian, when electromagnetic field is coupled to an atom, the second quantized expansion of three components transverse electromagnetic vector potential is used to avoid fictitious photons of helicity other than $\pm j$ (spin) and subsidiary state vector condition and to overcome the difficulty of vanishing amplitude for emission or absorption of photon as $p \to 0$. The selection rules, derived in the relativistically quantized manner, are identical with already known selec- tion rules for classical radiation fields, except that here the photon takes or supplies angular momentum to conserve the total angular momentum of the system.

Introduction

It has been shown by Koba, Tati & Tomonago (1947) and Schwinger (1948) that to pass over from the Heisenberg representation to the interaction representation, the supplementary condition due to Fermi for the electromagnetic field has to be modified by adding a charge term because this condition involves one difficulty that there is no normalized state which satisfies it, as shown by Ma (1949) and Belinfante (1949). To overcome this difficulty, Gupta (1950) has given a new treatment for the longitudinal part of the electromagnetic field where an indefinite metric has been used for scalar photons. Weinberg (1965) preferred to avoid indefinite metric and photons of helicity other than $\pm j$ (spin) by treating them as the rough conclusions of the fact that no symmetric tensor fields of rank j can be constructed from the creation and annihilation operators of massless particles of spin j. He further proved that the most general covariant field that can be constructed from such operators cannot represent real photon interaction because they give the amplitudes for emission and absorption of massless particles which vanish as p^j for momentum $p \to 0$.

The transformation of the first order Lorentz gauge formulation into the radiation gauge was done by Schwinger (1963) by decomposing the complementary fields into longitudinal and transverse fields and by eliminating the longitudinal fields (spin-zero components) from the physical quantities This elimination of longitudinal fields is always advantageous for the physical system containing photons. as proved by Weinberg (1964) that the zero mass has a special kind of dynamical self-consistency for spin-1 (transverse part) which it would not have for zero-spin (longitudinal part).

To avoid the use of fictitious photons of helicity other than j or the indefinite metric and subsidiary state-vector conditions and to overcome the difficulty of vanishing amplitude for emission or absorption of photons as $p \to 0$, we use here the three components transverse electromagnetic vector potential, curl of which gives the fields, for the study of interaction of electromagnetic fields For this purpose we use our results of reduction of electromagnetic fields, in linear (Rajput 1970a) and angular (Rajput 1969a) momentum basis, to the irreducible representation of proper, orthochronous, inhomogeneous Lorentz group These results have been derived by using our results for the reductions of antisymmetric tensor (Rajput 1969b, 1969c) scalar (Rajput 1969d) and three-components vector (Rajput 1969e) fields. Using these results, we also derived the reductions of generalized electromagnetic fields in presence of magnetic monopoles, for zero (Rajput 1970b) and nonzero (Rajput & Singh 1970) mass systems. In all these reduced expansions we decomposed the complementary fields into longitudinal and transverse parts, and omitted the longitudinal and scalar parts by setting them equal to zero for the physical systems.

To second quantize the electromagnetic fields the photon wavefunctions and their complex conjugates, in their reduced expansions on angular momentum basis, are replaced by annihilation and creation operators. Using these second quantized reduced expansions the interaction Hamiltonian, for the study of interaction of electromagnetic fields with atom, has been calculated. The selection rules derived here are identical to those derived by Blatt-Weiskopf (1952) and Rose (1957) for classical fields, except that here the photon takes or supplies the angular momentum in order to conserve the total angular momentum of the atomic system The probability of the emission of a photon by an atom is proved proportional to $(n+1)$ where n is the number of photons of a given kind in the interacting field. This explains the spontaneous emission, since the probability for no photon in the system is different from zero. Using similar procedure we have derived similar results for linear momentum representation in an earlier paper (Rajput 1970c). Our procedure, in contrast with that of Davydov (1965), is completely relativistic where photon wavefunctions are introduced explicitly.

Reduction of electromagnetic fields in angular momentum basis

In the angular momentum basis a wavefunction is given in terms of magnitude of linear momentum p, the total angular momentum quantum number k, the quantum number m of J_z (the z-components of angular momentum) and the helicity λ In this basis the reduced expansions of electric and magnetic fields are defined as (Rajput 1969a),

$$\vec{E}(\vec{x}, t) = E_1(\vec{x}, t) + E_1^*(\vec{x}, t)$$

$$\vec{H}(\vec{x}, t) = H_1(\vec{x}, t) + H_1^*(\vec{x}, t) \qquad \ldots \quad (1)$$

where

$$\vec{E_1}(\vec{x}, t) = -4\pi/(3\pi)^{\frac{1}{2}} \sum_{\lambda=\pm 1} \sum_{\beta=0,\pm 1} \vec{\chi}(\beta) \sum_{k=1}^{\alpha} \sum_{m=-k}^{k} (i)^{k+1+\beta-\lambda} \exp\{i\pi(\lambda - m/2)\}$$

$$\times Y_k^{m,\lambda}(\theta, \phi) Y_1^{\beta,\lambda *}(\theta, \phi)\, Y_k^{m0}(\hat{\theta}, \hat{\phi}) Y_k^{m,0*}(\theta, \phi)$$

$$\times \int dp/p.\, j_k(pr)\, F(p, k, m, \lambda) \exp(-ipt) \qquad \ldots \quad (2)$$

and

$$\vec{H_1}(\vec{x}, t) = -4\pi/(3\pi)^{\frac{1}{2}} \sum_{\lambda=\pm 1} \sum_{\beta=0,\pm 1} \vec{\chi}(\beta) \sum_{k=1}^{\alpha} \sum_{m=-l}^{k} \lambda(i)^{l-\lambda+\beta} \exp\{i\pi(\lambda - m/2)\}$$

$$\times Y_k^{m,\lambda}(\theta, \phi)\; Y_1^{\beta,\lambda *}(\theta, \phi) Y_k^{m,0}(\hat{\theta}, \hat{\phi})\; Y_l^{m,0*}(\theta, \phi)$$

$$\times \int dp/p.\, j_k(pr)\, F(p, k, m, \lambda) \exp(-ipt) \qquad \ldots \quad (3)$$

where $F(p, k, m, \lambda)$ is the wavefunction of the photon and $\vec{\chi}(\beta)$ is a vector having the following components

$$\vec{\chi}(\beta) = (2)^{\frac{1}{2}}(1, i\beta, 0) \quad \text{for } \beta = \pm 1$$

$$\vec{\chi}(0) = -i(0, 0, 1) \quad \text{for } \beta = 0$$

$Y_l^{m\lambda}(\theta, \phi)$ etc in equation (3) are the generalized spherical harmonics for θ, ϕ as the polar angles of the linear momentum vector $\vec{p}$ given by

$$\vec{p} = p(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta). \qquad \ldots \quad (5)$$

$j_k(pr)$, for $r = |\vec{x}|$, is spherical Bessel function of order k, and $\hat{\theta}, \hat{\phi}$ are the polar angles of the vector $\vec{x}$.

Using the transposition theorem of generalized spherical harmonics we have

$$Y_k^{m-\beta,0*}(\theta, \phi) = (i)^{2\beta-2m} Y_k^{0,m-\beta}(\theta, \phi)$$

$$Y_1^{\beta,\lambda}(\theta, \phi) = (i)^{2\lambda-2\beta} Y_1^{\lambda,\beta}(\theta, \phi),$$

and

$$Y_m^{\lambda,m}(\theta, \phi) = (i)^{2\lambda-2m} Y_k^{\lambda,m*}(\theta, \phi)$$

Substitusing these results in equation (2) we get

$$\overrightarrow{E_1(x},t) = -4(\pi/3)^{\frac{1}{2}} \sum_{\lambda=\pm1} \sum_{\beta=0,\pm1} \overrightarrow{\chi}(\beta) \sum_{k=1}^{\alpha} \sum_{m=-k}^{k} (i)^{k+\beta+\lambda+1-m}$$

$$\times Y_k^{\lambda,m*}(\theta,\phi)\ Y_1^{\lambda,\beta}(\theta,\phi)\ Y_k^{m-\beta,0}(\hat{\theta},\hat{\phi})\ Y_k^{0,m-\beta}(\theta,\phi)$$

$$\times \int dp/p.\ j_k(pr)\ F(p,k,m\ \lambda) \exp(-ipt) \quad \dots \quad (6)$$

On expanding the product and using the orthogonality relations for the generalized spherical harmonics, we get

$$Y_k^{0,m-\beta}(\theta,\phi)\ Y_1^{\lambda,\beta}(\theta,\phi) = \sum_{J=|k-1|}^{k+1} \left[\frac{3(2k+1)}{4\pi(2J+1)} \right]^{\frac{1}{2}} (k, m-\beta, 1, \beta \,|\, k, 1, J, m)$$

$$\times (k, 0, 1, \lambda \,|\, k, 1, J, \lambda)\, Y_J^{\lambda,m} \quad \dots \quad (7)$$

and

$$\int_0^{2\pi} d\phi \int_0^{2\pi} d\theta \sin\theta\ Y_j^{m,n*}(\theta,\phi)\ Y_{j'}^{m'n}(\theta,\phi) = \delta_{j,j'}\delta_{m,m'} \quad \dots \quad (8)$$

In equation (7) Clebsch-Gordan coefficients are used in the form $(j, m, j'. m' \,|\, j, j', J, M)$.

When equations (7) and (8) are substituted in equation (6) it is reduces to.

$$\overrightarrow{E_1(x},t) = (2)^{\frac{1}{2}} \sum_{\lambda} \sum_{k=1}^{\alpha} \sum_{m=-k}^{k} (i)^{k-m+1}$$

$$\times [Y_{k,k,m}(\theta,\phi) \int pdpj_k(pr)\ F(p,k,m,\lambda) \exp\{-ipt\}$$

$$-i\lambda\{k/(2k+1)\}^{\frac{1}{2}} Y_{k,k+1,m}(\theta,\phi) \int pdpj_{k+1}(pr) F(p,k.m,\lambda) \exp\{-ipt\}$$

$$+i\lambda\{(k+1)/(2k+1)\}^{\frac{1}{2}} Y_{k,k-1,m}(\theta,\phi) \int pdpj_{k-1}(pr) F(p\ k,m,\lambda) \exp\{-ipt\}] \quad \dots \quad (9)$$

where the vector spherical harmonics $Y_{k,k',m}(\theta,\phi)$ are defined as

$$Y_{k,k',M}(\theta,\phi) = \sum_{m,\beta} (i)^{\beta+1} \overrightarrow{\chi}(\beta)\ Y_{k'}^{m-\beta,0}(\hat{\theta},\hat{\phi})(k', m, 1, \beta \,|\, k', 1, k, M)$$

In deriving equation (10) we have used the values of Clebsch-Gordan coefficients. In a similar manner the reduction of magnetic field also can be derived as the following expression

$$H_1(\overline{x},t) = (2)^{\frac{1}{2}} \sum_{\lambda} \sum_{k=1}^{\alpha} \sum_{m=-k}^{k} (i)^{k-m}$$

$$\times [\lambda\ Y_{k,k,m}(\theta,\phi) \int pdpj_k(pr)\ F(p,k,m,\lambda) \exp\{-ipt\}$$

$$-i\{k/(2k+1)\}^{\frac{1}{2}} Y_{k,k+1,m}(\theta,\phi) \int dpj_{k+1}(pr) F(p,k,m,\lambda) \exp\{-ipt\},$$

$$+i\{(k+1)/(2k+1)\}^{\frac{1}{2}} Y_{k,k-1,m}(\theta,\phi) \int pdpj_{k-1}(pr)\ F(p,k,m,\lambda) \exp\{-ipt\}] \quad (10)$$

The three-dimensional vector potential $A(\vec{x}, t)$ of electromagnetic field is given by

$$E(\vec{x}, t) = -\frac{\partial}{\partial t} A(\vec{x}, t)$$

$$H(\vec{x}, t) = \text{curl}\, A(\vec{x}, t) \tag{11}$$

Using equations (9) and (10) in equation (11) the reduction of electromagnetic potential to the irreducible representation of inhomogeneous, orthochronous, proper Lorentz group in angular momentum basis can be derived as the following expansion

$$A_1(\vec{x}, t) = (2)^{\frac{1}{2}} \sum_{\lambda=\pm 1} \sum_{k=1}^{\infty} \sum_{m=-k}^{k} (i)^{k-m}$$

$$\times [Y_{k,k,m}(\theta, \phi) \int dp j_k(pr) F(p, k, m, \lambda) \exp(-ipt)$$

$$-i\lambda\{k/(2k+1)\}^{\frac{1}{2}} Y_{k,k+1,m}(\theta, \phi) \int dp j_{k+1}(pr) F(p, k, m, \lambda) \exp(-ipt)$$

$$+i\lambda \left\{\frac{k+1}{2k+1}\right\}^{\frac{1}{2}} Y_{k,k-1,m}(\theta, \phi) \int dp j_{k-1}(pr) F(p, k, m, \lambda) \exp(-ipt)] \quad \ldots (12)$$

where

$$A_1(\vec{x}, t) + A_1^*(\vec{x}, t) = A(\vec{x}, t)$$

The vector spherical harmonic $Y_{k,k,m}(\theta, \phi)$ in equations (9) and (10) which corresponds to the angular momentum quantum number k of total angular momentum J and the parity $(-1)^{J+1}$ can be considered as transverse magnetic vector spherical function. The transverse electrical vector spherical function which corresponds to angular quantum number $(k+1)$ and parity $(-1)^{J+1}$ can be considered as

$$Y_{k,k'm}((\theta, \phi) = 1/(2J+1)^{\frac{1}{2}}[J(J+1)^{\frac{1}{2}} Y_{k,k+1,m}(\theta, \phi) + (J+1)(J)^{\frac{1}{2}} Y_{k,k-1,m}(\theta, \phi)]$$

The longitudinal and scalar functions, which are derived from the scalar electromagnetic vector and the fourth component of vector electromagnetic potential corresponding to $\lambda = 0$ in the reduction, do not contribute at all so far so as physical effects are concerned.

Second quantization of electromagnetic fields in angular momentum basis

To second quantize the electromagnetic fields in the angular momentum representation; the photon wavefunction $F(p, k, m, \lambda)$ and its complex conjugate in the reduced expansions of $E(\vec{x}, t)$ and $H(\vec{x}, t)$ are replaced by annihilation and creation operators $b(p, k, m, \lambda)$ and $b^*(p, k, m, \lambda)$, respectively. These operators satisfy the following commutation rules

$$[b(s), b(s')] = [b^*(s), b^*(s')] = 0$$

$$[b(s), b^*(s)] = \delta(p-p')\delta_{k,k'}\delta_{m,m'} \quad \ldots (13)$$

where s denotes the collection of variables p, k, m and λ. In terms of these operators the Hamiltonian H and number of operator N are given as follows

$$H = 1/2 \sum_{\lambda} \int [b^*(s)\, b(s)+b(s)b^*(s)]dp$$

$$= \sum_{\lambda} \int [b^*(s)b(s)+1/2]dp$$

$$= \sum_{\lambda} \int [n(s)+1/2]dp \quad \ldots (14)$$

$$N = (2)^{-\frac{1}{2}} \sum_{\lambda} \int [b^*(s)b(s)+b(s)b^*(s)] \frac{dp}{p}$$

$$= \sum_{\lambda} \int [b^*(s)\, b(s)+1/2] \frac{dp}{p}$$

$$= \sum_{\lambda} \int [n(s)+1/2] \frac{dp}{p} \quad \ldots (15)$$

where $n(s) = b^*(s)b(s)$ is the operator of the number of photons with variables denoted by s. The poynting vector operator $\vec{P}$ can also be expressed in terms of annihilation and creation operators, as follows

$$\vec{P} = \vec{e}(8\pi)^{-3}\sum_{\lambda} 1/2 \int [b^*(s)b(s)+b(s)b^*(s)]dp$$

$$= \vec{e}(8\pi)^{-3}\sum_{\lambda} \int [b^*(s)b(s)+1/2]dp$$

$$= \vec{e}(8\pi)^{-3}\sum_{\lambda} \int [n(s)+1/2]dp$$

where $\vec{e}$ is unit vector in the direction of $\vec{P}$.

The n-particle basis vector for second quantization, in the angular momentum basis, is given by

$$|s_1, s_2, \ldots, s_n> = \frac{b^*(s_1)b^*(s_2) \ldots b^*(s_n)}{(n!)^{\frac{1}{2}}} |0> \quad \ldots (16)$$

where $|0>$ designates the vacuum state.

For the photons with well defined quantum state the equation (16) reduces to

$$|s_1, s_2, \ldots, s_n> = \frac{b^*n(s)}{(n!)^{\frac{1}{2}}} |0>$$

The annihilation and creation operators act upon these basis vectors (kets) in the following manner

$$b^*(s)\,|s_1, s_2, \ldots, s_n> = \{n(s)+1\}^{\frac{1}{2}}\,|s_1, s_2, \ldots, s_n, s> \qquad \ldots (17)$$

$$b(s)\,|s_1, s_2, \ldots, s_n> = \{n(s)\}^{\frac{1}{2}}\,p\,|s_1, s_2, \ldots, s_{n-1}> \qquad \ldots (18)$$

INTERACTION OF ELECTROMAGNETIC FIELD WITH ATOM

The number of photons in the system containing electrical charge is not constant as the photons can be emitted or absorbed Here we study the interaction between the electromagnetic fields and an atom assuming that the system is at rest.

Neglecting the interaction, the Hamiltonian H_0 of the system (atom and the field) is the sum of radiation and atomic Hamilonians

$$H_0 = H_a + H_{rad}$$

where H_a is the Hamiltonian of the atomic system and H_{rad} is field Hamiltonian operator given by equation (14).

The interaction Hamiltonian for the present case is of the form $\vec{A}(\vec{x}, 0).\vec{v}$, where $\vec{v}$ is a polar vector which is a function of atomic dynamical variables. The vector $\vec{v}$ may also be regarded as a first rank tensor, the average value of which for intitial and final atomic states gives current density. Using this value of interaction Hamiltonian operator form, we can study the emission and absorption of photon by an atomic system.

Emission Let the initial state $|\psi_I>$ of the system without interaction be considered as containing the atom and $n(s)$ photons, and the final state $|\psi_F>$ after the interaction as containing the atom and $\{n(s)+1\}$ photons Thus in the interaction the atom emits one photon with momentum p, other quantum numbers being k, m and parity π. Then

$$|\psi_I> = |V>\,|\psi_i> \qquad .. (19)$$

where $|V>$ is the field state containing $n(s)$ photons and $|\psi_i>$ designates the initial atomic state with quantum numbers k_i, m_i and π_i for the total angular momentum, z-component of angular momentum and parity, respectively.

$$|\psi_F> = |s>|\psi_f> \qquad \ldots (20)$$

where $|s>$ is the field state containing $\{n(s)+1\}$ photons and $|\psi_f>$ designates the final atomic state with corresponding quantum numbers k_f, m_f and π_f.

The matrix element of interest for emission is given by

$$< \psi_I |\ A(\vec{x}, 0).\vec{v}\ | \psi F >$$
$$= < \psi_f |\ < s | A_1^*(\vec{x}, 0).\vec{v} | V >\ | \psi_I >$$
$$= (2)^{\frac{1}{2}}(-i)^{k-m}\{n(s)+1\}^{\frac{1}{2}}[j_k(pr) < \psi_f |\ Y^*_{k,k,m}(\theta, \phi)\ .\ \vec{v}\ | \psi_I >$$
$$+ i\lambda\{k/(2k+1)\}^{\frac{1}{2}} j_{k+1}(pr) < \psi_f |\ Y^*_{k,k+1,m}(\theta, \phi).\ \vec{v}\ | \psi_I >$$
$$-i\lambda\{(k+1)/(2k+1)\}^{\frac{1}{2}} j_{k-1}(pr) < \psi_f |\ Y^*_{k,k-1,m}(\theta, \phi).\ \vec{v}\ | \psi_I > | \quad (21)$$

where we have used equation (17) from which it is clear that only $A^*_1(\vec{x}, 0)$ part of $A(\vec{x}, 0)$ contributes to interaction Hamiltonian for emission, while the other part, *i e.* $A_1(\vec{x}, 0)$ contributes to the Hamiltonian for absorption. The matrix element given by equation (21) consists of the terms like

$$< \psi_f |\ Y^*_{k,k',m}(\theta, \phi).\ \vec{v}\ | \psi_i > , \quad (k' = k, k \pm 1)$$

which can also be written in terms of quantum numbers of the initial and final states as follows

$$k_f, m_f, \pi_f\ |\ Y^*_{k,k',m(\pi)}(\theta, \phi).\ \vec{v}\ | k_i, m_i, \pi_i > \quad (22)$$

where

$Y^*_{k,k',m(\pi)}.\ \vec{v}$ is an irreducible tensor of rank k.

Applying Wigner-Eckart theorem, it is clear that only those matrix elements like (22) are nonvanishing for which following selection rules are satisfied

$$k_i = k_f+k,\ k_f+k-1,\ \ldots,\ |k_f-k| \quad \ldots \quad (23)$$
$$m_i = m_f+m \quad \ldots \quad (24)$$

The parity of irreducible tensor $Y^*_{k,k',m(\pi)}.v$ is $(-1)^{J+1}\pi_v$ for electric multipole and $(-1)^J\pi_v$ for magnetic multipole where π_v (the parity of the vector $\vec{v}$) is (-1) since it changes sign under reflection of coordinates and the operator for it anticommutes with parity operator Thus the parity selection rules for photon emission are derived as

$$\pi_f\pi_i = (-1)^J \text{ for electrical transition}$$
$$\pi_f\pi_i = (-1)^{J+1} \text{ for magnetic transition} \quad \ldots \quad (25)$$

The probability for the emission per unit time in the transition from $|\psi_I>$ to $|\psi_F>$ is proportional to the square of the matrix element (21). Hence, it is proportional to $\{n(s)+1\}$, which is nonvanishing even for $n(s) = 0$. The quantization of the electro-magnetic field thus explains the occurrence of spontaneous

Absorption. For absorption we consider the transition from the initial state $|\psi_I>$ given by equation (19) to final state $|\psi_{F'}>$ of the system containing the atom and $\{n(s)-1\}$ photons

$$|\psi_F'> = |s'> \, |\psi_f'> \qquad \ldots \quad (26)$$

where $|s'> = |s_1, s_2 \ldots s_{n-1}>$

The matrix element of interest in this case is

$$<\psi_f'| \; <s'|A_1(\vec{x}, 0).\vec{v}\,|V>|\psi_i>$$

$$= (2)^{1/2}\{n(s)\}^{1/2}p(i)^{k-m}[j_k(pr) < \psi_f|\, Y_{k,k,m}(\theta, \phi)\,.\vec{v}\,|\psi_i>$$

$$-i\lambda\{k/(2k+1)\}^{\frac{1}{2}}j_{k+1}(pr) < \psi_f|\; Y_{k,k+1,m}(\theta, \phi)\vec{v}\,|\psi_i>$$

$$+i\lambda\{(k+1)/(2k+1)\}^{\frac{1}{2}}j_{k-1}(pr) <\psi_f|\, Y_{k,k-1,m}(\theta, \phi).\vec{v}\,|\psi_i> \qquad \ldots \quad (27)$$

By a similar method as discussed for emission, we get the following selection rules for absorption

$$k_f = k_i+k,\ k_i+k-1,\ \ldots,\ |k_i-k| \qquad \ldots \quad (28)$$

$$m_f = m_i+m \qquad \ldots \quad (39)$$

The probability for absorption is proportional to the number of photons of a given kind in the initial state

Discussion

The reduction of electromagnetic fields to the irreducible representations of proper orthochronous inhomogeneous Lorentz group in angular momentum basis is given by equations (9) and (10) in terms of the wavefunctions of particles of zero mass and spin-1 (transverse photons). On replacing the photon wavefunctions and their complex conjugates in these reduced expansions by annihilation and creation operators, a covariant second quantized theory is obtained in purely relativistic manner. The second quantized operator $A(\vec{x}, t)$ derived in this manner is a covariant quantized analogue to the expansion in multipole of classical theory due to Blatt & Weiskopf (1952) This quantized reduced expansion of $A(\vec{x}, t)$ in terms of vector spherical harmonics, with annihilation and creation operator as the amplitudes, is used for calculating the interaction Hamiltonian to avoid the use of fictitious photons of helicity other than $\pm j$ (spin) and to overcome the difficulty of vanishing the amplitudes for photon emission and absorption as the momentum $p \rightarrow 0$ (Weinberg 1965).

The probability of photon emission is proportional to the square of the matrix element given by equation (21) and thus, consists of two terms The first term is independent of the number of photons in the electromagnetic field before emission and gives rise to spontaneous emission because it is nonvanishing even if there is no photon initially. The second term, which is proportional to the number of photons in the interacting field, gives rise to certain induced emission. The probability of absorption of a photon, given by the square of matrix element in equation (27), depends on the energy of absorbed photon and is proportional to the number of photons in the interacting electromagnetic field. The ratio of the probability of photon emission to that of its absorption is, therefore, proportional to $\{n(s)+1\}/\{n(s)\}$

The selection rules for emission and absorption of photons by atoms are identical to those derived by Blatt & Weisskopf (1952) classically and to those derived by Davydov (1965) non-relativistically, except that here the photon takes or supplies angular momentum, in order to conserve the total angular momentum of the system. We thus get the parallelism between the classical and quantum theories of radiation in angular momentum basis The similarity of the selection rules verifies the validity of reduction of electromagnetic fields given in equation (9) and (10) to the irreducible representation of proper, orthochronous inhomogeneous Lorentz group on angular momentum basis because in calculating the interaction Hamiltonian we have used the reduced expansion of $A(\vec{x}, t)$ derived from these expansions Moreover, this similarity of the selection rules for interaction of electromagnetic fields with atom suggests the use of this relativistic quantized procedure in the study of interactions of electromagnetic field with molecules, nuclei and elementary particles. The procedure being a relativistic one will prove itself more advantageous and straightforward.

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