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Analysis of observed grain density in nuclear emulsions

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Actually observed grain density, along the tracks of the charged particles in nuclear emulsions, is analysed in terms of primary, secondary and fog grains. An attempt has been made to estimate the contribution of the primary and secondary grain densities theoretically for various values of velocities. The results of our model are compared with those of Patrick & Barkas (1962) and Benton & Heckman (1964). It is concluded that the secondary grain density accounts for nearly 33.5% of the total grain density observed in G-5 emulsions for $0.08 < \beta < 0.14$ and 23% at minimum ionization. Our theoretical results agree well with the experimentally observed values.

1. INTRODUCTION

The signature of charged particles left in nuclear emulsions in the form of tracks can give sufficient information regarding their particulars *e.g.*, velocity, rate of energy loss, charge, kinetic energy and mass. The track parameters in use are grain density δ-ray density, tapering length, track width, range and scattering.

Kinoshita (1910) has defined the total grain density in emulsions by the following expression :

$$g = C(1-e^{-bI}) \quad \dots \quad (1)$$

Where C is defined as the saturation value of the grain density for heavily ionizing particles and is equal to the available number of silver halide grains ($g_{max} = N$) per hundred micron. The parameter b is defined as a function of grain sensitivity and its cross-sectional area and also includes the effect of development conditions. I denotes the specific energy loss which according to Blau (1949) is $(dE/dR)^{\frac{1}{2}}$ while according to Morand & Rossum (1951) is $(dE/dR)^{\frac{1}{2}}-a^{\frac{1}{2}}$ (a representing threshold energy). The magnitudes of exponents b and I are not well defined. Patrick & Barkas (1962), Benton & Heckman (1964) and Brown (1953) have given a similar expression for defining primary grain density with different constants.

Experimental observations show that the variation of grain density with specific energy loss for charged particle tracks is a characteristic curve (Fowler 1950, Fowler & Perkins 1951, Powell *et al* 1959, Sharma & Gaur 1968).

The variation at low energy losses has a direct proportionality but at higher values of specific ionization it deviates from linearity and the curve becomes almost flat.

Many workers (Della Corte *et al* 1953, Patrick & Barks 1962, Benton & Heckman 1964, Brown 1953) have pointed out that this actually measured (observed) grain density represents grains of the following types :

(i) Grains penetrated and affected by a charged particle to the extent that they are rendered developable during the process of development. The number of such grains per $100\mu m$ of track length is defined as the primary grain density.

(ii) Grains not directly traversed by the charged particle, but still made developable during the process of development due to some induced development created in them by the neighbouring grains, or due to the penetration of δ-rays projected from the path of the primary particle, are known as secondary grains.

(iii) Sharma & Gill (1962) have shown that few grains neither affected due to the process (i) nor due to process (ii) are also rendered developable due to the process of undesirable background development. Such grains have been referred as fog grains. They may be due to the radio-active contaminations and impurities etc.

In this paper we have tried to estimate the contribution of the primary and secondary grain densities towards observed grain density. A new scheme for calculating these grains densities is also given.

2. Experimental

The measurements were made on MBI-9 scattering microscope having an oil immersion objective of $90\times$ and a filar micrometer (attached with goniometer) eyepiece of $15\times$ carrying a fine scale attached with a small drum or rotating head with 100 divisions on its circular scale. The least count of each division for measurements was $0.1\mu m$. The turning stage arrangement for alignment of track was extremely fine. Emulsion stacks exposed to 1.5 Gev/C$\bar{K}$—beam (CERN) and 4 Gev/Cπ-beam (Berkeley) were used for this purpose.

For measurements well identified π-meson and proton tracks having a dip angle of less than 10° were chosen. Gap density and gap length measurements were made on these tracks. The values of β for various residual ranges were obtained with the help of the tabulated data of Trower (1966).

For determining the grain density, the following expression of Fowler & Perkins (1955) was used.

$$g = \frac{\ln\{H(l_1)/H(l_2)\}}{l_2 - l_1} \tag{2}$$

Where g is the actual grain density and $H(l_1)$ and $H(l_2)$ are the densities of gaps exceeding length l_1 and l_2 respectively.

The variation of observed grain density (g) with velocity (β) in G-5 emulsion is shown in figure 1. The figure indicates that the observed grain density increases rapidly with decreasing values of β at greater velocities but the curve tends to flatten below $\beta \sim 0.08$. The dependence of grain density on β is not linear for whole of the region of β, but for $0.08 < \beta < 0.14$, the grain density is nearly proportional to the velocity and can be represented by the following empirical relation :

$$g = 4.32\text{-}14.56\beta \quad \text{(per micron)} \tag{3}$$

A similar type of linear dependence of observed grain density on β is shown by Patrick & Barkas (1962) but with different constants for K-5 emulsions. Benton & Heckman (1964) have approximated from their experimental observations on heavily charged particles, an inverse square dependence of g on β.

3. Theoretical

3.1. *Calculation of Primary Grain Density* :

The development of a grain depends on the amount of latent images or the amount of ionization created in it. The maximum number of holes produced at some specific energy loss (dE/dR) in a grain of G-5 emuslion along its diameter (0.27 micron) can be given as : (Sharma & Gaur, 1969)

$$n_0 = 46.55\ (dE/dR) \qquad \ldots \tag{4}$$

where dE/dR is in Kev/μm.

The total number of holes given by the above relation is not utilized for latent image formation as a fraction of it recombines with electrons during the period of latent image formation Taking into account the recombination process, the effective number of positive holes (n) available for latent image formation in G-5 emulsions is given by the following relation :

$$n = \frac{46.55(dE/dR)}{1+0.0777(dE/dR)} \tag{5}$$

In an earlier communication (Sharma & Gaur 1968) it was shown that the probability of development of a grain can be expressed by the following expression:

$$\pi = 1-e^{-S'} \tag{6}$$

where

$$S' = \gamma e^{-\alpha}(1-\beta)^{-1}[1+(\alpha-\beta)(1-\beta)^{-1}+\tfrac{1}{2}(\alpha-\beta)(\alpha-2\beta)(1-\beta)^{-2}+$$
$$\frac{1}{6}(\alpha-\beta)(\alpha-2\beta)(\alpha-3\beta)(1-\beta)^{-3}+\frac{1}{24}(\alpha-\beta)(\alpha-2\beta)$$
$$(\alpha-3\beta)(\alpha-4\beta)(1-\beta)^{-4}+\ldots+\text{negligible terms}]$$

In the above expression,

$\alpha = \frac{n}{S}$, ratio of effective number of positive holes and total number of sensitivity centres in a grain.

$\beta = \frac{B}{S}$, ratio of limiting number of positive holes and total number of sensitivity centres.

and $r = \alpha/\beta = \frac{n}{B}$.

Substituting $S = 2000$, $B = 493$ (Sharma & Gill 1962), we get

$$S' = 1\,327ne^{-0.005n}[1+6\,635\times10^{-4}(n-B)+2.201\times10^{-7}(n-B)(n-2B)$$
$$+4.83\times10^{-11}(n-B)(n-2B)(n-3B)+8\,075\times10^{-15}(n-B)$$
$$(n-2B)(n-3B)(n-4B)+\ldots \text{ negligible terms}] \quad \ldots (7)$$

The primary grain density can be defined as the product of π, the probability of development and the number of grains per $100\mu m$ (*i.e.*, g_{max} or N) in the unprocessed emulsion.

Therefore, primary grain density, $g_p = \pi\times N = N(1-e^{-S'})$ ... (8)

The value of N for G-5 emulsion is around 275-300 (Voyvodic 1950 and Sharma & Gill 1962).

3.2 *Calculation of Secondary Grain Density* ·

For higher values of effective energy loss the primary grain density should approach a saturation value g_{max} which in case of G-5 emulsion is $\sim$275 per hundred micron (Voyvodic 1950) Fowler & Perkins (1955) have shown that the gap length coefficient for relativistic tracks of heavy charge in G-5 emulsion exceeds the maximum value g_{max} (considered $3/\mu$m) and approaches $5/\mu$m. This indicates that apart from primary grains *i.e.*, grains directly affected by the charged particles, few other grains are also developed which also contribute to the gap length coefficient and due to the presence of such grains, the actual grain density exceeds the saturation value g_{max} ($3/\mu$m). Many workers (Patrick & Barkas 1962, Benton & Heckman 1964, Brown 1953 and Della Corte *et al* 1953) have attempted a separation of the primary and secondary grain densities. These secondary grains are attributed to δ-rays.

The observed grain density, g can be represented as the summation of the three different grain densities *i.e.*,

$$g = g_p+g_s+g_f \quad (9)$$

Where g_p is the primary grain density, g_s, the secondary grain density and g_f, the grain density due to fog grains. According to the curves of Dodd & Waller

(1951) the value of fog grains is very small say around 5-10 fog grains per 100μm. If we ignore the effect of fog grains in comparison to other grain densities, then

$$g = g_p + g_s$$

or

$$g_s/g = 1 - g_p/g = \phi \qquad \ldots \quad (10)$$

where ϕ is known as the induction factor and represents the contribution due to the induced or secondary grains towards the observed grain density. The calculated values of ϕ from the above relation are shown in figure 4. From equation (10) we have :

$$g_s = \frac{\phi}{1-\phi} g_p. \qquad \ldots \quad (11)$$

Substituting the value of ϕ in this relation and knowing the value of g_p at particular specific energy loss or velocity of the particle from equation (8), one can easily calculate the value of secondary grain density.

We shall now calculate the density of such secondary or induced grains produced by δ-rays following the procedure considered by Patrick & Barkas (1962) and Benton & Heckman (1964). The range-velocity relation for electrons ($\beta < 0.3$) to a good approximation can be given by

$$R_e = 2.10^2 \beta^{10/3} \qquad \ldots \quad (12)$$

The grain density at different velocities according to experimental observations is expressed by equation (3) for a singly charged particle in G-5 emulsion. Thus the number of grains due to a δ-ray with an initial velocity β_e can be given by :

$$G(\beta_e) = \int_0^\beta g.\, dR_e = 6.66 \times 10^2 \int_0^\beta (4.32 - 14.56\beta)\beta^{7/3} d\beta$$

or

$$G(\beta_e) = 0.084 W^{5/3} - 1.36 \times 10^{-2} W^{13/6} \qquad \ldots \quad (13)$$

where W is electron energy in Kev. The δ-ray density between the energy interval W and $W+dw$ due to a particle of charge Z_e and velocity β is given by the following relation :

$$N(\delta)dw = \frac{2.55 \times 10^{-2} Z^2}{\beta^2} \times \frac{dw}{w^2} \qquad \ldots \quad (14)$$

The number of induced grains, g_s per hundred micron caused by δ-rays can be found by integrating the product of equations (13) and (14) over the energy interval of δ-rays from w_0 to w_m (w_0 and w_m are the energy limits of δ-rays which contribute towards the secondary grain density). The value of g comes out as :

$$g_s = \frac{0.32 Z^2}{\beta^2} (w_m^{2/3} - w_0^{2/3}) - \frac{2.97}{\beta^2} 10^{-2} Z^2 (w_m^{7/6} - w_0^{7/6}) \quad \ldots \quad (15)$$

The lower limit of δ-ray energy (w_0) is taken to be 2 Kev ((Shapiro 1952, Patrick & Barkas 1962) as discussed in section (3.3). The upper limit of δ-ray energy (w_m) is taken 22 Kev, as suggested by Barkas (1962) on the basis of their experimental observations on ^{16}O tracks. Substituting these values of w_0 and w_m in equation (15), we finally arrive at the following expression :

$$g = \frac{0.97\,Z^2}{\beta^2} \text{ (per hundred microns)} \tag{16}$$

The values of secondary grain density calculated from equation (16) are shown in figure 3.

3 3 *Calculation of Primary Grain Density on the Basis of Barkas Model*

The primary grain density has also been calculated by Patrick & Barkas (1962) and Benton & Heckman (1964) using the following expression :-

$$g_p = N(1-e^{-\lambda I'}) \quad \ldots \quad (17)$$

Where λ is a measure of emulsion sensitivity and I' is the restricted energy loss equal to $Z^{*2}i$, where i is the energy loss rate of singly charged particle and Z^{*2}, the mean square effective charge for an energy loss (Barkas 1963) From equation (17) we see that the value of the slope of the curve drawn in $-\ln(1-g_p/N)$ and I' will give us the value of λ. The value of g_p is taken to be the difference of observed and secondary grain density calculated from equation (16). We have found λ equal to $3\ 2\times10^{-2}\ g$/Mev cm^2 in case of G-5 emulsion while Benton & Heckman have found its value as 2.3×10^{-4} and $7.5\times10^{-4}g$/Mev cm^2 for K$-$1 and K$-$0 emulsions respectively. According to Patrick & Barkas (1962), $\lambda = 0.048\ g$/Mev cm^2 for K-5 emulsions. Hence equation (17) can be reduced to .

$$g_p = N(1-e^{-0\ 032I'}) \quad \ldots \quad (18)$$

Where I' is the restricted energy loss. We have calculated the restricted energy loss at various velocities with the help of the following relation (Barkas 1963) :

$$I' = \left(\quad\right)_{W_1} = \frac{A}{\beta^2}\left[\frac{\ln 2m_e C^2\beta^2\gamma^2 w_0}{I^2(Z)} - \beta^2 - C'\right. \tag{19}$$

Where (dE/dR) is the energy loss per unit path length (involving energy transfers of energies less than w_0 per incident collision), E is the energy of the ionizing particle and $v = \beta c$ is its velocity, $A = 0.06705$ Mev cm$^2/g$ for AgBr, m_e is the electron rest mass, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, C' is the density effect correction (depending on particle velocity) and has been tabulated by Barkas (1963). W_0 is the upper limit of δ-ray energy corresponding to the maximum energy deposited in a single grain, $I(Z)$ is the mean ionization potential of atoms in the medium (silver bromide) and its value is taken 434 ev as calculated by Sternheimer (1966).

There is some uncertainty about the best value of W_0 and $I(Z)$. This may be due to the fact that these constants have only a limited influence on the restricted energy loss and role of one is partially fulfilled by the other. Shapiro (1952, 1953) found that the ionization loss is not sensitive to the choice of W_0 between 2 and 5 Kev and assumed $W_0 = 2$ Kev in contradiction to Jongejan's (1959) value (100 Kev). Patrick & Barkas (1962) found a best fit to their data with $W_0 = 2$ Kev, considering the proposal of Messel & Ritson (1950) that for calculating energy loss the value of W_0 should be taken equal to the energy of the δ-ray which has a range equal to the size of the grain. According to Demers (1953) and Lozhkin (1957), the δ-rays of 2Kev energy are capable of sometimes causing development of a single grain near the track, hence it is reasonable to take 2 Kev as the minimum δ-ray energy capable of broadening the track. Keeping these points in view we have calculated energy loss taking $W_0 = 2$Kev as considered by these workers. The values of primary grain density calculated from relation (8) and (18) are shown in figure 2.

5. Results and Discussion

The variation of observed grain density and primary grain density with velocity β are show in figure 1 and figure 2 respectively. To check the validity of the former variation let us study first the latter one. Curve (a) of figure 2 is based on our calculations from equation (8) while the curve (b) is obtained from equation (18) derived according to the procedure similar to that o fPatrick & Barkas (1962).

The variation of secondary grain density g_s as a function of particle velocity is shown in figure 3. Curve (a) of this figure is generated from the theoretically calculated values of g (equation 16) assuming that the secondary grains are formed

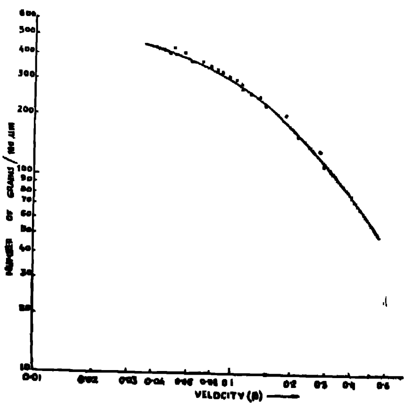

Figure 1. Variation of grain density with velocity β. Solid curve indicates the values of Fowler & Perkins (1951) and $\times$ points indicate the present work.

by δ-rays. Curves (b) and (c) of this figure indicate the variation of the secondary grain density based on the difference of the observed and primary grain densities, the later being calculated on the basis of our model (equation (8)) and that of Barkas (equation (18)) respectively. From figure 3 it is clear that the equation

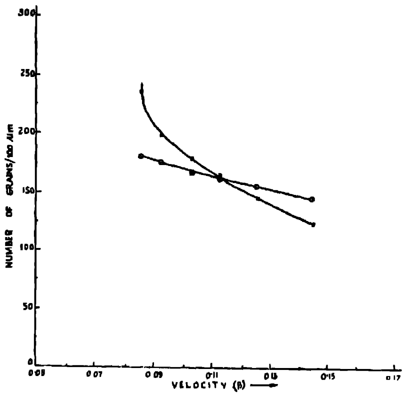

Figure 2. Variation of primary grain density with velocity β. × points indicate calculated primary grain density using Barkas model. O points indicate calculated primary grain density from our model.

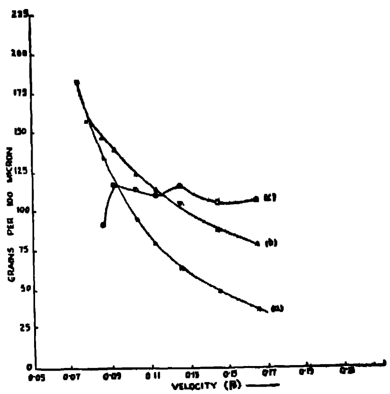

Figure 3. Variation of secondary grain density with velocity. Curve (a) shows secondary grain density calculated on the basis of δ-rays. Curves (b) and (c) show the difference of observed and primary grain densities, calculated on the basis of our model (equation 8) and that of Barkas (equation 18) respectively.

(16) fails to describe the production of δ-rays for velocities (β) < 0.08 and the secondary grain density continuos to increase for lower values of velocity (β). Similar results were found by Benton & Heckman (1964).

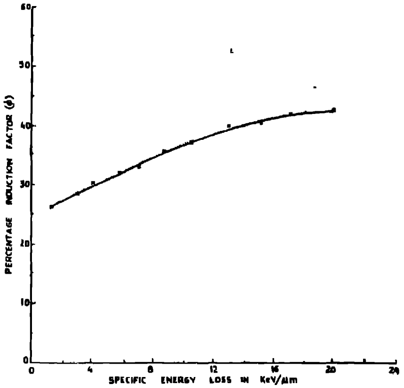

Figure 4. Variation of percentage induction factor with specific ionization.

The percentage contribution of g_s over g indicated by the induction factor (ϕ) for various values of (dE/dR) Kev/μm is plotted in figure 4. In order to calculate ϕ according to equation (13), the value of g_p are calculated from equation (8) and the values of g are taken from figure 1. This indicates that the contribution due to secondary ionization slightly increases at large values of specific energy losses and becomes almost constant. The average value of ϕ estimated from figure 4 is 35% (for $0.08 < \beta < .014$) with the consideration of the fog grains and 32% without fog grains. The mean of these variations is 33.5% At minimum ionization the contribution of secondary grains due to our model is 23%, which is in contradiction to the results of Nicoletta *et al* (1967) who have shown a contribution of only 10% at minimum ionization but in agreement with the results of Patrick & Barkas (1962) who have shown it as 25%. Benton & Heckman (1964) while studying the secondary grain density have found that the fraction of the observed grain density which is of secondary origin due to δ-rays for velocities $0.08 < \beta < 0.145$, is nearly constant and equal to $35 \pm 5\%$ for K$-$1 and K$-$0 emulsions and is independent of the atomic number of the charged particles

In figure 5 we have shown the variation of total grain density with β. Curve (a) shows the variation of observed (experimental) grain density in case of G-5 emulsions. Curve (b) shows the variation of total grain density represented as a sum of primary grain density (g_p) calculated on the basis of our model (equation

(8)) and secondary grain density calculated from equation (16). Curve (c) indicates the variation of total grain density represented as a sum of primary grain density calculated on the basis of Barkas model (equation 18) and secondary grain density due to δ-rays. These curves indicate that our theoretically calculated values are nearer to the experimental values.

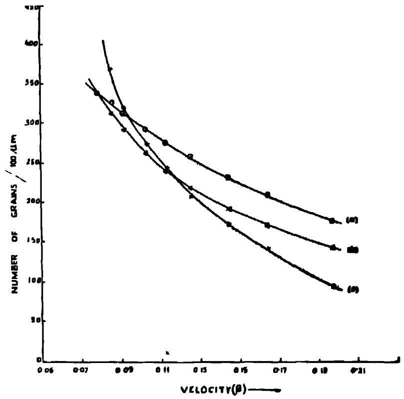

Figure 5. Variation of total grain density with velocity. Curve (a) indicates total observed grain density. Curves (b) and (c) indicate the total grain density, a sum of primary and secondary grain densities, the primary grain density being calculated from our model (equation 8) and from that of Barkas (equation 18) respectively.

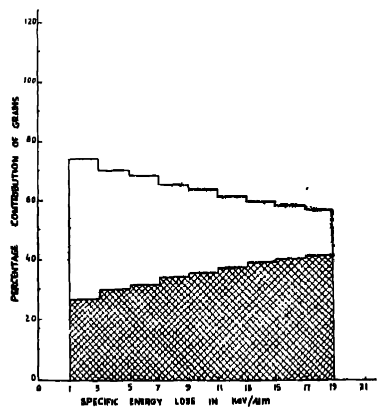

Figure 6. Percentage contribution of primary and secondary grain densities. The shaded area corresponds to the percentage contribution of secondary grain density.

The discrepancy in results may be due to some over-estimation in g_s calculated from equation (16), because a few secondary grains might have developed as a result of the joint action of two or more δ-rays (Powell *et al* 1959). We have assumed that all these δ-rays tend to lie along the trajectory of the particle but there may be a few such δ-rays which might go at a certain angle with the trajectory of the particle and may not contribute to the secondary grain density. The grains developed due to such δ-rays will not be considered as the part of the particle track.

The percentage contribution of primary and secondary grain density is shown in figure 6 and is in agreement with the results of Patrick & Barkas (1962) and Benton & Heckman (1964).

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References

Barkas W. H. 1961 *Phys. Rev.* **124**, 893.
Barkas W. H. 1963 *Nuclear Research Emulsions I* (Academy Press, New York).
Benton E. V. & Heckman H. H. 1964 *N. Cim.* **32**, 1457.
Blau M. 1949 *Phys. Rev.* **75**, 279.
Brown L. M. 1953 *Phys. Rev.* **90**, 95.
Della Corte M., Ramant M. & Ronchi L. 1953 *N Cim.*, **10**, 509.
Demers P & Lechno-Wosintynska Z. 1953 *Canad. J. Phys.*, **31**, 480.
Dodd E. C. & Waller C. 1951 *Fundamental Mechanism of Photographic Sensitivity* (Butterworths Scientific Publs. London) 266-271.
Fowler P. H. 1950 *Phil. Mag.*, **41**, 169.
Fowler P. H. & Perkins D.H. 1955 *Phil. Mag.* **46**, 587.
1951 *Fundamental Mechanism of Photographic Sensitivity* (Butterworths Scientific Publs. London) 340-345.
Jongejans B. 1959 *N. Cim.*, **16**, 625.
Kinoshita S. 1910 *Proc. Roy. Soc.* **A83**, 432.
Lozhkin O. V. 1957 *Soviet Physics JETP*, **5**, 293.
Messel H. & Ritson D. 1950 *Phil. Mag.* **41**, 1129.
Morand M. & Rossum L V. 1951 *Photographic Sensitivity* (Butterworths Scientific Publs London) 317.
Nicoletta C. & Menulty, P. J. & Jain P L. 1967 *Phy. Rev.* **164**, 1693.
Patrick J W. & Barkas W. H 1962 *Suppl. N. Cim.* **23**, 1.
Powel C. F, Fowler P. H & Perkins D. H. 1959 *Study of Elementary Particles by the Photographic Method* (Pergamon Press, London)
Sharma A. P. & Gaur R. K. 1969 *Indian J. Pure & Appl. Phys.* **7**, 325.
1968 *Indian J. of Phys* **42**, 656.
Sharma A. P. & Gill P. S. 1962 *Proc. National Inst. of Sciences of India* **28**, 166.
Sternheimer R. M 1966 *Phys. Rev.* **145**, 247.
Stiller B. & Shapiro M. M. 1952 *Phys. Rev.*, **87**, 682.
Stiller B. & Shapiro M. M. 1953 *Phys. Rev.* **92**, 735.
Trower W. P. 1966 *U.C R.L.* 2426, Vol. II.
Voyvodic L. 1950 *Cand. J. Res.*, **A28**, 315.