8

STATIONARY SPHERICALLY SYMMETRIC DUST DISTRIBUTION IN A STEADY STATE UNIVERSE

ASIT BANERJEE

JADAVPUR UNIVERSITY, CALCUTTA

(*Received February* 16, 1966)

As Hoyle and Narlikar (1964) (hereinafter referred to as H N) have shown, one can have, with the field equations of Pyree, a stationary spherically symmetric distribution of dust in an outside space which asymptotically approaches the steady state condition In their solution there is no creation in the stationary region but outside the creation does not vanish. The purpose of the present note is to point out some interesting features of this solution. We show first of all that the conditions of fit at the boundary set a possible upper limit to the dimension of the stationary dust distribution There is also a lower limit to the value of the density of the matter distribution.

From the boundary relation (equation (83) of H N) one obtains using equation (66) of H N

$$1 - \tfrac{9}{8}\pi G\rho r_b^2 - \frac{H^2}{\tfrac{8}{3}\pi G\rho}$$

$$= 2f/\rho$$

or

$$r_b = \sqrt{\frac{8}{9\pi G}} \cdot \frac{\sqrt{\rho - 2f}}{\rho} \qquad \text{(i)}$$

The above relation (i) shows that $\rho \geqslant 2f$ and further r_b has a maximum value given by

$$r_{b_{maxm}} = \frac{2}{3\sqrt{3}H} \qquad \dots \quad \text{(ii)}$$

and the maximum occurs at

$$\rho = 4f$$

The exterior solution is formally of the Schwarzschild empty-space type. It is of some interest to investigate whether a Schwarzschild like singularity ($g_{00} \rightarrow 0$) can occur at the boundary for sufficiently large finite concentrations leading to an Oppenheimer-Snyder (1939) cut-off of the light.

From equations (75) and (86) of H.N.

$$g_{00} = e^N = 1 - \frac{\gamma}{R} - H^2R^2$$

with
$$\gamma = 2G\left\{\frac{3}{4}M - \frac{27}{128}\left(\frac{H^2}{G}\right)r_b^3\right\}$$

where
$$M = \frac{4}{3}\pi r_b^3 s_c^3 \rho = \sqrt{\frac{2}{9\pi}}\frac{1}{G^{3/2}}\cdot\frac{(\rho-2f)^{3/2}}{\rho^2} \qquad \ldots \text{(iii)}$$

so that at the boundary

$$g_{00} = 1 - \frac{3}{2}\frac{GM}{R_b} - H^2R_b^2 + \frac{27}{64}\frac{H^2 r}{R_b}$$

Now since $R_b = 3/4r_b$ (equation (76) of H.N.)

$$g_{00} = 1 - \frac{2GM}{r_b} - \frac{9}{16}H^2r_b + \frac{9}{16}H^2r^2 = 1 - \frac{2GM}{r}$$

using equations (i) and (iii) one obtains

$$g_{00} = 2f/\rho$$

This is always positive For the limiting value $\rho = 2f$ $g_{00} = 1$; as ρ increases g_{00} monotonically decreases, tending to vanish as $\rho \to \infty$. For any significant condensation $\rho >> f$ and $g_{00} << 1$, so that one has an intense gravitational field at the surface although there is never a Oppenheimer-Snycer type cut-off of light.

The author expresses his gratitude to Professor A. K. Ray-Chowdhuri of Presidency College, Calcutta, for his guidance and valuable suggestions

REFERENCES

Hoyle, F, and Narlikar, J. V., 1964, *Proc. Roy. Soc.* A, **278**, 465.
Oppenheimer, J. R. and Snyder, H., 1939, *Phys. Rev.* **56**, 455.