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VALIDITY OF THE GENERALIZED RECIPROCITY EQUATION INVOLVING CIRCULAR POLARIZATION

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ABSTRACT A Generalized Reciprocity Equation expressing an algebraic relationship between the parameters of an Optical system and its reciprocal system, was formulated by the author and was verified with the help of data reported by various workers for plane-polarized light beams. This paper establishes within experimental error-limits, the validity of the equation in case of circularly-polarized beams also for light scattering by a set of oriented nylon fibres. Since the Generalized Reciprocity Equation follows from Mueller's Reciprocity Law, this study completes the experimental verification of Mueller's theorem also.

INTRODUCTION

R. S. Krishnan (1935) derived a reciprocity theorem in the form of an algebraic relation between the depolarization factors for unpolarized, horizontally-polarized and vertically-polarized incident beams of light. It has been experimentally verified by a large number of workers for random aggregation of colloidal particles. In case of oriented particles, it was found by Krishnan (1938) as well as Rao (1945), Subramanya and Rao (1949) and others, that the relation was true only for vertically oriented rod-like particles and failed for orientations in the horizontal plane along and perpendicular to the direction of the incident beams. Krishnan (1939) proposed another reciprocity relation where the electric vector of the incident beam of plane polarized light can assume any angle between the vertical and the horizontal axes. The relation however was found to lack generality because of phase relationship involved therein. Perrin (1942) extended Krishnan's work and proposed six reciprocity relations, which also included Krishnan's theorem. One of these relations involving circularly polarized beams was investigated by Ramanathan (1953) who established a phase reciprocity relation and verified it experimentally for circularly polarized light. Further study of reciprocity relations was undertaken by Krishnan, Narayanan and Sivarajan (1954), and Krishnan and Sivarajan (1956), for various cases. Subramanian (1963) proposed a reciprocity relation existing between the intensities of the scattered beams and verified it experimentally in the case of plane-polarized beams and the scatterers oriented along and perpendicular to the incident beams. Mueller (Parke-1949) trying to explain the cause of non-generality of the reciprocity relations of Krishnan and Perrin, found that the these reciprocity relations were in fact reversibility relations, and as such were valid for only reversible optical systems. He proposed a generalized reciprocity theorem of the form

$$M = M^{\neq} \qquad \ldots \quad (1)$$

where M and $M^{\neq}$ are the 4×4 Mueller matrices (Schuerliff 1962) of an optical system and its corresponding reciprocal system respectively. The reciprocal system being one in which the incident beam is replaced by the emergent beam and vice-versa, the beams being fairly parallel and the entrance and exit apertures being equal in area. The elements of the Mueller matrices are same as the sixteen scattering coefficients of Perrin. The present author (1965) derived a generalized recciprocity equation based on Mueller's theorem (1) and of the form

$$\frac{[1-(-)^{k}C_{ko}^{\neq}]C_{jk}-[1+(-)^{k}C_{ko}^{\neq}]C_{j\bar{k}}}{[1-(-)^{j}C_{jo}]C_{kj}^{\neq}-[1+(-)^{j}C_{jo}]C_{k\bar{j}}^{\neq}} = (-)^{j+k}. \qquad \ldots \quad (2)$$

which is an algebraic relation between the parameters of a natural optical system and its reciprocal system, where $C_{jk} = \cos 2\theta_{jk}$ and θ_{jk} is the angle between the vertical component of the electric vector of the scattered beam and the transmission axis of the Analyzer for equal intensity of the resolved components of H and V along it. The subscripts j, k refer to the types of analyzing and polarizing systems respectively, which are specified as follows .

Analyzer (j)	Polarizer (k)
1 = Plane horizontal;	0 = Unpolarized;
2 = Plane at 45°;	1 = Plane horizontal;
3 = Right-circular;	$\bar{1}$ = Plane vertical;
$\neq$ = Symbol superscripted on parameters of the Reciprocal systems.	2 = Plane at 45°;
	$\bar{2}$ = Plane at −45°
	3 = Right-circular;
	$\bar{3}$ = Left-circular;

The experimental validity of (2) was tested by the author (1965) with the help of data reported by various workers, for all possible cases involving plane polarized beams. This paper provides a test of the generalized equation (2) in case of circularly polarized beams, through a set of data obtained from a light scattering experiment using oriented nylon fibres as scatterers. Mueller's reciprocity theorem is therefore completely verified through this study in conjunction with the previous one (Tewarson 1965), for a set of oriented particles.

EXPERIMENTAL

The apparatus consisted of a 500 watt projection lamp with a yellow Wratten filter as the source. A set of condensing lenses was used for obtaining a fine parallel beam. The specimen consisted of a set of fine parallel nylon fibres stretched tightly and mounted at the centre of a specially designed holder capable of rotation through known angles in a vertical or horizontal plane. The holder was fitted into the prism-table shaft of a spectrometer. The collimator arm had a polaroid holder wherein the transmission axis of the polaroid could be set at any desired angle. The telescope arm carried an analyzing polaroid of the same type as the polarizer, having been cut from the same HN-38 sheet polaroid. A lens condensed the scattered beam on a photoelectric cell which was connected to a Leeds and Northrup mirror galvanometer having a sensitivity of 2.4×10^{-9} amp/mm. Adequate protection from stray light was ensured by enclosing the two arms of the spectrometer in blackened tubes, and a small shutter window helped in setting the photocell which was capable of being raised or lowered and also being moved back and forth. Cell biasing and a photomultiplier were not needed, since the cell was of photovoltaic type and provided a maximum deflection of about 20cms. on the scale. A priliminary check showed a linear response of the photocell to intensity variations within the range of the scale. A constant voltage stabilizer with $\pm1\%$ stability for 230 volts, 50 cycles A.C. and of 500 watts capacity was used with the lamp. The photocell arm could be set at any desired angle of scattering. Care was taken in cutting down reflected light from entering the photocell arm. The sample holder was enclosed in a blackened cylindrical chamber which had two holes for entrance and exit of the incident beam directly, and another hole for the scattered beam along 30°. All components including the specimen-holder where blackened and all experiments were performed in a dark room.

For the natural optical system the face of the sample was kept normal to the indicent beam, while for the reciprocal system, measurements were made after giving a rotation of 180° in the horizontal plane to the sample face and then setting it normally to the direction of the scattered beam. Quarter wave-plates used were also cut out from a single sheet. To avoid errors of centering and slight non-parallelism of the fibres, as well as slight ellipticity of the beams, readings of H and V of the scattered beam were taken for fibre orientations on both sides of the vertical, and only mean values were used in calculating the C_{jk} values by the relation

$$C_{jk} = \frac{H_{jk}-V_{jk}}{H_{jk}+V_{jk}} \qquad \ldots \quad (3)$$

RESULTS

The following five equations are obtained from (2) for all cases involving circulary polarized beams :

$$\frac{(1+C_{30}{}^{\prime})C_{13}-(1-C_{30})C_{1\bar{3}}}{(1+C_{10})C_{31}{}^{\prime}-(1-C_{10})C_{3\bar{1}}{}^{\prime}} = 1;\ (j=1,\ k=3) \qquad \ldots \quad (4)$$

$$\frac{(1+C_{10}{}^{\prime})C_{31}-(1-C_{10}{}^{\prime})C_{3\bar{1}}}{(1+C_{30})C_{13}{}^{\prime}-(1-C_{30})C_{1\bar{3}}{}^{\prime}} = 1;\ (j=3,\ k=1) \qquad \ldots \quad (5)$$

$$\frac{(1+C_{30}^{\neq})C_{23}-(1-C_{30}^{\neq})C_{2\bar{3}}}{(1-C_{20})C_{32}^{\neq}-(1+C_{20})C_{3\bar{2}}^{\neq}} = -1;\ (j=2,\ k=3) \qquad \ldots \quad (6)$$

$$\frac{(1-C_{20}^{\neq})C_{32}-(1+C_{20}^{\neq})C_{3\bar{2}}}{(1+C_{30})C_{23}^{\neq}-(1-C_{30})C_{2\bar{3}}^{\neq}} = -1;\ (j=3,\ k=2) \qquad \ldots \quad (7)$$

$$\frac{(1+C_{30}^{\neq})C_{33}-(1-C_{30}^{\neq})C_{3\bar{3}}}{(1+C_{30})C_{33}^{\neq}-(1-C_{30})C_{3\bar{3}}^{\neq}} = 1;\ (j=k=3) \qquad \ldots \quad (8)$$

The following Tables show the results of enumerations of the above equations The angle α^0 indicates the orientation of the fibres with respect to vertical, while LHS- implies left hand side of an equation :

TABLE I

α°	$-C^{\neq}_{30}$	$-C_{13}$	$-C_{1\bar{3}}$	$-C_{10}$	$-C^{\neq}_{31}$	$-C^{\neq}_{3\bar{1}}$	*LHS* (4)
0	.504	.520	623	512	514	555	1 065
30	505	597	605	515	516	557	1 019
60	.513	600	594	.510	555	.590	0 985
90	527	569	585	.525	586	580	1 005
120	513	.695	586	500	534	508	0 945

TABLE II

α°	$-C^{\neq}_{10}$	$-C_{31}$	$-C_{3\bar{1}}$	$-C_{30}$	$-C^{\neq}_{13}$	$-C^{\neq}_{1\bar{3}}$	*LHS* (5)
0	.517	.543	511	502	633	627	0 965
30	.524	.527	.548	520	660	613	0 980
60	500	464	561	505	656	611	1 021
90	508	.510	580	510	613	613	0 998
120	.515	531	513	495	656	600	0 920

TABLE III

α°	$-C^{\neq}_{30}$	$-C_{23}$	$-C_{2\bar{3}}$	$-C_{20}$	$-C^{\neq}_{32}$	$-C^{\neq}_{3\bar{2}}$	*LHS* (6)
0	.504	.791	.805	829	.500	522	−0 993
30	.505	.772	.759	779	505	481	−0 902
60	.513	.714	.765	.778	489	.467	−1.055
90	.527	.765	785	855	.485	.530	−1.018
120	.513	.750	759	.772	489	.467	−1.036

TABLE IV

α°	$-C^{\neq}_{20}$	$-C_{32}$	$-C_{3\bar{2}}$	$-C_{30}$	$-C^{\neq}_{23}$	$-C^{\neq}_{2\bar{3}}$	*LHS* (7)
0	.815	.513	.528	.502	802	780	−1.070
30	.810	.496	482	520	807	781	−1.009
60	.784	.500	.474	.500	744	742	−1.060
90	.796	.474	.478	510	833	.750	−1.040
120	813	.520	.460	495	.734	.818	−1.008

TABLE V

α°	$-C^{\neq}_{30}$	$-C_{33}$	$-C_{3\bar{3}}$	$-C_{30}$	$-C^{\neq}_{33}$	$-C^{\neq}_{3\bar{3}}$	*LHS* (8)
0	.504	.510	.510	.502	.514	.500	1.040
30	.505	506	.483	.520	.500	.500	0 935
60	.513	510	.488	505	515	482	1 021
90	.527	.422	466	510	495	.484	1 018
120	.513	510	.488	.495	.500	482	1.052

DISCUSSION

A glance at the last column of Table I through V reveals that the generalized reciprocity equation (2) is valid within about 5% experimental error limits for the cases in which circular polarization is involved, for the various angles of orientation between the vertical and the horizontal planes. Mean deviations of the last columns were also estimated and were found insignificant. The cases for linearly polarized beams having been already verified by the author in the previous paper, Mueller's reciprocity theorem stands completely verified for a set of oriented nylon fibres as the scattering medium. Considering the large volume of the data and the interinvolvement of the C-values, whereby errors would be propagated, the verifications appear fairly reliable. In Table V the *C*-values appear nearly equal, this being expected when both the analyzing and polarizing systems are alike, the polarizer and analyzer both being circular.

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