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# SCATTERING OF THE RADIATION FIELD OF A LOOP ANTENNA BY A CONDUCTING CYLINDER IMMERSED IN A COLD PLASMA

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**ABSTRACT.** The equations describing the scattered fields of a circular loop antenna and a conducting cylinder immersed in a homogeneous cold plasma are derived. It is assumed that the loop antenna is excited by a one-dimensional, uniform, inphase, sinusoidal current, i e, a current filament Solutions of Helmholtz's wave equation are formulated through an integral expansion of the product of cylindrical Hankel functions and transcendental functions. The coefficients in these solutions are evaluated by the application of the problem boundary conditions so that a solution for the scattered electric field is effected

## INTRODUCTION

Considerable interest has been shown by a number of authors, including Yeh (1964), Seshadri and Hessel (1964), and Seshadri (1964), in the scattering effect of plasmas and perfectly conducting surfaces on the radiation characteristics of various antennas. Seshadri (1964) considered the problem of the scattering of a plane wave due to the presence of a conducting cylinder immersed in a cold plasma The problem discussed in this paper is the scattering of the radiation field of a circular loop antenna immersed in a cold plasma in the presence of a perfectly conducting cylinder of infinite length (Fig. 1) It is assumed that the thin wire,

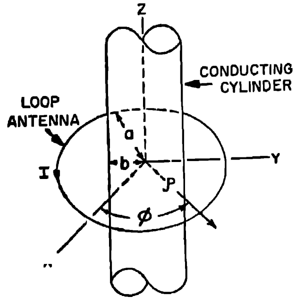


Figure 1.

single turn, loop antenna is located in the $xy$ plane with its center at the origin and that it is excited by a uniform, inphase sinusoidal current of the form $I_\varphi \exp(i\omega t)$

Since the plasma is assumed homogeneous and isotropic, its field characteristics may be described by a complex relative permittivity factor $K$

The equations governing propagation are Maxwell's equations and the equation of motion of the free charge existing in the plasma. For the sinusoidal steady state case, these equations may be expressed as

$$\nabla \times E = -i\omega\mu_0 H \qquad \ldots \quad (1)$$

$$\nabla \times H = i\omega\epsilon_0 K E \qquad \ldots \quad (2)$$

$$(i\omega + \nu) m v = qE \qquad \ldots \quad (3)$$

where $E$ is the electric field strength, $H$ is the magnetic field strength, $\omega$ is the angular wave frequency, $\mu_0$ is the permeability of free space, $\epsilon_0$ is the permittivity of free space, $K$ is the complex relative permittivity factor, $\nu$ is the average collisional frequency of electrons with neutral particles, $q$ is the particle charge, $m$ is the mass of the charged particles, and $v$ is the velocity of the charged particles For the plasma, $K$ is defined from(3) as

$$K = \left[\left(1 - \frac{\alpha^2}{1+\beta^2}\right) - i\left(\frac{\alpha^2\beta}{1+\beta^2}\right)\right] \qquad \ldots \quad (4)$$

where $\alpha^2 = (Nq^2/\epsilon_0 m)/\omega^2$, $\beta^2 = (\nu/\omega)^2$, and $N$ is the number density of charged particles in the plasma.

## FORMULATION OF THE WAVE POTENTIALS

Maxwell's equations (1) and (2), may be readily combined to yield Helmholtz's vector wave equation in terms of the wave potential $F$

$$\nabla^2 F + k^2 F = 0 \qquad \ldots \quad (5)$$

where $k^2 = \omega^2\mu_0\epsilon_0 K$ is complex. The components of the wave potential $F$ satisfy Helmholtz's scalar wave equation

$$\nabla^2\psi + k^2\psi = 0 \qquad \ldots \quad (6)$$

Harrington (1961) has shown that the wave potential $\psi$ may be expressed in the form

$$\psi_{n,k_z,k_\rho} = B_n(k_\rho\rho)\, g(n\varphi)\, h(k_z z) \qquad \ldots \quad (7)$$

where the $B_n(k_\rho\rho)$ are Bessel or Hankel functions, the $g(n\varphi)$ and $h(k_z z)$ are sinusoidal functions, and $k^2 = k_\rho^2 + k_z^2$. In general it is possible to formulate solutions to (6) as

$$\psi = \sum_n \int_{k_z} f_n(k_z) B_n(k_\rho\rho)\, g(n\varphi)\, h(k_z z)\, dk_z \qquad \ldots \quad (8)$$

with the integration over the complex plane and the function $f_n(k_z)$ to be determined from the boundary conditions.

An examination of Fig. 1 indicates that due to symmetry, there will be no component of $E$ in the $z$ direction. The stipulation of a uniform inphase current excitation of the loop antenna requires that $g(n\varphi)$ be a constant or that $n = 0$. Eq (8) may now be rewritten as

$$\psi_1 = \int_{k_z} [AH_0^{(1)}(k_\rho\rho)+BH_0^{(2)}(k_\rho\rho)]e^{ik_z z}dk_z$$

when $b < \rho < a$ and

$$\psi_2 = \int_{k_z} CH_0^{(2)}(k_\rho\rho)e^{ik_z z}dk_z \quad \text{when} \quad \rho > a \qquad \dots \quad (10)$$

Eq (9) represents a standing wave region (region 1) which exists between the conducting cylinder and a mathematical cylindrical surface containing the loop antenna. In region 2, $\rho > a$, only travelling waves exist. The constants $A$, $B$, and $C$ are the $f_0(k_z)$ to be determined from the boundary conditions and the $H_0^{(1)}(k_\rho\rho)$ and the $H_0^{(2)}(k_\rho\rho)$ are Hankel functions of zero order of the first and second kind. The electric field $E$ and the magnetic field $H$ are given respectively by

$$E = -\nabla\times F \qquad (11)$$

$$H = \frac{\nabla\times\nabla\times F}{i\omega\mu_0} \qquad (12)$$

where $$F = a_z\psi$$

DERIVATION OF THE SCATTERED FIELD

Application of the boundary conditions over the surface of the perfectly conducting cylinder and the loop of current yields the required $f_0(k_z)$. Since the tangential components of the $E$ field must be zero on the surface of the conducting cylinder ($\rho = b$), application of this boundary condition yields

$$B = -\left\{\frac{H_1^{(1)}(k_\rho b)}{H_1^{(2)}(k_\rho b)}\right\} A. \qquad \dots \quad (13)$$

In addition the tangential components of the $E$ field must be continuous over the cylindrical surface ($\rho = a$), so setting $E_{\varphi 1} = E_{\varphi 2}$ at $\rho = a$ gives

$$C = \left\{ \frac{H_1^{(1)}(k_\rho a)H_1^{(2)}(k_\rho b)-H_1^{(2)}(k_\rho a))H_1^{(1)}(k_\rho b)}{H_1^{(2)}(k_\rho a)H_1^{(2)}(k_\rho b)} \right\} A. \qquad \dots \quad (14)$$

Also at $\rho = a$, $z = 0$, the magnetic field must change in a discontinuous manner because of the current flowing in the loop antenna. That is

$$H_{z1}-H_{z2} = J_\varphi = \frac{I_\varphi}{a}\delta(z). \qquad \dots \quad (15)$$

The delta function may be represented by a complex integral of the form

$$\delta(z) = \frac{1}{2\pi} \int_{k_z} e^{k_{z}z} dk_z. \quad \ldots \quad (16)$$

Substitution of (16) into (15) allows evaluation of $A$ as

$$A = \frac{i\omega\mu_0 H_1^{(2)}(k_\rho a) I_\varphi}{2\pi a k_\rho^2 [H_0^{(1)}(k_\rho a) H_1^{(2)}(k_\rho a) - H_1^{(1)}(k_\rho a) H_0^{(2)}(k_\rho a)]} \quad \ldots \quad (17)$$

Utilization of the Wronskian for Hankel functions reduces (17) to

$$A = \frac{\omega\mu_0 H_1^{(2)}(k_\rho a) I_\varphi}{8k_\rho} \quad \ldots \quad (18)$$

The electric field $E$ (radiated and scattered) in regions 1 and 2 may now be written as

$$E_{\varphi 1} = \frac{-\omega\mu_0 I_\varphi}{8} \int_{k_z} \left\{ \frac{H_1^{(2)}(k_\rho b) H_1^{(1)}(k_\rho \rho) - H_1^{(1)}(k_\rho b) H_1^{(2)}(k_\rho \rho)}{H_1^{(2)}(k_\rho b)} \right\} H_1^{(2)}(k_\rho a) e^{ik_z z} dk_z \quad \ldots \quad (19)$$

$$E_{\varphi 2} = \frac{-\omega\mu_0 I_\varphi}{8} \int_{h_z} \left\{ \frac{H_1^{(1)}(k_\rho a) H_1^{(2)}(k_\rho b) - H_1^{(2)}(k_\rho a) H_1^{(1)}(k_\rho b)}{H_1^{(2)}(k_\rho b)} \right\} H_1^{(2)}(k_\rho \rho) e^{ik_z z} dk_z \quad \ldots \quad (20)$$

Since (19) and (20) represent the total field, radiated plus scattered, the field scattered by the cylinder may be found by subtracting the radiation field of the loop from the total field

Correspondingly, the radiation field of the loop antenna may be expressed as

$$E_{\varphi 1} = \frac{-\omega\mu_0 I_\varphi}{4} \int_{k_z} H_1^{(2)}(k_\rho a) J_1(k_\rho \rho) e^{ik_z z} dk_z \quad \ldots \quad (21)$$

$$E_{\varphi 2} = \frac{-\omega\mu_0 I_\varphi}{4} \int J_1(k_\rho a) H_1^{(2)}(k_\rho \rho) e^{ik_z z} dk_z \quad \ldots \quad (22)$$

Subtracting (21) from (19) and (22) from (20) yields the scattered field produced by the presence of the conducting cylinder. Thus the scattered field in region 2 may be written as

$$E_{\varphi 2}{}^{s} = \frac{-\omega\mu_0 I_\varphi}{8} \int_{k_z} \left[ \left\{ \frac{H_1{}^{(1)}(k_\rho a)H_1{}^{(2)}(k_\rho b) - H_1{}^{(2)}(k_\rho a)H_1{}^{(1)}(k_\rho b)}{H_1{}^{(2)}(k_\rho b))} \right\} \ldots \quad (23)$$

$$-2J_1(k_\rho a) \Big| \, H_1{}^{(2)}(k_\rho \rho) e^{ik_z z} dk_z.$$

It is also interesting to note that (23) reduces to a Fourier integral if $k_z$ is real. For the problem under consideration, this occurs when the plasma collisional frequency $\nu$ is zero since $K$ is then real. Integration limits on $k_z$ in (23) may then be written as $-\infty$ to $\infty$.

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