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RELAXATION METHOD APPLIED TO NETWORK PROBLEM INVOLVED IN ELECTRIC RAILWAY SYSTEM

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ABSTRACT. In this paper it has been shown how the network problem involving electric railway systems, can be solved using relaxation technique The present method utilises the set of linear simultaneous equations which are obtained with the help of Kirchoff's laws of electrical network, and shows how to solve them The relaxational solution as obtained is seen to be quite useful because it gives the values of the unknown voltages at all the nodal points simultaneously

INTRODUCTION

In the electric railway systems (Starr, 1946), a definite electrical problem is difficult to be formulated due to the fluctuating loads. But assuming the loads simulating the typical operating conditions the problem can be solved by different methods, which entail much more labour with the increase of nodal points in the corresponding network. But the method discussed in this paper shows its advantage in the sense that the increase in the number of nodal points does not generally bring about more complication in solving the problem.

In this network system as shown in Fig 1 the supply voltages, resistances of the trolley, feeders and rails, and ampere loads at the designated locations are known. The equivalent circuit diagram can be drawn as in Fig 2. Considering the nodal points of the Fig 2, a set of linear simultaneous equations can be obtained at each of them applying Kirchoff's laws of networks and these equations are then solved by relaxation method

THE METHOD

In this method the following linear simultaneous equations are obtained if the required nodal points of the equivalent circuit diagram are considered.

Hence :

$$\left.\begin{array}{lll} \text{At } A, & I_A-(V_A-V_B)g_{AB}-(V_A-V_D)g_{AD}-(V_A-V_C)g_{AC} & =0 \\ C, & (V_A-V_C)g_{AC}-(V_C-V_B)g_{CB}-(V_C-V_D)g_{CD} & =0 \\ B, & (V_A-V_B)g_{AB}+(V_C-V_B)g_{CB}-I_{BB'} & =0 \\ D, & (V_A-V_D)g_{AD}+(V_C-V_D)g_{CD}-I_{DD'} & =0 \\ B', & I_{BB'}+(V_{C'}-V_{B'})g_{C'B'}-(V_{B'}-V_{A'})g_{B'A'} & =0 \\ D', & I_{DD'}-(V_{D'}-V_C)'g_{D'C'}-(V_{D'}-V_{A'})g_{D'A'} & =0 \end{array}\right\} \quad (1)$$

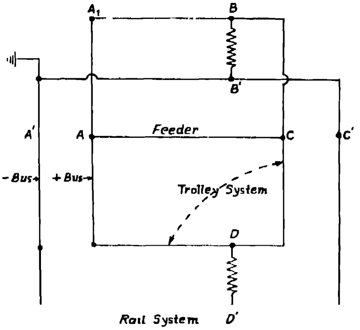

Fig. 1. Diagram for network in Electric Railway System

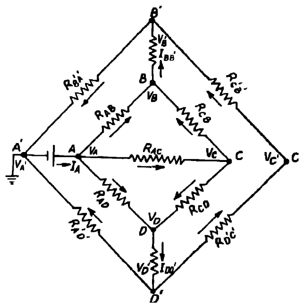

Fig. 2. Equivalent diagram for Network in Electric Railway System.

where

I_A is the current flowing towards A,

$I_{BB'}$ .... through the branch BB',

I_{DD} ....DD',

V_A ....potential at the nodal point A,

V_B .... B,

V_C .... C,

V_D .... D,

$V_{A'}$........... .... A' = zero (earth point)

$V_{B'}$ ....B'

$V_{C'}$ ....C',

$V_{D'}$ .... D',

$g_{AB} = 1/R_{AB}$, where R_{AB} is the resistance of the branch AB,

$g_{AC} = 1/R_{AC}$, ... R_{AC} ....AC,

$g_{AD} = 1/R_{AD}$, ... R_{AD} ....AD,

$g_{CB} = 1/R_{CB}$, ... R_{CB} ....CB,

$g_{CD} = 1/R_{CD}$, ... R_{CD} ....CD,

$g_{B'C'} = 1/R_{B\ C'}$ ... $R_{B'C'}$ ....$B'C'$,

$g_{B'A'} = 1/R_{B'A'}$ ... $R_{B'A'}$ ....$B'A'$,

$g_{D'C'} = 1/R_{D'C'}$ ... $R_{D'C'}$ .... $D'C'$, The above set of equations after necessary simplification and rearrangement can be written as shown below :

$$
\begin{aligned}
& I_A - V_A(g_{AB}+g_{AD}+g_{AC}) + V_B g_{AB} + V_C g_{AC} + V_D g_{AD} && = 0 = F_1 \\
& V_A g_{AC} + V_B g_{CB} - V_C(g_{AC}+g_{CB}+g_{CD}) + V_D g_{CD} && = 0 = F_2 \\
& V_A g_{AB} - V_B(g_{AB}+g_{CB}) + V_C g_{CB} - I_{BB'} && = 0 = F_3 \\
& V_A g_{AD} + V_C g_{CD} - V_D(g_{AD}+g_{CD}) - I_{DD'} && = 0 = F_4 \\
& I_{BB'} - V_{B'}(g_{C'B'}+g_{B'A'}) + V_{C'} g_{C'B'} && = 0 = F_5 \\
& I_{DD'} - V_{D'}(g_{D'C'}+g_{D'A'}) + V_{C'} g_{D'C'} && = 0 = F_6
\end{aligned}
\qquad (2)
$$

where F_1, F_2, F_3, F_4, F_5 and F_6 are the residuals The values of the unknowns shown in the relation (2) can be easily found out when the residuals are liquidated by relaxation method. To liquidate them the basic unit, block and group operations are carried out (Allen, 1954). In basic unit operation (Table I) the changes in the values of the residuals due to unit positive increment of the unknowns are found out. With the help of basic unit operations suitable block and group operations can be performed in which equal simultaneous, and unequal simultaneous increments are given respectively to more than one unknown to bring about the changes in some required residuals without affecting the rest In the relaxation table (Table II) the use of basic, block and group operations are shown in the liquidation steps, number 2 and 6; 1 and 5, and 3, 4 and 7 respectively. The liquidation is nearly complete in those seven steps yielding the values of the unknowns. The following illustration will clearly show the merit and utility of the method.

ILLUSTRATION

This illustrating example described hereafter is solved by Dawes (1952) by conventional method.

In Fig. 1 there is shown a simple railway system with a ring connected trolley and a single feeder connected to the busbars at A and to the trolley system at C. The station busbars at AA' are maintained at 600 volts, busbar A being positive and A' being negative and grounded The resistance of the busbar is negligible. The resistance of the overhead trolley is as follows : A_1 to $B = 0.30$ ohm, B to $C = 0.20$ ohm, C to $D = 0.20$ ohm, D to $A = 0.28$ ohm. A feeder connected from A to C and its resistance is 0.25 ohm. The resistance of the rail and the ground return is as follows : A' to $B' = 0.40$ ohm, B' to $C' = 0.25$ ohm, C' to $D' = 0.25$ ohm, D' to $A' = 0.36$ ohm. A trolley car at BB' takes 70 amps and a car at DD' takes 80 amps. It was desired to determine (a) Current in each section of trolley and in feeder, (b) Voltages at each car and at feeding point CC'.

Considering the equivalent circuit diagram shown in Fig. 2 of the railway system shown in Fig 1, and substituting the numerical values in relation (2) the following set of equations can be written :

$$\left.\begin{array}{ll}
3.333V_B+4V_C+3.571V_D-6392.4 & =0=F_1 \\
5V_B-14V_C+5V_D+2400 & =0=F_2 \\
-8.333V_B+5V_C+1930 & =0=F_3 \\
5V_C-8.571V_D+2062.6 & =0=F_4 \\
-6.5V_{B'}+4V_{C'}+70 & =0=F_5 \\
4V_{C'}-6.778V_{D'}+80 & =0=F_6
\end{array}\right\} \quad \ldots (3)$$

On liquidating the residuals of the relation (3) almost completely, the values of the potentials at the said nodal points are obtained correct to the required limit of accuracy. From those values of potentials and the supplied values of different resistances, the currents in the various branches of the network wanted in the illustration can be easily calculated as given below :

I_{AB} = 49.86 amps, where I_{AB} is the current flowing through AB,

I_{AD} = 55.91 ,, ... I_{AD} .... AD,

$I_{B'A'}$ = 70.75 ,, ... $I_{B'A'}$ .... $B'A'$

$I_{D'A'}$ = 79.44 ,, ... $I_{D'A'}$ .... $D'A'$,

I_{AC} = 43.48 ,, ... I_{AC} .... AC,

I_{CB} = 20.44 ,, ... I_{CB} .... CB,

I_{CD} = 23.93 ,, ... I_{CD} .... CD,

$I_{D'C'}$ = 0.60 ,, ... $I_{D'C'}$ .... $D'C'$,

$I_{C'B'}$ = 0.60 ,, ... $I_{C'B'}$ .... $C'B'$,

$V_{BB'}$ = 556.74 volts, $V_{BB'}$ being the voltage at the trolley car at BB',

$V_{DD'}$ = 555.74 ,, ... $V_{DD'}$ .... DD',

$V_{C'}$ = 28.45 ,, ... $V_{C'}$ .... feeding point C',

V_C = 589.13 ,, ... V_C .... C,

The above values as calculated by relaxation method are quite comparable with those found out by the other methods of network analysis (Dawes, 1952), shown in the table below (Table III).

DISCUSSION

This method is seen to yield the values of the voltages at different nodal points simultaneously, from which the calculations of the other desired quantities become very quick and easy. Although with the increase of the number of branches

TABLE I
Basic Unit Operation Table

δV_B	δV_C	δV_D	$\delta V_{B'}$	$\delta V_{C'}$	$\delta V_{D'}$	δF_1	δF_2	δF_3	δF_4	δF_5	δF_6
1	—	—	—	—	—	3 333	5 0	-8.333	0	0	0
—	1	—	—	—	—	4 000	-14 0	5 000	5 0	0	0
—	—	1	—	—	—	3.571	5 0	0	-8 571	0	0
—	—	—	1	—	—	0	0	0	0	-6 5	0
—	—	—	—	1	—	0	0	0	0	4 0	4 0
—	—	—	—	—	1	0	0	0	0	0	-6 778

TABLE II
Relaxation Table

Liquidation steps	δV_B	δV_C	δV_D	$\delta V_{B'}$	$\delta V_{C'}$	$\delta V_{D'}$	F_1	F_2	F_3	F_4	F_5	F_6	
	$V_B = V_C = V_D = V_{B'} = V_{C'} = V_{D'} = 0$						-6392 4	2400	1930	2062 6	70	80	
(1)	586 2436	586 2436	586 2436	—	—	—	0	55 0256	-23 9499	-30 8759	70	80	
(2)	—	3 9304	—	—	—	—	15 7216	0	-4 2979	-11 2239	70	80	
(3)	-0 5158	—	0 5158	—	—	—	15.8444	0	0	-15 6448	70	80	
(4)	-0 6859	-1 1433	-2 5153	—	—	—	0	0	0	0 19699	70	80	
(5)	—	—	—	28 0	28 0	28 0	0	0	0	0 19699	0	2 216	
(6)	—	—	—	—	—	0 3269	0	0	0	0 19699	0	0	
(7)	—	0 1	0 1	0 3	0 45	0 2731	0 7571	-0 9	0 5	-0 16011	-0 15	-0 05	
Values of the unknown	585 042	589 131	584 344	28 3	28 45	28 60	0 7571	-0 9	0 5	-0 16011	-0 15	-0 05	Residuals unliquidated

TABLE III
Comparison of values

Methods	I_{AB} amps	I_{AD} amps	$I_{B'A'}$ amps	$I_{D'A'}$ amps	I_{AC} amps	I_{CB} amps	I_{CD} amps	$I_{D'C'}$ amps	$I_{'B'}$ amps	$V_{BB'}$ volts	$V_{DD'}$ volts	V_C volts	$V_{C'}$ volts
Relaxation method	49 86	55 91	70 75	79.44	43 48	20 44	23 93	0.60	0 60	556 74	555 74	589 13	28 45
Conventional method	50 00	56 20	70 60	79 40	43 80	20 00	23 80	0 60	0 60	556 80	555 60	589 10	28 40

or nodal points the network becomes complicated, with some practice this method needs practically no extra labour in solving the problem. In the relaxation table (Table II) the residuals are not liquidated completely and consequently their values are reduced to narrowest possible limits so that the desired limit of accuracy of the values can be achieved.

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