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RECIPROCITY EQUATIONS FOR ISOTROPIC OPALESCENT SCATTERING MEDIA

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ABSTRACT Krishnan's reciprocity theorem was found to be restricted to random or vertical orientations of scatterers, and therefore non-general in validity. Perrin extended Krishnan's work to provide a number of reciprocity relations, which were also found to be restricted in validity. S. Subramanian formulated a reciprocity relation concerned with intensity relations and established it experimentally in a much more general form valid for the two cases of orientation in the horizontal plane of observation wherein Krishnan's theorem had previously failed. H. Mueller proposed another general form of reciprocity relation

$$M = M^{\neq},$$

where M and $M^{\neq}$ are the 4×4 matrices of the natural and its corresponding reciprocal optical system involved.

The present paper deals with the formulation of a generalized reciprocity relation, valid for seven types of polarized beams and for all possible orientations and types of scatterers. The equation follows from the Mueller law of reciprocity. Six equations follow from the generalized equation, three of which have been experimentally established through calculations on data from various experimenters available in the published literature. The equations are expected to find usefulness in colloid-optics, gamma-ray polarization studies and allied fields involving electromagnetic beams.

INTRODUCTION

R. S. Krishnan (1935a), derived the following Reciprocity theorem for Tyndall scattering

$$\rho_u = \frac{1+1/\rho_h}{1+1/\rho_v} \tag{1}$$

where ρ_u, ρ_h and ρ_v are depolarization factors for unpolarized, horizontally and vertically polarized incident beams respectively. This relation has been subjected to extensive experimental verifications by Krishnan (1935b), as well as a large number of other workers, mostly for random aggregations of colloidal particles of all shapes and sizes. For specifically oriented non-spherical particles it was experimentally shown by Krishnan (1938), Rao (1945), Subramanya and Rao (1949), etc., that (1) is true for only vertically-oriented particles and fails for particles oriented in the horizontal plane of observation. It was therefore concluded that the law of reciprocity is not a general law of optics, in the original form suggested by Lord Rayleigh (1877)

F. Perrin (1942) extended Krishnan's work to provide a number of additional reciprocity relations which were also found to have restricted validity confined to random orientations only.

In 1948, Muller (1948) reported the formulation of a phenomenological foundation of optics, having only operational bases and founded upon empirical laws of spectral decomposition, polarization, superposition and reciprocity. He gave a new interpretation to the concept of reciprocity based upon the Mueller Phenomenological Algebra (Parke 1949, Muller).

Among the recent workers, Subramanian's (1963) contribution is noteworthy. Under Krishnan's guidance, Subramanian formulated a general reciprocity relation of the form

$$I^A{}_B = I'^B{}_A; \qquad \ldots \quad (2)$$

where $I^A{}_B$ is the intensity of the component of the scattered light whose electric vector is inclined at an angle B to the vertical, with the external orienting field parallel to the incident beam. $I'^B{}_A$ is the corresponding intensity component, when the external field is parallel to the scattered beam The relation (2) was also verified by him experimentally The reasoning followed in the formulation of (2), is based upon Subramanian's observation that $\rho_v \times \rho_h$ is a constant for the two cases. However, Subramanian was concerned with intensity only and his result does not deal with the parameters of the optical system constituting the scattering medium.

The present paper deals with certain generalizations of reciprocity relations in light scattering media in terms of Muller's law of reciprocity, valid for all possible orientations of the scattering particles of all shapes and sizes, constituting an isotropic opalescent medium.

Perrin (1942) has defined an isotropic opalescent medium as one whose scattering elements are not very small compared to the wavelength of light, which is more or less turbid or opalescent, and which shows either absolute or statistical isotropyas a whole Examples of such media are suspensions, colloidal solutions, solutions of large molecules, smokes, fogs, fiberous matter, etc.

THE MUELLER LAW OF RECIPROCITY

In the phenomenological Mueller Algebra, an optical system (or instrument), is defined by a 4×4 Matrix of the form

$$M = t \begin{matrix} 1 & a_1 & a_2 & a_3 \\ b_1 & g_{11} & g_{12} & g_{13} \\ b_2 & g_{21} & g_{22} & g_{23} \\ b_3 & g_{31} & g_{32} & g_{33} \end{matrix} \qquad \ldots \quad (3)$$

where t is the transmission coefficient, a and b are the analyzing and polarizing vectors (3-dimensional), and g_{jk} $(j, k = 1, 2, 3)$ are the matrix elements of an optical Tensor G.

If L and L' are the Stokes' vectors of the incident and the corresponding emergent beams respectively,

$$L' = M \times L\,; \tag{4}$$

The reciprocal optical system

The reciprocal optical system (or instrument), $T^{\neq}$, corresponding to a natural system T, is defined as one in which the exit aperature is made the entrance aperture and the entrance aperture is made the exit aperture, and wherein the incident and emergent beams are interchanged in their places and reversed in directions. The parameters of the reciprocal system are indicated by superscripting with the sign $\neq$. The Matrix or the reciprocal system is given by

$$M^{\neq} = t^{\neq}\begin{pmatrix} 1 & b^{\neq}_1 & -b^{\neq}_2 & b^{\neq}_3 \\ a_1^{\neq} & g^{\neq}_{11} & -g^{\neq}_{21} & g^{\neq}_{31} \\ -a^{\neq}_2 & -g^{\neq}_{12} & g^{\neq}_{22} & -g^{\neq}_{32} \\ a^{\neq}_3 & g^{\neq}_{13} & -g^{\neq}_{23} & g^{\neq}_{33} \end{pmatrix} \quad \ldots \quad (5)$$

The Mueller law of reciprocity

Starting from the fundamental concepts of reciprocity propounded by Helmholtz and Rayleigh, that, if by any means one point can be seen from the other, the other should also be seen from the first, Mueller's theory states the law of reciprocity in the form

$$M = M^{\neq}; \quad \ldots \quad (6)$$

as the most general form of the law.

Reciprocity conditions

The following reciprocity conditions follow immediately from (3), (5) and (6):

$$t = t^{\neq} \tag{7}$$

$$a_i^{\neq} = (-)^{i+1} \cdot b_i\,; \qquad (i = 1, 2, 3); \tag{8}$$

$$a_i = (-)^{i+1}\, b_i^{\neq}\,; \qquad (i = 1, 2, 3); \tag{9}$$

and,

$$g_{jk} = (-)^{j+k} \cdot g^{\neq}_{kj}\,; \quad (j, k = 1, 2, 3); \tag{10}$$

III. RECIPROCITY AND REVERSIBILITY

The horizontal (H) and Vertical (V) intensity components of the scattered beam, corresponding to horizontally and vertically-polarized incident beams (indicated by subscripting H and V by h and v) are given by,

$$H = (I'+M')/2 \qquad \dots \quad (11)$$

$$V = (I'-M')/2 \qquad \dots \quad (12)$$

where I' and M' are stokes' parameters of the scattered beam. Hence in terms of matrix elements,

$$\begin{aligned} H_h &= (1+a_1+b_1+g_{11})/2 \\ V_h &= (1+a_1-b_1-g_{11})/2 \\ H_v &= (1-a_1+b_1-g_{11})/2 \\ V_v &= (1-a_1-b_1+g_{11})/2 \end{aligned} \qquad \dots \quad (13)$$

Krishnan's reciprocity relation $H_v = V_h$, yields the relation,

$$b_1 = a_1 \qquad \dots \quad (14)$$

This is true only when.

$$a_1^{\neq} = a_1; \text{ and } b_1^{\neq} = b_1; \qquad \dots \quad (15)$$

which implies that the system (instrument) is reversible, wherein the reciprocal of the instrument is also the instrument itself and has therefore a Hermitian matrix The complete requirements of reversibility are expressed by (15) and the additional condition,

$$g_{jk} = (-)^{j+k} \cdot g_{jk}^{\neq}, \qquad \dots \quad (16)$$

It can now easily be seen that all reciprocity relations put forth by Perrin (1942) and Krishnan (1935a), follow from the reversibility criteria expressed by (15) and (16). Thus in terms of the new theory, previous reciprocity relations of Perrin and Krishnan appear to be in fact reversibility relations. Since all optidal systems cannot be reversible, Krishnan and Perrin's reciprocity relations lack general validity.

IV THE RECIPROCITY EQUATIONS

A. *The scattering experiment*

Consider a general scattering experiment with depolarization factors,

$$\rho_u = \frac{H_v + H_h}{V_v + V_h} \qquad \dots \quad (17)$$

hence,

$$\rho_u = \frac{1+b_1}{1-b_1} \qquad \dots \quad (18)$$

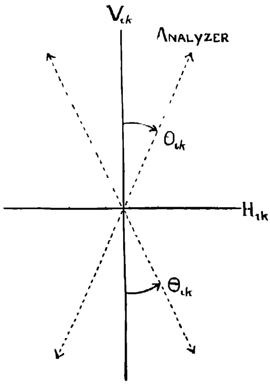
$$\rho_h = V_h/H_h = \frac{1-\left(\dfrac{b_1+g_{11}}{1+a_1}\right)}{\left(1+\dfrac{b_1+g_{11}}{1+a_1}\right)} \qquad \dots \quad (19)$$

and
$$\rho_v = H_v/V_v = \frac{1+\left(\dfrac{b_1-g_{11}}{1-a_1}\right)}{1-\left(\dfrac{b_1-g_{11}}{1-a_1}\right)} \qquad \ldots \quad (20)$$

Let the scattering experiment consist of seven types of polarizers and three types of analyzers as under .

Analyzer

Plane-Vertical ($i = 1$)
Plane at 45° to Vertical ($i = 2$)
Circular ($i = 3$)



Polarizer ,

None (Unpolarized beam)	...($k = 0$);
Plane-Vertical	...($k = \bar{1}$)
Plane-Horizontal	...($k = 1$)
Plane at +45° to vertical	...($k = 2$)
Plane at −45° to vertical	...($k = \bar{2}$)
Right circular	...($k = 3$)
Left circular	...($k = \bar{3}$)

Let θ_{ik} be the reading of the analyzer for equal intensities of the two halves of the field of view (H and V) which are orthogonally polarized; with the device like a Wollaston prism, etc. From the figure given above we get,

$$V_{ik} \cos^2 \theta_{ik} = H_{ik} \sin^2 \theta_{ik} \; ; \qquad \ldots \quad (21)$$

Hence, $$\rho_{ik} = H_{ik}/V_{ik} = \cot^2 \theta_{ik} = (1+\cos 2\theta_{ik})/(1-\cos 2\theta_{ik}) \qquad \ldots (22)$$

Using now the abbreviation $C_{ik} = \cos 2\theta_{ik}$; we have from (22),

$$b_i = C_{i0};\ (i = 1, 2, 3) \qquad \ldots (23)$$

$$\left.\begin{aligned} \rho_u &= (1+C_{10})/(1-C_{10}) \\ \rho_h &= (1-C_{11})/(1+C_{11}) \\ \rho_v &= (1+C_{11}^-)/(1-C_{11}^-) \end{aligned}\right\} \qquad \ldots (24)$$

Also, $$\left.\begin{aligned} C_{ik} &= \frac{b_i + g_{ik}}{1+a_k} \\ C_{ik}^- &= \frac{b_i - g_{ik}}{1-a_k} \end{aligned}\right. \qquad (25)$$

For brevity we express the sum and difference of the above two C values as,

$$S_{ik} = C_{ik} + C_{i\bar{k}}$$

and, $$d_{ik} = C_{ik} - C_{i\bar{k}} \qquad \ldots (26)$$

Thus we have,

$$a_k = (2b_i - S_{ik})/d_{ik} \qquad \ldots (27)$$

and, $$2g_{ik} = d_{ik} + a_k \cdot S_{ik} \qquad \ldots (28)$$

B. *The reciprocity equations*

Basing our arguments upon Mueller's proposed form of reciprocity law, it is now possible to formulate a generalized reciprocity equation as follows :

From (28) we can write,

$$2g_{jk} = d_{jk} + a_k \cdot S_{jk}$$

and, $$2g^{\neq}_{kj} = d^{\neq}_{kj} + a^{\neq}_j \cdot S^{\neq}_{kj}$$

hence, $$2(g_{jk} - g^{\neq}_{kj}) = (d_{jk} - d^{\neq}_{kj}) + (a_k \cdot S_{jk} - a^{\neq}_j \cdot S^{\neq}_{kj}) \qquad \ldots (29)$$

Substitution of reciprocity conditions (8), (9) and (10) in (29) gives,

$$2[(-)^{j+k} g^{\neq}_{kj} - g^{\neq}_{kj}] = [d_{jk} - d^{\neq}_{kj}] + [(-)^{k+1} \cdot b^{\neq}_k \cdot S_{jk} - a^{\neq}_j \cdot S^{\neq}_{kj}]$$

i.e., $$[d^{\neq}_{kj} + a^{\neq}_j \cdot S^{\neq}_{kj}](-)^{j+k} = [d_{jk} + (-)^{k+1} b^{\neq}_k \cdot S_{jk}]$$

But $$a^{\neq}_j = (-)^{j+1} \cdot b_j$$

hence, $$\frac{d_{jk} + (-)^{k+1} \cdot b^{\neq}_k \cdot S_{jk}}{d^{\neq}_{kj} + (-)^{j+1} \cdot b_j \cdot S^{\neq}_{jk}} = (-)^{j+x}; \qquad \ldots (30)$$

In terms of C_{jk} values from (26) we have,

$$\frac{(C_{jk} - C_{j\bar{k}}) + (-)^{k+1} \cdot b^{\neq}_k (C_{jk} + C_{j\bar{k}})}{(C^{\neq}_{kj} - C^{\neq}_{k\bar{j}}) + (-)^{j+1} b_j (C^{\neq}_{kj} + C_{k\bar{j}})} = (-)^{j+k} ; \qquad \ldots (31)$$

which can finally be written in the form,

$$\frac{[1-(-)^{k}C^{\neq}_{k0}]C_{jk}-[1+(-)^{k}C^{\neq}_{k0}]C_{j\bar{k}}}{[1-(-)^{j}C_{j0}]C^{\neq}_{kj}-[1+(-)^{j}C_{j0}]C_{kj}}=(-)^{j+k}; \qquad \ldots \quad (32)$$

(where : $j, k=1, 2, 3$)

Equation (32) is the most general form of reciprocity relation existing between the parameters of the natural and its corresponding reciprocal system, measured in terms of $\cos 2\theta_{jk}$ values. Some special cases of (32) are of great interest, in asmuch as their validity can be tested through available data in literature especially that of Krishnan, Subramanya and Rao, Subramaniam, A. Mueller etc. We consider them as follows :

Case I : $(j=k=1, 2, 3)$

It follows immediately from the general reciprocity equation (32), that,

(a) *For* $j=k=1$:

$$\frac{[1+C^{\neq}_{10}]C_{11}-[1-C^{\neq}_{10}]C_{1\bar{1}}}{[1+C_{10}]C^{\neq}_{11}-[1-C_{10}]C^{\neq}_{1\bar{1}}}=1 \qquad \ldots \quad (33)$$

In terms of depolarization factors, (33) can be expressed with the help of (24) and (25) as,

$$\left[\frac{\rho^{\neq}_{u}\left(\frac{1-\rho_h}{1+\rho_h}\right)+\left(\frac{1-\rho_v}{1+\rho_v}\right)}{\rho_u\left(\frac{1-\rho_h}{1+\rho_h}\right)^{\neq}+\left(\frac{1-\rho_v}{1+\rho_v}\right)^{\neq}}\right]\times\left[\frac{1+\rho_u}{1+\rho_u^{\neq}}\right]=1, \qquad \ldots \quad (34)$$

Writing,

$$\left.\begin{aligned} R_n &= \frac{1-\rho_u}{1+\rho_u} \\ R_h &= \frac{1-\rho_h}{1+\rho_h} \\ \text{and,} \quad R_v &= \frac{1-\rho_v}{1+\rho_v} \end{aligned}\right\} \qquad \ldots \quad (35)$$

Eq. (34) can be reduced to the simple form :

$$\frac{(1+R^{\neq}_{u})R_v+(1-R^{\neq}_{u})R_h}{(1+R_u)R^{\neq}_{v}+(1-R_u)R^{\neq}_{h}}=1, \qquad \ldots \quad (36)$$

(b) *For* $j=k=2, 3$:

We obtain similarly the following two relations,

$$\frac{[1+C^{\neq}_{20}]C_{22}-[1-C^{\neq}_{20}]C_{2\bar{2}}}{[1+C_{20}]C^{\neq}_{22}-[1-C_{20})C^{\neq}_{2\bar{2}}}=1; \qquad (37)$$

and,
$$\frac{[1+C^{\neq}_{30}]C_{33}-[1-C^{\neq}_{30}]C_{3\bar{3}}}{[1+C_{30}]C^{\neq}_{33}-[1-C_{30}]C^{\neq}_{3\bar{3}}}=1; \quad \ldots \quad (38)$$

Case II

(a) $j=1,\ k=2$

We have from (32),

$$\frac{[1-C^{\neq}_{20}]C_{12}-[1+C^{\neq}_{20}]C_{1\bar{2}}}{[1+C_{10}]C^{\neq}_{21}-[1-C_{10}]C^{\neq}_{2\bar{1}}}=-1; \quad \ldots \quad (39)$$

(b) $j=2,\ k=1$

$$\frac{[1+C^{\neq}_{10}]C_{21}-[1-C^{\neq}_{10}]C_{2\bar{1}}}{[1-C_{20}]C^{\neq}_{12}-[1+C_{20}]C^{\neq}_{1\bar{2}}}=-1; \quad \ldots \quad (40)$$

We can combine (39) and (40) into a single package,

$$\frac{[1-C^{\neq}_{20}]C_{12}-[1+C^{\neq}_{20}]C_{1\bar{2}}}{[1-C_{20}]C^{\neq}_{12}-[1+C_{20}]C^{\neq}_{1\bar{2}}}\times\frac{[1+C^{\neq}_{10}]C_{21}-[1-C^{\neq}_{10}]C_{2\bar{1}}}{[1+C_{10}]C^{\neq}_{21}-[1-C_{10}]C^{\neq}_{2\bar{1}}}=1; \quad \ldots \quad (41)$$

Case III. ($j=1,\ k=3$) and ($j=3,\ k=1$).

In the same manner as in (41) the following relation is obtained for this case,

$$\frac{[1+C^{\neq}_{30}]C_{13}-[1-C^{\neq}_{30}]C_{1\bar{3}}}{[1+C_{30}]C^{\neq}_{13}-[1-C_{30}]C^{\neq}_{1\bar{3}}}\times\frac{[1+C^{\neq}_{10}]C_{31}-[1-C^{\neq}_{10}]C_{3\bar{1}}}{[1+C_{10}]C^{\neq}_{31}-[1-C_{10}]C^{\neq}_{3\bar{1}}}=1; \quad \ldots \quad (42)$$

Case IV. ($j=2,\ k=3$) and ($j=3,\ k=2$).

$$\frac{[1+C^{\neq}_{30}]C_{23}-[1-C^{\neq}_{30}]C_{2\bar{3}}}{[1+C_{30}]C^{\neq}_{23}-[1-C_{30}]C^{\neq}_{2\bar{3}}}\times\frac{[1-C^{\neq}_{20}]C_{32}-[1+C^{\neq}_{20}]C_{3\bar{2}}}{[1-C_{20}]C^{\neq}_{32}-[1+C_{20}]C^{\neq}_{3\bar{2}}}=1; \quad \ldots \quad (43)$$

Thus, six specific reciprocity relations follow from the general reciprocity equation (32). The experimental verification of all of them requires elaborate and extensive experimental work. Nevertheless, it is possible to verify (34) and (37) with the help of available data in the existing literature.

V. EXPERIMENTAL VALIDITY

The experimental validity of some cases of the general reciprocity equation (32) is established through available data in the following Tables

a) *Krishnan's* (1938) *data for Megnetically oriented particles*

Krishnan performed three experiments on graphite sols, using a magnetic orienting field and with incident and emergent beams at right angles in the horizont l plane of observation, as follows.

(i) Particles vertically oriented

(ii) Particles horizontally oriented and parallel to incident beam

(iii) Particles horizontally oriented and perpendicular to incident beam and parallel to scattered beam.

In terms of the previous discussion it is easy to see that the system in case (iii) is the reciprocal of the system in case (ii) and vice-versa. The result of calculations for reciprocity *Eq*.(34) are tbulated below .

TABLE I

Test of reciprocity Eq. (34) through Krishnan's data

H (Gauss)	Case (ii) Natural Instrument			Case (iii) Reciprocal Instrument			(34) (Left-hand side)
	ρ_h	ρ_v	ρ_u	ρ_h	ρ_u	ρ_u	
0	260	045	198	.260	045	198	1 000
1120	410	.086	217	755	045	170	1.010
4060	472	.124	205	1 342	045	.125	1.005
5730	.537	150	.228	2 198	.040	.099	1 030
6860	.537	.163	.238	3 000	037	082	1.050
7620	537	163	254	3 000	.037	082	1 035

b) *Subramanya and Rao* (1949) *data for Electrically oriented particles*

The authors repeated Krishnan's experiments with the only difference, that they used an electric field for orienting the particles, given below is their data

TABLE II

Subramanya and Rao's data

Concentration of Sols = 0.0008%

H (Volts)	Natural Instrument Case (ii)			Reciprocal Instrument Case (iii)			Rec Eq. (34) (Left hand side)
	ρ_v	ρ_h	ρ_u	ρ_v	ρ_h	ρ_u	
0	.068	.260	.355	.068	260	355	1.000
60	.102	350	.283	.068	344	.262	0.940
120	.124	.419	.285	.068	476	.255	0 930
180	.131	435	285	077	.548	.247	1 060

Conc, = 0.0004%

E (Volts)	ρ_v	ρ_h	ρ_u	ρ_v	ρ_h	ρ_u	Rec Eq. (34) (Left hand side)
0	070	.236	.331	.070	.236	.331	1 000
60	078	255	.331	.074	.390	312	1.010
120	.092	.307	.328	.079	.394	.305	1.010
180	.108	350	315	083	448	295	0 991

The agreement with the proposed reiprocity equation is much more excellent for the lower concentration of 0.0004% than for the higher oncentration of 0 0008% This is because of much better orientation effect and much lower amount of multiple-scattering for lower concentrations

3. *Subramanian's data* (1963)

Subramanian repeated Krishnan's experments, using the magnetic fielp also for verification of his reciprocity relation in terms of Intensities. His data for depolarization factors for the last two cases has been used for verifying the equation (34)

TABLE III

S. Subramanian's Data for oriented particles

H (Gauss)	Field parallel to the incident beam			Field perpendicular to incident beam			Rec. Eq. (34) (Left hand side)
	ρ_u	ρ_v	ρ_h	ρ_u	ρ_v	ρ_h	
0	.20	.09	.69	20	.09	.69	1.000
3600	.17	10	.90	20	09	70	0 984
3000	.16	.10	90	.202	.09	1 00	0 980
2000	—	—	93	—	095	1.00	—

Looking at the last column it is apparent, the agreement in this case is also of the order of 98%.

4. *A. Mueller's data on oriented Nylon fibers*

Miss A. Mueler's data is recorded in the present Author's (1964) previous work She performed a set of extensive experments using extremely fine parallel nylon fibers as scatterers. The detection technique was developed by H. Mueller utilizing a highly sensitive photoelectric method. Experiments were conducted in an air-conditioned chamber and the angle of scattering was kept at 41°. Table IV gives the result of calculations for the data.

TABLE IV

A Mueller's data on oriented Nylon fibers

Orientation Angle degrees A	C_{10}	$C\neq_{10}$	C_{11}	$C\neq_{11}$	$-C_{1I}$	$-C\neq_{1I}$	Rec. Eq. (33)
0	.0324	0196	2232	.1898	1552	1826	1 021
30	0256	0239	1286	1192	1205	1184	1.050
60	0399	0544	1855	2043	1211	1234	0 946
90	.0674	0728	4490	4157	3638	3751	1 002
120	0554	0399	3517	2860	.2738	.2367	1 185
150	.0188	0123	1014	1048	0611	0780	1 060
180	.0300	.0175	2000	.1976	1466	1812	0 913

TABLE V

A Mueller's data for verification of Reciprocity Eq.(37)

A Degrees	C_{20}	$C\neq_{21}$	$C\neq_{22}$	$C\neq_{20}$	C_{2I}	C_{22}	Rec Eq (37)
0	− 0039	0838	− 0188	− 0008	.0774	− 0148	1 090
30	−.0088	.2124	− 1826	− 0060	2155	−.1883	0 980
60	−.0144	3377	− 3287	− 0366	.3086	− 3065	1 077
90	+ 0035	1741	− 1743	− 0049	1847	− 1468	1 047
120	+.0274	.3067	−.2326	+ 0171	.3075	− 2169	1.035
150	+.0022	2315	− 1753	+ 0019	2331	−.1736	1 001
180	+ 0076	0853	− 0585	− 0088	0825	− 0372	1 199

TABLE VI

A. Mueller's data for verification of Reciprocity Eq. (41)

(all C values are to be multiplied by 10^{-3})

A (Degrees)	C_{2I}	$C\neq_{2I}$	C_{21}	$C\neq_{21}$	C_{10}	$C\neq_{10}$	C_{20}	$C\neq_{20}$	C_{12}	$C\neq_{12}$	C_{14}	$C\neq_{14}^{*}$	Rec. Eq (41)
0	011	066	015	009	032	020	−004	−001	076	052	−021	−035	1 130
45	031	−028	−019	016	022	025	−011	−025	−019	023	061	060	1.090
90	016	010	009	009	067	073	004	−005	159	161	−012	−032	0.890
135	−034	−008	078	041	027	027	016	013	000	−036	055	093	1.175

The agreement of the data with (41), In Table VI, is fairly good considering the number of small parameters involved in the equation and also the greatest difficulty of maintaining the same experimental conditions over a length of time for the 12 sets of readings involved.

Error Analysis of the Reciprocity Equations

Consider X and Y as the correct values of the numerator and the denominator for the general form of the reciprocity equation

$$X/Y = 1$$

If d_x and d_y are the net amounts of error in the experimental values of X and Y respectively, it follows that,

$$\frac{X \pm d_x}{Y \pm d_y} = \frac{X(1 \pm d_x/X)}{Y(1 \pm d_y/Y)}$$

$$= \frac{X(1 \pm d_x/X)(1 \pm d_y/Y)}{Y} \; ; \; (\text{If } X = Y)$$

$$= X/Y \pm (d_x \pm d_y)/Y$$

Since d_x and d_y are bound to be rather small quantities compared to X and Y, the error term would be very small. Any consistent and appreciable divergence from the value $X/Y = 1$ would therefore naturally be due to real significant disparity with the law, and would signify the non-validity of the Mueller's form of the reciprocity law in that case.

CONCLUSION

The validity of the proposed reciprocity equation (34) has been conclusively established through the data of Krishnan, Subramanya and Rao, S. Subramanian and A. Muller, as shown in the last columns of tables I to IV. The divergences from the predicted value unity are well within about 5% experimental error limits. Reciprocity Equations (37) and (41) have also been established through the data of A. Mueller in Tables V and VI respectively, though the agreement for these two is not so good as for the previous ones. This is partly because of rather small parameters involved and being for all sorts of arrangements of the scatterers and the apparatus, having been taken over a length of time. It is very difficult to maintain exact experimental conditions over a long period ot time, nevertheless the average agreement within about 10% is fairly reasonable. The verification of the equations involving circularly polarized beams is left for further work. The validity of (34), (37) and (41), seems to provide a strong evidence in support of the general reciprocity eq.(32), and as such of Mueller's reciprocity law.

The utility of the proposed form of reciprocity relations is in their efficiency and elegance of providing reliable means of testing the Mueller theory in a straightforward and compact manner. In general matrix elements in ordinary scattering experiments are very small, as such direct comparisons of matrix elements entail numerous calculations on small quantities yielding inconclusive results. The proposed equations may find usefulness in Colloid-optics and allied fields Matrix representation of polarized electromagnetic beams is being increasingly used in case of gamma-ray polarization studies as shown by McMaster (1954, 1961) etc

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