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On longitudinal disturbances in a semi-infinite piezoelectric rod with a body-force in a magnetic field.

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This article presents the solution of the problem of longitudinal disturbances in a semi-infinite piezoelectric rod being acted by a magnetic field in presence of a body-force.

1. Introduction

The elastic problems on piezoelectric bodies have been thoroughly discussed in the recent papers of Paria (1960), Paul (1961), Sinha (1963, 1965), Giri (1966) and of Das (1967). A few of these papers have been mentioned here as references. This kind of discussion can be conveniently extended to the corressponding cases in a magnetic field. Banes & Soska (1964) have, perhaps for the first time, studied the effect of a magnetic field on the disturbances in a piezoelectric material in their recent paper (1964). Recently, Sinha (1967) has considered the problem of similar nature. The object of the present paper is to deal with the simple problem of longitudinal disturbances in a semi-infinite piezoelectric bar caused by a body-force and a magnetic field. Body-force dependent on the dimension as well as time has been assumed here. The method of Laplace transform has been found effective in making the problem amenable to solution.

2. Problem, equations and boundary conditions

Let a magnetic field represented by the magnetic induction B run in a direction perpendicular to the direction of a semi-infinite piezo-eletric bar $0 \leqslant x \leqslant +\infty$. Let a time-dependent displacement be applied at the finite end of the bar $x=0$. The problem is to investigate the longitudinal displacement $u(x,t)$ of the bar that stems from the interaction of mechanical and electromagnetic fields with a body force $\boldsymbol{F}$.

The basic equations are, therefore, given by Benes & Soska (1964) as follows,

$$C_{ijkl}\frac{\partial u_{kl}}{\partial x_j}+\epsilon_{ijk}\left[e_{jrs}\left(\frac{\partial u_r}{\partial t}+\frac{\partial u_{rs}}{\partial x_m}\cdot\frac{\partial x_m}{\partial t}\right)+\epsilon_0 k_{jm}\frac{\partial E_m}{\partial t}\right]=\rho\frac{\partial^2 u_i}{\partial t^2} \quad (1)$$

$$2u_{kl}=\frac{\partial u_k}{\partial x_l}+\frac{\partial u_l}{\partial x_k} \quad \ldots(2)$$

Where the symbols have their usual meanings as in Benes & Soska's (1964) For present discussion, the equation of motion is given by

$$C_{22}\frac{\partial^2 u}{\partial x^2}-e_{1,2}B\frac{\partial^2 u}{\partial x\partial t}+\rho F=\rho\frac{\partial^2 u}{\partial t^2} \quad \ldots(3)$$

where F, being the body-force which is assumed to be $H(t)e^{-\lambda x}$, λ, being a costant and $H(t)$ being Heaviside's unit function equal to unity when $t>0$ and equal to zero then $t>0$.

Therefore (3) becomes

$$C_{22}\frac{\partial^2 u}{\partial x^2}-e_{1,2}B\frac{\partial^2 u}{\partial x\partial t}-\rho\frac{\partial^2 u}{\partial t^2}=-\rho H(t)e^{-\lambda x} \quad \ldots(4)$$

The boundary conditions are

$$u\to 0 \text{ as } x\to\infty \quad \ldots(5)$$

$$u=P_0\left(1-e^{-kt}\right);\ k>0 \text{ and } P_0 \text{ a constant, at } x=0 \quad \ldots(6)$$

3. Solution of the problem

The present problem, being one of the boundary value problems encountered in partial differential equations, the principles of Laplace transform can be used effectively to obtain $u\,(x,\,t)$ from the equation (4). Defining

$$\bar{u}\,(x,p)=\int_0^\infty e^{-pt}\,u\,(x,t)\,dt, \qquad Re(p)>0,\ t>0$$

as the Laplace transform of $u\,(x,\,t)$, the equation (4) is transformed to

$$C_{22}\frac{\partial^2\bar{u}}{\partial x^2}-e_{1,2}\,B,p\,\frac{\partial\bar{u}}{\partial x}-\rho\,p^2\bar{u}=-\frac{\rho\,e^{-\lambda x}}{p} \quad \ldots(7)$$

Its solution is given by

$$\bar{u} = C_1 e^{m_1 x} + C_2 e^{m_2 x} - \frac{\rho e^{-\lambda x}}{p (C_{22} \lambda^2 - e_{1,2} Bp \lambda - \rho p^2)} \quad \ldots (8)$$

where m_1, and m_2 are the roots of

$$C_{22} m^2 - e_{1,2} Bpm - \rho p^2 = 0 \quad \ldots (9)$$

The conditions (5) and (6) give

$$\bar{u} \to 0 \text{ as } x \to \infty \quad \ldots (10)$$

$$\bar{u} = -\frac{P_0 K}{p (p + K)} \quad at\ x = 0 \quad \ldots (11)$$

Because of (10), (8) is given by

$$\bar{u} = C_2 e^{-px[\sqrt{e_{1,2}^2 B^2 + 4\rho C_{22}} + e_{1,2} B]/2C_{22}} - \frac{\rho e^{-\lambda x}}{p(C_{22}\lambda^2 - e_{1,2} Bp\lambda - \rho p^2} \quad \ldots (12)$$

using (11), we have

$$C_2 = -\frac{P_0 K}{p (p + K)} + \frac{\rho}{p (C_{22} \lambda^2 - e_{1,2} B p \lambda - \rho p^2)}$$

Therefore

$$\bar{u} = \left[-\frac{P_0 K}{p (p + K)} + \frac{\rho}{p (C_{22} \lambda^2 - e_{1,2} Bp\lambda - \rho p^2)} \right]$$

$$\times e^{-px [\sqrt{e_{1,2}^2 B^2 - 4\rho C_{22}} - e_{1,2} B]/2C_{22}}$$

$$- \frac{\rho e^{-\lambda x}}{p (C_{22} \lambda^2 - e_{1,2} Bp\lambda - \rho p^2)} \quad \ldots (13)$$

Taking the inverse transform of (13), we obtain

$$u = \frac{e^{-\lambda x}}{-2\beta (\beta^2 - \delta^2)} \left[2\beta - (\beta + \delta) e^{-(\beta - \delta)t} + (\beta - \delta) e^{+(\beta + \delta)t} \right]$$

when $0 < t < x[\sqrt{e_{1,2}^2 B^2 + 4\rho C_{22}} + e_{1,2} B]/2C_{22}$.

and

$$u=P_0\left[1-e^{-k\{t-(\sqrt{e_{1,2}^2B^2+4\rho C_{22}}+e_{1,2}B)/2C_{22}\}}\right]+$$

$$+\frac{1}{2\beta(\beta^2-\delta^2)}\left[2\beta-(\beta+\delta)e^{-(\beta-\delta)\{t-x(\sqrt{e_{1,2}B^2+4\rho C_{22}}+e_{1,2}B)/2C_{22}\}}\right.$$

$$\left.+(\beta-\delta)e^{+(\beta+\delta)\{t-x(\sqrt{e_{1,2}^2B^2+4\rho C_{22}}+e_{12}B)/2C_{22}\}}\right]$$

$$-\frac{e^{-\lambda x}}{2\beta(\beta^2-\delta^2)}\left[2\beta-(\beta+\delta)e^{-(\beta-\delta)t}+(\beta-\delta)e^{-(\beta+\delta)t}\right]$$

when

$$t>x\left[\sqrt{e_{1,2}^2B^2+4\rho c_{22}}+e_{1,2}B\right]/2c_{22}$$

where

$$\beta=\frac{\lambda}{2\rho}\sqrt{e_{1,2}^2B^2+4\rho c_{22}}\ ,\ \delta=\frac{e_{1,2}B\lambda}{2\rho}$$

Thus we find that the displacement is partly transient in nature for a certain period of time and is partly constant and partly transient after it exceeds a certain period.

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