Indian J. Phys. **43**, 550—554 (1969)

A note on the linear flow of a viscous incompressible conducting fluid past an infinite flat plate with constant suction in the presence of a transverse magnetic field

By S. N. DUBE

Department of Mathematics, Institute of Technology, Banaras Hindu University, Varanasi-5, India

(*Received* 7 *July* 1969)

Analytical solution has been obtained for the momentum equations of the linear flow of a viscous incompressible electrically conducting fluid past an infinite porous flat plate in the presence of a transverse magnetic field when the suction velocity normal to the plate is constant. It is observed that the velocity in the boundary layer increases with the increase of the intensity of the magnetic field.

INTRODUCTION

The study of the response of laminar two-dimensional boundary layers to the fluctuations in the oncoming stream, initiated by Lighthill (1954), attracted the attention of many research workers in the past few years. Stuart (1955) investigated the flow of a viscous incompressible fluid past an infinite flat plate with constant suction at the surface when the free-stream velocity fluctuates about a mean value. Reddy (1964) extended the work of Stuart to the case when the fluid is moderately rarefied by introducing the slip boundary conditions. Pandey (1968) studied the same problem when the free-stream velocity varies exponentially with time. The corresponding magneto-hydrodynamic problem, when the fluid is electrically conducting and a transverse magnetic field is present, has been studied by Suryaprakaso Rao (1962, 1963). Kelly (1965) and Messiha (1966) have investigated the problem with an incompressible non-conducting fluid when the suction velocity is oscillatory. Recently Mehta & Radha Krishna (1968) studied the corresponding magnetohydrodynamic problem with a viscous incompressible electrically conducting fluid in the presence of a transverse magnetic field.

In the present note an attempt has been made to study the effects of the magnetic field and constant suction on the flow of an incompressible electrically conducting fluid when the free-stream velocity varies linearly with time. The magnetic field of strength B'_0 is imposed normal to the plate.

BASIC EQUATIONS

We consider a two-dimensional viscous incompressible electrically conducting fluid flow along an infinite porous flat plate in the presence of a transverse magnetic field. The flow is independent of the distance parallel to the plate and the suction velocity normal to the plate is directed towards it and is constant. The x'-axis is taken along the plate, y'-axis normal to the plate. Dashes denote dimensional quantities. Neglecting the induced magnetic field which is usually small and assuming the electric field $\vec{E}$ to be equal to zero, the equations of motion and continuity (Pai 1962) are

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho'} \frac{\partial p'}{\partial x'} + \nu' \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma_e'}{\rho'} B_0'^2 u', \qquad \text{.. (1)}$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'} \frac{\partial p'}{\partial y'}, \qquad \text{(2)}$$

$$\frac{\partial v'}{\partial y'} = 0, \qquad \text{...(3)}$$

where σ_e' is the electrical conductivity of the fluid.

The boundary and initial conditions are

$$\left.\begin{array}{l} t' = 0, \quad u' = 0 \quad \text{for all } y' \\ t' > 0: \ u' = 0 \text{ at } y' = 0 \text{ and } u' \to U'(t') \text{ at } y' \to \infty \end{array}\right\}, \qquad \text{...(4)}$$

where $U' = a' t'$ is the velocity at a large distance from the wall.

Although the equation (3) shows that v' is a function of time only, we now further restrict consideration to the case of v' equal to a negative constant $(-v_0')$, from which it follows that p' is independent of y'. Now outside the boundary layer equation (1) gives

$$\frac{dU'}{dt'} = -\frac{1}{\rho'} \frac{\partial p'}{\partial x'} - \frac{\sigma_e'}{\rho'} B_0'^2 U'. \qquad \text{. (5)}$$

Substituting for the pressure term in equation (1) from (5), we get

$$\frac{\partial u'}{\partial t'} - v'_0 \frac{\partial u'}{\partial y'} = \nu' \frac{\partial^2 u'}{\partial y'^2} + \frac{dU'}{dt'} + \frac{\sigma'^2_e}{\rho'} B'^2_0 (U' - u'). \qquad \text{.. (6)}$$

We now introduce the non-dimensional quantities defined by

$$y = \frac{y' v'_0}{\nu'}, \quad t = \frac{v'^2_0 t'}{4\nu'}, \quad u = \frac{u'}{U'_0}, \quad U = \frac{U'}{U'_0}, \quad M = \frac{B_0'}{U_0'} \sqrt{\frac{\nu' \sigma_e'}{\rho'}}, \qquad \text{. (7)}$$

where U_0' is a reference velocity and M is the Hartmann number. Equation (6) takes the non-dimensional form as

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \frac{1}{4} \frac{\partial u}{\partial t} = -\frac{1}{4} \frac{dU}{dt} - M^2 (U - u), \qquad \text{...(8)}$$

subject to the boundary conditions

$$\left.\begin{array}{l} t = 0,\ u = 0 \text{ for all } y \\ t > 0 : u = 0 \text{ at } y = 0 \text{ and } u \to U(t) \text{ at } y \to \infty \end{array}\right\}. \quad \ldots(9)$$

The dimensionless form of the free-stream velocity is

$$U = ct,$$

where $c = \dfrac{4a'\nu'}{U_0' v_0'^2}$ is clearly a dimensionless constant. Substituting the above value of U in (8), we get

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \frac{1}{4}\frac{\partial u}{\partial t} = -\frac{c}{4} - M^2(ct-u). \quad \ldots(10)$$

Multiplying equation (10) by e^{-st} and then integrating the resulting equation with respect to t between the limits 0 to ∞, we get

$$\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial \bar{u}}{\partial y} - \left(M^2 + \frac{s}{4}\right)\bar{u} = -c\left[\frac{1}{4s} + \frac{M^2}{s^2}\right], \quad \ldots(11)$$

where $\bar{u}$ is the Laplace transform of u defined by

$$\bar{u} = \int_0^\infty e^{-st}.\, u\, dt.$$

The boundary conditions (9) reduce to

$$\bar{u} = 0 \text{ at } y = 0 \text{ and } \bar{u} \to \frac{c}{s^2} \text{ at } y \to \infty. \quad (12)$$

The solution of equation (11) under the conditions (12) is

$$\bar{u} = \frac{c}{s^2}\left[1 - e^{-\frac{1+\sqrt{1+s+4M^2}}{2}y}\right]. \quad \ldots(13)$$

Now applying Laplace inversion theorem, we get

$$u = ct\left[1 - e^{-\frac{1+\sqrt{1+4M^2}}{2}y}\right]$$

$$+ \frac{c}{4\sqrt{1+4M^2}}\, y.\, e^{-\frac{1+\sqrt{1+4M^2}}{2}} \quad \ldots(14)$$

The non-dimensional skin-frictfon τ_0 is given by

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1+\sqrt{1+4M^2}}{2}.\, ct + \frac{c}{4\sqrt{1+4M^2}} \quad \ldots(15)$$

Figure 1 has been obtained by plotting the velocity distribution u against y for $c = 4$, $t = 1$ and $M = 0, \frac{1}{2}, 1$. From the figure it is clear that initially u increases with the increase of y upto about $y = 3.5$, but beyond $y = 3.5$, u is constant. This means that the velocity profiles beyond $y = 3.5$ are straight lines normal to the plate. This graph also indicates that the velocity in the boundary layer increases with the increase of the intensity of the magnetic field.

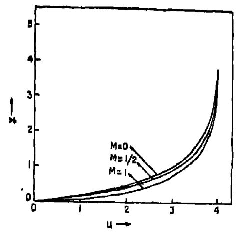

Figure 1

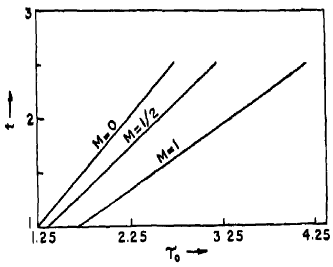

Figure 2

Figure 2 has been obtained by plotting τ_0 against t for $c = 1$ and $M = 0, \frac{1}{2}, 1$. The expression (15) and the figure 2 show that the skin-friction τ_0 varies linearly with time. From this figure it is also evident that the skin-friction τ_0 increases with the increase of the intensity of the magnetic field.

For $M = 0$, the results transform to the results obtained by the author (1969) for the two-dimensional viscous incompressible non-conducting fluid flow past an infinite flat plate with constant suction at the surface when the free-stream velocity varies linearly with time.

The authar wishes to express his gratitude to Professor P. L. Bhatnagar Vice-Chancellor, Rajasthan University, for his kind guidance in the preparation of this note.

REFERENCES

Dube, S. N. 1969 (*Accepted for publication in J. Indian Math. Soc.*)

Kelly, R. E., 1965 *Quart. J. Mech. Appl. Math.*, **18** .287.

Lighthill, M. J. 1954 *Proc. Roy. Soc*, **A**, **224**, 1.

Mehta, K. N. & Radha Krishna, C. V. 1968 *Proc. Nat. Inst. Sci. India*, A, **34**, 28.

Messiha, S. A. S. 1966 *Proc. Camb. Phil. Soc.*, **62**, 329.

Pai, S. 1962 *Magneto gas-dynamics and Plasma Dynamics*, Springer-Verlag Berlin

Pandey, K. S. 1968 *Indian J. Phys.*, **42**, 213.

Reddy, K. C. 1964 *Quart. J. Mech. Appl. Math.*, **17**, 381.

Stuart, J. T. 1955 *Proc. Roy. Soc.*, A. **231**, 116.

Suryaprakaso Rao, U. 1962 *Z. Angew. Math. Mech.*, **42**, 133.

1963 *Z. Angew. Math. Mech.*, **43**, 127.